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дубна

E2 - 12469

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Submitted to ЯФ

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E2 - 12469

Простейшая группа супергравитации Эйнштейна

Обсуждается простейшая супергруппа супергравитации Эйнштейна - комплексная супергруппа общих преобразований координат в левом и правом киральных сопряженных суперпространствах с условием сохранения левого и правого суперобъемов. Вещественная часть векторной координаты суперпространства отождествляется с пространственно-временной координатой x^m, а мнимая - с аксиальным гравитационным суперполем $\mathcal{H}^m(x,\theta,\bar{\theta})$. Подробно анализируются преобразования полевых компонент $\mathcal{H}^m$. Излагаемый подход является геометрической основой так называемого "тензорного исчисления".

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1979

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The Simplest Group of Einstein Supergravity

The simplest supergroup of Einstein supergravity is considered. It is the complex supergroup of general coordinate transformations in left- and right-handed chiral conjugated superspaces restricted by the condition of left- and right- supervolume-preservation. The real part of the vector coordinate of the superspace is identified with the space-time coordinate x^m, and the imaginary one, with the axial gravitational superfield $\mathcal{H}^m(x,\theta,\bar{\theta})$. The transformations of the field components of $\mathcal{H}^m$ are studied in detail. The approach described is the geometrical basis of the so-called "tensor calculus"

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1979

I. Introduction

Supergravity is a gauge version of supersymmetry which includes necessarily the theory of gravitational field. At present there is no common supergravity formalism. Einstein's gravity theory is built on a beautiful geometrical basis, the concept of curved space-time. An analogous pattern would be most desirable for supergravity. However, the main results of supergravity (the renormalizability properties, the possibility of unification with internal symmetries, etc.) have been obtained in the framework of the so-called "practical" or "component" approach /1/. This approach has nothing to do with geometry. There the supergravity group has been realized as a group of complicated transformations of a set of ordinary fields. The form of the transformations and of the invariant action has (happily) been guessed by great computational skill but without any geometrical ideas /1/. The difficulty with the nonclosing transformation algebra has been solved /2/ by introducing an appropriate minimal set of auxiliary fields *). Further attempts to simplify and systematize the complicated technique of the component approach have led to "tensor calculus" rules /3/. These rules for handling sets of physical and auxiliary fields strongly resemble the composition laws for the superfield components. In other words, the practical approach is gradually going close to a superfield one. However, its geometrical meaning remains unclear, and the component notation used is not compact and manifestly covariant. Therefore a manifestly supercovariant formulation based on a transparent geometrical idea is desirable.

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*) This set of fields has been prompted by the field content of the axial superfield which we had earlier proposed /4/ as the minimal adequate gravitational superfield.

It should be pointed out that the search for geometrical approaches to supergravity began even before the appearance of the component approach. Many authors have tried to solve this problem (see Refs. /5-12/ and references therein).

The common point in those attempts was the use of the concept of superspace (SS below) $\{(x^m, \theta^\mu, \bar{\theta}^{\dot\mu})\}$ *). In the SS the supergroup of general coordinate transformations was considered. However, such approaches were found to be too noneconomical. Firstly, the symmetry group was enormously large. Secondly, the fundamental objects in such theories (e.g., the "supervierbeins" $E_M^A(x,\theta,\bar\theta)$ and the connections $\omega_{AB}{}^C(x,\theta,\bar\theta)$) were very complicated superfields (SF below) containing fields with spins 3, 5/2, etc. Therefore, in order to obtain physical information, the gauge had to be fixed strongly and certain properly chosen algebraic constraints had to be imposed (as it was skilfully done by Wess and Zumino /6/).

The most adequate formulation of supergravity has to be not only geometrical but also minimal (i.e., the simplest possible groups and SF have to be used).

The outlines of such a formulation were given in 1976 when we proposed /4/ to consider supergravity as the theory of an axial SF $\mathcal{H}^m(x,\theta,\bar\theta)$ (the simplest SF containing spin 2) generated by the supercurrent. Following to this idea we succeeded to find the minimal group of supergravity /13/ and then developed a formalism for constructing invariants of this group out of the single SF $\mathcal{H}^m$.

In the present paper the minimal group of pure supergravity will be described and its action on the components of the SF $\mathcal{H}^m$ in the case of Einstein supergravity will be studied in detail. The main geometrical ideas are the following. The complex supergroup of general coordinate transformations in the complex left-handed chiral 4+2-dimensional SS $\{(x_L^m, \theta_L^\mu)\}$ and the conjugated supergroup in the conjugated right-handed SS $\{(x_R^m, \bar\theta_R^{\dot\mu})\}$ are considered (here conjugation means: $x_R^m = (x_L^m)^*$, $\bar\theta_R^{\dot\mu} = (\theta_L^\mu)^+$). Then the real (i.e., self-conjugated) physical SS $\{(x^m, \theta^\mu, \bar\theta^{\dot\mu})\}$ is introduced as a 4+4-dimensional supersurface in the 8+4-dimensional SS $\{(x_L^m, x_R^m, \theta_L^\mu, \bar\theta_R^{\dot\mu})\}$ with complex structure. This supersurface is given by the equations

$$\frac{1}{2i}(x_L^m - x_R^m) = \mathcal{H}^m(x^n, \theta^\nu, \bar\theta^{\dot\nu}),$$

where $\mathcal{H}^m$ is an arbitrary superfunction of the arguments

$$x^n \equiv \frac{1}{2}(x_L^n + x_R^n), \quad \theta^\nu \equiv \theta_L^\nu, \quad \bar\theta^{\dot\nu} \equiv \bar\theta_R^{\dot\nu}.$$

The coordinates $x^n, \theta^\nu, \bar\theta^{\dot\nu}$ of the physical SS together with the SF $\mathcal{H}^m(x^n, \theta^\nu, \bar\theta^{\dot\nu})$ transform under the initial left- and right-handed chiral supergroups (but not under the larger general coordinate transformation supergroup in the 4+4-dimensional real SS). Thus the supergravity group is realized in the physical SS nonlinearly with the help of the SF $\mathcal{H}^m(x,\theta,\bar\theta)$.

This group as a whole corresponds to Weyl supergravity, and a subgroup of it (singled out by the requirement that the supervolume in the left and right SS be preserved) corresponds to Einstein supergravity. A somewhat weaker condition (when only the product of the left and right supervolumes is preserved) adds global chiral transformations to the Einstein supergravity group. The simplest action for Einstein supergravity possesses in fact such an additional chiral invariance, and this leads to certain important selection rules for the possible counterterms.

Further on in the paper it is shown that in the Einstein-supergravity case after a partial gauge fixing the gravitational SF $\mathcal{H}^m$ contains the graviton field $e^{am}(x)$, the gravitino field $\psi_\alpha^m(x)$ and the auxiliary fields $S(x)$, $P(x)$ and $A^m(x)$. The action of the supergroup on these fields reduces to general coordinate, local Lorentz and local supersymmetry transformations. The closure of the algebra of these transformations is almost obvious. Finally, the transformations obtained are identified with those in the component approach.

In Appendix A the notations are listed. In Appendix B the Einstein supergravity group including global γ_5 -transformations is considered.

In a forthcoming paper a differential geometry formalism in terms of the single SF $\mathcal{H}^m$ will be developed on the ground of the supergroup found.

Let us make some remarks concerning the literature on this subject.

An interesting paper of Siegel and Gates /14/ should be pointed out. They have proposed a formulation of supergravity using both an axial and a spinor SF (the latter seems to be unessential).

*) We label the vector (spinor) coordinates of SS by indices $l, m, n, \ldots (\lambda, \mu, \nu, \ldots)$ as it is done, e.g., in Ref. /5/.

Though being less geometrical their approach has much in common with ours (especially in the way of use of differential geometry), and the final results are equivalent.

It is worthwhile mentioning that chiral coordinates have also been discussed in Ref.[8] although we think the authors have underestimated their relevance.

After the appearance of our preprint [13] in Ref.[15] the transformations of the chiral scalar SF were rederived starting with the tensor calculus results[3].

