объединенный
институт
ядерных
исследований
дубна

E2 - 12496

R.Lednický, V.Tzeitlin

THE PROBLEM OF NEUTRAL CURRENTS IN THE GRAND-UNIFIED E_7-THEORY

1979

E2 - 12496

R.Lednický, V.Tzeitlin*

THE PROBLEM OF NEUTRAL CURRENTS IN THE GRAND-UNIFIED E_7-THEORY

Submitted to ЯФ

*P.N.Lebedev Physical Institut, Moscow.

E2 - 12496

Ледницки Р., Цейтлин В.

Проблема нейтральных токов в объединенной E_7 - теории

Показано , что в E_7 - теории гранд-объединения существует возможность описания слабых и электромагнитных взаимодействий с помощью калибровочной группы $SU(2) \otimes [U(1)]^3$. В этом случае E_7 - теория находится в хорошем согласии со всеми имеющимися экспериментальными данными и имеет ряд специфических предсказаний, которые могут быть проверены в ближайшее время в готовящихся экспериментах по $\bar{p}p$- и e^+e^--аннигиляциям и рассеянию мю-мезонов на нуклонах.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1979

E2 - 12496

Lednický R., Tzeitlin V.

The Problem of Neutral Currents in the Grand-Unified E_7-Theory

We show that in the E_7-theory of grand unification there is a possibility of describing the weak and electromagnetic interactions in the framework of the gauge group $SU(2) \otimes [U(1)]^3$. In this case the E_7-theory is in good agreement with all the available experimental data. Some of its specific predictions are discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1979

Introduction

At the present time all the available experimental data are in a brilliant agreement with the predictions of the Weinberg-Salam (WS in what follows) theory of weak and electromagnetic interactions provided $\sin^2\theta_W = 0.23$, except for the atomic-physics experiments where some clarification of the situation is needed. The common belief in the validity of the WS-model has grown up substantially especially after the recent fine SLAC experiment on polarized electron-nucleon scattering has been fulfilled (see ref. /1/). Due to the successes of the WS theory it is commonly assumed that the more ambitious schemes which try to unify weak, elektromagnetic and strong interactions must coincide with that model in the sector of weak interactions. Among the known models of grand unification the E_7-theory seems to be the most consistent one in view of general ideology since all the fundamental fermions are placed here in a single irreducible representation, i.e., there is a total symmetry among quarks and leptons up to spontaneous breakdown. Besides, the E_7-theory is the most attractive one as it admits natural definitions of electric charge and color $SU^c(3)$ -group /2/. But the attempt to describe the weak and electromagnetic interactions with the help of WS-model meets severe troubles here. The main of them is the large value of $\sin^2\theta_W = 3/4$ (2/3 after renormalization /3/) fixed by the $SU(2) \otimes U(1)$ embeding into E_7 and being in apparent contradiction with experiment. However, we may try to go beyond the traditional WS-scheme to avoid this problem in describing neutral-current phenomena in the E_7-theory.

In the present paper we show that there is a possibility to describe the weak and electromagnetic interactions in E_7-theory in the framework of the gauge group $SU(2)\otimes[U(1)]^3$. In this case the E_7-theory is in good agreement with all available experimental data. Some of its specific predictions are discussed.

The paper is organized as follows. In Section I we describe the neutral-current phenomena under experimental investigation at the present time. In Section II we give the general properties of the E_7-theory of grand unification. In Section III we show that Higgs mechanism gives possibilities to have $SU(2)\otimes[U(1)]^2$ and $SU(2)\otimes[U(1)]^3$ as gauge groups of weak and electromagnetic interactions in the E_7-theory. We also discuss the problem of flavor-changing neutral currents in this Section. In Section IV the E_7-theory is compared with the experimental data on neutral-current phenomena. It is shown that good agreement of the E_7-scheme with experiment may be achieved. In Section V we discuss the results obtained and some specific predictions of the E_7-theory. In Appendix A the second possible definition of the electric charge in E_7-theory is discussed and some remarks on E_8-theory are presented.

I. The Phenomenology of Neutral Currents

In what follows we shall use the experimental data on neutral-currents. So, below we describe briefly their phenomenology.

Up to the date, the three types of netral-current phenomena in weak interactions are under experimental investigation. The first-type processes are those involving different νN-scattering phenomena described with the help of the model-independent effective Lagrangian

$$\mathcal{L} = \frac{G}{\sqrt{2}}\bar{\nu}\gamma^{\mu}(1+\gamma_5)\nu[u_L^{\nu}\bar{u}\gamma_{\mu}(1+\gamma_5)u + u_R^{\nu}\bar{u}\gamma_{\mu}(1-\gamma_5)u + d_L^{\nu}\bar{d}\gamma_{\mu}(1+\gamma_5)d + d_R^{\nu}\bar{d}\gamma_{\mu}(1-\gamma_5)d]. \quad (1)$$

The values of the quark coupling constants u_L^{ν}, u_R^{ν}, d_L^{ν}, d_R^{ν} in Eq. (1), up to the common sign, may be directly determined by using the full set of data [4]. Processes of the second type involve the νe-scattering. The corresponding effective

Lagrangian is as follows:

$$\mathcal{L} = \frac{G}{\sqrt{2}}\bar{\nu}\gamma^{\mu}(1+\gamma_5)\nu[e_L^{\nu}\bar{e}\gamma_{\mu}(1+\gamma_5)e + e_R^{\nu}\bar{e}\gamma_{\mu}(1-\gamma_5)e]. \quad (2)$$

For this kind of phenomena the values of $\sigma_{el}(\bar{\nu}_{\mu}e)$, $\sigma_{el}(\nu_{\mu}e)$ and $\sigma_{el}(\bar{\nu}_e e)$ are known.

