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$SU(4) \times SU(4)$ нарушение с углом Кабиббо для нелинейного лагранжиана адронов

В работе обсуждается возможность связи между нарушением группы $SU(4) \times SU(4)$ и возникновением структуры Кабиббо у слабого заряженного тока, а также соотношениями ЧСАТ в нелинейной киральной модели. Рассмотренная схема является обобщением метода Оакса, предложенного впервые для описания нарушения группы $SU(3) \times SU(3)$. Получены массовые формулы для 15-плета 0^--мезонов и 20-плета $1/2^+$-барионов.

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$SU(4) \times SU(4)$ -Breaking and the Cabibbo Angle in a Nonlinear Hadron Lagrangian

The possibility of an internal relationship between $SU(4) \times SU(4)$ symmetry breaking, the Cabibbo structure of the hadronic charged weak current and the PCAC relation is investigated within a nonlinear chiral lagrangian. The symmetry-breaking scheme used is a generalization of the $SU(3) \times SU(3)$ -breaking mechanism proposed by Oakes. It provides us with mass formulas for the 15-plet of 0^--mesons and 20-plet of $1/2^+$-baryons.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

It is well established that strong interactions show an approximate chiral-symmetry. This symmetry manifests itself most transparently in pion-nucleon interactions (chiral group $SU(2) \times SU(2)$), where it is broken only weakly by the small pion mass. A considerably larger symmetry breaking is needed for the group $SU(3) \times SU(3)$ which characterizes the strong interaction of both non-strange and strange hadrons. Also in this case, however, the concept of chiral symmetry turns out to be a useful guide for determining the basic structure of the underlying Lagrangian /1-3/. In order to introduce into the theory the recently observed charmed particles, it seems to be reasonable to further enlarge the chiral group from $SU(3) \times SU(3)$ to $SU(4) \times SU(4)$. Of course, this enlarged symmetry must be broken far much stronger than the group $SU(3) \times SU(3)$ because of the appearance of the heavy charmed mesons D, F, η_c. The problem of finding suitable symmetry breaking schemes for this group has been discussed in some recent works /4/.

In this paper, we shall continue the investigation of the $SU(4) \times SU(4)$ breaking mechanism. In particular, we shall study a symmetry breaking scheme which satisfies the following conditions: It contains a minimal number of arbitrary parameters, reproduces a qualitatively correct picture of the masses of the 15-plet of 0^--mesons and 20-plet of $1/2^+$-baryons (with an uncertainty of, say, 10%) and yields the Cabibbo structure of the charged weak current including its PCAC condition. Moreover, we want to get a relationship between the strength of the $SU(2) \times$ $\times SU(2)$ breaking (the finiteness of the pion mass), the nonconservation of strangeness in weak interactions, and the Cabibbo angle.

A symmetry breaking scheme satisfying an analogous set of requirements has been proposed some years ago for the group $SU(3) \times SU(3)$ by Oakes /5/. There exist also attempts of modifying and improving this scheme /6/. However, as has been shown /2,3/ the original version is sufficient in order to get a satisfactory description of the low energy interactions of hadrons. In the following we shall give a suitable generalization of Oakes' scheme to the group $SU(4) \times SU(4)$. Throughout we shall work with the nonlinear meson-baryon Lagrangian used in ref./7/ to construct a chiral unified gauge model of weak and electromagnetic interactions.

The paper is organized as follows. In Sec. 2 we introduce the nonlinear chiral $SU(4) \times SU(4)$ meson-baryon Lagrangian. Sec. 3 contains a discussion of the symmetry breaking terms and the mass formulas. In Sec. 4 we derive the charged weak current of the group $SU(2)_L \times U(1)$, the PCAC condition, and clarify the role of the Cabibbo angle. Finally, Sec. 5 deals with a brief discussion of the results.

2. CHIRAL $SU(4) \times SU(4)$ MESON-BARYON LAGRANGIAN

Let us consider the 15-plet of 0^--mesons and 20-plet of $1/2^+$-baryons of the group $SU(4)$ described by the fields $\xi_i = \Phi_i / F$ $(i = 1,2,\dots,15)$ and $B_c^{[a,b]}$ $(a, b, c = 1,2,\dots,4)$, respectively, F being a dimensional parameter (for the explicit expression of the hadron fields in terms of ξ_i, $B_c^{[a,b]}$ see Appendix A).