II. Superspace and Supergravity Group

II.1. The manifestly covariant formalism of global (flat) supersymmetry is based on the concept of SS. Usually an 4+4-dimensional real*) SS $\{(x^m, \theta^\mu, \bar\theta^{\dot\mu})\}$ is considered where x^m is a real space-time coordinate and $\theta^\mu, \bar\theta^{\dot\mu}$ are left- and right-handed conjugated Weyl spinors (Grassman variables). The supersymmetry group is realized on functions $\Phi(x,\theta,\bar\theta)$ (SF) as follows**)

$$\Phi'(x',\theta',\bar\theta') = \Phi(x,\theta,\bar\theta). \qquad (1)$$

Here the coordinate transformations besides the Poincaré group transformations include also supertranslations

$$\begin{aligned} x'^m &= x^m + i\,\theta\sigma^m\bar\lambda - i\,\lambda\sigma^m\bar\theta \\ \theta'^\mu &= \theta^\mu + \lambda^\mu \\ \bar\theta'^{\dot\mu} &= \bar\theta^{\dot\mu} + \bar\lambda^{\dot\mu} \end{aligned} \qquad (2)$$

with constant (infinitesimal) Grassman parameters λ^μ and $\bar\lambda^{\dot\mu}$.

The most straightforward generalization of the supersymmetry group to the nonflat case is based on an analogy with the theory of gravitation. There the Poincaré group is replaced by the general coordinate transformation group. Similarly, in a number of geometrical approaches to supergravity [5-12] the global transformations (2) are replaced by general transformations of the coordinates of SS

$$\begin{aligned} x'^m &= x^m + \lambda^m(x,\theta,\bar\theta) \\ \theta'^\mu &= \theta^\mu + \lambda^\mu(x,\theta,\bar\theta) \\ \bar\theta'^{\dot\mu} &= \bar\theta^{\dot\mu} + \bar\lambda^{\dot\mu}(x,\theta,\bar\theta) \end{aligned} \qquad (3)$$

with arbitrary superfunctions-parameters λ.

However, as it was stressed in the Introduction, such a supergroup is excessively large and includes too many superfluous gauge parameters. Indeed, in the decompositions of the superfunctions $\lambda(x,\theta,\bar\theta)$ in powers of θ and $\bar\theta$ there are 128 independent components (4×2^4 in λ^m, 2×2^4 in λ^μ and 2×2^4 in $\bar\lambda^{\dot\mu}$) while only 14 of them have physical meaning (4 parameters of general transformations of the coordinates x^m, 6 for the local Lorentz an 4 for the local supersymmetry transformations). To get rid of the remaining 114 ones the gauge has to be strongly fixed.

II.2. In what follows we are going to propose another, more adequate way to generalize supersymmetry in the nonflat case. The basic idea is prompted by the existence of the so-called chiral (fundamental) representations of flat supersymmetry. They can be realized in SS more simply than the general ones (1), (2). Instead of the 4+4-dimensional real SS $\{(x^m, \theta^\mu, \bar\theta^{\dot\mu})\}$ one can consider a complex 4+2-dimensional left-handed chiral SS $\{(x^m_L, \theta^\mu_L)\}$ and its conjugate right-handed one $\{(x^m_R, \bar\theta^{\dot\mu}_R)\}$ (i.e., $x^m_R = (x^m_L)^*$, $\bar\theta^{\dot\mu}_R = (\theta^\mu_L)^+$). In these SS left-handed $\varphi_L(x_L,\theta_L)$ and right-handed $\varphi_R(x_R,\bar\theta_R)$ chiral scalar SF can be defined on which global supersymmetry is realized as follows

$$\varphi'_{L,R}(x'_{L,R};\theta'_{L,R}) = \varphi_{L,R}(x_{L,R};\theta_{L,R}).$$

Here the coordinate transformations include supertranslations:

Left SS	Right SS
$x'^m_L = x^m_L + 2i\,\theta_L\sigma^m\bar\lambda$	$x'^m_R = x^m_R - 2i\,\lambda\sigma^m\bar\theta_R$
$\theta'^\mu_L = \theta^\mu_L + \lambda^\mu$	$\bar\theta'^{\dot\mu}_R = \bar\theta^{\dot\mu}_R + \bar\lambda^{\dot\mu}$.

(4)

It is clear that an attempt to generalize supersymmetry group just in the simpler chiral SS will lead to gauge groups smaller than (3). However, the vector coordinate x^m_L (x^m_R) in the chiral SS is complex while the physical coordinate x^m is real. How to imbed a physical SS into the chiral ones? In the global

*) Here "real" means self-conjugated under complex conjugation in the bosonic sector and hermitian one in the fermionic sector.

**) In this paper we consider only scalar SF. SF with Lorentz indices will be defined in a paper on the differential geometry formalism.

supersymmetry case the answer is simple. Consider a 8+4-dimensional SS $\{(x_L^m, x_R^m, \theta_L^\mu, \bar\theta_R^{\dot\mu})\}$ and define in it an 4+4-dimensional supersurface. This means that four of the space-time coordinates have to be excluded by four equations covariant (or in this particular case just invariant) under the transformations (4)

$$x_L^m - x_R^m = 2i\,\theta_L \sigma^m \bar\theta_R . \tag{5}$$

This supersurface can be parameterized as follows:

$$x_L^m = x^m + i\,\theta\sigma^m\bar\theta \qquad x_R^m = x^m - i\,\theta\sigma^m\bar\theta$$
$$\theta_L^\mu = \theta^\mu \qquad \bar\theta_R^{\dot\mu} = \bar\theta^{\dot\mu} . \tag{6}$$

Thus one obtains a 4+4-dimensional SS with coordinates

$$x^m = \tfrac{1}{2}(x_L^m + x_R^m),\ \theta^\mu,\ \bar\theta^{\dot\mu} \tag{7}$$

transforming just according to Eq.(2).

So we conclude that global supersymmetry can be realized as the group of motions of the 4+4-dimensional supersurface (5) in the 8+4-dimensional SS with complex structure $\{(x_L^m, x_R^m, \theta_L^\mu, \bar\theta_R^{\dot\mu})\}$ (Fig. 1a).

Note that the chiral coordinates (6) have been earlier considered in the SS $\{(x^m, \theta^\mu, \bar\theta^{\dot\mu})\}$ /16/. However, they have been regarded just as some bases in SS suitable to describe the corresponding chiral SF. We do not consider Eq.(6) as definitions of certain bases in the 4+4-dimensional SS. Instead, we take them as equations of a supersurface in the 8+4-dimensional SS. Such an interpretation can be nontrivially generalized in the nonflat case.

II.3. This paragraph contains the basic ideas of our geometric construction. Define in the left- and right-handed SS conjugated supergroups of general coordinate transformations. Their infinitesimal form is

$$\begin{array}{ll} \text{Left SS} & \text{Right SS} \\ x'^m_L = x_L^m + \lambda^m(x_L, \theta_L) & x'^m_R = x_R^m + \bar\lambda^m(x_R, \bar\theta_R) \\ \theta'^\mu_L = \theta_L^\mu + \lambda^\mu(x_L, \theta_L) & \bar\theta'^{\dot\mu}_R = \bar\theta_R^{\dot\mu} + \bar\lambda^{\dot\mu}(x_R, \bar\theta_R) \end{array} \tag{8}$$

where λ^m, λ^μ are arbitrary left-handed superfunctions and $\bar\lambda^m, \bar\lambda^{\dot\mu}$ are their conjugates (i.e., for instance, $\bar\lambda^m(x_R, \bar\theta_R) = (\lambda^m(x_L, \theta_L))^+$).