Processes of the third kind are the effects of parity non-conservation of the electron-nucleon interactins. Till recently the only source of these data has been the atomic-physics experiments where the different and hardly consistent results have been obtained by several groups. However, after the recent SLAC experiment has been carried out, the more reliable data have become available. In this experiment the asymmetry in polarized e scattering on nucleons was measured.

The set of data on all three types of phenomena is in remarkable agreement with the WS-theory, provided $\sin^2\theta_W \simeq 0.23$, ≈ 0.23, see ref. [1] and below.

II. General Properties of the E_7-Theory of Grand Unification

On the other hand, an idea attractive from the aesthetical and theoretical points of view of the grand unification of strong, weak, and electromagnetic interactions in a unique gauge theory has been intensively investigated for the last time. The theory of such a kind is based on some grand symmetry group which contains the group of strong, weak, and electromagnetic interactions. This grand-group containing the Weinberg-Salam group $SU(2)\otimes U(1)$ fixes the value of $\sin^2\theta_W$. This "symmetric" value of $\sin^2\theta_W$ correspond to the physical value of this parameter in a range of energies where the breakdown of the initial grand-symmetry may be neglected. To obtain the value of $\sin^2\theta_W$ at the present energies the certain renormalization procedure must be carried out

In the minimal possible $SU(5)$- and also in $SO(10)$- and E_6-grand unification models where $\sin^2\theta_W$ takes a symmetric value of 3/8 the renormalization leads to reasonable value of this parameter at present energies [5-8]. However, it is necessary to use reducible representations of the grand-group to embrace all the fundamental fermions in these theories that seems unsatisfactory. The most consistent in this view are the models based on exceptional groups

E_7 and E_8. Besides, these schemes are attractive as they admit natural definitions of the colour SU(3) and electric charge operator. The E_8-model possesses a very big number (248) of fundamental fermions and has not been yet under detailed investigation owing to this redundancy. The choice of E_8 as a grand group may be justified by the ability of E_8-theory to reproduce the WS-model with a reasonable symmetric value of $\sin^2\theta_W$ (see Appendix A). However, here we analyze the more economic E_7-theory.

The fundamental fermions, quarks and leptons, are placed in 56-plet in the E_7-theory while the vector gauge fields form 133-plet. According to the maximal subgroup $SU(6)\otimes SU^c(3)$, where $SU^c(3)$ is the gauge group of QCD, SU(6) is flavor group, these representations decompose as follows

$$\underline{56} = (20.1^c) + (6.3^c) + (\bar{6}.\bar{3}^c), \tag{3}$$

$$\underline{133} = (35.1^c) + (\overline{15}.3^c) + (15.\bar{3}^c) + (1.8^c). \tag{4}$$

Hence, leptons form 20-plet and quarks and anti-quarks form sextet and anti-sextet respectively. There is total symmetry among all quarks and leptons up to spontaneous breakdown. The vector fields responsible for weak and electromagnetic interactions are the members of SU(6) 35-plet, the colour octet corresponds to gluons. The members of representations $(\overline{15}.3^c)$ and $(15.\bar{3}^c)$ in Eq. (4) are the lepto-quarks. They enter both quark-, anti-quark and quark-lepton vertices and, hence, lead to the proton decay. There are two ways to provide the observable proton stability in the theory of such a kind. In the first of them it is supposed that the lepto-quark fields acquire super-large masses under spontaneous symmetry breakdown. In the second, the proton stability is provided via some additional global symmetry generalizing the baryon-number conservation (owing to the mechanism proposed in ref. /2/), i.e., in principle, no superlarge masses are needed. In the E_7-model of ref. /9/ the first method is used while the models of ref. /10/ utilize the second. Note, however, that the small ratio of QED to QCD coupling constants at present energies /3/ leads to the necessity to introduce the superlarge masses in the E_7-models of ref. /10/ as well.

In the present paper we adopt the simplest two-stage pattern of symmetry breakdown. The initial symmetry is broken at the first stage down to $SU^c(3)\otimes G_W$, where G_W is responsible for presently observed weak and electromagnetic interactions, and at the second stage G_W is broken

down to U(1)-group of the electric charge. In this case the difference between the E_7-models of refs. /9/ and /10/ is unessential in view of our investigation (the renormalized value of $\sin^2\theta_W$ being 2/3 /3/) and in what follows we shall not refer to the differences of the two approaches*.