In the following we shall deal with the nonlinear field transformations given by /1,7/

$$g\, e^{i\xi\cdot A} = e^{i\xi'\cdot A}\, e^{iu'(\xi,g)\cdot V},$$

$$B'^{[m,n]}_{a} = (U^+)^m_i (U^+)^n_j U^k_a B^{[i,j]}_k ; \quad U = e^{iu'(\xi,g)\cdot V}, \tag{1}$$

where

$$\xi \cdot A = \sum_{i=1}^{15} \xi_i A_i , \quad u' \cdot V = \sum_{i=1}^{15} u'_i V_i .$$

Here g is an arbitrary element of the chiral group $SU(4) \times SU(4)$, $V_i = \frac{\lambda_i}{2} I$, $A_i = \frac{\lambda_i}{2}\gamma^5$ $(i=1,2,\dots,15)$ are the generators of its Lie algebra, and λ_i are the usual Gell-Mann matrices. A meson-baryon Lagrangian invariant with respect to the chiral transformations (1) can easily be constructed by using the Cartan forms /1/

$$D_\mu \xi_i = -\frac{i}{2} \mathrm{Sp}\{A_i\, e^{-i\xi\cdot A}\, \partial_\mu\, e^{i\xi\cdot A}\},$$

$$\theta^i_\mu(\xi) = -\frac{i}{2} \mathrm{Sp}\{V_i\, e^{-i\xi\cdot A} \partial_\mu\, e^{i\xi\cdot A}\}, \tag{2}$$

where the trace is taken over internal and Lorentz indices, and we used the normalization $\mathrm{Sp}\, A_i A_k = \mathrm{Sp}\, V_i V_k = 2\delta_{ik}$, $\mathrm{Sp}\, A_i V_k = 0$. The effective strong-interaction Lagrangian may be written as a sum of an $SU(4) \times SU(4)$ invariant part $\mathcal{L}_{inv}(D_\mu \xi; B, D_\mu B)$ and a symmetry breaking term $\Delta\mathcal{L}$

$$\mathcal{L} = \mathcal{L}_{inv}(D_\mu \xi, B, D_\mu B) + \Delta\mathcal{L}, \tag{3}$$

where /7/

$$\mathcal{L}_{inv}(D_\mu \xi; B, D_\mu B) = \frac{F^2}{2} D^\mu \xi_i D_\mu \xi_i + \bar{B}(i\gamma^\mu D_\mu - m_0)B -$$
$$- g_A[\alpha(\bar{B}\gamma^\mu D_\mu \xi_i A_i B)_d + (1-\alpha)(\bar{B}\gamma^\mu D_\mu \xi_i A_i B)_f] \tag{4}$$

(Sum over repeated indices is understood). Here $g_A \approx 1.25$ determines the renormalization of the axial vector coupling, m_0 is an averaged baryon mass. The subscripts d and f in expressions of the type $(\bar{B}A_i B)_{d,(f)}$ denote d or f couplings defined by

$$(\bar{B}A_i B)_{d,(f)} = \frac{1}{2} \bar{B}^a_{[m,n]} (A_i)^b_a B^{[m,n]}_b + (-)\bar{B}^m_{[b,n]} (A_i)^b_a B^{[a,n]}_m \tag{5}$$

and α is the mixing parameter of d–f couplings. Finally, the covariant derivative of baryon tensor is given by

$$\bar{B} i\gamma^\mu D_\mu B = \frac{1}{2} \bar{B}^a_{[m,n]} i\gamma^\mu \partial_\mu B^{[m,n]}_a - (\bar{B}\gamma^\mu V_i B)_f\, \theta^i_\mu(\xi), \tag{6}$$

$\theta^i_\mu(\xi)$ being the Cartan form introduced in eq. (2). The $SU(4) \times SU(4)$ breaking term $\Delta\mathcal{L}$ will be discussed in the subsequent section.