Further, when considering the 8+4-dimensional SS$\{(x_L^m, x_R^m, \theta_L^\mu, \bar\theta_R^{\dot\mu})\}$ we have to get rid of a four-vector coordinate. For this purpose let us define once again a 4+4-dimensional real supersurface to play the role of the physical SS. Equations (5) are not more appropriate. They describe a particular supersurface (corresponding

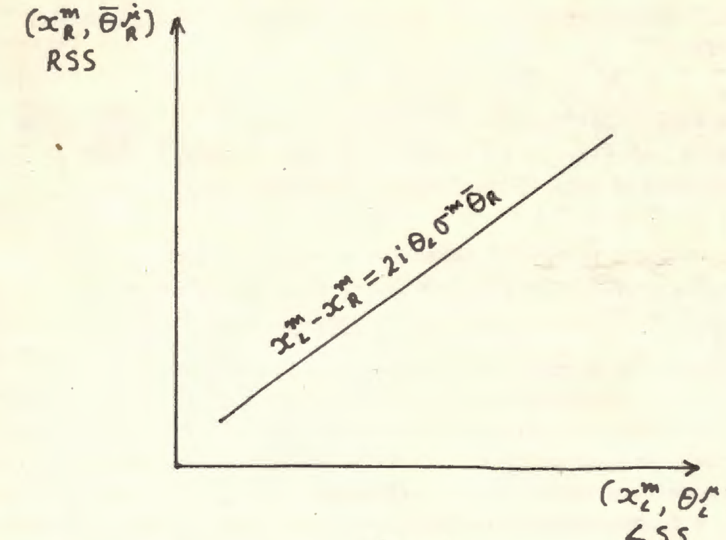

Fig. 1a

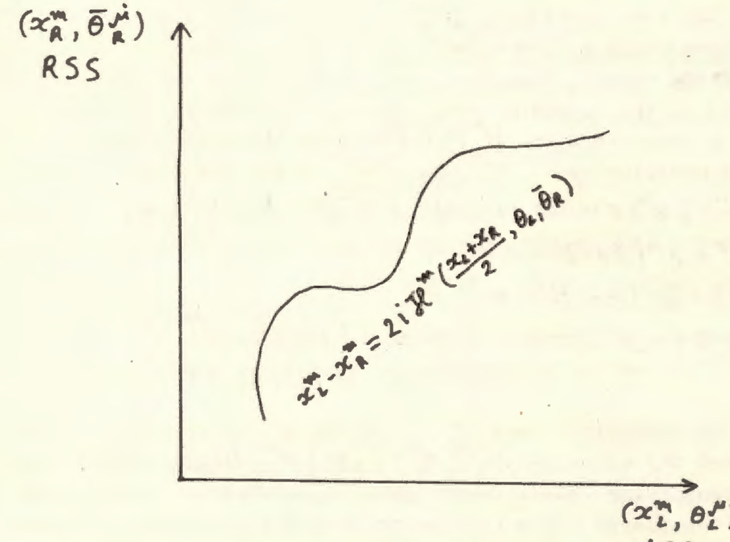

Fig. 1b

to the flat SS) and are not covariant under the new transformations (8). Therefore we shall define a general 4+4-dimensional supersurface

$$x_L^m - x_R^m = 2i\,\mathcal{H}^m\left(\tfrac{1}{2}(x_L+x_R),\theta_L,\bar\theta_R\right). \tag{9}$$

Here the real superfunction $\mathcal{H}^m$ is not fixed (it is represented as a "curve" in Fig. 1b in contrast to the "straight line" in Fig. 1a). Introducing real space-time coordinates

$$x^m = \tfrac{1}{2}(x_L^m + x_R^m) \tag{10}$$

the supersurface equations take the parametric form

$$\begin{aligned} x_L^m &= x^m + i\,\mathcal{H}^m(x,\theta,\bar\theta) \qquad & x_R^m &= x^m - i\,\mathcal{H}^m(x,\theta,\bar\theta) \\ \theta_L^\mu &= \theta^\mu & \bar\theta_R^{\dot\mu} &= \bar\theta^{\dot\mu}\,. \end{aligned} \tag{11}$$

It should be stressed once more that the superfunction $\mathcal{H}^m$ in Eq. (9) is completely arbitrary. This is due to two reasons. Firstly, we want full freedom in the choice of the supersurface (9). Thereby the geometry of the 4+4-dimensional physical SS (which is determined by the superfunction $\mathcal{H}^m$, as it will be shown in a forthcoming paper) is not restricted a priori. Secondly, the form of the superfunction $\mathcal{H}^m$ contains some gauge freedom. Indeed, let some particular (although arbitrary) supersurface be fixed. Nevertheless, its equations (9) still depend on the coordinate frame chosen, i.e., they will change under the transformations (8) of the coordinates of the 8+4-dimensional SS. Consequently, the superfunction $\mathcal{H}^m(x,\theta,\bar\theta)$ has to transform together with the coordinates x^m, θ^μ and $\bar\theta^{\dot\mu}$ of the 4+4-dimensional SS:

$$x'^m = x^m + \tfrac{1}{2}\lambda^m[x+i\mathcal{H}(x,\theta,\bar\theta),\theta] + \tfrac{1}{2}\bar\lambda^m[x-i\mathcal{H}(x,\theta,\bar\theta),\bar\theta] \tag{12a}$$

$$\theta'^\mu = \theta^\mu + \lambda^\mu[x+i\mathcal{H}(x,\theta,\bar\theta),\theta] \tag{12b}$$

$$\bar\theta'^{\dot\mu} = \bar\theta^{\dot\mu} + \bar\lambda^{\dot\mu}[x-i\mathcal{H}(x,\theta,\bar\theta),\bar\theta] \tag{12c}$$

$$\begin{aligned}\mathcal{H}'^m(x',\theta',\bar\theta') = \mathcal{H}^m(x,\theta,\bar\theta) &- \tfrac{i}{2}\lambda^m[x+i\mathcal{H}(x,\theta,\bar\theta),\theta] + \\ &+ \tfrac{i}{2}\bar\lambda^m[x-i\mathcal{H}(x,\theta,\bar\theta),\bar\theta]\,.\end{aligned} \tag{12d}$$

Let us summarize the different steps of our construction. We introduced two chiral SS $\{(x_L^m,\theta_L^\mu)\}$ and $\{(x_R^m,\bar\theta_R^{\dot\mu})\}$ together with the corresponding general coordinate transformation supergroups. Then we considered these complex SS as a 8+4-dimensional SS with complex structure and identified the real part of its complex vector coordinate with the physical space-time coordinate. The imaginary part was transferred into an axial (see p.III.1) super-

field $\mathcal{H}^m(x,\theta,\bar\theta)$ giving a 4+4-dimensional real supersurface (the physical SS)*). The initial complex supergroup (8) was realized nonlinearly (12) in the physical SS with the help of the SF $\mathcal{H}^m$. In what follows we shall identify this group with the supergravity group and the SF $\mathcal{H}^m$ with the gravitational superfield. This step will be justified by the detailed analysis of the transformations (12) in terms of components of $\mathcal{H}^m$ carried out in Section III.

Note that besides the gravitational SF $\mathcal{H}^m$ general and chiral scalar SF's can also be defined in the physical SS. Under the transformations (12) the general ones transform as follows

$$\varphi'(x',\theta',\bar\theta') = \varphi(x,\theta,\bar\theta) \tag{13a}$$

and the chiral ones

$$\begin{aligned}\varphi_L'(x_L',\theta') &= \varphi_L(x_L,\theta) \\ \varphi_R'(x_R',\bar\theta') &= \varphi_R(x_R,\bar\theta)\end{aligned} \tag{13b}$$

with x_L and x_R from Eq.(11). The group character of these laws is obvious. SF with external indices will be defined in a forthcoming paper.

II.4. Comparing the transformation supergroup (3) underlying the straightforward generalizations of supersymmetry and our supergroup (8) (or, which is the same, (12)), one can see that the latter is rather simpler. Indeed, the chiral superfunctions - parameters λ and $\bar\lambda$ in Eq.(8) contain 48 components (4×2^2 in λ^m, 2×2^2 in λ^μ and the same number in $\bar\lambda^m$ and $\bar\lambda^{\dot\mu}$) instead of the 128 ones in the general superfunctions in Eq.(3) (see the end of p. II.1). However, the supergroup (8) can be narrowed further.

