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Гравитационное аксиальное суперполе и формализм дифференциальной геометрии

На основе простейшей группы супергравитации развивается формализм дифференциальной геометрии. В нем все геометрические объекты - супертетрады, связности и так далее - выражаются через аксиальное гравитационное суперполе.

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The Gravitational Axial Superfield and the Formalism of Differential Geometry

On the basis of the simplest supergravity group a differential-geometry formalism is developed. There all the geometrical objects - supervierbeins, connections, etc. - are expressed in terms of the axial gravitational superfield.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Earlier[1,2] we have described the simplest supergravity group. It is the complex supergroup of general coordinate transformations in the left-handed $\{(x^m_L, \theta^\mu_L)\}$ and right-handed $\{(x^m_R, \bar\theta^{\dot\mu}_R)\}$ 4+2-dimensional conjugated chiral superspaces (SS below):

$$\begin{array}{ll} \text{Left SS} & \text{Right SS} \\ x'^m_L = x^m_L + \lambda^m(x_L, \theta_L) & x'^m_R = x^m_R + \bar\lambda^m(x_R, \bar\theta_R) \\ \theta'^\mu_L = \theta^\mu_L + \lambda^\mu(x_L, \theta_L) & \bar\theta'^{\dot\mu}_R = \bar\theta^{\dot\mu}_R + \bar\lambda^{\dot\mu}(x_R, \bar\theta_R) . \end{array} \tag{1.1}$$

Here $x^m_R = (x^m_L)^*$, $\bar\theta^{\dot\mu}_R = (\theta^\mu_L)^\dagger$ and $\lambda^m(x_L, \theta_L), \lambda^\mu(x_L, \theta_L)$ and their conjugates $\bar\lambda^m(x_R, \bar\theta_R)$, $\bar\lambda^{\dot\mu}(x_R, \bar\theta_R)$ are arbitrary infinitesimal chiral superfunctions-parameters.

The complex 4+2-dimensional SS can be regarded as an 8+4-dimensional SS $\{(x^m_L, x^m_R, \theta^\mu_L, \bar\theta^{\dot\mu}_R)\}$ with complex structure. Into this SS a real physical SS $\{(x^m, \theta^\mu, \bar\theta^{\dot\mu})\}$ is imbedded as a 4+4-dimensional supersurface given by four equations

$$x^m_L - x^m_R = 2i\,\mathcal{H}^m\left(\tfrac{1}{2}(x^n_L + x^n_R), \theta^\nu_L, \bar\theta^{\dot\nu}_R\right). \tag{1.2}$$

Here $\mathcal{H}^m$ is an arbitrary real superfunction. The coordinates of the physical SS are

$$x^m = \tfrac{1}{2}(x^m_L + x^m_R), \quad \theta^\mu = \theta^\mu_L, \quad \bar\theta^{\dot\mu} = \bar\theta^{\dot\mu}_R . \tag{1.3}$$

Under the transformations (1.1) the coordinates (1.3) as well as the superfunction $\mathcal{H}^m$ change (because the supersurface equations (1.2) must be covariant):

$$x'^m = x^m + \tfrac{1}{2}\lambda^m\left[x^n + i\mathcal{H}^n(x,\theta,\bar\theta), \theta^\nu\right] + \tfrac{1}{2}\bar\lambda^m\left[x^n - i\mathcal{H}^n(x,\theta,\bar\theta), \bar\theta^{\dot\nu}\right] \tag{1.4a}$$

$$\theta'^{\mu} = \theta^{\mu} + \lambda^{\mu}[x^{n} + i\mathcal{H}^{n}(x,\theta,\bar\theta), \theta^{\nu}] \tag{1.4b}$$

$$\bar\theta'^{\dot\mu} = \bar\theta^{\dot\mu} + \bar\lambda^{\dot\mu}[x^{n} - i\mathcal{H}^{n}(x,\theta,\bar\theta), \bar\theta^{\dot\nu}] \tag{1.4c}$$

$$\mathcal{H}'^{m}(x',\theta',\bar\theta') = \mathcal{H}^{m}(x,\theta,\bar\theta) - \tfrac{i}{2}\lambda^{m}[x^{n} + i\mathcal{H}^{n}(x,\theta,\bar\theta), \theta^{\nu}] + \tfrac{i}{2}\bar\lambda^{m}[x^{n} - i\mathcal{H}^{n}(x,\theta,\bar\theta), \bar\theta^{\dot\nu}] . \tag{1.4d}$$

Thus the supergravity group is realized nonlinearly in the SS $\{(x^{m}, \theta^{\mu}, \bar\theta^{\dot\mu})\}$ and on the (axial) superfield $\mathcal{H}^{m}(x,\theta,\bar\theta)$. In the case of Einstein supergravity the parameters λ are restricted by the condition of supervolume preservation in the left- and right-handed SS:

$$\frac{\partial}{\partial x^{m}_{L}} \lambda^{m}(x_L,\theta_L) - \frac{\partial}{\partial \theta^{\mu}_{L}} \lambda^{\mu}(x_L,\theta_L) = 0$$

$$\frac{\partial}{\partial x^{m}_{R}} \bar\lambda^{m}(x_R,\bar\theta_R) - \frac{\partial}{\partial \bar\theta^{\dot\mu}_{R}} \bar\lambda^{\dot\mu}(x_R,\bar\theta_R) = 0. \tag{1.5}$$

As shown in Ref./2/, after a partial gauge-fixing (in order to avoid the nonpolynomiality of the law (1.4)) the gravitational superfield (SF below) $\mathcal{H}^{m}(x,\theta,\bar\theta)$ contains the physical fields of graviton $e^{am}(x)$ (the vierbein field) and gravitino $\psi_{\alpha}{}^{m}(x)$ (the Rarita-Schwinger field), and the auxiliary fields $S(x)$, $P(x)$, $A^{m}(x)$. The transformations (1.4), (1.5) in terms of these fields reduce to general transformations of the coordinates x^{m}, to local Lorentz transformations, and to local supersymmetry ones. So, the supergroup (1.4), (1.5) describes the whole gauge freedom inherent to Einstein supergravity, and the SF $\mathcal{H}^{m}$ contains in a minimal way all the necessary gauge fields.

Now we have to build up a dynamical theory on this minimal ground, i.e., to write down covariant equations of motion, invariant action, and counterterms in terms of our single dynamical variable $\mathcal{H}^{m}$. For this purpose the differential-geometry formalism is convenient*).

In our approach the concepts and objects of this formalism (world and local Lorentz groups, supervierbeins, connections, etc.) are more specific than in the approaches of Akulov, Volkov and Soroka/3/, Wess and Zumino/4,5/, Brink, Gell-Mann, Ramond and Schwarz/6/, Bedding, Downes-Martin and Taylor/7/, etc. Our world supergroup (1.4), (1.5) is simpler than the supergroup of general coordinate transformations in the physical SS considered in Refs. /3-7/. Further, in Refs./3-7/ the local Lorentz group (acting on the SF indices) is independent of the world supergroup, whereas in our case it is completely induced by the world one. Finally, we shall express all the supervierbeins (which are independent variables in Refs./3-7/) and connections in terms of the single SF $\mathcal{H}^{m}(x,\theta,\bar\theta)$.

The contents of the paper is as follows. In Sect. 2 some brief information about differential geometry in SS is given. In Sect. 3 the general and chiral scalar SF are defined, and their derivatives are written down in left, right, and symmetric coordinate bases. The analysis of the transformation properties of the spinor-coordinate derivatives carried out in Sect. 4 leads to a natural definition of the local Lorentz group for SF with external indices and of the covariant derivatives of scalar SF. In Sect. 5 connections are introduced, and the vector covariant derivative is defined as the anticommutator of the spinor ones. In Sect. 6 the supervierbeins are extracted out of the expressions for the covariant derivatives of the scalar SF. The expressions obtained contain, besides $\mathcal{H}^{m}$, some additional quantities. In order to find the latter, in Sect. 7 left and right supervierbeins (connected with the covariant derivatives of the chiral scalar SF) are introduced. With their help in Sect. 8 the derivation of the supervierbeins and connections in terms of $\mathcal{H}^{m}$ is completed. In Sect. 9 the simplest Lagrangian for pure supergravity is discussed. A summary of the main results of the paper is given in Sect. 10. Section 11 contains concluding remarks.

2. Basic concepts of differential geometry. The points of the world SS can be denoted by z^{M}:

$$z^{M} = (x^{m}, \theta^{\mu}, \bar\theta^{\dot\mu}) . \tag{2.1}$$

Consider some (world) supergroup acting in this SS

$$z'^{M} = z'^{M}(z^{N}) . \tag{2.2}$$

In Refs./3-7/ it is the supergroup of general coordinate transformations, whereas in our case it is the supergroup (1.4), (1.5). Define scalar SF as a superfunction $\varphi(z)$ transforming according to

$$\varphi'(z') = \varphi(z) . \tag{2.3}$$

Then

$$(\partial_M \varphi)' \equiv \frac{\partial \varphi'(z')}{\partial z'^{M}} = \frac{\partial z^{N}}{\partial z'^{M}} \partial_N \varphi(z) ; \tag{2.4}$$

*) Notice that Siegel and Gates/8/ develop a formalism of the same kind. However, they use both vector and spinor SF.

generalizing, one can define co- and contravariant world tensor SF, e.g.

$$\varphi'_M(z') = \frac{\partial z^N}{\partial z'^M}\varphi_N(z); \quad \varphi'^M(z') = \varphi^N \frac{\partial z'^M}{\partial z^N}. \tag{2.5}$$

Two such objects can be contracted invariantly

$$\varphi^M\psi_M \equiv \varphi^m\psi_m + \varphi^\mu\psi_\mu + \varphi_{\dot\mu}\psi^{\dot\mu}. \tag{2.6}$$

Note that the transformations (2.5) mix the world vector indices (m) of the SF with the spinor ones ($\mu, \dot\mu$). In flat supersymmetry the external SF indices transform under the Lorentz group which does not mix them. In nonflat SS this can be achieved by introducing a new kind of SF indices and a local Lorentz group acting (infinitesimally) on them as follows

$$\varphi'_A(z) = \left(1 + i\,\Omega^{cd}(z)\Lambda_{cd}\right)_A{}^B\varphi_B(z) \equiv \varphi_A(z) + L_A{}^B\varphi_B(z). \tag{2.7}$$

Here $\Omega^{cd}(z)$ are antisymmetric tensor superfunctions-parameters, and $(\Lambda_{cd})_A{}^B$ are the Lorentz group generators. The new indices will be called Lorentz indices (to be distinguished from the world ones (2.5)).

Let us stress that owing to the block-diagonal structure of the Lorentz generators in Eq.(2.7) the vector (a) and spinor (α, $\dot\alpha$) indices are not mixed. Nevertheless, for convenience we shall usually denote the whole set $(a, \alpha, \dot\alpha)$ by a single letter A:

$$\varphi_A = \begin{pmatrix} \varphi_a \\ \varphi_\alpha \\ \varphi^{\dot\alpha} \end{pmatrix}. \tag{2.8}$$

The indices A are raised and lowered by the matrix η^{AB}

$$\varphi^A = \eta^{AB}\varphi_B, \quad \eta^{AB} = \begin{pmatrix} \eta^{ab} & & 0 \\ & \varepsilon^{\alpha\beta} & \\ 0 & & \varepsilon_{\dot\alpha\dot\beta} \end{pmatrix}. \tag{2.9}$$

We shall deal, as a rule, with the invariant contraction

$$\varphi^A\psi_A = \varphi^a\psi_a + \varphi^\alpha\psi_\alpha + \varphi_{\dot\alpha}\psi^{\dot\alpha} \tag{2.10}$$

although $\varphi^a\psi_a$, $\varphi^\alpha\psi_\alpha$ and $\varphi_{\dot\alpha}\psi^{\dot\alpha}$ are, of course, invariant separately too.