The generators of E_7 corresponding to the intermediate vector bosons of weak interaction and to the photon are contained in the set of SU(6)-generators. If the following basis is chosen for the latter

$$\lambda_m \otimes \sigma_a \;, \lambda_m \otimes 1 \;, \; 1 \otimes \sigma_a \;, \tag{5}$$

where λ_m, m = 1,2,...,8 are Gell-Mann matrices, σ_a, a = =1,2,3 are Pauli matrices, 1 is unit matrix, the standard (and most natural) definition of electic charge operator is /9/:

$$Q = T_3 + \frac{1}{\sqrt{3}} T_8 = \begin{pmatrix} \lambda_3 + \frac{1}{\sqrt{3}}\lambda_8 & 0 \\ 0 & \lambda_3 + \frac{1}{\sqrt{3}}\lambda_8 \end{pmatrix} = \mathrm{diag}(2/3,-1/3,-1/3,2/3,-1/3,-1/3).$$

The following generators correspond to standard charged weak currents:

$$T^{\pm} = \frac{1}{\sqrt{2}} \begin{pmatrix} \lambda_1 \mp i\lambda_2 & 0 \\ 0 & \lambda_1 \mp i\lambda_2 \end{pmatrix}. \tag{7}$$

They form, together with T_3, the weak group $SU(2)_W$. Using the known formula /5/ for the symmetric value of $\sin^2\theta_W$

$$\sin^2\theta_{W_0} = \sum_i T_{3i}^2 / \sum_i Q_i^2 \;, \tag{8}$$

where the sum is over all members of 56-plet, we find that $\sin^2\theta_{W_0} = 3/4$ in E_7-theory. This value corresponds to the region of distances comparable with the inverse scale of the first-stage breakdown of E_7-symmetry. At the distances achieved at present energies $\sin^2\theta_W$ takes the value 2/3 /3/.

*It may be shown that even in the case of multi-stage break-down in the E_7-theory the super-large masses have to appear, i.e., the renormalization effects for the quantities like Weinberg angle are essential.

Hence, the generator corresponding to standard Z_0-boson of Weinberg-Salam is

$$\Gamma_0 \equiv \frac{1}{\cos\theta_W}[T - \sin^2\theta_W Q] = \frac{\sqrt{3}}{18}\,\mathrm{diag}(1,-5,4,1,-5,4). \qquad (9)$$

The lepton 20-plet contains the following $SU(2)_W$-representations

triplets:

$$\begin{pmatrix} L^{+}_{134} \\ \frac{1}{\sqrt{2}}(L^{\circ}_{234}+L^{\circ}_{135}) \\ L^{-}_{235} \end{pmatrix}, \quad \begin{pmatrix} L^{+}_{146} \\ \frac{1}{\sqrt{2}}(L^{\circ}_{246}+L^{\circ}_{156}) \\ L^{-}_{256} \end{pmatrix}$$

(10)

doublets:

$$\begin{pmatrix} L^{\circ}_{346} \\ L^{-}_{356} \end{pmatrix}, \begin{pmatrix} L^{\circ}_{136} \\ L^{-}_{236} \end{pmatrix}, \begin{pmatrix} L^{+}_{124} \\ L^{\circ}_{125} \end{pmatrix}, \begin{pmatrix} L^{+}_{145} \\ L^{\circ}_{245} \end{pmatrix}$$

singlets:

$$L^{\circ}_{123}, L^{\circ}_{456}, L^{\circ}_{126}, L^{\circ}_{345}, \frac{1}{\sqrt{2}}(L^{\circ}_{234}-L^{\circ}_{135}), \frac{1}{\sqrt{2}}(L^{\circ}_{246}-L^{\circ}_{156}).$$

Here we denote the components of 20-plet as $L^{\mathrm{charge}}_{ijk}$, $i \neq j \neq k$, $i,j,k = 1,\ldots,6$ what corresponds to the representation of 20 as the totally anti-symmetric direct product of three sextets. With the help of such a representation all the quantum numbers of each 20-plet-component may be easily obtained. The quark sextet contains two doublets with the electric charges (2/3,-1/3) and two singlets with charge -1/3. The following quark (anti-quark) assignment in sextet (anti-sextet) is being in agreement with the known properties of charged weak currents

$$\begin{pmatrix} u \\ d(\theta) \\ b(\theta) \\ c \\ s(\theta) \\ h(\theta) \end{pmatrix}, \quad \begin{pmatrix} \bar{u} \\ \bar{h}(\phi) \\ \bar{d}(\phi) \\ \bar{c} \\ \bar{b}(\phi) \\ \bar{s}(\phi) \end{pmatrix} \qquad (11)$$

(we remind that 56-plet ψ of fermions and anti-fermions in the E_7-theory is left-handed, i.e., $\frac{1+\gamma_5}{2}\psi = \psi,\ \frac{1-\gamma_5}{2}\psi = 0$ whereas the right-handed anti-fermions and fermions are their charge-conjugates).