It is easy to see that it has nontrivial subgroups. To show this, consider the analogue of Jacobian (superdeterminant or Berezinian[18]) of the coordinate transformations in the left (or right) SS

$$\mathrm{Ber}\left\|\frac{\partial(x_L',\theta_L')}{\partial(x_L,\theta_L)}\right\| = \mathrm{Det}\left\|\frac{\partial x_L'^m}{\partial x_L^n} - \frac{\partial x_L'^m}{\partial\theta_L^\mu}\frac{\partial\theta_L^\mu}{\partial\theta_L'^\nu}\frac{\partial\theta'^\nu}{\partial x_L^n}\right\|\cdot\mathrm{Det}\left\|\frac{\partial\theta_L^\mu}{\partial\theta_L'^\nu}\right\|.$$

It has the multiplicative property

*) Note that in their pioneering paper Volkov and Akulov[17] identified the Grassman coordinate of SS with the neutrino field.

$$Ber\left\|\frac{\partial(x''_L,\theta''_L)}{\partial(x'_L,\theta'_L)}\right\|\cdot Ber\left\|\frac{\partial(x'_L,\theta'_L)}{\partial(x_L,\theta_L)}\right\| = Ber\left\|\frac{\partial(x''_L,\theta''_L)}{\partial(x_L,\theta_L)}\right\| . \qquad (14)$$

Owing to this property the transformations (8) restricted by the condition

$$Ber\left\|\frac{\partial(x'_L,\theta'_L)}{\partial(x_L,\theta_L)}\right\| = 1 \qquad (15)$$

form a subgroup of the supergroup of general transformations of the left SS. This subgroup has clear geometrical meaning: It preserves the "supervolume" in the left SS*):

$$d^4x'_L\, d^2\theta'_L = Ber\left\|\frac{\partial(x'_L,\theta'_L)}{\partial(x_L,\theta_L)}\right\|\cdot d^4x_L\, d^2\theta_L = d^4x_L\, d^2\theta_L .$$

In infinitesimal form Eq.(15) reads

$$\frac{\partial}{\partial x_L^m}\lambda^m(x_L,\theta_L) - \frac{\partial}{\partial\theta_L^\mu}\lambda^\mu(x_L,\theta_L) \equiv \partial_M{}^L\lambda^M(-1)^{p(M)} \qquad (15')$$

(here the index M takes values m and μ ; $p(m)=0, p(\mu)=1$). The analogous condition holds for the conjugated parameters $\bar\lambda^m, \bar\lambda^{\dot\mu}$ of the right supergroup. The constraint (15)and its conjugate reduce the number of the independent gauge parameters to 40.

Two more subgroups are worth mentioning. They are given by conditions weaker than (15):

$$Ber\left\|\frac{\partial(x'_L,\theta'_L)}{\partial(x_L,\theta_L)}\right\|\cdot Ber\left\|\frac{\partial(x'_R,\bar\theta'_R)}{\partial(x_R,\bar\theta_R)}\right\| = 1 \quad \text{or} \qquad (16)$$

$$Ber\left\|\frac{\partial(x'_L,\theta'_L)}{\partial(x_L,\theta_L)}\right\|\cdot Ber^{-1}\left\|\frac{\partial(x'_R,\bar\theta'_R)}{\partial(x_R,\bar\theta_R)}\right\| = 1 ; \qquad (17)$$

in infinitesimal form they read

$$(\partial_M^L\lambda^M + \partial_M^R\bar\lambda^M)(-1)^{p(M)} = 0 \qquad (16')$$

$$(\partial_M^L\lambda^M - \partial_M^R\bar\lambda^M)(-1)^{p(M)} = 0. \qquad (17')$$

II.5. Now we shall formulate the main statements which have to be proved.

A. The transformation supergroup (8) is the group of Weyl (conformal) supergravity.

B. The transformation supergroup (8) with left (and right)-supervolume-preservation condition (15) is the group of Einstein (N=1) supergravity.

C. The transformation supergroup (8) with conditions (16) or

*) An analogy: in space-time $\{(x^m)\}$ the group of general coordinate transformations $\delta x^m = f^m(x)$ has a subgroup singled out by the constraint $Det\|\partial x'/\partial x\| = 1 \to \partial/\partial x^m f^m = 0$. Note that on the way towards the theory of general relativity Einstein /19/ has intensively discussed this volume-preserving subgroup.

(17) is the Einstein supergravity group plus global chiral or scale transformations.

Case A has been considered in our paper /20/. Case B will be investigated in detail in Section III of the present paper and case C in Appendix B.

III. The Einstein supergravity group and the components of the superfield $\mathcal{H}^m$

In the present section it will be demonstrated that the axial SF $\mathcal{H}^m(x,\theta,\bar\theta)$ describes the gravitational supermultiplet, and the group (8), (12), (15) (case B) is the Einstein supergravity group. To this end we shall establish the physical meaning and the transformation properties of the field components entering into the decomposition of $\mathcal{H}^m$ in powers of θ and $\bar\theta$:

$$\begin{aligned}\mathcal{H}^m(x,\theta,\bar\theta) &= B^m(x) + i\theta^\mu\chi_\mu^m(x) - i\bar\theta_{\dot\mu}\bar\chi^{m\dot\mu}(x) + \theta\sigma_a\bar\theta\, e^{am}(x) + \\ &+ \varkappa\,\theta\theta\big(P^m(x) + iS^m(x)\big) + \varkappa\,\bar\theta\bar\theta\big(P^m(x) - iS^m(x)\big) + \\ &+ i\varkappa\,\bar\theta\bar\theta.\theta^\mu\psi_\mu^m(x) - i\varkappa\,\theta\theta.\bar\theta_{\dot\mu}\bar\psi^{m\dot\mu}(x) + \varkappa\,\theta\theta.\bar\theta\bar\theta\, C^m(x).\end{aligned} \qquad (18)$$

In this decomposition the tensor field $e^{am}(x)$ will describe graviton (the vierbein field)*) and the Rarita-Schwinger field $\psi_\mu^m(x), \bar\psi^{m\dot\mu}(x)$ will correspond to gravitino. The constant $\varkappa$ of dimension cm $(\hbar = c = 1)$ will be identified with the gravitational constant. All the other fields in Eq.(18) will be either auxiliary ones or purely gauge degrees of freedom.

This section is planned as follows. First, we have to get rid of the nonpolynomiality of the transformation law (12). This can be achieved by such a partial gauge fixing in which the first several terms in Eq.(18) vanish. Afterwards there remains certain class of transformations (12) which preserve the fixed gauge. Their action on the remaining components of $\mathcal{H}^m$ is established. This class of transformations turns out to consist of Einstein general coordinate transformations, local Lorentz, and local supersymmetry ones. The evident group property of these transformations is demonstrated. Finally, we prove the equivalence of the results obtained to those of the component approach /2/.

*) For this reason we call the superfield $\mathcal{H}^m$ "axial": the tensor field $e^{am}(x)$ is the coefficient of the axial combination $\theta\sigma_a\bar\theta$.

III.1. We begin with a discussion of the functions – parameters in the decomposition of the superfunctions $\lambda^{M}(x_L,\theta_L)$:

$$\lambda^{m}(x_L,\theta)=-a^{m}(x_L)+ib^{m}(x_L)+\theta^{\nu}\varphi_{\nu}^{m}(x_L)+\theta\theta(s^{m}(x_L)+i\rho^{m}(x_L)) \qquad (19)$$

$$\lambda^{\mu}(x_L,\theta)=\varepsilon^{\mu}(x_L)+\theta^{\nu}\omega_{\nu}{}^{\mu}(x_L)+\theta\theta\eta^{\mu}(x_L)$$

and their conjugates $\bar{\lambda}^{M}(x_R,\bar\theta)$ (the parameters $a^{m}, b^{m}, s^{m}, \rho^{m}$ are real functions, e.g. $(a^{m}(x_L))^{*}=a^{m}(x_R)$). In Einstein supergravity (case B) there is the condition of supervolume preservation (15) which leads to the following restrictions

$$\partial_m^L \rho^{m}(x_L)=\partial_m^L s^{m}(x_L)=0 \qquad (20a)$$

$$\eta^{\mu}(x_L)=-\tfrac{1}{2}\partial_m^L\varphi^{m\mu}(x_L) \qquad (20b)$$

$$\omega_{\nu}{}^{\mu}(x_L)=\tfrac{1}{2}(-\partial_m^L a^{m}(x_L)+i\partial_m^L b^{m}(x_L))\delta_{\nu}^{\mu}-\tfrac{i}{2}\Omega^{ab}(x_L)(\sigma_{ab})_{\nu}{}^{\mu} \qquad (20c)$$

(and to the corresponding equalities for the conjugated quantities). Without loss of generality one can regard the antisymmetric tensor $\Omega^{ab}(x_L)$ in Eq.(20c) as a real function*).