In Refs./3-7/ the local (in SS) Lorentz group is regarded as an independent one, i.e., its parameters $\Omega^{cd}(z)$ (2.7) are not supposed to be connected to the world parameters (2.2). Correspondingly, the SF's $\varphi_A(z)$ behave as scalars under the world supergroup transformations (2.2):

$$\varphi'_A(z') = \varphi_A(z). \tag{2.11}$$

We shall see later on that in our case, on the contrary, the local (in SS) Lorentz-group-transformations will be induced by the world ones.

The simplicity of the tensor analysis for the Lorentz group is the main advantage when dealing with Lorentz-type objects. To be able to turn world indices into Lorentz ones, new objects - supervierbeins $E_A{}^M(z)$, $E_M{}^A(z)$ are introduced:

$$\varphi_A(z) = E_A{}^M(z)\varphi_M(z); \quad \varphi^A(z) = E^A{}_M(z)\varphi^M(z), \tag{2.12}$$

$E_M{}^A(z)$ being the inverse matrix of $E_A{}^M$:

$$E_A{}^M E_M{}^B = \delta_A{}^B, \quad E_M{}^A E_A{}^N = \delta_M{}^N. \tag{2.13}$$

Comparing Eq.(2.5) with Eq.(2.7) one finds the transformation laws

$$\begin{aligned} E'_A{}^M(z') &= \left(\delta_A{}^B + L_A{}^B\right)E_B{}^N(z)\frac{\partial z'^M}{\partial z^N} \\ E'_M{}^A(z') &= \frac{\partial z^N}{\partial z'^M}E_N{}^B(z)\left(\delta_B{}^A - L_B{}^A\right). \end{aligned} \tag{2.14}$$

We would like to point out an interesting fact. Owing to the block-diagonal structure of the generators $(\Lambda_{cd})_A{}^B$ there exist, besides $E_M{}^A E_A{}^N (= \delta_M{}^N)$, three more world tensors

$$(\Pi^1)_M{}^N = E_M{}^a E_a{}^N, \quad (\Pi^2)_M{}^N = E_M{}^\alpha E_\alpha{}^N, \quad (\Pi^3)_M{}^N = E_{M\dot\alpha}E^{\dot\alpha N}.$$

They form a complete set of orthogonal projection operators (see Eq.(2.13)):

$$(\Pi^1 + \Pi^2 + \Pi^3)_M{}^N = E_M{}^A E_A{}^N = \delta_M{}^N$$

$$(\Pi^1)_M{}^N(\Pi^1)_N{}^P = E_M{}^a E_a{}^N E_N{}^b E_b{}^P = E_M{}^a E_a{}^P = (\Pi^1)_M{}^P$$

$$(\Pi^1)_M{}^N(\Pi^2)_N{}^P = E_M{}^a E_a{}^N E_N{}^\alpha E_\alpha{}^P = 0, \quad etc.$$

One can say that they decompose the space of world indices M into three independent sectors reflecting, in fact, the vector and spinor sectors in the space of Lorentz indices A.

The Lorentz-covariant derivative of a scalar SF is defined with the help of $E_A{}^M$:

$$\mathcal{D}_A\varphi(z) = E_A{}^M(z)\partial_M\varphi(z). \tag{2.15}$$

This definition can be extended to include SF with external indices

$$\mathcal{D}_A\varphi_B = E_A{}^M\partial_M\varphi_B + \omega_{AB}{}^C\varphi_C. \tag{2.16}$$

Here

$$\omega_{AB}{}^{C}(z) = i\omega_{A}{}^{de}(z)(\Lambda_{de})_{B}{}^{C} \tag{2.17}$$

are Lorentz-algebra-valued connection coefficients. To provide tensor transformation law for $\mathcal{D}_A \varphi_B$ (2.16) they must transform as follows:

$$\omega'_{AB}{}^{C}(z') = \omega_{AB}{}^{C}(z) + L_A{}^{\mathcal{D}}\omega_{\mathcal{D}B}{}^{C} + L_B{}^{\mathcal{D}}\omega_{A\mathcal{D}}{}^{C} + \\ - \omega_{AB}{}^{C} L_C{}^{\mathcal{D}} - E_A{}^{M}\partial_M L_B{}^{C}. \tag{2.18}$$

So, the supervierbeins and connections enable us to deal with Lorentz-covariant SF and their derivatives. The quantities $E_A{}^M$, $E_M{}^A$, $\omega_{AB}{}^C$ themselves are not Lorentz tensors. However, the (anti) commutator of two covariant derivatives (2.16) gives tensors constructed out of E and ω *)

$$[\mathcal{D}_A, \mathcal{D}_B\} \varphi_C = T_{AB}{}^{\mathcal{D}} \mathcal{D}_{\mathcal{D}} \varphi_C + R_{ABC}{}^{\mathcal{D}} \varphi_{\mathcal{D}} ; \tag{2.19}$$

$T_{AB}{}^{\mathcal{D}}$ is called "torsion tensor" and

$$R_{ABC}{}^{\mathcal{D}} \equiv R_{AB}{}^{ef}(\Lambda_{ef})_C{}^{\mathcal{D}} \quad - \tag{2.20}$$

"curvature tensor". Combining these tensors, matter SF and their covariant derivatives according to the simple Lorentz rules, one can construct Lagrangians, counterterms, etc.

So far we have considered the supervierbeins $E_A{}^M(z)$ and the connections $\omega_{AB}{}^C(z)$ as independent quantities. In a straightforward approach to supergravity /3-7/ $E_A{}^M(z)$ are the basic potentials. However, they are too complicated SF, and contain a great number of superfluous fields (for instance, the SF $E_\alpha{}^m(z)$ has in its decomposition spins 3, 5/2, etc.). To eliminate these extra degrees of freedom, one has to fix severely the gauge and to impose a set of (happily guessed) algebraic constraints on the components of the torsion tensor T. So the straightforward approach begins with the very general framework of differential geometry and then gradually reduces it to the essential physical content.

We are also going to utilize the formalism described above. However, in our case it will be built on a more concrete and economical ground. This means, first, that all the quantities ($E_A{}^M$, $\omega_{AB}{}^C$ and afterwards $T_{AB}{}^C$, $R_{ABC}{}^{\mathcal{D}}$) will be expressed in terms of the single SF $\mathcal{H}^m$. Thus we shall automatically get rid of the most of the superfluous components. Second, our group from the very beginning is much smaller. Instead of the general coordinate transformations in the physical SS we consider the simpler transformations in chiral SS; moreover, an independent local Lorentz group is not needed. So, we reduce essentially the initial gauge freedom in the theory.

*) $[\mathcal{D}_A, \mathcal{D}_B\} = \mathcal{D}_A\mathcal{D}_B - (-1)^{p(A)p(B)}\mathcal{D}_B\mathcal{D}_A \quad (p(a)=0,\ p(\alpha)=p(\dot\alpha)=1)$

Let us begin to realize this programme.

3. <u>Scalar superfields and their derivatives</u>. The general scalar SF are naturally defined in our scheme as objects transforming under the supergroup (1.4) as follows

$$\varphi'(x', \theta', \bar\theta') = \varphi(x, \theta, \bar\theta). \tag{3.1}$$

Now recall that the initial structure of our SS and supergroup was chiral. This meant, in particular, that in the left and right SS chiral SF could be defined, i.e., functions of x_L^m and θ_L^μ or x_R^m and $\bar\theta_R^{\dot\mu}$ only. The concept of chiral SF remains meaningful in the real (physical) SS too. To see this it is convenient to introduce chiral coordinate bases into the physical SS. Let us call "symmetric" the basis in which the points of the SS are parametrized by the coordinates x^m, θ^μ, $\bar\theta^{\dot\mu}$. The "left chiral" basis will be the one where independent variables are

$$x_L^m = x^m + i\mathcal{H}^m(x, \theta, \bar\theta),\ \theta^\mu,\ \bar\theta^{\dot\mu} \tag{3.2}$$

and in the "right" one

$$x_R^m = x^m - i\mathcal{H}^m(x, \theta, \bar\theta),\ \theta^\mu,\ \bar\theta^{\dot\mu}. \tag{3.3}$$

Let us stress that introducing new bases we do not go into left or right SS but just change the variables in the physical SS itself.

Now one can easily define left (right) chiral SF as a SF dependeing only on x_L^m and θ^μ (or x_R^m and $\bar\theta^{\dot\mu}$), and transforming according to the law

$$\varphi_L'(x_L', \theta') = \varphi_L(x_L, \theta),\quad \bar\varphi_R'(x_R', \bar\theta') = \bar\varphi_R(x_R, \bar\theta) \tag{3.4}$$

where (see Eq.(1.4))

$$x_L'^m = x_L^m + \lambda_L^m(x_L, \theta);\quad x_R'^m = x_R^m + \bar\lambda_R^m(x_R, \bar\theta) \\ \theta'^\mu = \theta^\mu + \lambda_L^\mu(x_L, \theta);\quad \bar\theta'^{\dot\mu} = \bar\theta^{\dot\mu} + \bar\lambda_R^{\dot\mu}(x_R, \bar\theta). \tag{3.5}$$

The chiral SF can also be written down in the symmetric basis, using Eqs.(3.2), (3.3):

$$\varphi_L(x_L, \theta) = \varphi_L(x + i\mathcal{H}(x, \theta, \bar\theta), \theta) \equiv \varphi(x, \theta, \bar\theta) \tag{3.6a}$$

$$\bar{\varphi}_R(x_R,\bar{\theta}) = \bar{\varphi}_R(x - i\mathcal{H}(x,\theta,\bar{\theta}),\bar{\theta}) \equiv \bar{\varphi}(x,\theta,\bar{\theta}). \tag{3.6b}$$

The labels L and R in Eqs. (3.5), (3.6) (and in what follows) show that the corresponding SF is <u>written down</u> in a chiral basis (but not that the SF itself is chiral). In the symmetric notation (3.6) the transformation laws of chiral SF coincide with Eq.(3.1).