In Eq.(11) $d_L(\theta)$, etc., denote a mixing of four quarks with electric charge -1/3 which depends on six parameters θ_i:

$$(h(\theta)\,b(\theta)\,s(\theta)\,d(\theta))_L = (h\,b\,s\,d)_L \cdot O(4), \qquad (12)$$

where $d(\theta)$, etc., are the states entering the interaction vertex, d, s, b, h are the fermionic-mass-matrix eigenstates (the right-handed-quark mixings are determined analogously but depend on the other angles ϕ owing to the non-symmetric fermionic mass-matrix /9,10/. The $O(4)$ is a 4x4 orthogonal matrix which can be represented as follows*

$$O(4) = g_1(\theta_1)\cdot g_2(\theta_2)\cdot g_3(\theta_3)\cdot g_1(\theta_4)\cdot g_2(\theta_5)\cdot g_1(\theta_6), \qquad (13)$$

where $g_a(\theta_i)$ are rotations in $(a, a+1)$ plane, $a = 1,2,3,4$. Here θ_i are Cabibbo-type angles, θ_3 is the original Cabibbo angle θ_c. The value of $\cos\theta_2 \sin\theta_3$ corresponds to $\sin\theta_c$ in the usual scheme and we get from experiment /5/ $\cos\theta_2 \geq 0.96$ and $\theta_2 \leq 16°$. There are no strict limits on the values of other angles θ_i at present time. The just mentioned quark mixing will play an important role in what follows.

There is a lot of possible lepton assignments in 20-plet as far as neutral currents are not concerned /9/.

---

*In principle, the mixing matrix is complex and it leads to the CP-violation in the theory. But here we neglect these effects, which are the subject of a separate investigation.

From all the above-stated it is evident that an attempt to describe the weak interactions in the E_7-theory with the help of the WS-scheme meets severe troubles. First of all, $\sin^2\theta_W = 2/3$ is in apparent contradiction with experiment. Besides, there are no charged leptonic $SU(2)_W$-singlets in the theory. That contradicts the WS-prescription for e^-_R and μ^-_R. We should note that there is yet another possible definition of electric charge operator /9/. We discuss it in Appendix A and show that in the framework of E_7-theory it does not lead to a reasonable result.

III. Weak Interactions and Neutral Currents in E_7-Theory

As was mentioned above, we assume the two-stage pattern of spontaneous symmetry breakdown. To avoid the contradiction with the well-defined universal $SU(2)$-structure of the charged weak currents, we require that the group G_W remaining unbroken after the first stage, does not contain any charge generator except $T^{\pm}$. Moreover, we require all the gauge fields corresponding to the neutral nondiagonal $SU(6)$-generators, as well as all leptoquarks, to acquire superlarge masses at the first stage of breakdown. Hence, in general case, after the first stage of symmetry breakdown the group $SU^c(3) \otimes SU(2)_W \otimes [U(1)]^n$ survives, where $n \leq 3$ since there are only two diagonal orthonormal generators of $SU(6)$ commuting with those of $SU(2)_W$, besides T_8:

$$\Gamma_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} \lambda_8 & 0 \\ 0 & -\lambda_8 \end{pmatrix}. \tag{14}$$

The spontaneous breakdown of E_7-theory occurs via Higgs mechanism. We shall limit ourselves to the minimal set of Higgs fields, i.e., to the representations of E_7 containing in the direct product $\underline{56} \otimes \underline{56}$ /9,10/. Only colour singlets may acquire nonzero v.e.v. Hence, we get

$$\text{v.e.v.} \in \underline{6} \otimes \underline{\bar{6}} + \underline{20} \otimes \underline{20}. \tag{15}$$

Then it is easy to see, studying representations given in Eq. (15), that three different situations for G_W may take place depending on what Higgs fields acquire superlarge v.e.v. at the first stage of breakdown. They are a) $G_W = SU(2)_W \otimes U(1)$ of Weinberg-Salam, b) $G_W = SU(2)_W \otimes [U(1)]^2$, where $U(1)$'s correspond to T_8 and some fixed combination of Γ_1 and Γ_2, c) $G_W = SU(2)_W \otimes [U(1)]^3$, where $U(1)$'s corresponds to T_8, Γ_1 and Γ_2. The case a) is of no interest owing to the above-mentioned difficulties. The cases b) and c) must be investigated, but before their studying we have to dwell on the question of flavor-changing neutral currents.

As is known, the conditions of natural flavor conservation formulated in ref. /11/ are not satisfied in the E_7-thoery. To suppress the strangeness-changing neutral-currents (at least in the lowest order) the quark mixings must obey some restrictions. Let us denote by $\tilde{Z}_A$, $Z = 0, 1, \ldots$, the eigenstates of the neutral-field mass matrix, $\tilde{\Gamma}_A$ are corresponding generators of G_W. Then to cancel all the vertices $\bar{d}s\tilde{Z}_A$, the following conditions should be imposed.

$$\sum_f O(4)_{fd}\, O(4)_{fs}\, z_{fA} = 0, \quad A = 0, 1, \ldots, \tag{16}$$

where sum is over all quark flavors with electric charge - 1/3 (see Eq. (12)) and z_{fA} are eigenvalues of $\tilde{\Gamma}_A$ on the states d_L, s_L, b_L, h_L of $\underline{56}$-plet. The analogous conditions are imposed also on the right-handed-quark mixings.