III.2. Several gauge functions in Eq. (19) can be used to exclude some components of $\mathcal{H}^{m}$ (18). To show this let us write down the transformation law (12d) as a form-variation of $\mathcal{H}^{m}$

$$\begin{aligned}\delta^{*}\mathcal{H}^{m}(x,\theta,\bar\theta)&\equiv\mathcal{H}'^{m}(x,\theta,\bar\theta)-\mathcal{H}^{m}(x,\theta,\bar\theta)=-\tfrac{i}{2}[\lambda^{m}(x+i\mathcal{H},\theta)-\bar\lambda^{m}(x-i\mathcal{H},\bar\theta)]-\\&-\tfrac{1}{2}[\lambda^{n}(x+i\mathcal{H},\theta)+\bar\lambda^{n}(x-i\mathcal{H},\bar\theta)]\partial_n\mathcal{H}^{m}(x,\theta,\bar\theta)-\\&-[\lambda^{\nu}(x+i\mathcal{H},\theta)\tfrac{\partial}{\partial\theta^{\nu}}+\bar\lambda^{\dot\nu}(x-i\mathcal{H},\bar\theta)\tfrac{\partial}{\partial\bar\theta^{\dot\nu}}]\mathcal{H}^{m}(x,\theta,\bar\theta).\end{aligned} \qquad (21)$$

Now decompose the right-hand side in powers of $\mathcal{H}^{m}$ singling out the terms without $\mathcal{H}^{m}$

$$\begin{aligned}\delta^{*}\mathcal{H}^{m}(x,\theta,\bar\theta)&=-\tfrac{i}{2}\lambda^{m}(x,\theta)+\tfrac{i}{2}\bar\lambda^{m}(x,\bar\theta)+\ldots=\\&=b^{m}(x)-\tfrac{i}{2}\theta^{\nu}\varphi_{\nu}^{m}(x)+\tfrac{i}{2}\bar\theta_{\dot\nu}\bar\varphi^{m\dot\nu}(x)-\\&-\tfrac{i}{2}\theta\theta(s^{m}(x)+i\rho^{m}(x))+\tfrac{i}{2}\bar\theta\bar\theta(s^{m}(x)-i\rho^{m}(x))+\ldots\end{aligned}$$

We see that the components $B^{m}, \chi_{\nu}^{m}, \bar\chi_{\dot\nu}^{m}$ of $\mathcal{H}^{m}$ get completely arbitrary additive contributions at infinitesimal level. This fact indicates that they are pure gauge degrees of freedom which can be removed by an appropriate gauge choice.

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*) Indeed, $(\Omega_1^{ab}+i\Omega_2^{ab})\sigma_{ab}=(\Omega_1^{ab}-\tfrac{1}{2}\varepsilon^{abcd}\Omega_{2cd})\sigma_{ab}\equiv\Omega^{ab}\sigma_{ab}\;((\Omega^{ab})^{*}=\Omega^{ab})$ due to the identity $\sigma_{ab}=\tfrac{i}{2}\varepsilon_{abcd}\sigma^{cd}$.

So, there exists such a gauge in which the axial superfield decomposition is reduced to

$$\begin{aligned}\mathcal{H}^{m}(x,\theta,\bar\theta)&=\varkappa\theta\theta(P^{m}(x)+iS^{m}(x))+\varkappa\bar\theta\bar\theta(P^{m}(x)-iS^{m}(x))+\theta\sigma_a\bar\theta e^{am}(x)+\\&+i\varkappa\bar\theta\bar\theta.\theta^{\mu}\psi_{\mu}^{m}(x)-i\varkappa\theta\theta.\bar\theta_{\dot\mu}\bar\psi^{m\dot\mu}(x)+\varkappa\theta\theta.\bar\theta\bar\theta C^{m}(x).\end{aligned} \qquad (22)$$

In this gauge the transformation law (21) acquires a polynomial form because the third power of $\mathcal{H}^{m}$ vanishes. In the supersymmetric Yang-Mills theory Wess and Zumino used a similar gauge in order to get rid of nonpolynomialities/21/.

III.3. It is very important that after such a partial gauge fixing there remains a set of transformations of the type (21) preserving the form (22) of $\mathcal{H}^{m}$. These transformations have the following specific parameters

$$b^{m}=0 \qquad (23a)$$

$$\varphi_{\mu}^{m}=2i(\sigma^{m}\bar\varepsilon)_{\mu}+4i\varepsilon_{\mu}(P^{m}+iS^{m}) \qquad (23b)$$

$$\bar\varphi^{m\dot\mu}=2i(\tilde\sigma^{m}\varepsilon)^{\dot\mu}-4i\bar\varepsilon^{\dot\mu}(P^{m}-iS^{m}). \qquad (23c)$$

$$(\sigma^{m}=\sigma_a e^{am})$$

Here we come across a peculiar phenomenon: Due to the partial gauge fixing the transformation parameters become dependent on the fields. Just for this reason the bracket parameters ("structure constants") of local supersymmetry depend on the fields. We shall discuss and use this property in what follows.

III.4. Now we consider separately the role of the functions-parameters $\rho^{m}(x)$ and $s^{m}(x)$ in Eq.(19). Under the transformations (21) they change the component fields $P^{m}(x)$, $S^{m}(x)$ and $C^{m}(x)$ only:

$$\delta^{*}P^{m}(x)=\tfrac{1}{2\varkappa}\rho^{m}(x) \qquad (24a)$$

$$\delta^{*}S^{m}(x)=-\tfrac{1}{2\varkappa}s^{m}(x) \qquad (24b)$$

$$\delta^{*}C^{m}(x)=P^{n}\partial_n s^{m}+S^{n}\partial_n\rho^{m}-s^{n}\partial_n P^{m}-\rho^{n}\partial_n S^{m} \qquad (24c)$$

In Einstein supergravity the longitudinal parts of the gauge functions ρ^{m} and s^{m} vanish owing to the restriction (20a). Then equations (24a), (24b) tell us that the transversal parts of the fields $P^{m}(x)$ and $S^{m}(x)$ are gauge degrees of freedom. Because of this it is expedient to introduce fields

$$P=\partial_m P^{m}\,,\quad S=\partial_m S^{m} \qquad (25a)$$

$$D^{m}=C^{m}+2\varkappa(P^{n}\partial_n S^{m}-S^{n}\partial_n P^{m}) \qquad (25b)$$

that do not change under transformations (24). Now one can forget about the parameters $\rho^{m}(x)$ and $s^{m}(x)$ as they have already played their role.

III.5. Thus after the partial gauge fixing at our disposal there remain the functions-parameters $a^m(x)$, $\Omega^{ab}(x)$ and $\varepsilon_\mu(x)(\bar\varepsilon^{\dot\mu}(x))$. In this paragraph it will be shown that they serve as parameters of general transformations of coordinates x^m, of local Lorentz transformations, and of local supersymmetry, respectively.