The chirality condition for a SF $\varphi(x,\theta,\bar{\theta})$ can also be written down in differential form. In the corresponding chiral basis it has the obvious form

$$\bar{\partial}^L_{\dot{\mu}} \varphi_L(x_L,\theta,\bar{\theta}) \equiv \frac{\partial}{\partial\bar{\theta}^{\dot{\mu}}} \varphi_L(x_L,\theta,\bar{\theta}) = 0 \tag{3.7a}$$

$$\partial^R_\mu \bar{\varphi}_R(x_R,\theta,\bar{\theta}) \equiv \frac{\partial}{\partial\theta^\mu} \bar{\varphi}_R(x_R,\theta,\bar{\theta}) = 0 . \tag{3.7b}$$

In the symmetric basis it is somewhat more complicated. We shall derive, for instance, the condition of left chirality. Regarding x, θ and $\bar{\theta}$ as independent variables, let us take the derivatives of Eq.(3.6a) with respect to $\bar{\theta}^{\dot{\mu}}$ and x^m:

$$\frac{\partial\varphi}{\partial\bar{\theta}^{\dot{\mu}}} = i\frac{\partial\mathcal{H}^m}{\partial\bar{\theta}^{\dot{\mu}}}\frac{\partial\varphi_L}{\partial x^m_L} , \tag{3.8}$$

$$\frac{\partial\varphi}{\partial x^m} = (1+i\mathcal{H})_m{}^n \frac{\partial\varphi_L}{\partial x^n_L} \quad \text{or} \quad \frac{\partial\varphi_L}{\partial x^m_L} = (1+i\mathcal{H})^{-1}{}_m{}^n\frac{\partial\varphi}{\partial x^n} , \tag{3.9}$$

where

$$(1+i\mathcal{H})_m{}^n \equiv \delta_m{}^n + i\frac{\partial\mathcal{H}^n}{\partial x^m} . \tag{3.10}$$

Substituting Eq.(3.9) into Eq.(3.8), one obtains the form of the left-chirality condition in the symmetric basis

$$\bar{\Delta}_{\dot{\mu}}\varphi \equiv [-\bar{\partial}_{\dot{\mu}} + i\,\bar{\partial}_{\dot{\mu}}\mathcal{H}^m(1+i\mathcal{H})^{-1}{}_m{}^n\partial_n]\,\varphi = 0. \tag{3.11}$$

Analogously one derives the right-chirality condition as

$$\Delta_\mu\bar{\varphi} \equiv [\partial_\mu + i\partial_\mu\mathcal{H}^m(1-i\mathcal{H})^{-1}{}_m{}^n\partial_n]\,\bar{\varphi} = 0. \tag{3.12}$$

The differential operators $\Delta_\mu, \bar{\Delta}_{\dot{\mu}}$ defined in Eqs.(3.11), (3.12) will play an important role in what follows. It is easy to check that they can be rewritten as

$$\Delta_\mu = \partial_\mu + i\Delta_\mu\mathcal{H}^m\,\partial_m , \quad \bar{\Delta}_{\dot{\mu}} = -\bar{\partial}_{\dot{\mu}} - i\bar{\Delta}_{\dot{\mu}}\mathcal{H}^m\,\partial_m . \tag{3.13}$$

Several useful identities for Δ_μ, $\bar{\Delta}_{\dot{\mu}}$ are proved in Appendix A.

The operators Δ_μ and $\bar{\Delta}_{\dot{\mu}}$ have also the meaning of spinor derivatives rewritten from the chiral bases into the symmetric one:

$$\bar{\partial}^L_{\dot{\mu}}\varphi_L = \bar{\partial}_{\dot{\mu}}\varphi - i\bar{\partial}_{\dot{\mu}}\mathcal{H}^m(1+i\mathcal{H})^{-1}{}_m{}^n\partial_n\varphi = -\bar{\Delta}_{\dot{\mu}}\varphi ; \tag{3.14}$$

$$\partial^R_\mu\bar{\varphi}_R = \partial_\mu\bar{\varphi} + i\partial_\mu\mathcal{H}^m(1-i\mathcal{H})^{-1}{}_m{}^n\partial_n\bar{\varphi} = \Delta_\mu\bar{\varphi}.$$

In Appendix A formulae are given to transfer derivatives from one basis into another.

Finally, note that the differential chirality conditions (Eq.(3.7)or Eqs. (3.11), (3.12))must remain valid when the coordinates are transformed, because the concept of chirality is invariant, see Eq.(3.4). This important property will be used in Sect. 4 to define covariant derivatives and local Lorentz group.

4. <u>Local Lorentz group and spinor covariant derivatives</u>.

In this section one of the principal ideas of our construction is explained. This is the introduction of a local (in SS) Lorentz group induced by the world supergroup.

As was pointed out in Sect. 1, the transformations (1.4) describe the whole necessary gauge freedom of the field components of $\mathcal{H}^m$. In particular, they include local (in x -space) Lorentz transformations. Therefore it would be undesirable to introduce a new <u>independent</u> local group (2.7): this would lead to superfluous gauge freedom. On the other hand, the main idea of the formalism described in Sect. 2 is to use quantities that transform under a local (in SS) Lorentz group. Introducing such a group one gains significant (technical) advantages, which we are not going to give up. However, in contrast with some other papers[3-7], in our case the parameters of Lorentz-group transformations (2.7) will completely be determined by the parameters λ of the world supergroup (1.4).

Such a possibility is suggested by the fact that the differential chirality conditions must be covariant (see the end of Sect. 3). Analysing the transformation properties of the corresponding operators we shall find out how to define the local Lorentz group.

Consider in the left basis the derivative $\bar{\partial}^L_{\dot{\mu}}\varphi_L(x_L,\theta,\bar{\theta})$. Under infinitesimal transformations (1.4) (see Eq.(3.1))

$$\bar{\partial}'^L_{\dot{\mu}}\varphi'_L(x'_L,\theta',\bar{\theta}') = \bar{\partial}'^L_{\dot{\mu}}\varphi_L(x_L,\theta,\bar{\theta}) =$$

$$= \frac{\partial x^n_L}{\partial\bar{\theta}'^{\dot{\mu}}}\partial^L_n\varphi_L + \frac{\partial\theta^\nu}{\partial\bar{\theta}'^{\dot{\mu}}}\partial^L_\nu\varphi_L + \frac{\partial\bar{\theta}^{\dot{\nu}}}{\partial\bar{\theta}'^{\dot{\mu}}}\bar{\partial}^L_{\dot{\nu}}\varphi_L . \tag{4.1}$$

Keeping only the terms of first order in λ one finds (see Eqs. (1.4), (3.2), (3.3), (3.5))

$$\frac{\partial x^n_L}{\partial\bar\theta'^{\dot\mu}} = 0, \quad \frac{\partial\theta^\nu}{\partial\bar\theta'^{\dot\mu}} = 0, \quad \frac{\partial\bar\theta^{\dot\nu}}{\partial\bar\theta'^{\dot\mu}} = \delta_{\dot\mu}^{\dot\nu} - \bar\partial_{\dot\mu}{}^L\,\bar\lambda_L^{\dot\nu} \tag{4.2}$$

(In the left basis the right-chiral superfunction $\bar\lambda^{\dot\nu}(x_R, \bar\theta)$ depends on the arguments $x_L, \theta, \bar\theta$; recall Eq.(1.2)). So, the quantity $\bar\partial_{\dot\mu}{}^L\varphi_L$ transforms homogeneously

$$\bar\partial'^{\,L}_{\dot\mu}\varphi'_L = \bar\partial^L_{\dot\mu}\varphi_L - (\bar\partial^L_{\dot\mu}\bar\lambda^{\dot\nu}_L)\,\bar\partial^L_{\dot\nu}\varphi_L \quad . \tag{4.3}$$

Going into the symmetric basis (see Eq.(A.11)), one obtains

$$\delta(\bar\Delta_{\dot\mu}\varphi) = -(\bar\Delta_{\dot\mu}\bar\lambda^{\dot\nu})\bar\Delta_{\dot\nu}\varphi \, . \tag{4.4}$$

Here and below the symbol δ means "variation of the function"

$$\delta(\bar\Delta_{\dot\mu}\varphi) \equiv \bar\Delta'_{\dot\mu}\varphi'(x', \theta', \bar\theta') - \bar\Delta_{\dot\mu}\varphi(x, \theta, \bar\theta). \tag{4.5}$$

Similarly,

$$\delta(\Delta_\mu\varphi) = -(\Delta_\mu\lambda^\nu)\Delta_\nu\varphi \, . \tag{4.6}$$

The transformation laws (4.4) and (4.6) differ from the Lorentz one (2.7) only in that the matrices $\Delta_\mu\lambda_\nu$ and $\bar\Delta_{\dot\mu}\bar\lambda_{\dot\nu}$ have nonvanishing trace (i.e., they do not belong to the Lorentz algebra). This can easily be corrected by multiplying $\bar\Delta_{\dot\mu}$ (3.11) and Δ_μ (3.12) by some factors $\bar F$ and F ,

$$\nabla_\mu = F\Delta_\mu \, , \quad \bar\nabla_{\dot\mu} = \bar F\bar\Delta_{\dot\mu} \, . \tag{4.7}$$

To compensate for the traces of $\Delta_\mu\lambda_\nu$ and $\bar\Delta_{\dot\mu}\bar\lambda_{\dot\nu}$, these factors have to transform as follows

$$\delta F = -\tfrac{1}{2}(\Delta^\mu\lambda_\mu)F \, , \quad \delta\bar F = -\tfrac{1}{2}(\bar\Delta_{\dot\mu}\bar\lambda^{\dot\mu})\bar F \, . \tag{4.8}$$

Explicit expressions for F and $\bar F$ in terms of $\mathcal{H}^m$ will be given later on (see Eq.(8.4)).

So, we obtained operators (4.7) with transformation law that imitates the local-Lorentz law (2.7). In order to keep the (formal) difference between world indices and local-Lorentz ones, we shall denote the latter by letters α and $\dot\alpha$ (according to the convention of Sect. 2). Then

$$\nabla_\alpha = \delta_\alpha{}^\mu\nabla_\mu = (F\delta_\alpha{}^\mu)\Delta_\mu \, , \quad \bar\nabla_{\dot\alpha} = \delta_{\dot\alpha}{}^{\dot\mu}\bar\nabla_{\dot\mu} = (\bar F\delta_{\dot\alpha}{}^{\dot\mu})\bar\Delta_{\dot\mu} \, . \tag{4.9}$$

Emphasize that the role of such a notation is just to remind of the (formal) Lorentz character of the corresponding indices.

The transformation laws of the new derivatives coincide in form with the local Lorentz law (2.7)

$$\delta(\nabla_\alpha\varphi) = \frac{i}{2}\,\Omega^{ab}(\sigma_{ab})_\alpha{}^\beta\,\nabla_\beta\varphi \tag{4.10a}$$

$$\delta(\bar\nabla_{\dot\alpha}\varphi) = \frac{i}{2}\,\Omega^{ab}(\tilde\sigma_{ab})_{\dot\alpha\dot\beta}\,\bar\nabla^{\dot\beta}\varphi \tag{4.10b}$$

$$\Omega^{ab} = \frac{i}{4}(\Delta\sigma^{ab}\lambda + \bar\Delta\tilde\sigma^{ab}\bar\lambda) \tag{4.10c}$$

(Here the identities (C.1) are used). However, in Eq.(2.7) the parameters Ω^{ab} were completely arbitrary, while in Eq.(4.10) they are expressed in terms of the parameters λ^μ and $\bar\lambda^{\dot\mu}$ of the world group (besides, Ω^{ab} depend nonlinearly on the gravitational SF $\mathcal{H}^m$ through the arguments x_L, x_R of the parameters $\lambda^\mu, \bar\lambda^{\dot\mu}$ and through the operators $\Delta_\mu, \bar\Delta_{\dot\mu}$). So, we do not have a separate, independent local Lorentz group acting on the SF indices. Our Lorentz group is "locked" to the world one, i.e.,the transformations of the former are induced by the transformations of the latter. Of course, for the field-components of the SF there is a local (in x -space) Lorentz group independent of the general (x-space) coordinate group. This x -local Lorentz group rotates the component indices that are contracted with the indices of θ and $\bar\theta$ in the decomposition of the SF, as well as the component indices corresponding to the external index of the SF. This can be seen inserting the decomposition of the parameters $\lambda^\mu, \bar\lambda^{\dot\mu}$ (see Ref.[2]) into Eq.(4.10). Thus, our formulation provides all the necessary transformations of the component fields without introducing an additional independent gauge Lorentz group.