3. SU(4) × SU(4) BREAKING AND MASS FORMULAS

The chiral-invariant part of the Lagrangian (4) contains only massless (Goldstone) mesons and equal-mass baryons. In order to get massive mesons as well as baryons with physical masses we have yet to add a suitable symmetry breaking term $\Delta\mathcal{L}$. To this end, let us apply the symmetry breaking scheme of Oakes [5] by extending it from the group $SU(3)\times SU(3)$ to $SU(4)\times SU(4)$. The idea behind this approach is the following. The group $SU(4)\times SU(4)$ (or $SU(3)\times SU(3)$ [5,6]) is first broken down to $SU(2)\times SU(2)$. The breaking of $SU(2)\times SU(2)$ and thus a finite pion mass is then achieved by a mixing of the strange and nonstrange hadrons where the mixing angle turns out to be the Cabibbo angle.

a) mesons

Let us start with a general symmetry breaking term transforming as a $(4,4^*)+(4^*,4)$ representation of $SU(4)\times SU(4)$ $\left(V_0 = \frac{1}{2\sqrt{2}}\,\mathbf{I}\right)$

$$\Delta\mathcal{L}_m = \frac{\mu^2}{2\sqrt{8}} F^2\, \mathrm{Sp}\left[(V_0 + aV_8 + bV_{15})\, e^{i2\xi\cdot A}\right]. \tag{7}$$

In order to preserve the $SU(2)\times SU(2)$ symmetry, we express the parameters μ^2, a, b by the masses of the neutral K and D meson as follows:

$$\mu^2 = m^2_{K^\circ} + m^2_{D^\circ}, \qquad a = -\sqrt{\frac{8}{3}}\, m^2_{K^\circ}/\mu^2,$$

$$b = -(3m^2_{D^\circ} - m^2_{K^\circ})/\sqrt{3}\mu^2 .$$

Then

$$\Delta\mathcal{L}_m = \frac{F^2}{4}\,\mathrm{Sp}\begin{pmatrix} 0 & & & \\ & 0 & & \\ & & m^2_{K^\circ} & \\ & & & m^2_{D^\circ}\end{pmatrix} e^{i2\xi\cdot A} . \tag{7'}$$

Next, the manifest $SU(2)\times SU(2)$ invariance of (7′) will be broken by rotating the "bare" fields ξ_i by means of the unitary transformation $U_c = e^{i2\theta V_7}$. This yields

$$\Delta \mathcal{L}_m = \frac{F^2}{4} Sp\,[U_c^+ \begin{pmatrix} 0 & & \\ & 0 & & \\ & & m^2_{K^\circ} & \\ & & & m^2_{D^\circ} \end{pmatrix} U_c e^{i2\tilde{\xi}\cdot A}]_{\Delta S=0} =$$

$$= -\frac{F^2}{2} \{ m^2_{K^\circ} \sin^2\theta (\tilde{\xi}\cdot\lambda)^2_{22} + m^2_{K^\circ} \cos^2\theta (\tilde{\xi}\cdot\lambda)^2_{33} + \tag{8}$$

$$+ m^2_{D^\circ} (\tilde{\xi}\cdot\lambda)^2_{44} \} + O(\tilde{\xi}^4),$$

where $\tilde{\xi}_i$ are new "physical" meson fields defined by $\exp i2\tilde{\xi}\cdot A = U_c^+ \exp i2\xi\cdot A \;\, U_c$, and we have retained only the strangeness-conserving parts in (8). Then, the masses of the 15-plet of pseudoscalar mesons may be expressed by the masses of the K°, D° meson and the angle θ as follows

$$m^2_{\pi^+} = m^2_{K^\circ} \sin^2\theta, \qquad m^2_{K^+} = m^2_{K^\circ} \cos^2\theta,$$

$$m^2_{D^+} = m^2_{D^\circ} + m^2_{\pi^+}, \qquad m^2_{F^+} = m^2_{D^\circ} + m^2_{K^+}, \tag{9}$$

$$m^2_{\eta_8} = m^2_{K^\circ} (\frac{1}{3} + \cos^2\theta), \quad m^2_{\eta_{15}} = \frac{1}{6} m^2_{K^\circ} + \frac{3}{2} m^2_{D^\circ}.$$

In Table 1 we have listed the meson masses obtained from eq. (9) by using the values m_{K° =498 MeV, m_{D° =1863 MeV and $\theta \approx 0.26$. There is further a mixing term

$$-\frac{m^2_{\pi^+}}{\sqrt{3}} \pi^\circ (\eta_8 + \frac{\eta_{15}}{\sqrt{2}}) - \frac{(3m^2_{K^+} - m^2_{K^\circ})}{3\sqrt{2}} \eta_8 \eta_{15} \tag{10}$$

leading to $\pi^\circ - \eta_8 - \eta_{15}$ mixing. Instead of diagonalizing the full $\pi^\circ - \eta_8 - \eta_{15}$ mass matrix it suffices for our purposes to diagonalize the $\pi^\circ - \eta_8$ mixing only (the $\pi^\circ - \eta_{15}$ and $\eta_8 - \eta_{15}$ mixing terms yield very small corrections due to the large mass value m_{η_8} and $m_{\eta_{15}}$. Introducing the new fields $\pi^{\circ\prime}$, η_8'