The mass matrices of quark- and gauge fields are determined by the same v.e.v. of Higgs fields. Hence, their parameters are connected and, in principle, the validity of Eqs. (16) may be checked if the detailed investigation of the fermionic mass-generation mechanism is carried. However, this is a rather complicated problem and lies beyond the scope of this paper. We should only note that if Eqs. (16) are not valid the E_7-theory comes into conflict with experiment. Thus, in what follows we assume their validity.

Summing up all the above-stated we come to the following picture. The initial E_7-symmetry at the first stage is broken down to its subgroup $SU^c(3) \times SU(2)_W \times [U(1)]^n$, $n = 2$ or $n = 3$. Getting ahead, we should mention that the good description of the data is impossible in the case $n = 2$ (the Higgs mechanism restricts the generator connected with additional Z-bozon in this case to be $\pm \mathrm{diag}(0,0,1,0,0,-1)$, $\pm \frac{1}{2\sqrt{3}} \mathrm{diag}(-1,-1,\mp 2,1,1,\pm 2)$ or $\pm \frac{1}{2} \mathrm{diag}(-1,-1,0,1,1,0)$; corresponding Confidence Levels are less than 0.4%) and we pass directly to the more general case $n = 3$. In this case the weak and electromagnetic interactions of quarks and leptons at present energies are governed by the following covariant derivative:

$$D_\mu = \partial_\mu - ieQA_\mu - ig_W[(T^+W^+_\mu + T^-W^-_\mu) + \Gamma_0 Z_{0\mu} + \sqrt{\tfrac{2}{3}}(\Gamma_1 Z_{1\mu} + \Gamma_2 Z_{2\mu})]. \quad (17)$$

Here g_W is the $SU(2)_W$ gauge coupling constant, e the electromagnetic constant

$$e = g_W \sin\theta_W = g_W \sqrt{\tfrac{2}{3}} . \quad (18)$$

The generators Γ_0, Γ_1 and Γ_2 are given in Eqs. (9), (15), A_μ is a photon field, $W^\pm_\mu$, $Z_{0\mu}$ are standard intermediate vector bosons of Weinberg-Salam, $Z_{1\mu}$, $Z_{2\mu}$ are gauge fields of Γ_1 and Γ_2, and $\sqrt{\tfrac{2}{3}}$ appears because of different renormalization of SU(2) and U(1) gauge coupling constants /3/.

At the second stage of symmetry breakdown all the vector fields in Eq. (20), but photon acquire masses. At this stage the breakdown is provided by all electrically neutral components of Eq. (16) that did not obtain v.e.v. at the first stage. Since there is a lot of $SU(2)_W$ -singlets, doublest, triplets, quadruplets and pentaplets, this stage of symmetry breakdown is very complicated in E_7-theory. Mixing of all of three neutral bosons $Z_{0\mu}$, $Z_{1\mu}$, $Z_{2\mu}$ occurs and no simple relation of WS-type between charged- and neutral-boson masses exists. The free parameters of the theory are the v.e.v.'s of Higgs fields. They determine the neutral boson-mass matrix. In terms of the mass-matrix eigenstates $\tilde{Z}_{A\mu}$ with eigenvalues m^2_A, A = 0,1,2 we rewrite Eq. (17) as follows:

$$D_\mu = \partial_\mu - ieQA_\mu - ig_W[(T^+W^+_\mu + T^-W^-_\mu) + \sum_A \tilde{\Gamma}_A \tilde{Z}_{A\mu}], \quad (19)$$

where

$$\tilde{\Gamma}_A = M_{AB}(\theta, \phi, \psi)\Gamma_B , \quad A, B = 0,1,2, \quad (20)$$

$\sqrt{\tfrac{2}{3}}$ being absorbed into Γ_A, and $M_{AB}(\theta,\phi,\psi)$ is the 3x3 Euler rotation matrix in the following parametrization: $M_{00} = \cos\theta$, , $M_{01} = \sin\theta \sin\phi$, $M_{20} = \sin\theta\sin\psi$, etc. We shall regard m^2_A and θ, ϕ, ψ as free parameters to be determined from the neutral-current experiments.