This situation reminds two ways of describing spinors in the theory of general relativity. In the standard (vierbein) approach, at the expense of introducing an independent vierbein Lorentz group and using more complicated gauge fields (the vierbeins $e^{am}(x)$ instead of the metric $g^{mn}(x)$), the spinors transform as a standard, linear representation of this group. In the alternative approach[9] the vierbein degrees of freedom are fixed, and the spinors transform nonlinearly (with respect to the gravitational field). Although both approaches are ultimately equivalent,in our case, where the group is sophisticated enough, any possible reduction of the number of the extra parameters is strongly desirable.

The operators Δ_μ and $\bar\Delta_{\dot\mu}$ are direct generalizations of the spinor derivatives $\mathcal{D}^{\circ}_\alpha$ and $\bar{\mathcal{D}}^{\circ}_{\dot\alpha}$ in flat supersymmetry. It is clear in the appropriate bases (right and left), and in the symmetric one it can easily be shown inserting $\mathcal{H}^m_0 = \theta\sigma^m\bar\theta$. Now we have all the reasons to call ∇_α and $\bar\nabla_{\dot\alpha}$ covariant derivatives of the scalar SF:

$$\mathcal{D}_\alpha \varphi \equiv \nabla_\alpha \varphi, \quad \bar{\mathcal{D}}_{\dot\alpha} \varphi \equiv \bar\nabla_{\dot\alpha} \varphi . \tag{4.11}$$

Let us briefly summarize Sect. 4. The analysis of the transformation properties of the operators Δ_μ and $\bar\Delta_{\dot\mu}$ (i.e., the derivatives ∂_μ^R and $\partial_{\dot\mu}^L$ in the corresponding chiral bases) suggested the definition of the local Lorentz group as a group induced by the world supergroup. We also defined the simplest covariant derivatives (those of the scalar SF). In what follows the remaining geometrical objects described in Sect. 2 have to be introduced and expressed completely in terms of the gravitational SF $\mathcal{H}^m$.

5. Local-Lorentz law for SF with external indices. We adopt the definition (2.7)

$$\delta \varphi_A = i\,\Omega^{cd} (\Lambda_{cd})_A{}^B \varphi_B \tag{5.1}$$

with parameters Ω^{cd} from Eq.(4.10c). Further, as in Eq.(2.16), one can introduce covariant derivatives of these SF's:

$$\mathcal{D}_\alpha \varphi_B = \nabla_\alpha \varphi_B + \omega_{\alpha B}{}^C \varphi_C \tag{5.2a}$$

$$\bar{\mathcal{D}}_{\dot\alpha} \varphi_B = \bar\nabla_{\dot\alpha} \varphi_B + \bar\omega_{\dot\alpha B}{}^C \varphi_C, \tag{5.2b}$$

where

$$\omega_{\alpha B}{}^C = i\omega_\alpha{}^{de} (\Lambda_{de})_B{}^C, \quad \bar\omega_{\dot\alpha B}{}^C = i\bar\omega_{\dot\alpha}{}^{de} (\Lambda_{de})_B{}^C . \tag{5.2c}$$

The connections ω_α and $\bar\omega_{\dot\alpha}$ will be expressed in terms of $\mathcal{H}^m$ (Eqs. (8.6)), but for the time being we need only their transformation laws which can be extracted from Eq.(2.18).

In Eq. (2.16) there is the vector covariant derivative also. We shall define it (as in the flat case) as the anticommutator of the spinor ones

$$\mathcal{D}_a = \frac{i}{4} (\tilde\sigma_a)^{\dot\alpha\alpha} \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} . \tag{5.3}$$

Note that Eq.(5.3)fixes automatically the vector connection present in Eq.(2.16):

$$\omega_a{}^{bc} = \frac{i}{4} (\tilde\sigma_a)^{\dot\alpha\alpha} (\nabla_\alpha \bar\omega_{\dot\alpha}{}^{bc} + \bar\nabla_{\dot\alpha} \omega_\alpha{}^{bc}) - \\ -\frac{1}{8}\left[(\sigma_a \tilde\sigma_{de})^{\alpha\dot\alpha} \omega_\alpha{}^{de} \bar\omega_{\dot\alpha}{}^{bc} + (\tilde\sigma_a \sigma_{de})^{\dot\alpha\alpha} \bar\omega_{\dot\alpha}{}^{de} \omega_\alpha{}^{bc}\right]. \tag{5.4}$$

6. Expressions for the supervierbeins $E_A{}^M$ in terms of $\mathcal{H}^m$ and the quantities $F, \bar F, \omega_\alpha, \bar\omega_{\dot\alpha}$.

To find the supervierbeins $E_\alpha{}^M$ and $E_{\dot\alpha}{}^M$ one has to represent the spinor covariant derivatives of the scalar SF (4.11)in the form (2.15). Using Eqs.(4.7), (3.13), one obtains

$$\mathcal{D}_\alpha \varphi = F \delta_\alpha{}^\mu (\partial_\mu + i\Delta_\mu \mathcal{H}^m \partial_m)\varphi = E_\alpha{}^M \partial_M \varphi$$

$$\bar{\mathcal{D}}_{\dot\alpha} \varphi = \bar F \delta_{\dot\alpha}{}^{\dot\mu} (-\bar\partial_{\dot\mu} - i\bar\Delta_{\dot\mu} \mathcal{H}^m \partial_m)\varphi = E_{\dot\alpha}{}^M \partial_M \varphi .$$

This gives

$$E_\alpha{}^\mu = F\delta_\alpha{}^\mu ; \; E_\alpha{}^{\dot\mu} = 0 ; \; E_\alpha{}^m = i\nabla_\alpha \mathcal{H}^m \tag{6.1a}$$

$$E_{\dot\alpha}{}^\mu = 0 ; \; E_{\dot\alpha}{}^{\dot\mu} = -\bar F \delta_{\dot\alpha}{}^{\dot\mu} ; \; E_{\dot\alpha}{}^m = -i\bar\nabla_{\dot\alpha} \mathcal{H}^m . \tag{6.1b}$$

Further, the vector derivative (5.3) gets the form

$$\mathcal{D}_a = \frac{i}{4}(\tilde\sigma_a)^{\dot\alpha\alpha}\left[\nabla_\alpha \bar\nabla_{\dot\alpha}\varphi + \omega_{\alpha\dot\alpha\dot\beta}\bar\nabla^{\dot\beta}\varphi + \bar\nabla_{\dot\alpha}\nabla_\alpha\varphi + \bar\omega_{\dot\alpha\alpha}{}^\beta \nabla_\beta \varphi\right] = \\ = \frac{i}{4}(\tilde\sigma_a)^{\dot\alpha\alpha} F\bar F \{\Delta_\alpha, \bar\Delta_{\dot\alpha}\}\varphi + \frac{i}{4}\left[(\tilde\sigma_a \nabla)^{\dot\beta} \bar F - \bar F (\tilde\sigma_a)^{\dot\alpha\alpha} \omega_{\alpha\dot\alpha}{}^{\dot\beta}\right] \delta_{\dot\beta}{}^{\dot\mu} \bar\Delta_{\dot\mu} \varphi + \\ + \frac{i}{4}\left[-(\sigma_a \bar\nabla)^\beta F + F(\tilde\sigma_a)^{\dot\alpha\alpha} \bar\omega_{\dot\alpha\alpha}{}^\beta\right] \delta_\beta{}^\mu \Delta_\mu \varphi = E_a{}^M \partial_M \varphi$$

where

$$\omega_{\alpha\dot\alpha\dot\beta} = \frac{i}{2}\omega_\alpha{}^{ab} (\tilde\sigma_{ab})_{\dot\alpha\dot\beta}, \quad \bar\omega_{\dot\alpha\alpha\beta} = \frac{i}{2}\bar\omega_{\dot\alpha}{}^{ab} (\sigma_{ab})_{\alpha\beta} . \tag{6.2}$$

Taking into account Eqs.(3.13), (A.5), one obtains

$$E_a{}^\mu = \frac{i}{4}\left[-(\sigma_a \bar\nabla)^\beta F + F(\tilde\sigma_a)^{\dot\alpha\alpha} \bar\omega_{\dot\alpha\alpha}{}^\beta\right] \delta_\beta{}^\mu \tag{6.1c}$$

$$E_a{}^{\dot\mu} = \frac{i}{4}\left[-(\tilde\sigma_a \nabla)^{\dot\beta} \bar F + \bar F (\tilde\sigma_a)^{\dot\alpha\alpha} \omega_{\alpha\dot\alpha}{}^{\dot\beta}\right] \delta_{\dot\beta}{}^{\dot\mu} \tag{6.1d}$$

$$E_a{}^m = \frac{1}{4} F\bar F (\Delta\sigma_a \bar\Delta - \bar\Delta \tilde\sigma_a \Delta)\mathcal{H}^m + iE_a{}^\mu \Delta_\mu \mathcal{H}^m + iE_a{}^{\dot\mu} \bar\Delta_{\dot\mu} \mathcal{H}^m . \tag{6.1e}$$

The way of derivation of these supervierbeins guarantees that they transform in accordance with Eq.(2.4), i.e.,infinitesimally

$$\delta E_A{}^\mu = L_A{}^B E_B{}^\mu + E_A{}^N \partial_N \lambda^\mu \\ \delta E_A{}^{\dot\mu} = L_A{}^B E_B{}^{\dot\mu} + E_A{}^N \partial_N \bar\lambda^{\dot\mu} \\ \delta E_A{}^m = L_A{}^B E_B{}^m + E_A{}^N \partial_N \tfrac{1}{2}(\lambda^m + \bar\lambda^m), \tag{6.3}$$

where $L_A{}^B$ is defined in Eq.(2.7), and Ω^{ab} in Eq.(4.10c).

The supervierbeins obtained are not completely expressed in terms of $\mathcal{H}^m$ because in Eq.(6.1) the unknown yet quantities $F, \bar F, \omega_\alpha{}^{ab}, \bar\omega_{\dot\alpha}{}^{ab}$ are present. To find them we shall introduce new and very useful objects - the left and right supervierbeins. Their existence emphasizes once more the chiral structure of our supergroup.