Table 1

	m_{π°	m_{π^+}	m_{K^+}	m_{D^+}	m_{F^+}	m_{η_8}	$m_{\eta_{15}}$
(MeV)	127	128	481	1867	1924	559	2291
$\frac{m^{th}-m^{exp}}{m^{exp}}\cdot 100\%$	-6%	-8%	-2.6%	0%	-5.2%	1.8%	

$$\pi^\circ = \pi^{\circ\prime}\cos\phi + \eta_8'\sin\phi,$$

$$\eta_8 = -\pi^{\circ\prime}\sin\phi + \eta_8'\cos\phi \qquad (11)$$

we obtain $m_{\pi^{\circ\prime}} = 127$ MeV*, $m_{\eta_8'} = 559$ MeV, and

$$\mathrm{tg}\,2\phi = -\frac{2}{\sqrt{3}}\;\frac{m^2_{\pi^+}}{(m^2_{\eta_8} - m^2_{\pi^\circ})} = -0.064, \quad \phi \approx -0.03. \qquad (12)$$

The mass value $m_{\eta_{15}} \approx 2291$ MeV (cf. Tab. 1) is rather small when confronted with the mass around 2.8 GeV of a recently reported particle ψ_P which is considered to be a possible candidate of the η_c meson. The correct treatment of the masses of the η, η' and η_c mesons requires the additional inclusion of the SU(4) -singlet η°. A detailed study of this $\eta - \eta' - \eta_c$ puzzle can be found in the work of Kazi et al.[9] and in ref.[10]. It should be mentioned that the identification $\psi_P \equiv \eta_c$ is, however, not yet completely confirmed.

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*Note that the $\pi^+ - \pi^\circ$ mass splitting is determined almost completely by the electromagnetic interactions ($\Delta^{(elm)}_{m_\pi} \sim 5$ MeV, $\Delta^{(\Delta\mathcal{L})}_{m_\pi} \sim 1$ MeV). On the other hand, the $K^+ - K^\circ$ mass difference is dominated by $\Delta\mathcal{L}_m$ (see ref.[8] and the discussion in Sec. 5).

b) baryons

To get baryon mass splittings we choose the following symmetry breaking term which is close to eq. (8)

$$\Delta\mathcal{L}_b = \frac{1}{2}\bar{\tilde{B}}^{i}_{[m,n]}\begin{pmatrix}0 & & \\ & a\sin^2\theta & \\ & & a\cos^2\theta\end{pmatrix}_{b\,i}^{\;j}\tilde{B}^{[m,n]}_{j} + \tag{13}$$

$$+\ \bar{\tilde{B}}^{m}_{[i,n]}\begin{pmatrix}0 & & \\ & c\sin^2\theta & \\ & & c\cos^2\theta\end{pmatrix}_{d\,j}^{\;i}\tilde{B}^{[j,n]}_{m} .$$

Here $\tilde{B}^{[m,n]}_{\ell} = (U_c^+)^m_i\,(U_c^+)^n_j(U_c)^k_\ell\, B^{[ij]}_k$ are the "physical" baryon fields, U_c being the matrix of the Cabibbo rotation. In writing eq. (13) we have taken into account the two possible couplings (d, f) of the baryons. Eq. (13) yields the following mass formulas*

$$m_P = (m_0 - d) - c\cos^2\theta, \qquad m_{\Sigma^+} = (m_0 - d) - c\sin^2\theta,$$

$$m_N = m_P - a\sin^2\theta, \qquad m_{\Sigma^0} = (m_0 - d) - \frac{a+c}{2}\sin^2\theta$$

$$m_\Lambda = (m_0 - d) - \frac{a+c}{6}(1 + 3\cos^2\theta), \qquad m_{\Sigma^-} = (m_0 - d) - a\sin^2\theta,$$

$$m_{\Xi^0} = (m_0 - d) - c\sin^2\theta - a\cos^2\theta$$

$$m_{\Xi^-} = (m_0 - d) - a\cos^2\theta,$$

*Due to the asymmetric form of eq. (13) the parameter m_0 ceases having the meaning of an averaged baryon mass.