III.5.1. We begin with the parameter $a^m(x)$. Under the transformations (21) with this parameter one has

$$\delta^*_G P = \partial_n a^n \cdot P + a^n \partial_n P \qquad (a)$$
$$\delta^*_G S = \partial_n a^n \cdot S + a^n \partial_n S \qquad (b)$$
$$\delta^*_G e^{am} = \partial_n a^n \cdot e^{am} + a^n \partial_n e^{am} - \partial_n a^m \cdot e^{an} \qquad (c) \quad (26)$$
$$\delta^*_G \psi^m_\alpha = \tfrac{3}{2}\partial_n a^n \cdot \psi^m_\alpha + a^n \partial_n \psi^m_\alpha - \partial_n a^m \cdot \psi^n_\alpha \qquad (d)$$
$$\delta^*_G \mathcal{D}^m = 2\,\partial_n a^n \cdot \mathcal{D}^m + a^n \partial_n \mathcal{D}^m - \partial_n a^m \cdot \mathcal{D}^n \quad . \qquad (e)$$

Here the Weyl spinors $\psi^m_\mu, \bar\psi^{m\dot\mu}$ are combined into the Majorana spinor (in this section we shall use 4-component notation)

$$\psi^m_\alpha = \begin{pmatrix} \psi^m_\mu \\ \bar\psi^{m\dot\mu} \end{pmatrix} . \qquad (27)$$

We see that $a^m(x)$ is a general coordinate transformation parameter, the index m of the fields e^{am}, ψ^m_α and $\mathcal{D}^m$ being a world contravariant one. The indices a of the field e^{am} and α of the field ψ^m_α are not affected by transformations (26). We shall see in the next paragraph that they are local Lorentz indices. To distinguish such indices from the world ones we shall denote them by the first letters of Latin (vectors) and of Greek (spinors) alphabet.

Note that all the fields have nonzero weight (1 for P, S and e^{am}, $\frac{3}{2}$ for ψ^m_α, 2 for $\mathcal{D}^m$). Of course, these weights can be eliminated multiplying the fields by appropriate powers of $\mathrm{Det}\|e^{am}\|$.

III.5.2. The transformations with parameter $\Omega^{ab}(x)$ have the following form

$$\delta^*_L P = \delta^*_L S = 0 \qquad (a)$$
$$\delta^*_L e^{am} = -2\,\Omega^{ab}\, e_b^{\ m} \qquad (b) \quad (28)$$
$$\delta^*_L \psi^m_\alpha = \tfrac{i}{2}\,\Omega^{ab}(\sigma_{ab}\psi^m)_\alpha \qquad (c)$$
$$\delta^*_L \mathcal{D}^m = -\tfrac{1}{2}\,\partial_n \Omega^{ab} \cdot \varepsilon_{abcd}\, e^{cm} e^{dn} \quad . \qquad (d)$$

These are local Lorentz transformations. The fields P and S are scalars. The field $e^{am}(x)$ with respect to index a transforms as a Lorentz vector (its second index m is a world one, so e^{am} is the vierbein field). ψ^m_α is a Lorentz spinor

(α) and a world vector (m). Only the field $\mathcal{D}^m$ has a non-standard transformation law (28d). So it is convenient to introduce a new field variable

$$A^m = \mathcal{D}^m - \tfrac{1}{4}\,\varepsilon^{mnlr}\, e_{la}\,\partial_n e_r^{\ a} , \qquad (29)$$

where

$$\varepsilon^{mnlr} = \varepsilon_{abcd}\, e^{am} e^{bn} e^{cl} e^{dr}$$

and e_{ma} is the inverse vierbein matrix $(e_{ma} e^{an} = \delta_m^{\ n})$. The contravariant field A^m obtained has now standard transformation laws

$$\delta^*_G A^m = 2\,\partial_n a^n \cdot A^m + a^n \partial_n A^m - \partial_n a^m \cdot A^n \qquad (26a')$$
$$\delta^*_L A^m = 0. \qquad (28d')$$

III.5.3. Finally, consider the transformations with parameter $\varepsilon_\alpha = \begin{pmatrix} \varepsilon_\mu \\ \bar\varepsilon^{\dot\mu} \end{pmatrix}$ which have to be the local supersymmetry transformations. One obtains

$$\delta^*_S P = -\tfrac{1}{2}\,\tilde\nabla_m(\bar\varepsilon\gamma_5\psi^m)$$
$$\delta^*_S S = \tfrac{1}{2}\,\tilde\nabla_m(\bar\varepsilon\,\psi^m)$$
$$\delta^*_S e^{am} = i\varkappa\,\bar\varepsilon\gamma^a\psi^m \qquad (30)$$
$$\delta^*_S \psi^m_\alpha = 2(\gamma_5\varepsilon)_\alpha A^m - 2i(\sigma^{mn}\tilde\nabla_n\varepsilon)_\alpha + 2i\left[(S + P\gamma_5)(\gamma^m\varepsilon)\right]_\alpha$$
$$\delta^*_S A^m = -\tfrac{i}{4}\,\varepsilon^{mnlr}\tilde\nabla_n(\bar\varepsilon\gamma_l\psi_r) + i(\tilde\nabla_n\bar\varepsilon)\gamma^m\gamma_5\psi^n - \tfrac{i}{2}\,\bar\varepsilon\gamma^n\gamma_5\tilde\nabla_n\psi^m -$$
$$- \tfrac{3i}{2}(\tilde\nabla_n\bar\varepsilon)\gamma^n\gamma_5\psi^m - 2\,\bar\varepsilon(P + S\gamma_5)\psi^m$$

where $\gamma^m = \gamma_a e^{am}$, $\psi_n = \psi^m e_{ma} e^a_{\ n}$. The symbol $\tilde\nabla_m$ denotes the covariant derivative for fields of nonzero weight. (Its definition is given in Appendix A, Eq.(A.2)).

III.5.4. At first sight formulas (30) differ significantly from those in the component approach /2/. However, these differences are superficial and can be eliminated. For this purpose one has to do the following. Firstly, one has to remove the weights multiplying the fields by appropriate powers of $e = \mathrm{Det}\|e_{ma}\|$. Secondly, some redefinition of the fields ψ^m_α, S, P and A^m is needed. The new field variables are the following

$$e'^{am} = e^{1/3} e^{am}$$
$$\psi'^m_\alpha = e^{1/2}(\psi^m - \tfrac{1}{3}\gamma^m \gamma_n \psi^n)_\alpha$$
$$P' = e^{1/3} P - \tfrac{i}{8} \bar{\psi}'^m \sigma'_{mn} \gamma_5 \psi'^n$$
$$S' = e^{1/3} S + \tfrac{i}{8} \bar{\psi}'^m \sigma'_{mn} \psi'^n$$
$$A'^m = e^{2/3} A^m + \tfrac{i}{4} \bar{\psi}'^m \gamma'_n \gamma_5 \psi'^n - \tfrac{i}{4} \bar{\psi}'^n \gamma'^m \gamma_5 \psi'_n +$$
$$+ \tfrac{9i}{16} \varepsilon'^{mnlk} \bar{\psi}'_n \gamma'_l \psi'_k$$
where $\gamma^m = \gamma_a e^{am}$, $\gamma'^m = \gamma_a e'^{am}$, $\psi'_m = e'_{ma} e'^a_n \psi'^n$.
Thirdly, the indices of all the contravariant vectors have to be lowered with the help of the metric tensor $g'_{mn} = e'_{ma} e'^a_n$.
Finally, the transformations (30) have to be modified by combining them with Lorentz transformations (20) with the parameter
$\Omega^{\Delta L}_{ab} = -\frac{i}{2} \varepsilon_{abmn} \bar{\varepsilon}' \gamma'^m \gamma_5 \psi'^n \quad (\varepsilon'_\alpha = e^{-1/6} \varepsilon_\alpha)$.
The resulting local supersymmetry transformations of the new field variables have the form(primes are implied everywhere)