7. Left and right supervierbeins. Consider the covariant derivatives of the left chiral scalar SF

$$\mathcal{D}_A \varphi = E_A{}^M \partial_M \varphi . \tag{7.1}$$

Let us rewrite them in the left basis where the chirality condition has the form

$$\bar{\partial}_{\dot\mu}{}^L \varphi_L = 0. \tag{3.7a}$$

To this end one has to replace the derivatives ∂_M in Eq.(7.1) by the derivatives in the left basis. Using Eqs. (A.10), (A.13), (A.16), and taking into account Eq.(3.7a), one finds

$$\mathcal{D}_A^L \varphi_L = E_A{}^m (1+i\mathcal{H})_m{}^n \partial_n^L \varphi_L + E_A{}^\mu (\partial_\mu^L \varphi_L + i\partial_\mu \mathcal{H}^n \partial_n^L \varphi_L) +$$
$$+ E_A{}^{\dot\mu} (i\bar\partial_{\dot\mu} \mathcal{H}^n \partial_n^L \varphi_L) = (E_A{}^m + i E_A{}^N \partial_N \mathcal{H}^m)\partial_m^L \varphi_L + E_A{}^\mu \partial_\mu^L \varphi_L \equiv$$
$$\equiv \ell_A{}^M \partial_M^L \varphi_L \tag{7.2}$$

(the derivatives of $\mathcal{H}^m$ are always taken with respect to its natural arguments $x^n, \theta^\nu, \bar\theta^{\dot\nu}$). The coefficients $\ell_A{}^M$ in the last line of Eq.(7.2) will be called left supervierbeins. In fact the index M takes the values m and μ only ($\ell_A{}^{\dot\mu}$ vanish because $\bar\partial_{\dot\mu}^L \varphi_L = 0$). The explicit form of $\ell_A{}^M$ is derived from Eq. (7.2)with the help of Eq. (6.1):

$$\ell_\alpha{}^\mu = E_\alpha{}^\mu = F\delta_\alpha{}^\mu \tag{7.3a}$$
$$\ell_\alpha{}^m = E_\alpha{}^m + iE_\alpha{}^N \partial_N \mathcal{H}^m = 2i\nabla_\alpha \mathcal{H}^m \tag{7.3b}$$
$$\left.\begin{aligned} &\ell_{\dot\alpha}{}^\mu = 0 \\ &\ell_{\dot\alpha}{}^m = E_{\dot\alpha}{}^m + i(E_{\dot\alpha}{}^n\partial_n + E_{\dot\alpha}{}^{\dot\nu}\bar\partial_{\dot\nu})\mathcal{H}^m = -i\bar\nabla_{\dot\alpha}\mathcal{H}^m + i\bar\nabla_{\dot\alpha}\mathcal{H}^m = 0 \end{aligned}\right\} \tag{7.3c}$$
$$\ell_a{}^\mu = E_a{}^\mu = \tfrac{i}{4}\left[-(\sigma_a\bar\nabla)^\beta F + F(\tilde\sigma_a)^{\dot\alpha\alpha}\bar\omega_{\dot\alpha\alpha}{}^\beta\right]\delta_\beta{}^\mu \tag{7.3d}$$
$$\ell_a{}^m = E_a{}^m + i(E_a{}^n\partial_n + E_a{}^\nu\partial_\nu + E_a{}^{\dot\nu}\bar\partial_{\dot\nu})\mathcal{H}^m =$$
$$= \left[\tfrac14 F\bar F(\Delta\sigma_a\bar\Delta - \bar\Delta\tilde\sigma_a\Delta)\mathcal{H}^n + iE_a{}^\mu\Delta_\mu\mathcal{H}^n + iE_a{}^{\dot\mu}\bar\Delta_{\dot\mu}\mathcal{H}^n\right](1+i\mathcal{H})_n{}^m +$$
$$+ iE_a{}^\nu\Delta_\nu\mathcal{H}^n(1-i\mathcal{H})_n{}^m - iE_a{}^{\dot\nu}\bar\Delta_{\dot\nu}\mathcal{H}^n(1+i\mathcal{H})_n{}^m =$$

$$= -\tfrac12 F\bar F\hat\ell_a{}^m + 2iE_a{}^\mu\Delta_\mu\mathcal{H}^m. \tag{7.3e}$$

Here the notation

$$\hat\ell_a{}^m \equiv \bar\Delta\tilde\sigma_a\Delta\mathcal{H}^m \tag{7.3f}$$

and the identities (3.13) and (A.6) are used. From Eq. (7.3c) it is clear that the index A of $\ell_A{}^M$ takes the values a, α only, so $\ell_A{}^M$ form a square matrix

$$\|\ell_A{}^M\| = \begin{pmatrix} \ell_a{}^m & \ell_a{}^\mu \\ \ell_\alpha{}^m & \ell_\alpha{}^\mu \end{pmatrix}. \tag{7.4}$$

Similarly, considering the covariant derivatives of the right chiral scalar SF in the right basis one can obtain a set of right supervierbeins

$$\|\tau_A{}^M\| = \begin{pmatrix} \tau_a{}^m & \tau_a{}^{\dot\mu} \\ \tau_{\dot\alpha}{}^m & \tau_{\dot\alpha}{}^{\dot\mu} \end{pmatrix}; \tag{7.5}$$

$$\tau_{\dot\alpha}{}^{\dot\mu} = E_{\dot\alpha}{}^{\dot\mu} = -\bar F\delta_{\dot\alpha}{}^{\dot\mu} \tag{7.6a}$$
$$\tau_{\dot\alpha}{}^m = 2E_{\dot\alpha}{}^m = -2i\bar\nabla_{\dot\alpha}\mathcal{H}^m \tag{7.6b}$$
$$\tau_a{}^{\dot\mu} = E_a{}^{\dot\mu} = \tfrac{i}{4}\left[-(\tilde\sigma_a\nabla)^{\dot\beta}\bar F + \bar F(\tilde\sigma_a)^{\dot\alpha\alpha}\omega_{\alpha\dot\alpha}{}^{\dot\beta}\right]\delta_{\dot\beta}{}^{\dot\mu} \tag{7.6c}$$
$$\tau_a{}^m = \tfrac12 F\bar F\hat\tau_a{}^m + 2iE_a{}^{\dot\mu}\bar\Delta_{\dot\mu}\mathcal{H}^m \tag{7.6d}$$
$$\hat\tau_a{}^m \equiv \Delta\sigma_a\bar\Delta\mathcal{H}^m. \tag{7.6e}$$

The transformation laws for $\ell_A{}^M$ and $\tau_A{}^M$ follow from the equations

$$\mathcal{D}_A^L\varphi_L = \ell_A{}^M\partial_M^L\varphi_L \quad , \quad \mathcal{D}_A^R\bar\varphi_R = \tau_A{}^M\partial_M^R\bar\varphi_R . \tag{7.7}$$

One finds

$$\delta\ell_A{}^M = L_A{}^B\ell_B{}^M + \ell_A{}^N\partial_N^L\lambda_L^M \tag{7.8a}$$
$$\delta\tau_A{}^M = L_A{}^B\tau_B{}^M + \tau_A{}^N\partial_N^R\bar\lambda_R^M . \tag{7.8b}$$

A few words about the interpretation of $\ell_A{}^M$ and $\tau_A{}^M$. In the initial chiral SS $\{(x_L^m, \theta_L^\mu)\}$ and $\{(x_R^m, \bar\theta_R^{\dot\mu})\}$ the chiral SF's are the most general ones. So, the formulae (7.7) could be regarded there as definitions of supervierbeins, i.e., $\ell_A{}^M$ and $\tau_A{}^M$ could be thought of as the supervierbeins of the left and right SS separately. However, $\ell_A{}^M$ and $\tau_A{}^M$ do not transform with the help of the left (λ^M) and, correspondingly, the right $(\bar\lambda^M)$ parameters only. The Lorentz transformations mix them up (the parameters Ω^{ab} (4.10c) depend both on λ and $\bar\lambda$). Nevertheless, the role of the "left and right supervierbeins" proves to be very constructive.

8. Explicit form of the factors $F, \bar F$ and connections $\omega, \bar\omega$. The explicit expressions in terms of $\mathcal{H}^m$ for these quantities will be found with the help of the left and right supervierbeins.

Consider the Berezinians (superdeterminants) of ℓ_A^M and τ_A^M :

$$Ber(\ell_A^M) = Det(\ell_a^m - \ell_a^\mu \ell^{-1}{}_\mu^\alpha \ell_\alpha^m).\, Det^{-1}(\ell_\alpha^\mu) =$$

$$= 2^{-4} F^2 \bar F^4 Det(\hat\ell_a^m) \qquad (8.1)$$

$$Ber(\tau_A^M) = 2^{-4} \bar F^2 F^4 Det(\hat\tau_a^m).$$

They are scalars with respect to the supergroup (1.4), (1.5). Indeed (ℓ_M^A are defined by the relation $\ell_M^A \ell_A^N = \delta_M^N$)

$$\delta(Ber(\ell_A^M)) = Ber(\ell).\, \ell_M^A \delta\ell_A^M.\, (-1)^{P(M)} =$$

$$= Ber(\ell).\,[\ell_M^A L_A^B \ell_B^M + \ell_M^A \ell_A^N \partial_N^L \lambda_L^M](-1)^{P(M)} =$$

$$= Ber(\ell).\,[\partial_M^L \lambda_L^M.\, (-1)^{P(M)}]\,. \qquad (8.2)$$

The last term vanishes owing to the Einstein condition (1.5). Let us stress that this is the only place in the present paper where we do use the Einstein condition. A natural question arises:could this condition be weakened in some way? One such possibility, connected with the γ_5 -invariance of the action for supergravity, is discussed in Sect. 11.

Thus, without loss of generality, we can set the scalars (8.1) equal to some constant:

$$Ber(\ell_A^M) = Ber(\tau_A^M) = 2^{-4} \qquad (8.3)$$

and obtain explicit expressions for F and $\bar F$ in terms of $\mathcal{H}^m$:

$$F = Det^{-\frac13}(\hat\tau_a^m).\, Det^{\frac16}(\hat\ell_a^m)\,, \quad \bar F = Det^{\frac16}(\hat\tau_a^m).\, Det^{-\frac13}(\hat\ell_a^m)\,. \qquad (8.4)$$

Finally, we have to find the explicit form of $\omega_{\alpha ab}$ and $\bar\omega_{\dot\alpha ab}$. The only requirement they must fulfil is to transform according to Eq.(2.18). We shall show now that the following quantities do transform correctly

$$\omega_{\alpha ab} = \frac14(\nabla_\alpha \tau_a^M.\, \tau_{Mb} - \nabla_\alpha \tau_b^M.\, \tau_{Ma}) \qquad (8.5a)$$

$$\bar\omega_{\dot\alpha ab} = \frac14(\bar\nabla_{\dot\alpha} \ell_a^M.\, \ell_{Mb} - \bar\nabla_{\dot\alpha} \ell_b^M.\, \ell_{Ma})\,. \qquad (8.5b)$$

Indeed (see Eqs. (7.8), (4.10) and the definition $\tau_A^M \tau_M^B = \delta_A^B$),

$$\delta\omega_{\alpha ab} = \frac14 \Omega^{cd} \frac{i}{2} (\sigma_{cd}\nabla)_\alpha \tau_a^M.\, \tau_{Mb} +$$

$$+\frac14 \nabla_\alpha(-2\Omega_{ac}\tau^{cM} + \tau_a^N \partial_N^R \bar\lambda_R^M).\, \tau_{Mb} +$$

$$+\frac14 \nabla_\alpha \tau_a^M.\,(-2\Omega_{bc}\tau_M^c - \partial_M^R \bar\lambda_R^N.\, \tau_{Nb}) - (a\leftrightarrow b) =$$

$$= \frac{i}{2}\Omega^{cd}(\sigma_{cd})_\alpha{}^\beta \omega_{\beta ab} - 2\Omega_{ac}\omega_\alpha{}^c{}_b + 2\omega_{\alpha a}{}^c \Omega_{cb} - \nabla_\alpha \Omega_{ab} +$$

$$+\frac14 (-1)^{P(N)} \{ \tau_a^N [\nabla_\alpha \partial_N^R \bar\lambda_R^M] \tau_{Mb} - \tau_b^N [\nabla_\alpha \partial_N^R \bar\lambda_R^M] \tau_{Ma} \}\,.$$

The terms in the braces vanish because the operator ∇_α in the left basis is proportional to ∂_μ^R (see Eq.(3.14)), and $\bar\lambda_R$ does not depend on θ^μ.