$$m_{A_1^+} = m_0 - \frac{2}{3}(b+d) - \frac{a+c}{6}\sin^2\theta - c\cos^2\theta,$$

$$m_{A_2^+} = m_0 - \frac{2}{3}(b+d) - \frac{a+c}{6}\cos^2\theta - c\sin^2\theta,$$

$$m_{A_2^\circ} = m_0 - \frac{2}{3}(b+d) - \frac{a+c}{6},$$

$$m_{B_3^{++}} = m_0 - c, \qquad m_{B_3^+} = m_0 - c - \frac{a-c}{2}\sin^2\theta,$$

$$m_{B_3^\circ} = m_0 - c\cos^2\theta - a\sin^2\theta,$$

$$m_{B_2^+} = m_0 - c - \frac{a-c}{2}\cos^2\theta, \qquad m_{B_2^\circ} = m_0 - \frac{a+c}{2},$$

$$m_{B_1^\circ} = m_0 - c\sin^2\theta - a\cos^2\theta,$$

$$m_{C_2^{++}} = m_0 - b - c, \quad m_{C_2^+} = m_0 - c\cos^2\theta - b,$$

$$m_{C_1^+} = m_0 - c\sin^2\theta - b. \tag{14}$$

Because $\Delta\mathcal{L}_b$ contains the mixing term

$$\frac{c+a}{2\sqrt{3}}\cos^2\theta\{\bar{B}_2^+A_2^+ + \bar{B}_2^\circ A_2^\circ + \text{h.c.}\} -$$

$$- \frac{c+a}{2\sqrt{3}}\sin^2\theta\{\bar{B}_2^\circ A_2^\circ + \bar{B}_3^+ A_1^+ + \bar{\Sigma}^\circ\Lambda + \text{h.c.}\}, \tag{15}$$

some of the states appearing in (14) have yet to be diagonalized.

Using the three relations following from eq. (14)

$$m_P + m_{\Sigma^+} = 2m - c, \quad m_{\Xi^-} + m_{\Sigma^-} = 2m - a, \quad m_{\Xi^\circ} + m_N = 2m - a - c,$$

where $m = m_0 - d$, as well as the mass formulas for the A_1^+ and B_3^{++} baryons with the recently obtained mass values $m_{A_1^+} =$ 2260 MeV, $m_{B_3^{++}} =$ 2426 MeV [11], we may fix the 5 parameters m_0, a, b, c and d as follows

Table 2

notations		SU(3)	(I, I_3)	S	C	mass(MeV)
P		8	$(\frac{1}{2}, \frac{1}{2})$	0	0	949
N		8	$(\frac{1}{2}, -\frac{1}{2})$	0	0	958
Λ		8	(0, 0)	-1	0	1109
Σ^+		8	(1, 1)	-1	0	1179
Σ^0		8	(1, 0)	-1	0	1191
Σ^-		8	(1, -1)	-1	0	1204
Ξ^0		8	$(\frac{1}{2}, \frac{1}{2})$	-2	0	1297
Ξ^-		8	$(\frac{1}{2}, -\frac{1}{2})$	-2	0	1315
A_1^+ [17]	C_0^+ [18]	$\bar{3}$	(0, 0)	0	1	2260
A_2^+	A^+	$\bar{3}$	$(\frac{1}{2}\ \ \frac{1}{2})$	-1	1	2460
A_2^0	A^0	$\bar{3}$	$(\frac{1}{2} - \frac{1}{2})$	-1	1	2476
B_3^{++}	C_1^{++}	6	(1, 1)	0	1	2426
B_3^+	C_1^+	6	(1, 0)	0	1	2439
B_3^0	C_1^0	6	(1, -1)	0	1	2452
B_2^+	S^+	6	$(\frac{1}{2}, \frac{1}{2})$	-1	1	2618
B_2^0	S^0	6	$(\frac{1}{2}, -\frac{1}{2})$	-1	1	2629
B_1^0	T^0	6	(0, 0)	-2	1	2791
C_2^{++}	X_u^{++}	3	$(\frac{1}{2}, \frac{1}{2})$	0	2	3647
C_2^+	X_d^+	3	$(\frac{1}{2}, -\frac{1}{2})$	0	2	3664
C_1^+	X_s^+	3	(0, 0)	-1	2	3894

$$a = -127 \text{ MeV}, \quad b = -1221 \text{ MeV}$$
$$c = 264 \text{ MeV}, \quad d = 1494 \text{ MeV} \qquad (16)$$
$$m_0 = 2690 \text{ MeV}.$$

Table 2 contains the masses of the 20-plet of $1/2^+$-baryons obtained from eqs. (14), (16) after having performed the necessary diagonalizations.

