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Факторизация и формфактор пиона в квантовой хромодинамике

В рамках квантовой хромодинамики установлено, что асимптотика формфактора пиона, трактуемого как связанное состояние кварка и антикварка, определяется взаимодействием кварков на малых расстояниях. Выведена формула, в которой асимптотика формфактора пиона выражается через фундаментальные константы теории.

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Factorization and Pion Form Factor in QCD

In the framework of quantum chromodynamics it is established that the asymptotical behaviour of electromagnetic form factor of pion treated as a quark-antiquark bound state is controlled by the short-distance properties of the theory. The formula is derived which expresses the asymptotical behaviour of the pion form factor in terms of fundamental constants of the theory.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Introduction

It is widely hoped now that quantum chromodynamics (QCD) is a true basis for the theory of strong interactions. A very attractive feature of QCD is its asymptotic freedom /1/ which justifies the use of perturbation theory at short distances, i.e., at large space-like momenta $q^2=-Q^2$. However, in any physical process small momentum scales p^2 related to initial and/or final states are also involved. As a result, there appear effects due to long-distance interactions which can invalidate a straightforward application of perturbation theory.

But as it has been shown recently /2-6/, the short- and long-distance effects can be separated from each other for hard (i.e., involving high momentum transfer q) inclusive processes, such as deep inelastic scattering, massive lepton-pair production, etc. The key issue is the factorization of the relevant cross section

$$d\sigma(Q^2,p^2) = d\sigma_{sd}(Q^2,\mu^2) \otimes f_{ld}(\mu^2,p^2) + R(Q^2,p^2) \, , \quad (1)$$

where $d\sigma_{sd}$ describes a short-distance subprocess and f_{ld} accumulates information about the long-distance interactions, the parameter $1/\mu$ being the boundary between "short" and "long" distances. The regular term $R(Q^2,p^2)$ which, in general, does not factorize, is damped by powers of $Q^{-n}, n\geq 2$ as compared to $d\sigma_{sd}$ and may be neglected in the high-Q^2 limit. The short-distance cross section $d\sigma_{sd}$

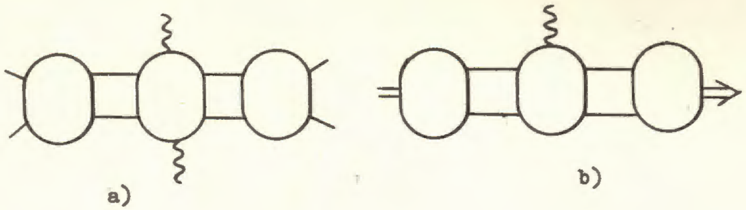

Fig.1

can be calculated in QCD perturbation theory whereas the long-distance factors $f_{\ell d}$ (i.e., parton densities) must be inferred from experiment.

All attempts to prove the factorization for hard inclusive processes start with analysis of the corresponding forward amplitude. See fig. 1a, where that for massive lepton-pair production is shown. Note a similarity between figs. 1a and 1b, the latter describing the EM form factor of pion treated as a bound state in the $q\bar{q}$-system.

The natural question is whether the high-Q^2 behaviour of the pion EM form factor can be calculated in QCD just in the same way as the cross-section of massive lepton-pair production.

Our main goal here is to argue that the answer is yes/7/. In particular, we show that at very large Q^2 the pion EM form factor can be expressed in terms of the fundamental constants of the theory:/8/

$$F_{(\pi)}^{(as)}(Q) = 8\pi\alpha_s(Q)\,\frac{f_\pi^2}{Q^2}\,, \tag{2}$$

where $\alpha_s(Q)$ is the running QCD coupling constant and $f_\pi =$ 133 MeV is the pion decay constant . In this formula the factor $8\pi\alpha_s/Q^2$ is due to the short-distance interaction whereas f_π^2 absorbes all the long-distance effects.

2. Technique of Factorization

To illustrate the problems one is faced with trying to apply the ordinary perturbation theory for amplitudes involving both large and small momentum invariants, let us consider the forward Compton amplitude $T(q,p)$, where p is the quark momentum, $p^2<0$, and q - that of the virtual photon $-q^2 = Q^2 \gg |p^2|$. To simplify the study, we treat quarks as massless.