So, the quantities (8.5) can serve as connections. However, they are not yet expressed in terms of $\mathcal{H}^m$ only, because ℓ_a^M and ℓ_M^b themselves depend on $\bar\omega_{\dot\alpha}$ (see, e.g., Eq. (7.3d)). Therefore formulae (8.5) are equations rather than definitions for ω_α and $\bar\omega_{\dot\alpha}$. Their solutions are

$$\omega_{\alpha ab} = \frac12(\nabla_\alpha \hat\tau_a^m.\, \hat\tau_{mb} - \nabla_\alpha \hat\tau_b^m.\, \hat\tau_{ma}) - \frac{i}{2}(\sigma_{ab}\nabla)_\alpha \ln F\bar F \qquad (8.6a)$$

$$\bar\omega_{\dot\alpha ab} = \frac12(\bar\nabla_{\dot\alpha} \hat\ell_a^m.\, \hat\ell_{mb} - \bar\nabla_{\dot\alpha} \hat\ell_b^m.\, \hat\ell_{ma}) - \frac{i}{2}(\tilde\sigma_{ab}\bar\nabla)_{\dot\alpha} \ln F\bar F \qquad (8.6b)$$

where $\hat\tau_a^m \hat\tau_m^b = \delta_a^b$, $\hat\ell_a^m \hat\ell_m^b = \delta_a^b$. The derivation of Eq.(8.6) as well as formulae for the connections applied to spinors are given in Appendix B.

So, all the basic objects of differential geometry are defined in a reasonable way, and moreover, they are expressed in terms of the gravitational SF $\mathcal{H}^m$. The formalism developed can now be applied for constructing and investigating the invariants of the supergroup. This will be the subject of a separate paper. Here we will give the invariant action only.

9. Invariant action for Einstein supergravity. First of all, a scalar density has to be defined because the volume element $d^4x d^4\theta$ in the physical SS is not invariant. This is just the superdeterminant

$$E = Ber\, E_M^A = (Ber\, E_A^M)^{-1} \qquad (9.1)$$

(the analogue of $\sqrt{-g}$ in ordinary gravity). Indeed (see Eq.(6.3)),

$$\delta E = -E \,.\, E_M{}^A \, \delta E_A{}^M (-1)^{p(M)} =$$
$$= E . \left[-\tfrac{1}{2} \partial_m (\lambda^m + \bar\lambda^m) + \partial_\mu \lambda^\mu + \bar\partial_{\dot\mu} \bar\lambda^{\dot\mu} \right] .$$

Then the action

$$S = \frac{1}{\varkappa^2} \int d^4x \, d^4\theta . E . \mathcal{L} \tag{9.2}$$

will be invariant with any matter Lagrangian $\mathcal{L}(x,\theta,\bar\theta)$ ($\mathcal{L}$ being a scalar with respect to the supergroup (1.4)):

$$S' = \frac{1}{\varkappa^2} \int d^4x' d^4\theta' \, E'(x',\theta',\bar\theta') \, \mathcal{L}'(x',\theta',\bar\theta') =$$
$$= \frac{1}{\varkappa^2} \int \left[d^4x \, d^4\theta \left(1 + \tfrac{1}{2} \partial_m (\lambda^m + \bar\lambda^m) - \partial_\mu \lambda^\mu - \partial_{\dot\mu} \bar\lambda^{\dot\mu} \right) \right] \times$$
$$\times \left[E(x,\theta,\bar\theta) \left(1 - \tfrac{1}{2} \partial_m (\lambda^m + \bar\lambda^m) + \partial_\mu \lambda^\mu + \bar\partial_{\dot\mu} \bar\lambda^{\dot\mu} \right) \right] \mathcal{L}(x,\theta,\bar\theta) = S .$$

In the case of pure supergravity the action is just the "invariant volume"

$$S_{SG} = \frac{1}{\varkappa^2} \int d^4x \, d^4\theta \, E(x,\theta,\bar\theta) . \tag{9.3}$$

This remarkable fact was pointed out by Wess and Zumino [5]. However, in their approach $E_M{}^A$ are the basic potentials, and E does not contain their derivatives. So, the variation of the action (9.3) is not straightforward: certain (previously guessed) "kinematic" constraints on the torsion components must be taken into account.

In our approach E is expressed in terms of the derivatives of the gravitational SF

$$E = 2^4 \left(\mathrm{Det} \| \Delta \sigma_a \bar\Delta \mathcal{H}^m \| \right)^{-\frac{1}{6}} \left(\mathrm{Det} \| \bar\Delta \tilde\sigma_a \Delta \mathcal{H}^m \| \right)^{-\frac{1}{6}} \times$$
$$\times \left(\mathrm{Det} \| \delta_m^n + \partial_m \mathcal{H}^k \partial_k \mathcal{H}^n \| \right)^{\frac{1}{2}} \tag{9.4}$$

(see the derivation in Appendix B). Therefore in our case the variation of Eq.(9.3) with respect to $\mathcal{H}^m$ leads directly to the equations of motion (more details will be given in a forthcoming paper).

The matter Lagrangian in Eq.(9.2) is obtained from the corresponding flat Lagrangian $\mathcal{L}^0$ by replacing the spinor derivatives by the covariant ones (defined in the present paper). Such an action (9.2) gives in the linearized limit the equations we postulated in 1976 [10]. Thus the idea of supergravity as a theory of an axial SF generated by the supercurrent [10] is confirmed once again.

10. Summary of the main results. For convenience of the reader we list here the explicit expressions in terms of $\mathcal{H}^m$ for the basic quantities.

$$\Delta_\mu \mathcal{H}^m = \partial_\mu \mathcal{H}^n (1 - i\mathcal{H})^{-1}{}_n{}^m , \quad \mathcal{H} = \| \partial_m \mathcal{H}^n \|$$
$$\bar\Delta_{\dot\mu} \mathcal{H}^m = -\bar\partial_{\dot\mu} \mathcal{H}^n (1 + i\mathcal{H})^{-1}{}_n{}^m ;$$
$$\nabla_\alpha = F \delta_\alpha{}^\mu \Delta_\mu , \quad \bar\nabla_{\dot\alpha} = \bar F \delta_{\dot\alpha}{}^{\dot\mu} \bar\Delta_{\dot\mu} , \tag{4.9}$$
$$F = \mathrm{Det}^{-\frac{1}{3}} (\hat r_a{}^m) \mathrm{Det}^{\frac{1}{6}} (\hat l_a{}^m), \quad \bar F = \mathrm{Det}^{\frac{1}{6}} (\hat r_a{}^m) . \mathrm{Det}^{-\frac{1}{3}} (\hat l_a{}^m) , \tag{8.4}$$

where

$$\hat r_a{}^m = \Delta \sigma_a \bar\Delta \mathcal{H}^m , \quad \hat l_a{}^m = \bar\Delta \tilde\sigma_a \Delta \mathcal{H}^m .$$

Connections:

$$\omega_{\alpha a b} = \tfrac{1}{2} (\nabla_\alpha \hat r_a{}^m . \hat r_{mb} - \nabla_\alpha \hat r_b{}^m . \hat r_{ma}) - \tfrac{i}{2} (\sigma_{ab} \nabla)_\alpha \ln F \bar F$$
$$\bar\omega_{\dot\alpha a b} = \tfrac{1}{2} (\bar\nabla_{\dot\alpha} \hat l_a{}^m . \hat l_{mb} - \bar\nabla_{\dot\alpha} \hat l_b{}^m . \hat l_{ma}) - \tfrac{i}{2} (\tilde\sigma_{ab} \bar\nabla)_{\dot\alpha} \ln F \bar F , \tag{8.6}$$

where $\hat r_a{}^m \hat r_m{}^b = \delta_a{}^b , \quad \hat l_a{}^m \hat l_m{}^b = \delta_a{}^b .$

Supervierbeins:

$$E_\alpha{}^\mu = F \delta_\alpha{}^\mu , \quad E_\alpha{}^{\dot\mu} = 0 , \quad E_\alpha{}^m = i \nabla_\alpha \mathcal{H}^m$$
$$E_{\dot\alpha}{}^\mu = 0 , \quad E_{\dot\alpha}{}^{\dot\mu} = -\bar F \delta_{\dot\alpha}{}^{\dot\mu} , \quad E_{\dot\alpha}{}^m = -i \bar\nabla_{\dot\alpha} \mathcal{H}^m$$
$$E_a{}^\mu = \tfrac{i}{4} \left[-(\sigma_a \bar\nabla)^\beta F + F (\tilde\sigma_a)^{\dot\alpha\alpha} \bar\omega_{\dot\alpha\alpha}{}^\beta \right] \delta_\beta{}^\mu \tag{6.1}$$
$$E_a{}^{\dot\mu} = \tfrac{i}{4} \left[-(\tilde\sigma_a \nabla)^{\dot\beta} \bar F + \bar F (\tilde\sigma_a)^{\dot\alpha\alpha} \omega_{\alpha\dot\alpha}{}^{\dot\beta} \right] \delta_{\dot\beta}{}^{\dot\mu}$$
$$E_a{}^m = \tfrac{1}{4} F \bar F (\hat r_a{}^m - \hat l_a{}^m) + i E_a{}^\mu \Delta_\mu \mathcal{H}^m + i E_a{}^{\dot\mu} \bar\Delta_{\dot\mu} \mathcal{H}^m ,$$

where

$$\omega_{\alpha\dot\beta\dot\mu} = \tfrac{i}{2} \omega_{\alpha a b} (\tilde\sigma^{ab})_{\dot\beta\dot\mu} , \quad \bar\omega_{\dot\alpha\beta\mu} = \tfrac{i}{2} \bar\omega_{\dot\alpha a b} (\sigma^{ab})_{\beta\mu}$$

$$E = \mathrm{Ber}(E_M{}^A) = 2^4 \mathrm{Det}^{-\frac{1}{6}} (\hat r_a{}^m) . \mathrm{Det}^{-\frac{1}{6}} (\hat l_a{}^m) . \mathrm{Det}^{\frac{1}{2}} (1 + \mathcal{H}^2) . \tag{9.4}$$

11. Concluding remarks. So, the aim of the paper has been achieved. The formalism of differential geometry in the physical 4+4-dimensional SS is developed on the basis of the complex supergroup of general transformations of the coordinates of the left and right chiral SS. In this formalism the only independent dynamical variable is the axial gravitational SF $\mathcal{H}^m(x,\theta,\bar\theta)$. All

the geometrical structures, the supervierbeins $E_A{}^M$ and the connections $\omega_{AB}{}^C$ are expressed in terms of $\mathcal{H}^m$. A simple calculation shows that this approach is much more economical than the straightforward one. The latter uses $E_A{}^M$ as potentials and is based on the general-coordinate-transformation supergroup in the real SS $\{(x^m, \theta^\mu, \bar\theta^{\dot\mu})\}$ (see, e.g., Refs./3-7/).