4. CURRENTS, PCAC AND THE CABIBBO ANGLE

Let us now show that the angle θ appearing in the mass terms of the effective strong interaction Lagrangian coincides with the Cabibbo angle of the charged weak hadronic current. To this end, we shall first derive the expression of the charged weak current associated with the group $G_w = SU(2)_L \times U(1)$ of the standard Weinberg-Salam model /12/. We use the following 4x4 matrix representation of the generators of G_w

$$SU(2)_L: \quad \hat{G}_i = \frac{1+\gamma^5}{2}\frac{G_i}{2}, \quad G_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_1\sigma_i\sigma_1^{-1} \end{pmatrix} \quad (i = 1,2,3)$$
$$U(1): \quad \frac{\hat{Y}}{2} = \frac{Y_w}{2} + \frac{1-\gamma^5}{2}\frac{G_3}{2}, \qquad (17)$$

where σ_i are usual Pauli matrices and Y_w is the weak hypercharge.*

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* The matrix σ_1 in G_i is required by our choice of SU(4) λ-matrices corresponding to a quark spinor $q = \begin{pmatrix} \binom{u}{d} \\ \binom{s}{c} \end{pmatrix}$, but not $\begin{pmatrix} \binom{u}{d} \\ \binom{c}{s} \end{pmatrix}$ (Comp. also eq. (23)).

We now consider field variations generated by a group transformation $h \in G_w$.* The currents and their divergencies can be then calculated from the corresponding variation $\delta\mathcal{L}$ of Lagrangian (3) by standard methods [18]. The relevant field variations for deriving the charged weak current and the PCAC condition are $(\epsilon_+ = \frac{1}{\sqrt{2}}(\epsilon_1 - i\epsilon_2))$

$$\delta_+ e^{i\tilde{\xi}\cdot A} = \frac{i}{\sqrt{2}} \epsilon_+ \frac{1+\gamma^5}{2} G_+(\theta) e^{i\tilde{\xi}\cdot A}, \tag{18}$$

$$\delta_+ \vec{B} \sim 0,$$

where

$$G_+(\theta) = U_c^+ G_+ U_c, \quad G_\pm = \frac{1}{2}(G_1 \pm iG_2).$$

The sign $\sim$ in eq. (18) means "up to terms of order $O(u' \cdot V)$". Such terms can be dropped because they either cancel each other in $\delta\mathcal{L}_{inv}$ or contribute only to the divergence of the vector current in $\delta\Delta\mathcal{L}_b$.

In order to obtain $\delta\Delta\mathcal{L}_m$ it is more convenient to use the transformation law for the square of the meson matrix $\exp i2\xi\cdot A$ [1]

$$e^{i2\xi'\cdot A} = h\, e^{i2\xi\cdot A}\, \bar{h}^{-1}, \quad \bar{h} = h(\gamma^5 \to -\gamma^5), \tag{19}$$

In terms of physical fields, the infinitesimal version of eq. (19) reads

$$\delta_+ e^{i2\tilde{\xi}\cdot A} = \frac{i}{2\sqrt{2}} \epsilon_+ ([G_+(\theta), e^{i2\tilde{\xi}\cdot A}]_- + \gamma^5 \{G_+(\theta), e^{i2\tilde{\xi}\cdot A}\}_+). \tag{20}$$

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* Since the "electro-weak" group G_w may be embedded into the "strong" group $SU(4) \times SU(4)$ (the unit matrix in $\hat{Y}/2$ yields an irrelevant phase factor), the hadron fields transform nonlinearly under G_w. The transformation law has the form of eq. (1), with g being replaced by $h = \exp i(\epsilon_i \hat{G}_i + \eta \hat{Y}/2) \in G_w$.