$$\delta^*_{S+\Delta L} e_{ma} = -i \bar{\varepsilon} \gamma_a \psi_m$$
$$\delta^*_{S+\Delta L} \psi_{m\alpha} = -2(\nabla_m \varepsilon)_\alpha - \tfrac{2i}{3} \gamma_m [(S - P\gamma_5)\varepsilon]_\alpha + 2[(A_m - \tfrac{1}{3}\gamma_m \gamma^n A_n)\gamma_5 \varepsilon]_\alpha$$
$$\delta^*_{S+\Delta L} S = \tfrac{i}{2} \bar{\varepsilon} \sigma^{mn} \nabla_m \psi_n + \tfrac{i}{2} (\bar{\varepsilon} \gamma^m \psi_m) S -$$
$$- \tfrac{i}{2} (\bar{\varepsilon} \gamma^m \gamma_5 \psi_m) P + \tfrac{1}{2} (\bar{\varepsilon} \gamma_5 \psi_m) A^m$$
$$\delta^*_{S+\Delta L} P = -\tfrac{i}{2} \bar{\varepsilon} \gamma_5 \sigma^{mn} \nabla_m \psi_n + \tfrac{i}{2} (\bar{\varepsilon} \gamma^m \gamma_5 \psi_m) S + \qquad (31)$$
$$+ \tfrac{i}{2} (\bar{\varepsilon} \gamma^m \psi_m) P + \tfrac{1}{2} (\bar{\varepsilon} \psi_m) A^m$$
$$\delta^*_{S+\Delta L} A_m = -\tfrac{i}{2} \bar{\varepsilon} \gamma^n \gamma_5 \nabla_n \psi_m + \tfrac{i}{2} \bar{\varepsilon} \gamma^n \gamma_5 \nabla_m \psi_n + \tfrac{i}{4} \varepsilon_{mnlk} \bar{\varepsilon} \gamma^n \nabla^l \psi^k -$$
$$- \tfrac{1}{2} \bar{\varepsilon} (P + S\gamma_5) \psi_m + \tfrac{i}{2} (\bar{\varepsilon} \gamma^n \psi_n) A_m$$
$$- i (\bar{\varepsilon} \gamma^n \psi_m) A_n + \tfrac{i}{4} \varepsilon_{mnlk} A^n (\bar{\varepsilon} \gamma^l \gamma_5 \psi^k).$$

where ∇_m is the covariant derivative (see its definition in Appendix A, Eq.(A.2); note that terms bilinear in ψ^m_α appear in the connection coefficients (A.3) just because of the changes of variables made). The transformations (31) coincide with those earlier guessed by other authors in the component approach[2].

Stress once more that the representations of local supersymmetry (30) and (31) are equivalent.

Note also that in the component approach[2] transformations of the so-called "scalar" and "vector" multiplets of matter fields have been found. These multiplets correspond to the chiral and general scalar SF defined in Eq.(13). It can be shown (we shall not do this here) that the SF laws (13) in terms of component fields give the same results as the ones of Ref.[2].

III.6. Now we have to check that the transformations (12) and (15) in the gauge (22) (i.e.,those with specific functions-parameters (23)) form a subgroup of the group (8), (15).

In the component approach the only method to prove this fact is a straightforward calculation of the commutators of transformations (31) between themselves and with transformations δ^*_G and δ^*_L. Then one must succeed in identifying the different parts of the commutators obtained with the initial transformations having some new ("bracket") parameters. This is a nontrivial technical task. The main difficulty is due to the field dependence of the bracket parameters.

In our approach we can proceed much more simply and effectively because we know the clear group structure. We have a field dependence of the parameters from the very beginning as a consequence of the gauge fixing (23). The only question is what happens to this dependence when one commutes the transformations.

Consider symbolically the general situation. Take a set of fields that transform infinitesimally according to

$$\varphi \to \varphi + G(a)\varphi, \qquad (32)$$

where a is a set of parameters and G are the generators of the transformations. Let the transformations (32)form a group. This means that the commutator of two transformations with parameters a_1 and a_2 respectively is a new transformation of the same type with some bracket parameter

$$[G(a_2), G(a_1)]\varphi = G(a_3(a_1, a_2))\varphi. \qquad (33)$$

Now let the parameters depend on the fields, $a = a(\varphi)$:

$$\varphi \to \varphi + G(a(\varphi))\varphi. \qquad (34)$$

Then two successive transformations give (taking into account the infinitesimality of the parameters)

$$\varphi \to \varphi + G(a_1(\varphi))\varphi \to \varphi + G(a_1(\varphi))\varphi + G[a_2(\varphi + G(a_1(\varphi))\varphi)].G[\varphi + G(a_1(\varphi))\varphi] =$$

$$= \varphi + G(a_1(\varphi))\varphi + G(a_2(\varphi))\varphi + G\left[\frac{\delta a_2}{\delta\varphi} G(a_1(\varphi))\varphi\right]\varphi +$$
$$+ G(a_2(\varphi))\, G(a_1(\varphi))\,\varphi .$$

The commutator of these transformations is equal to

$$[G_2, G_1]\varphi = G\left[\frac{\delta a_2}{\delta\varphi} G(a_1(\varphi))\varphi\right]\varphi - G\left[\frac{\delta a_1}{\delta\varphi} G(a_2(\varphi))\varphi\right]\varphi +$$
$$+ [G(a_2(\varphi)), G(a_1(\varphi))]\varphi .$$

We conclude that due to the group law (33) and to the evident property $G(a')+G(a'')=G(a'+a'')$ this commutator is again a transformation (34) with bracket parameters

$$a^{br}(\varphi) = \frac{\delta a_2}{\delta\varphi} G(a_1(\varphi))\varphi - \frac{\delta a_1}{\delta\varphi} G(a_2(\varphi))\varphi + a_3[a_1(\varphi), a_2(\varphi)] . \qquad (35)$$

The last question is whether the bracket parameters depend on the set of fields, $a^{br} = a^{br}(\varphi)$, in the same way as the initial ones do, $a = a(\varphi)$. In our case the answer is yes. Indeed, the field dependence (23) is a consequence of gauge fixing and the commutator of two gauge preserving transformations preserves the gauge too.

So, the knowledge of the supergravity group structure makes evident the closure of the transformation algebra of field components. Moreover, it gives a simple expression (35) for the bracket parameters. We shall not use their explicit form.

IV. Conclusions

The analysis carried out in Section III corroborated that our group is in fact the supergravity group. The structure of this group proved to be very simple and transparent although unusual. Two features should be stressed especially. First, the initial SS where the group is defined has complex structure. Second, in order to introduce the physical SS, the imaginary part of the vector coordinate is identified with the axial gravitational SF, i.e., it is converted into a dynamical variable.

The dynamics is based on the symmetry group. To describe dynamics, it is necessary to express invariantly the action and possible counterterms via the dynamical variables. In a forthcoming publication we shall adapt the formalism of differential geometry for construction and investigation of invariants. It will essentially differ from the one applied in Ref. /5/ in two aspects. First, all the geometrical quantities will be expressed in terms of the single dynamical variable, just the axial SF $\mathcal{H}^m(x,\theta,\bar\theta)$. Second, our local Lorentz group acting on the external indices of the superfields will not be independent. Instead, its transformations will be induced by the world transformations.

The authors are grateful to E.A.Ivanov, L.Mezincescu, P. van Nieuwenhuizen, Ya.A.Smorodinsky, K.S. Stelle, D.V.Volkov and B.M.Zupnik for useful discussions.