<u>In our approach</u>:	<u>In the straightforward approach</u>:
Variables:	
$\mathcal{H}^m(x,\theta,\bar\theta)$ or $4 \times 16 = \boxed{64}$ field components	$E_A{}^M(x,\theta,\bar\theta)$ or $(8 \times 8) \times 16 = \boxed{1024}$ field components
Parameters of the supergroup:	
Chiral superfunctions $\lambda^m(z_L), \lambda^\mu(z_L), \bar\lambda^m(z_R), \bar\lambda^{\dot\mu}(z_R)$ or (4+2+4+2)x4=48 functions.	General superfunctions $\lambda^m(z), \lambda^\mu(z), \bar\lambda^{\dot\mu}(z)$ or (4+2+2)x16=128 functions.
8 supervolume-preservation conditions leave $48-8=\boxed{40}$ parameter-functions.	Parameters $\Omega^{ab}(z)$ of the local Lorentz group or 6x16= 96 functions. Total number of parameter-functions: $128+96=\boxed{224}$.

In fact, to describe supergravity only 38 variables are needed (16 components of the graviton field $e_a^m(x)$, 16 of gravitino $\psi_\alpha^m(x)$ and 6 auxiliary fields $P(x), S(x), A^m(x)$; 14 of them are gauge degrees of freedom (4 are connected with the general coordinate transformations, 6 with the local Lorentz ones, and 4 with the local-supersymmetry ones). In our case 40-14=26 gauge functions are used to exclude 64-38=26 superfluous field components when the gauge is partially fixed (see Eq. (22)in Ref./2/).

In the straightforward approach the gauge transformations cannot remove all the extra degrees of freedom (224<< 1024). Therefore algebraic constraints have to be imposed on the components of torsion tensor. This constraints are guessed by intuition,by analogy with the flat case, etc.

In the framework of our approach the torsion and curvature tensors are evaluated directly in terms of $\mathcal{H}^m$ (details will be given in a following paper). It turns out that the constraints of Wess and Zumino/4/are fulfilled automatically. The concrete form of these constraints depends on the definition of the vector covariant derivative and connection $\omega_a{}^{bc}$. From our natural choice (5.3) (and, correspondingly, (5.4)) it follows that the component $R_{\alpha\dot\alpha}{}^{bc}$ of the curvature tensor vanishes (see Eq.(2.19)). At this point our results slightly differ from the ones of Ref./5/. However, it is a purely formal difference and can be removed by adding appropriate components of the torsion tensor to $\omega_a{}^{bc}$ (5.4).

The last remark concerns the global chiral invariance of the Einstein-supergravity action/11/. In our formulation this fact is easily proved. In Ref./2/ we considered a supergroup that, compared with the supergroup (1.4), (1.5), contained just extra γ_5-transformations. It was given by a condition weaker than Eq. (1.5): the condition of preservation of the product of left and right supervolumes

$$(\partial^L_M \lambda^M_L + \partial^R_M \bar\lambda^M_R)(-1)^{P(M)} = 0. \qquad (11.1)$$

Now recall that in the present paper we used the condition (1.5) at one point only: in the derivation of F and $\bar F$ (see Eq.(8.2)). If Eq.(1.5)is replaced by the weaker one (11.1), then only the product Ber($\ell_A{}^M$).Ber($\tau_A{}^M$) will be a scalar. Correspondingly, the arguments of the beginning of Sect. 8 will allow one to find $F\bar F$ but not F and $\bar F$. However, in the derivation of the explicit expression (9.4) for E=Ber(E_M^A) just this product $F\bar F$ took part (see Eq.(B.8)). So, the Einstein supergravity action (9.3)with E from Eq.(9.4)proves to be invariant under the larger supergroup (1.4), (11.1), i.e.,under the global γ_5-transformation. This property is very important in the investigation of the higher counterterms /12,13/.

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Appendix A

Here we prove some useful identities for the differential operators

$$\Delta_\mu = \partial_\mu + i\partial_\mu \mathcal{H}^m (1-i\mathcal{H})^{-1}{}_m{}^n \partial_n \equiv \partial_\mu + i\Delta_\mu \mathcal{H}^m \partial_m \qquad (A.1)$$

$$\bar\Delta_{\dot\mu} = -\bar\partial_{\dot\mu} + i\bar\partial_{\dot\mu} \mathcal{H}^m (1+i\mathcal{H})^{-1}{}_m{}^n \partial_n \equiv -\bar\partial_{\dot\mu} - i\bar\Delta_{\dot\mu} \mathcal{H}^m \partial_m . \qquad (A.2)$$

A. $\{\Delta_\mu, \Delta_\nu\} = 0 \;, \quad \{\bar\Delta_{\dot\mu}, \bar\Delta_{\dot\nu}\} = 0 .$ (A.3)

Proof:

$$\{\Delta_\mu, \Delta_\nu\} = \{\partial_\mu + i\Delta_\mu \mathcal{H}^m \partial_m, \partial_\nu + i\Delta_\nu \mathcal{H}^n \partial_n\} =$$

$$= i\{\Delta_\mu, \Delta_\nu\}\mathcal{H}^m \partial_m \quad . \qquad \text{(A.4)}$$

Then

$$\{\Delta_\mu, \Delta_\nu\}\, \mathcal{H}^m . (1 - i\mathcal{H})_m{}^n = 0$$

and

$$\{\Delta_\mu, \Delta_\nu\}\mathcal{H}^m = 0 .$$

Putting this into Eq.(A.4) we obtain Eq.(A.3).

B. $\{\Delta_\mu, \bar\Delta_{\dot\mu}\} = -i[\Delta_\mu, \bar\Delta_{\dot\mu}]\mathcal{H}^m \partial_m \quad .$ (A.5)

Proof:

$$\{\Delta_\mu, \bar\Delta_{\dot\mu}\} = \{\partial_\mu + i\Delta_\mu \mathcal{H}^m\partial_m, -\bar\partial_{\dot\mu} - i\bar\Delta_{\dot\mu}\mathcal{H}^n\partial_n\} =$$

$$= -i[\Delta_\mu, \bar\Delta_{\dot\mu}]\mathcal{H}^m \partial_m \quad .$$

C. $[\Delta_\mu, \bar\Delta_{\dot\mu}]\mathcal{H}^m . (1 + i\mathcal{H})_m{}^n = -2\bar\Delta_{\dot\mu}\Delta_\mu \mathcal{H}^n$ (A.6)

D. $[\Delta_\mu, \bar\Delta_{\dot\mu}]\mathcal{H}^m . (1 - i\mathcal{H})_m{}^n = 2\Delta_\mu \bar\Delta_{\dot\mu}\mathcal{H}^n .$ (A.7)

Proof: with the help of Eq. (A.5) one finds

$$[\Delta_\mu, \bar\Delta_{\dot\mu}]\mathcal{H}^m . (1 + i\mathcal{H})_m{}^n = [\Delta_\mu, \bar\Delta_{\dot\mu}]\mathcal{H}^n + i[\Delta_\mu, \bar\Delta_{\dot\mu}]\mathcal{H}^m\partial_m\mathcal{H}^n =$$

$$= [\Delta_\mu, \bar\Delta_{\dot\mu}]\mathcal{H}^n - \{\Delta_\mu, \bar\Delta_{\dot\mu}\}\mathcal{H}^n = -2\bar\Delta_{\dot\mu}\Delta_\mu \mathcal{H}^n .$$

Now we list formulae for transferring derivatives of SF from chiral bases into the symmetric one and vice versa. Note that the derivatives of $\mathcal{H}^m$ are always taken with respect to its natural arguments x^n, θ^ν, $\bar\theta^{\dot\nu}$.

$$\partial_\mu^L \varphi_L = \partial_\mu \varphi - i\partial_\mu \mathcal{H}^m (1 + i\mathcal{H})^{-1}{}_m{}^n \partial_n \varphi \qquad \text{(A.8)}$$

$$\partial_\mu^R \varphi_R = \partial_\mu \varphi + i\partial_\mu \mathcal{H}^m (1 - i\mathcal{H})^{-1}{}_m{}^n \partial_n \varphi \equiv \Delta_\mu \varphi \qquad \text{(A.9)}$$

$$\partial_\mu \varphi = \partial_\mu^L \varphi_L + i\partial_\mu \mathcal{H}^m \partial_m^L \varphi_L = \partial_\mu^R \varphi_R - i\partial_\mu \mathcal{H}^m \partial_m^R \varphi_R \qquad \text{(A.10)}$$

$$\bar\partial_{\dot\mu}^L \varphi_L = \bar\partial_{\dot\mu}\varphi - i\bar\partial_{\dot\mu}\mathcal{H}^m (1 + i\mathcal{H})^{-1}{}_m{}^n \partial_n \varphi \equiv -\bar\Delta_{\dot\mu}\varphi \qquad \text{(A.11)}$$

$$\bar\partial_{\dot\mu}^R \varphi_R = \bar\partial_{\dot\mu}\varphi + i\bar\partial_{\dot\mu}\mathcal{H}^m (1 - i\mathcal{H})^{-1}{}_m{}^n \partial_n \varphi \qquad \text{(A.12)}$$

$$\bar\partial_{\dot\mu}\varphi = \bar\partial_{\dot\mu}^L \varphi_L + i\bar\partial_{\dot\mu}\mathcal{H}^m \partial_m^L \varphi_L = \bar\partial_{\dot\mu}^R \varphi_R - i\bar\partial_{\dot\mu}\mathcal{H}^m \partial_m^R \varphi_R \qquad \text{(A.13)}$$

$$\partial_m^L \varphi_L = (1 + i\mathcal{H})^{-1}{}_m{}^n \partial_n \varphi \qquad \text{(A.14)}$$

$$\partial_m^R \varphi_R = (1 - i\mathcal{H})^{-1}{}_m{}^n \partial_n \varphi \qquad \text{(A.15)}$$

$$\partial_m \varphi = (1 + i\mathcal{H})_m{}^n \partial_n^L \varphi_L = (1 - i\mathcal{H})_m{}^n \partial_n^R \varphi_R \; . \qquad \text{(A.16)}$$

Appendix B

Here we derive the formulae (8.6) for the connections and (9.4) for the Berezinian of the supervierbeins.