IV. E_7-Theory Versus Experiment

The interaction of neutral currents generated by Eq. (19) at the momenta transferred small in comparison with the intermediate boson masses, leads to the effective Lagrangian of the type given in Eqs. (1), (2), where for Fermi constant we have

$$\frac{G}{\sqrt{2}} = \frac{g^2_W}{8m^2_W} \quad (21)$$

and the quark and lepton form factors in neutrino scattering are defined as follows:

$$f^\nu_{L(R)} = \sum_A \xi_A \nu_A f_{L(R)A} , \quad f = u, d, e. \quad (22)$$

Here f_A, ν_A are $\tilde{\Gamma}_A$ eigenvalues on fermionic states; it is easy to obtain them with the help of Eqs. (9), (11), (14), (20), $\xi_A = 2m^2_W/m^2_A$ is a parameter convenient to use instead of , m_A (we remind that the value of m_W is immediately derived from Eqs. (18), (21) and equals $37.3/\sin\theta_W$ GeV). The various asymmetries in lepton-isoscalar target scattering are defined as follows /12/:

$$A^\mp = -k(\pm E_{AV}(\ell, q) + E_{VA}(\ell, q)g(t)) \cdot |\lambda|,$$

$$B = k(E_{AA}(\ell, q) - |\lambda| E_{VA}(\ell, q)g(y), \quad (23)$$

$$D = k(E_{AA}(\ell, q)g(y) + \lambda E_{AV}(\ell, q)), \quad C = D(\lambda = 0),$$

where $k = 0.3G/\sqrt{2}\pi a$, λ is the longitudinal polarization of lepton, $g(y) = \frac{1-(1-y)^2}{1+(1-y)^2}$,

$$E_{AV}(\ell, q) = 2\epsilon_{AV}(\ell, u) - \epsilon_{AV}(\ell, d), \text{ etc.}, \quad (24)$$

while

$$\epsilon_{\substack{AV\\VA\\AA}} = \sum \xi_A (\ell_{R_A} \mp \ell_{L_A}) \cdot (q_{R_A} \pm q_{L_A}) . \quad (25)$$

In comparison of E_7-theory with experiment we use all the available set of data except for yet uncertain atomic-physics results. But since the full set of data on neutrino-nucleon scattering allows one to determine the direct values of quark form factors (up to the common sign) we, in fact, have eight experimental points f_k^{exp} , $k = 1,2,\dots,8$, namely, experimental values of the following quantities:

$$u_L^{\nu_\mu},\ u_R^{\nu_\mu},\ d_L^{\nu_\mu},\ d_R^{\nu_\mu},\ \frac{\sigma_{el}(\bar\nu_\mu e)}{E_{\bar\nu}},\ \frac{\sigma_{el}(\nu_\mu e)}{E_\nu},\ \frac{\sigma_{el}(\bar\nu_e e)}{E_{\bar\nu}},\ \frac{A^-(y)}{Q^2}\Big|_{0.21},$$

where $A^-(y)/Q^2|_{0.21}$ was measured in the famous SLAC experiment. The theoretical expressions for the first four of them are given in Eq. (22). The formulae for asymmetries have been just given and the expressions for elastic νe - cross sections are well-known:

$$\sigma(\bar\nu_\mu e)/E_{\bar\nu} = \frac{2}{\pi} G^2 m_e \left[\frac{(e_L^{\nu_\mu})^2}{3} + (e_R^{\nu_\mu})^2\right],$$

$$\sigma(\nu_\mu e)/E_\nu = \frac{2}{\pi} G^2 m_e \left[\frac{(e_R^{\nu_\mu})^2}{3} + (e_L^{\nu_\mu})^2\right], \qquad (26)$$

$$\sigma(\bar\nu_e e)/E_{\bar\nu} = \frac{2}{\pi} G^2 m_e \left[\frac{(1+e_L^{\nu_e})^2}{3} + (e_R^{\nu_\mu})^2\right].$$

Using the experimental values f_k^{exp} , $k=1,\dots,8$, see, refs. [1,4]

$u_L^{\nu_\mu} = 0.35 \pm 0.07$ $\qquad d_L^{\nu_\mu} = -0.40 \pm 0.07$

$u_R^{\nu_\mu} = -0.19 \pm 0.06$ $\qquad d_R^{\nu_\mu} = 0.0 \pm 0.11$

$\sigma(\bar\nu_\mu e)/E_{\bar\nu} = (1.8 \pm 0.9)\cdot 10^{-42}$ cm^2/GeV

$\sigma(\nu_\mu e)/E_\nu = (1.7 \pm 0.5)\cdot 10^{-42}$ cm^2/GeV

$\sigma(\bar\nu_e e)/E_{\bar\nu} = (5.7 \pm 1.2)\cdot 10^{-42}$ cm^2/GeV

$A^-(y{=}0.21)/Q^2 = (-9.5 \pm 1.6)\cdot 10^{-5}$ (GeV/c)$^{-2}$

we can determine the values of parameters ξ_A, θ, ϕ, ψ, by minimizing the χ^2-functional

$$\chi^2 = \sum_{k=1}^{8} \left[\frac{f_k(\xi_A, \theta, \phi, \psi) - f_k^{exp}}{\sigma_k}\right]^2$$

and estimate the level of agreement of the theory with experiment according to the χ^2-criterium.