The charged weak current now follows from the definition

$$J_\mu^{ch} = -\frac{\delta \mathcal{L}}{\delta \partial_\mu \epsilon_+ / 2\sqrt{2}} = -\frac{\delta \mathcal{L}_{inv}}{\delta \partial_\mu \epsilon_+ / 2\sqrt{2}} \tag{21}$$

and eqs. (4.18). We have

$$\begin{aligned} J_\mu^{ch} = & -F^2 D_\mu \tilde{\xi}_i \, Sp(\hat{G}_+(\theta) e^{i\tilde{\xi}\cdot A} A_i e^{-i\tilde{\xi}\cdot A}) + \\ & + (\bar{\tilde{B}} \gamma_\mu V_i \tilde{B})_f \, Sp(\hat{G}_+(\theta) e^{i\tilde{\xi}\cdot A} V_i e^{-i\tilde{\xi}\cdot A}) + \\ & + g_A [\alpha (\bar{\tilde{B}} \gamma_\mu A_i \tilde{B})_d + (1-\alpha)(\bar{\tilde{B}} \gamma_\mu A_i \tilde{B})_f] \times \\ & \times Sp(\hat{G}_+(\theta) e^{i\tilde{\xi}\cdot A} A_i e^{-i\tilde{\xi}\cdot A}). \end{aligned} \tag{22}$$

Furthermore, by using the decomposition

$$\begin{aligned} \hat{G}_+(\theta) = & \frac{1}{2}\cos\theta [(V^{1+i2} + A^{1+i2}) + (V^{13-i14} + A^{13-i14})] + \\ & + \frac{1}{2}\sin\theta [(V^{4+i5} + A^{4+i5}) - (V^{11-i12} + A^{11-i12})], \end{aligned} \tag{23}$$

where

$$V^{k\pm i\ell} = V^k \pm iV^\ell, \quad A^{k\pm i\ell} = A^k \pm iA^\ell$$

the current (22) can easily be shown to possess the generalized Cabibbo structure of the GIM-scheme /14/

$$J_\mu^{ch} = \cos\theta [J_\mu^{1+i2} + J_\mu^{13-i14}] + \sin\theta [J_\mu^{4+i5} - J_\mu^{11-i12}]. \tag{24}$$

From eq. (24) we see that the angle θ originally introduced in order to break chiral $SU(2)\times SU(2)$ really coincides with the Cabibbo angle of the weak current.

Finally, taking into account the prescription

$$\partial^{\mu} J_{\mu}^{ch} = -\frac{\delta \mathcal{L}}{\delta \epsilon_{+}/2\sqrt{2}} = -\frac{\delta \Delta \mathcal{L}}{\delta \epsilon_{+}/2\sqrt{2}} \qquad (25)$$

and eqs. (8), (13) and (19) we obtain the PCAC condition

$$\partial^{\mu} J_{\mu,ax}^{ch} = -i \frac{F^{2}}{4} \mathrm{Sp}\, [\begin{pmatrix} 0 & & & \\ & m_{\pi^{+}}^{2} & & \\ & & m_{K^{+}}^{2} & \\ & & & m_{D^{\circ}}^{2} \end{pmatrix} ([G_{+}(\theta), e^{i2\tilde{\xi}\cdot A}]_{-} +$$

$$+ \gamma^{5}\{G_{+}(\theta), e^{i2\tilde{\xi}\cdot A}\}_{+})] \qquad (26)$$

or

$$\partial^{\mu} J_{\mu,ax}^{ch} = \sqrt{2} F\, [\cos\theta (m_{\pi^{-}}^{2}\, \pi^{-} + m_{F^{-}}^{2} F^{-}) +$$

$$+ \sin\theta (m_{K^{-}}^{2} K^{-} - m_{D^{-}}^{2} D^{-})] + O(\tilde{\xi}^{3}).$$

In a similar way one can calculate the remaining weak neutral and the electromagnetic currents of the group $G_{W} = SU(2)_{L} \times U(1)$. In particular, by considering local gauge transformations $h(x) \in G_{W}$ one may construct a nonlinear realization of the Weinberg-Salam model in terms of hadron fields /7/.

5. SUMMARY AND DISCUSSIONS

There exists an extensive literature on symmetry breaking schemes for the group $SU(4) \times SU(4)$ /4/ or $SU(4)$ /9,15/ and their respective mass formulas. In this paper, we have dealt with a special $SU(4) \times SU(4)$ - breaking scheme which generalizes the $SU(3) \times SU(3)$ -breaking scheme proposed by Oakes /5/. As is mentioned in the introduction, this scheme enables one to understand from a unique point of view the appearance of a finite pion mass, the mass splittings in isotopic multiplets, and the nonconservation of strangeness in weak interactions. All these effects turn out to be closely related to the Cabibbo angle.