In the initial formula (8.5b) for $\bar\omega_{\dot\alpha ab}$, besides the left supervierbeins $\ell_a{}^M$, their inverses $\ell_M{}^a$ are also present $(\ell_a{}^M \ell_M{}^b = \delta_a{}^b)$. One can easily check that (see Eq.(7.3))

$$\ell_m{}^a = -\frac{2}{F\bar F}\hat\ell_m{}^a \;, \quad \ell_\mu{}^a = \frac{2}{F\bar F}\ell^{-1}_\mu{}^\alpha \ell_\alpha{}^m \hat\ell_m{}^a \qquad \text{(B.1)}$$

where $\hat\ell_a{}^m \hat\ell_m{}^b = \delta_a{}^b$, $\ell^{-1}_\mu{}^\alpha = \frac{1}{F}\delta_\mu{}^\alpha$. Let us insert Eq.(B.1) into Eq. (8.5b) taking into account Eq. (7.3):

$$\bar\omega_{\dot\alpha ab} = \frac{1}{4}\bar\nabla_{\dot\alpha}\left(-\frac{1}{2}F\bar F \hat\ell_a{}^m + 2iE_a{}^\nu \Delta_\nu \mathcal{H}^m\right).\left(-\frac{2}{F\bar F}\right)\hat\ell_{mb} +$$

$$+ \frac{1}{4}(\bar\nabla_{\dot\alpha}E_a{}^\mu)\left(\frac{4i}{F\bar F}\Delta_\mu \mathcal{H}^m \hat\ell_{mb}\right) - (a \leftrightarrow b) =$$

$$= \frac{1}{4}\bar\nabla_{\dot\alpha}\hat\ell_a{}^m . \hat\ell_{mb} + \frac{i}{2F}(\sigma_b)_{\mu\dot\alpha}\ell_a{}^\mu - (a \leftrightarrow b) =$$

$$= -\frac{i}{16}(\tilde\sigma_a \sigma_{cd}\sigma_b - \tilde\sigma_b \sigma_{cd}\sigma_a)_{\dot\alpha\dot\beta}\bar\omega^{\dot\beta cd} +$$

$$+ \frac{1}{4}(\bar\nabla_{\dot\alpha}\hat\ell_a{}^m . \hat\ell_{mb} - \bar\nabla_{\dot\alpha}\hat\ell_b{}^m . \hat\ell_{ma}) - \frac{i}{4}(\tilde\sigma_{ab}\bar\nabla)_{\dot\alpha}\ln F . \qquad \text{(B.2)}$$

From Eq. (B.2) the following expression for $\bar\omega_{\dot\alpha ab}$ can be derived

$$\bar\omega_{\dot\alpha ab} = \varphi_{\dot\alpha ab} - \frac{i}{8}(\tilde\sigma_a \sigma_{cd}\sigma_b - \tilde\sigma_b \sigma_{cd}\sigma_a)_{\dot\alpha\dot\beta}\varphi^{\dot\beta cd} \qquad \text{(B.3)}$$

where

$$\varphi_{\dot\alpha ab} = \frac{1}{4}(\bar\nabla_{\dot\alpha}\hat\ell_a{}^m . \hat\ell_{mb} - \bar\nabla_{\dot\alpha}\hat\ell_b{}^m . \hat\ell_{ma}) - \frac{i}{4}(\tilde\sigma_{ab}\bar\nabla)_{\dot\alpha}\ln F .$$

Finally, using the identity (see Eqs. (A.3), (A.6), (A.7))

$$\bar\nabla_{\dot\alpha}\hat\ell_a{}^m = \bar F \delta_{\dot\alpha}{}^{\dot\mu}\bar\Delta_{\dot\mu}\bar\Delta\tilde\sigma_a \Delta \mathcal{H}^m = -\frac{1}{2}\bar F \bar\Delta_{\dot\mu}\bar\Delta^{\dot\mu}(\tilde\sigma_a \Delta)_{\dot\alpha}\mathcal{H}^m \qquad \text{(B.4)}$$

and the identity (see Eq. (8.4))

$$\bar\nabla_{\dot\alpha}\hat\ell_a{}^m . \hat\ell_m{}^a = \bar\nabla_{\dot\alpha}\ln \mathrm{Det}(\hat\ell_a{}^m) = -2\bar\nabla_{\dot\alpha}\ln(F\bar F^2) \qquad \text{(B.5)}$$

one obtains Eq. (8.6b).

The expressions for the connections applied to spinors are frequently used. They are:

$$\omega_{\alpha\dot\beta\dot\mu} \equiv \frac{i}{2}\,\omega_{\alpha ab}(\tilde\sigma^{ab})_{\dot\beta\dot\mu} = \frac{i}{2}\,(\nabla_\alpha \hat\tau_a^{\ m}\cdot \hat\tau_{mb})(\tilde\sigma^{ab})_{\dot\beta\dot\mu} \tag{B.6a}$$

$$\bar\omega_{\dot\alpha\beta\mu} \equiv \frac{i}{2}\,\bar\omega_{\dot\alpha ab}(\sigma^{ab})_{\beta\mu} = \frac{i}{2}\,(\bar\nabla_{\dot\alpha} \hat l_a^{\ m}\cdot \hat l_{mb})(\sigma^{ab})_{\beta\mu}\,. \tag{B.6b}$$

Here the identity (C.1) is taken into account. Further,

$$\omega_{\alpha\beta\mu} \equiv \frac{i}{2}\,\omega_{\alpha ab}(\sigma^{ab})_{\beta\mu} = (\varepsilon_{\alpha\beta}\varepsilon_{\mu\delta} + \varepsilon_{\alpha\mu}\varepsilon_{\beta\delta})\nabla^\delta \ln F \tag{B.7a}$$

$$\bar\omega_{\dot\alpha\dot\beta\dot\mu} \equiv \frac{i}{2}\,\bar\omega_{\dot\alpha ab}(\tilde\sigma^{ab})_{\dot\beta\dot\mu} = -(\varepsilon_{\dot\alpha\dot\beta}\varepsilon_{\dot\mu\dot\delta} + \varepsilon_{\dot\alpha\dot\mu}\varepsilon_{\dot\beta\dot\delta})\bar\nabla^{\dot\delta} \ln \bar F. \tag{B.7b}$$

In the derivation of Eq. (B.7) the identities (B.4), (B.5) and their conjugates, and the Fierz rules are employed.

Now let us obtain Eq. (9.4). Using Eq. (6.1) one finds

$$E^{-1} = \mathrm{Ber}\, E_A^{\ M} = \mathrm{Det}\,(E_a^{\ m} - E_a^{\ \mu} E^{-1\ \alpha}_{\mu} E_\alpha^{\ m} - E_{a\dot\mu} E^{-1\ \dot\mu}_{\dot\alpha} E^{\dot\alpha m}) \times$$
$$\times\, \mathrm{Det}^{-1}(E_\alpha^{\ \mu})\,.\,\mathrm{Det}^{-1}(E_{\dot\alpha}^{\ \dot\mu}) =$$
$$= \mathrm{Det}\Big[\tfrac{1}{4} F\bar F(\Delta\sigma_a\bar\Delta - \bar\Delta\tilde\sigma_a\Delta)\mathcal{H}^m + i\,E_a^{\ \mu}\Delta_\mu\mathcal{H}^m + i\,E_a^{\ \dot\mu}\bar\Delta_{\dot\mu}\mathcal{H}^m -$$
$$- E_a^{\ \mu}F^{-1}\delta_\mu^{\ \alpha}\, i\nabla_\alpha\mathcal{H}^m - E_{a\dot\mu}\bar F^{-1}\delta_{\dot\alpha}^{\ \dot\mu}\, i\bar\nabla^{\dot\alpha}\mathcal{H}^m\Big](F\bar F)^{-2} =$$
$$= (F\bar F)^2\,\mathrm{Det}\Big[\tfrac{1}{4}(\Delta\sigma_a\bar\Delta - \bar\Delta\tilde\sigma_a\Delta)\mathcal{H}^m\Big].$$

This expression can be rewritten with the help of Eqs. (A.6), (A.7), (7.3f), (7.6g):

$$E^{-1} = (F\bar F)^2\,\mathrm{Det}\,\Big[-\tfrac{1}{2}\,\hat l_a^{\ n}(1+i\mathcal{H})^{-1\ m}_{\ \ n}\Big] =$$
$$= 2^{-4}(F\bar F)^2\,\mathrm{Det}\,(\hat l_a^{\ m})\,.\,\mathrm{Det}^{-1}(1+i\mathcal{H}) = \tag{B.8}$$
$$= 2^{-4}(F\bar F)^2\,\mathrm{Det}\,(\hat\tau_a^{\ m})\,.\,\mathrm{Det}^{-1}(1-i\mathcal{H}) =$$
$$= 2^{-4}(F\bar F)^2\,\mathrm{Det}^{\frac12}(\hat l_a^{\ m})\,.\,\mathrm{Det}^{\frac12}(\hat\tau_a^{\ m})\,.\,\mathrm{Det}^{-\frac12}(1+\mathcal{H}^2).$$

Finally, inserting the explicit form of F and $\bar F$ (8.4) into Eq. (B.8) we obtain Eq. (9.4).

Appendix C

In the paper the two-component Van-der-Waerden notation is used.

$$\bar\Psi_{\dot\alpha} = (\Psi_\alpha)^+,\quad \Psi^\alpha = \varepsilon^{\alpha\beta}\Psi_\beta\,,\quad \bar\Psi^{\dot\alpha} = \varepsilon^{\dot\alpha\dot\beta}\bar\Psi_{\dot\beta}\,;$$
$$\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha},\quad \varepsilon^{\dot\alpha\dot\beta} = -\varepsilon^{\dot\beta\dot\alpha},\quad \varepsilon^{12} = -\varepsilon_{12} = \varepsilon^{\dot1\dot2} = -\varepsilon_{\dot1\dot2} = 1,$$
$$\varepsilon^{\alpha\beta}\varepsilon_{\beta\mu} = \delta^\alpha_\mu\,;\quad \Psi\varphi \equiv \Psi^\alpha\varphi_\alpha\,,\quad \bar\Psi\bar\varphi = \bar\Psi_{\dot\alpha}\bar\varphi^{\dot\alpha}\,;$$

$$(\sigma_a)_{\alpha\dot\alpha} = (\mathbb{1}, \vec\sigma)_{\alpha\dot\alpha}\,,\quad (\tilde\sigma_a)^{\dot\alpha\alpha} = (\mathbb{1}, -\vec\sigma)^{\dot\alpha\alpha} = \varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\alpha\beta}(\sigma_a)_{\beta\dot\beta}\,;$$
$$(\sigma_{ab})_{\alpha\beta} = \frac{i}{2}(\sigma_a\tilde\sigma_b - \sigma_b\tilde\sigma_a)_{\alpha\beta}\,,\quad (\tilde\sigma_{ab})^{\dot\alpha\dot\beta} = \frac{i}{2}(\tilde\sigma_a\sigma_b - \tilde\sigma_b\sigma_a)^{\dot\alpha\dot\beta}.$$

Some important identities:

$$(\sigma_a)_{\alpha\dot\alpha}(\tilde\sigma^a)^{\beta\dot\beta} = 2\,\delta_\alpha^{\ \beta}\delta_{\dot\alpha}^{\ \dot\beta}\,;\quad (\sigma_{ab})_{\alpha\beta} = (\sigma_{ab})_{\beta\alpha}\,;$$
$$\sigma_a\tilde\sigma_b = \eta_{ab} - i\,\sigma_{ab}\,,\quad \eta_{ab} = \mathrm{diag}\,(+---)\,;$$
$$\left.\begin{aligned}&(\sigma_{ab})_\alpha^{\ \beta}(\sigma^{ab})_\rho^{\ \tau} = 4(2\,\delta_\alpha^{\ \tau}\delta_\rho^{\ \beta} - \delta_\alpha^{\ \beta}\delta_\rho^{\ \tau})\\ &(\sigma_{ab})_{\alpha\beta}(\tilde\sigma^{ab})^{\dot\alpha\dot\beta} = 0\end{aligned}\right\} \tag{C.1}$$

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