In the course of this investigation we have to study, evidently all the possible lepton assignment in 20-plet compatible with the known structure of the charged weak currents. As a result of our analysis, we find that there are two variants of lepton assignment which provide good agreement of E_7-theory with experiment*. In both of these variants e_L^- has quantum numbers of L_{256}^- (the component of $SU(2)_W$-triplet, see Eq. (10)). ν_e corresponds to L_{246}° or L_{156}°, e_L^+ corresponds to L_{124}^+ (the component of $SU(2)_W$ -doublet). ν_μ and μ_L^- form $SU(2)_W$ -doublet. In the first variant it is $(L_{346}^\circ, L_{356}^-)$ whereas in the second $(L_{136}^\circ, L_{236}^-)$. The quantum numbers of μ_L^+ cannot be determined from the available data, hence μ_L^+ may be associated with L_{145}^+ ($(SU(2)_W$-doublet) or with L_{134}^+ ($SU(2)_W$ -triplet). The remaining unoccupied places in 20-plet may be used for τ-lepton assignment; there are several possibilities.

The results of our fit for both the variants are given in Table 1. For comparison we give in Table 2 the results of the analogous analysis for WS-model, where there is the single free parameter $\sin^2\theta_W$, note that the doublet assignment for e_R^- is not rejected by the present data, whereas the triplet one gives unsatisfactory χ^2. For the sake of completeness we also give in Tables 1,2 the predicted (for the values of parameters given in Tables) value of Q_W

$$Q_W = 584(\epsilon_{AV}(e,u) + 1.15\epsilon_{AV}(e,d)) \qquad (28)$$

which measures the parity nonconservation in bismuth. The predictions of y-dependence for various asymmetries in muon-isoscalar target scattering are shown in the Figure.

V. Discussion and Conclusions

It is evident from Table 1 that E_7-theory can provide a good description of all the available data. It is interesting that predictions of the E_7-theory for Q_W are consistent with the last result of the Novosibirsk group[13]

---

*We have also tried to fit the data with the opposite sign of quark form factors in Eq. (27) but have not find any solution with reasonable χ^2.

Table 1

Variant	m_W (GeV)	m_0 Γ_0 (GeV)	m_1 Γ_1 (GeV)	m_2 Γ_2 (GeV)	$\cos\theta$	$\cos\phi$	$\cos\psi$	Q_W	χ^2/ND	CL %
I	45.7	65±6 .92±.09	22±2 .31±.03	111±22 1.84±.37	-.65±.05	.49±07	.69±04	-81±45	2.7/2	27
II		44±3 .62±.04	118 1.68	54±6 .90±.10	-.32±.31	-.83±.06	.82±.05	-184±58		

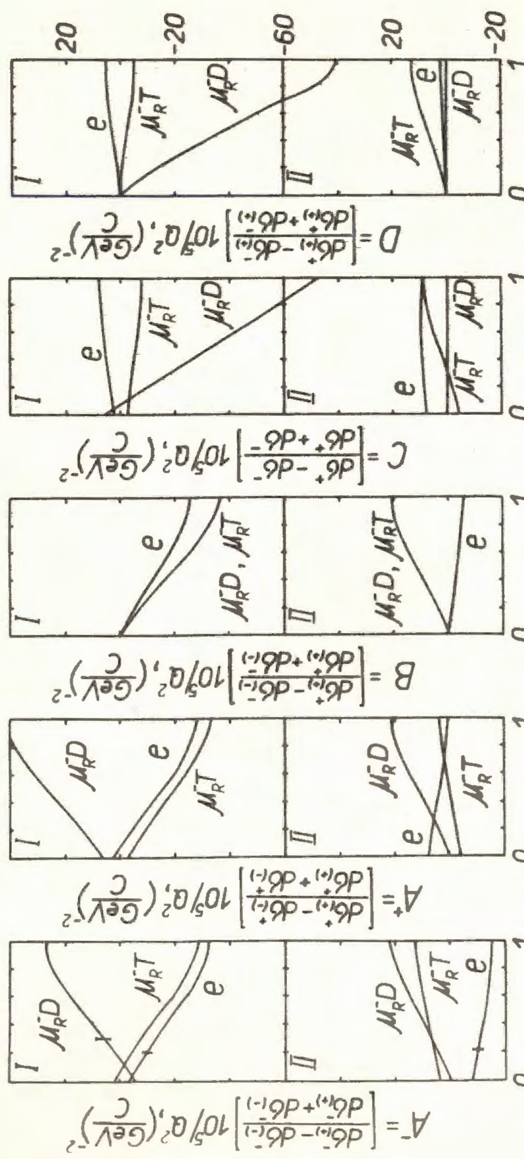
Comment to Table. Due to the strong correlations of the parameters in Var. I we give their errors obtained with one of the engles being fixed. In Var. II we give the results with the value of m_1 being fixed and consistent with the Higgs mechanism (with m_1 not fixed the fit gives a too big value for m_1 with the same value of χ^2). We give also the upper bounds for $\tilde{Z}_A$-bosons' total widths, assuming all the members of 56-plet have masses $\ll m_A$.