The original work of Oakes contains also some imperfections, taken over into our approach: This concerns first of all the adhoc rejection of a term $\sim V_6$ in eq. (8) in order to avoid strong interaction transitions with $\Delta S \neq 0$. Furthermore, the resulting nonelectromagnetic mass splittings are in general too large (with the exception of the $\pi^{\pm}\pi^{\circ}$ mass difference), although their signs are correct.

On the other hand, it is worth remarking that pure electromagnetic corrections are not able to reproduce even the correct sign of the mass splittings /8/.* If we combine the mass splittings of Tabl. 1 with the electromagnetic corrections of ref. /8/ the $\pi^{\pm}\pi^{\circ}$ mass difference turns out to be almost completely of electromagnetic nature ($\Delta^{(\Delta\mathcal{P})}_{m_\pi} \approx 1$ MeV, $\Delta^{(elm)}_{m_\pi} \approx 5$ MeV). Moreover, the other too large mass differences are reduced, although not yet sufficiently.

It is not difficult to see that the nonelectromagnetic mass splittings can, in principle, be further reduced by introducing additional parameters into the above scheme of symmetry breaking. This goes, however, beyond the scope of our paper.

APPENDIX A

Let us quote, for completeness, the explicit expressions for the 4x4 meson matrix $P = \sqrt{2}\,F \sum_{i=1}^{15} V_i \xi_i$ and the tensor representation of the baryon wave functions. We have

$$P = \begin{pmatrix} \frac{\pi^\circ}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_{15}}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^\circ \\ \pi^- & -\frac{\pi^\circ}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_{15}}{\sqrt{12}} & K^\circ & D^- \\ K^- & \bar{K}^\circ & -\frac{2}{\sqrt{6}}\eta_8 + \frac{\eta_{15}}{\sqrt{12}} & F^- \\ D^\circ & D^+ & F^+ & -\frac{\sqrt{3}}{2}\eta_{15} \end{pmatrix}$$

* This discussion does not concern models with sophisticated tadpole terms related to the exchange of hypothetical scalar mesons /16/.

where $\pi^{+} = (\Phi_1 - i\Phi_2)/\sqrt{2}$, etc., and [17]

$$B_1^{[12]} = \frac{\overset{+}{A}_2}{\sqrt{6}} - \frac{\overset{+}{B}_2}{\sqrt{2}}, \qquad B_1^{[13]} = \frac{\overset{+}{A}_1}{\sqrt{6}} + \frac{\overset{+}{B}_3}{\sqrt{2}}, \qquad B_1^{[14]} = \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^{\circ}}{\sqrt{2}},$$

$$B_1^{[23]} = B_3^{++}, \qquad B_1^{[24]} = \Sigma^{+}, \qquad B_1^{[34]} = P,$$

$$B_2^{[12]} = \frac{A_2^{\circ}}{\sqrt{6}} - \frac{B_2^{\circ}}{\sqrt{2}}, \qquad B_2^{[13]} = B_3^{\circ} \qquad B_2^{[14]} = \Sigma^{-},$$

$$B_2^{[23]} = \frac{\overset{+}{A}_1}{\sqrt{6}} - \frac{\overset{+}{B}_3}{\sqrt{2}}, \qquad B_2^{[24]} = \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^{\circ}}{\sqrt{2}}, \qquad B_2^{[34]} = N,$$

$$B_3^{[12]} = -B_1^{\circ}, \qquad B_3^{[13]} = \frac{A_2^{\circ}}{\sqrt{6}} + \frac{B_2^{\circ}}{\sqrt{2}}, \qquad B_3^{[14]} = -\Xi^{-},$$

$$B_3^{[23]} = \left(\frac{\overset{+}{A}_2}{\sqrt{6}} + \frac{\overset{+}{B}_2}{\sqrt{2}}\right), \qquad B_3^{[24]} = \Xi^{\circ}, \qquad B_3^{[34]} = -\sqrt{\frac{2}{3}}\Lambda,$$

$$B_4^{[12]} = -C_1^{+}, \qquad B_4^{[13]} = C_2^{+}, \qquad B_4^{[14]} = -\sqrt{\frac{2}{3}}A_2^{\circ},$$

$$B_4^{[23]} = -C_2^{++}, \qquad B_4^{[24]} = \sqrt{\frac{2}{3}}\overset{+}{A}_2, \qquad B_4^{[34]} = \sqrt{\frac{2}{3}}\overset{+}{A}_1 .$$

$$(B_i^{[ik]} = 0, \ (k = 1,2,..,4)) .$$

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