Appendix A

In the present paper the Van-der-Waerden two-component formalism is mainly used. The basic notation is as follows:

$$\bar\psi_{\dot\alpha} = (\psi_\alpha)^+ ,\quad \psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta ,\quad \bar\psi^{\dot\alpha} = \varepsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta}$$

$$\varepsilon^{12} = \varepsilon^{\dot1\dot2} = -\varepsilon_{12} = -\varepsilon_{\dot1\dot2} = \varepsilon_{21} = \varepsilon_{\dot2\dot1} = 1$$

$$\psi\varphi = \psi^\alpha\varphi_\alpha ,\quad \bar\psi\bar\varphi = \bar\psi_{\dot\alpha}\bar\varphi^{\dot\alpha}$$

$$(\sigma_a)_{\alpha\dot\alpha} = (\mathbb{1}, \vec\sigma)_{\alpha\dot\alpha} ,\quad (\tilde\sigma_a)^{\dot\alpha\alpha} = (\mathbb{1}, -\vec\sigma)^{\dot\alpha\alpha}$$

$$\sigma_{ab} = \frac{i}{2}(\sigma_a\tilde\sigma_b - \sigma_b\tilde\sigma_a) ,\quad \tilde\sigma_{ab} = \frac{i}{2}(\tilde\sigma_a\sigma_b - \tilde\sigma_b\sigma_a) .$$

At the end of Section III four-component notation is employed. The Majorana spinor is written down as

$$\Psi_\alpha = \begin{pmatrix}\psi_\alpha \\ \bar\psi^{\dot\alpha}\end{pmatrix} ,\quad \Psi^\alpha = C^{\alpha\beta}\Psi_\beta ,\quad C = \begin{pmatrix}\varepsilon_{\alpha\beta} & 0 \\ 0 & \varepsilon^{\dot\alpha\dot\beta}\end{pmatrix} ;\quad \varepsilon^{0123} = 1$$

$$\gamma_a = \begin{pmatrix}0 & \sigma_a \\ \tilde\sigma_a & 0\end{pmatrix} ,\quad \gamma_5 = \begin{pmatrix}i\mathbb{1} & 0 \\ 0 & -i\mathbb{1}\end{pmatrix} ,\quad \sigma_{ab} = \frac{i}{2}[\gamma_a, \gamma_b] .$$

In Eq.(30) the vierbein e^{am} has weight 1 and for fields of nonzero weight the covariant derivative $\tilde\nabla_m$ has the form

$$\tilde\nabla_m\varphi^n_\alpha = \partial_m\varphi^n_\alpha + w\Gamma_m\varphi^n_\alpha + \Gamma_{mk}^{\ \ n}\varphi^k_\alpha + \frac{i}{2}\omega_m^{\ ab}(\sigma_{ab}\varphi^n)_\alpha . \qquad (A.1)$$

Here w is the weight of the field φ^n_α (i.e. $\delta^*_G\varphi^n_\alpha = w\,\partial_k a^k\cdot\varphi^n_\alpha + a^k\partial_k\varphi^n_\alpha - \partial_k a^n\cdot\varphi^k_\alpha$); $\Gamma_m = \frac{1}{3}\partial_m e_{na}\cdot e^{an}$;

$$\Gamma_{mk}^{\ \ n} = \frac{1}{2}(-\partial_m e_{ka}\cdot e^{an} + \partial^n e_{ka}\cdot e^a_m - g^{nl}\partial_m e_{la}\cdot e^a_k)$$
$$+\Gamma_m\delta^n_k - \frac{1}{2}\Gamma^n g_{mk} + (m\leftrightarrow k) ;\quad g^{mn} = e^m_a e^{an} ;$$

$$\omega_m^{\ ab} = \frac{1}{4}(\partial_m e^a_n\cdot e^{bn} - \partial_n e^a_m\cdot e^{bn} + \partial_k e_{nc}\cdot e^c_m e^{ak}e^{bn}) -$$
$$- \frac{1}{2}e^a_m e^{bk}\Gamma_k - (a\leftrightarrow b) .$$

In Eq.(31) the vierbein and all the other fields have zero weight and the covariant derivative ∇_m takes the form

$$\nabla_m \varphi^n_\alpha = \partial_m \varphi^n_\alpha + \Gamma_{mk}{}^n \varphi^k_\alpha + \frac{i}{2}\omega_m^{ab}(\sigma_{ab}\varphi^n)_\alpha , \qquad \text{(A.2)}$$

where

$$\Gamma_{mk}^n = \frac{1}{2}(-\partial_m e_{ka}\cdot e^{an} + \partial^n e_{ka}\cdot e^a_m - g^{nl}\partial_m e_{la}\cdot e^a_k) + (m\leftrightarrow k)$$

$$\omega_m^{ab} = \frac{1}{4}(\partial_m e^a_n\cdot e^{bn} - \partial_n e^a_m\cdot e^{bn} + \partial_k e_{nc}\cdot e^c_m e^{ak}e^{bn}) + \qquad \text{(A.3)}$$

$$+\frac{1}{32}(2\bar\Psi_m\gamma_n\Psi_k - 2\bar\Psi_m\gamma_k\Psi_n + \bar\Psi_n\gamma_m\Psi_k)e^{an}e^{bk} - (a\leftrightarrow b).$$

As a consequence of the field redefinition (p.II.5.4) there appears a gravitino-field-dependent term in the expression for connection ω_m^{ab}.

Appendix B

Here we analyse the subgroup of the group (8) preserving the product of left and right- supervolumes (restriction (16)). Using the decomposition (19) we write down condition (16') for the parameters-superfunctions as

$$\partial^L_m a^m(x_L) + i\partial^L_m b^m(x_L) + \theta^\mu\partial^L_m\varphi_\mu^m(x_L) + \theta\theta(\partial^L_m s^m(x_L) + i\partial^L_m p^m(x_L)) -$$

$$-\omega_\mu^\mu(x_L) + 2\theta^\mu\eta_\mu(x_L) + \partial^R_m a^m(x_R) - i\partial^R_m b^m(x_R) +$$

$$+\bar\theta_{\dot\mu}\partial^R_m\bar\varphi^{m\dot\mu}(x_R) + \bar\theta\bar\theta(\partial^R_m s^m(x_R) - i\partial^R_m p^m(x_R)) -$$

$$-\bar\omega^{\dot\mu}_{\dot\mu}(x_R) + 2\bar\theta_{\dot\mu}\bar\eta^{\dot\mu}(x_R) = 0 , \qquad \text{(B.1)}$$

where

$$x^m_L = x^m + i\mathcal{H}^m(x,\theta,\bar\theta), \quad x^m_R = x^m - i\mathcal{H}^m(x,\theta,\bar\theta). \qquad \text{(B.2)}$$

The partial gauge fixing is again given by Eq.(22). It is easy to check that transformations with parameters b^m violate this gauge, so we have again

$$b^m = 0. \qquad \text{(B.3)}$$

Now we substitute Eq.(B.2) into Eq.(B.1) taking into account Eqs. (22)and (B.3). The decomposition (22) of $\mathcal{H}^m$ begins with terms bilinear in θ and (or) $\bar\theta$. Therefore the coefficients of zero and first powers of θ and $\bar\theta$ in Eq.(B.1) must vanish

$$\eta_\mu(x) = -\frac{1}{2}\partial_m\varphi^m_\mu(x), \quad \bar\eta_{\dot\mu}(x) = -\frac{1}{2}\partial_m\bar\varphi^m_{\dot\mu}(x) \qquad \text{(B.4)}$$

$$\omega_\mu^\mu(x) + \bar\omega^{\dot\mu}_{\dot\mu}(x) = 2\partial_m a^m(x). \qquad \text{(B.5)}$$

Equation (B.4) coincides with Eq.(20b) while Eq.(B.5) is somewhat weaker than Eq.(20c). Further, an analysis of the coefficients of $\theta\theta$, $\bar\theta\bar\theta$ and $\theta\sigma_a\bar\theta$ gives the conditions

$$\partial_m p^m(x) = \partial_m s^m(x) = 0$$

which are the same as (20a) and

$$\partial_m(\omega_\mu{}^\mu(x) - \bar\omega^{\dot\mu}{}_{\dot\mu}(x)) = 0. \qquad \text{(B.6)}$$

The coefficients of the higher powers of $\theta, \bar\theta$ vanish automatically.

Conditions (B.5)and (B.6) lead to

$$\omega_\mu{}^\nu(x) = (\frac{1}{2}\partial_m a^m(x) + ib)\mathbb{1}_\mu{}^\nu + \frac{i}{2}\Omega^{ab}(x)(\sigma_{ab})_\mu{}^\nu. \qquad \text{(B.7)}$$

Eq.(B.7) is the same as Eq.(20c) except for the extra <u>constant</u> real parameter b. It is not hard to verify that this parameter generates global γ_5-transformations of the field-components of $\mathcal{H}^m$. So, we conclude that global chiral transformations can easily be introduced into Einstein supergravity just by replacing the volume-preserving condition (15) by a slightly weaker one (16). The implications of this fact will be discussed elsewhere.

Similarly, condition (17) can be shown to allow for extra global scale transformations (instead of the γ_5-ones).

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Received by Publishing Department
on May 21,1979