Table 2

WS-model	$\sin^2\theta_W$	m_W (GeV)	m_Z (GeV)	$1Q_W$	χ^2/ND	CL
e_R^- singlet	.23±.03	79±4	90±3	-120±8	4.1/7	77%
e_R^- doublet	.21±.03	83±6	93±5	0	13.2/7	9%

and with those of the WS-model. However, one may be disappointed by the fact that good agreement of E_7-theory with the data is achieved by adjusting as much as six parameters. We should note in this connection that the big number of Higgs fields and, therefore, a big number of v.e.v.'s is intrinsic in grand-unified theories /10/ and is the necessary price we have to pay for the beauty of the basic idea of total unification and total lepton-hadron symmetry.

The E_7-theory has a number of specific predictions. First of all, in this scheme the W-boson and some neutral intermediate bosons are substantially lighter than the W- and Z_0-ones in the WS-model (see Table 1).This property of the E_7-theory may be verified in the near future in the experiments on $\bar{p}p$- and e^+e^--annihilation. The predictions of the E_7-theory for asymmetries in μ-isoscalar target-scattering also differ essentially from those of the traditional WS-model and may be checked soon in the CERN-Dubna experiment under preparation. Hence, in the near future we shall be able to give a final judgement on applicability of the E_7-theory. But, even would the E_7-theory turn to be wrong, it is worth remembering that the most general and extreme choice in the framework of grand-unification ideology is to take E_8 as a grand group. Due to the advantage of the possible replication of the WS-model with the fixed symmetric value of 0.3 for $\sin^2\theta_W$ (see Appendix A) this choice evidently needs thorough examination.



The asymmetries in lepton-isoscalar target scattering for variants I and II of the E_7-theory according to Table 1. We display the results for both possible μ_R^--assignments. $\sigma^+_{(-)}$ denotes here the cross section of positively charged leptons with longitudinal polarization $\lambda=-1$, etc. The characteristic error is shown in the figure for asymmetry A^-. In the same figure the experimental SLAC point is shown.

Acknowledgements

We are grateful to the members of the CERN-Dubna NA4 collaboration for stimulating discussions and constant interest in this work. We are much indebted to Prof.V.I.Ogievetsky for numerous valuable discussions and critical reading of the manuscript.

Appendix A

The second possible definition of electric charge operator in E_7-theory is [9]:

$$Q = \mathrm{diag}(2/3, -1/3, 2/3, 2/3, -1/3, -4/3). \quad (A1)$$

Our main remark is that using Eq. (8) we obtain in this case

$$\sin^2\theta_{W_0} = 0.3. \quad (A2)$$

After renormalization we get

$$\sin^2\theta_W = 2/9 \quad (A3)$$

which is just the value favored by experiment, see Table 2. With such a definition of electrin charge we find that doubly-charged leptons appear in $SU(2)_W$ -triplets in $\underline{20}$-plet, whereas the charges of $SU(2)_W$ -doublets remain the same as in Eq. (10). It is important that the charged $SU(2)_W$ -singlets appear in $\underline{20}$-plet giving a possibility to fulfill the WS-prescription for the known leptons. Unfortunately, in this case we loose the possibility to place the right-handed d-quark in $SU(2)_W$-singlet while doublet d_R -assignment is inconsistent with experiment [4]. Besides, in this case we gave only two -1/3-charged quarks and, therefore, b-quark should have electric charge 2/3 or -4/3 which is not favored by the data [14]. We should mention right a way that we can avoid all this difficulties by passing to the E_8-grand-unified theory and preserving the definition of Eq. (12) for the electric charge. The detailed discussion of the E_8-theory is out of the scope of our investigation and will be given elsewhere. Now we only mention that due to the too big number (248) of fermionic fields in the E_8-model the more economic description in the framework of the E_7-theory is more preferable at the moment.

References

1. Baltay C. In: Proceedings of the 19 Intern. HEP Conf. Tokyo, 1979.
2. Gell-Mann M., Ramond P., Slansky R. Rev.Mod.Phys., 1978, 50, p.721.
3. Djakonov D.I. Yad. fiz., 1977, 26, p.845.
4. Abbot L.F., Barnett R.M. Phys.Rev., 1978, D18, p.3214.
5. Harari H. Phys.Reports, 1978, 42, p.235.
6. Gursey F., Ramond P., Sikivie P. Phys.Lett., 1976, 60B, 177.
7. Buras A.S. et al. Nucl.Phys., 1978, B135, p.66.
8. Chanowitz M., Ellis J., Gaillard M.K. Nucl.Phys., 1977, B128, p.506,
9. Sikivie P., Gursey F. Phys.Rev., 1977, D16, p.816.
10. Ogievetsky V.I., Tzeitlin V. Yad.fiz., 1978, 28,p.1616.
11. Glashow S.L., Weinberg S. Phys.Rev., 1977, D15, p.1958.
12. Klein M., Riemann T. JINR, E2-1127, Dubna, 1978.
13. Barkov L.M., Zolotarev M.S. JETP Letters.,1978, 27, p.379; 27, p.379.
14. Hara Y. In: Proceedings of the 19 Intern. HEP Conf. Tokyo, 1979.

Received by Publishing Department
on May 29 1979.