A straightforward calculation shows that in higher orders there appear logarithmical factors $\left[g^2(\ln q^2/p^2)\right]^N$ which spoil the perturbation expansion in the high-Q^2 region. These logarithms (which "connect" short ($\sim 1/Q$) distances with a very long ($\sim 1/p$) ones) indicate that the initial-state interactions must be taken into account.

The progress in solving this problem is based essentially on a simple trick: one divides $\ln q^2/p^2$ into short- and long-distance parts

$$\ln q^2/p^2 = \ln Q^2/\mu^2 + \ln \mu^2/|p^2|\,, \tag{3}$$

$1/\mu$ by definition being the boundary between short and long distances. It can be proved then that the $\ln Q^2/\mu^2$ and $\ln\mu^2/p^2$ - terms factorize:

$$T(q,p) = t\left(\ln Q^2/\mu^2\right) \otimes f\left(\ln |p^2|/\mu^2\right) + O(1/Q^2). \tag{4}$$

In our proof of the factorization property/4/ we have utilized the α-parametric representation of the Feynman diagrams based on the following formula for a propagator

$$(m^2-p^2)^{-1} = i\int_0^\infty d\alpha \ \exp i\alpha(p^2-m^2). \tag{5}$$

The parameter α_σ may be considered as a measure of virtuality of a momentum flowing through the σ-line: $\alpha \sim 1/p^2$.

Applying eq. (5) to all the propagators and taking Feynman integrals over d^4p (which are Gaussian in this case) one obtains the α-representation for a particular Feynman diagram in terms of the α_σ-parameters related to each line σ of the diagram

$$T(\{p_i\},m) \sim \int_0^\infty \prod_\sigma d\alpha_\sigma\, D^{-2}(\alpha_\sigma)\, G(\alpha,\{p_i\},m)\,.$$
$$\exp i\left\{\sum A_{i_1\ldots i_n}(\alpha)\,(p_{i_1}+\ldots+p_{i_n})^2/D(\alpha) - \sum_\sigma \alpha_\sigma m_\sigma^2\right\}, \tag{6}$$

where the functions D, A are universal for all theories (they depend only on topology of the diagram), whereas $G(\alpha,\{p_i\},m)$ depends also on the spinor structure of the diagram.

Using invariant momentum variables one can rewrite the exponential factor for the kinematic situation we are interested in as

$$\exp i\left\{\frac{Q^2 A(\alpha,\omega_k)}{D(\alpha)} + p^2\,\frac{I(\alpha)}{D(\alpha)}\right\}, \tag{7}$$

where ω_k are ratios of large invariants, e.g. $\omega = 2(Pq)/Q^2$ in deep inelastic scattering.

Sometimes it is worth introducing dimensionless $\tilde{\alpha}$ -parameters $\tilde{\alpha} = \alpha p^2$. Then eq. (7) is

$$\exp i \left\{ \frac{Q^2}{p^2} \frac{A(\tilde{\alpha}, \omega_k)}{D(\tilde{\alpha})} + \frac{I(\tilde{\alpha})}{D(\tilde{\alpha})} \right\}. \qquad (8)$$

From eq.(8) it follows that the leading large- Q^2 contribution is dominated by integration over a region of the $\tilde{\alpha}$ -space, where $A(\tilde{\alpha})/D(\tilde{\alpha}) \sim p^2/Q^2 \to 0$.

If this region gives a logarithmic contribution ($\ln q^2/p^2)^N$, then it is responsible also for a mass singularity at $p^2 = 0$. Hence, to find the logarithms, one can study the singularities of the amplitude $T(q,p)$ as $p^2 \to 0$. However this "mass singularity" (MS) approach has a disadvantage in that it tells nothing about terms with $N=0$ (i.e., constants) which are equally important. These terms are simply assumed to be given by the parton model approximation. In other words, in the MS-approach one must take the parton model as a starting point. But if our goal is to derive the parton picture from QCD, we must forget the very existence of the parton model. That is why we prefer to study large-Q^2 behaviour of the amplitude $T(q,p)$ rather than their singularities at small p^2.

There exist three main possibilities to get $A/D=0$: i) short distance regime: $A/D=0$ at the origin of some set of α -parameters $\alpha_{\sigma_1}, \dots \alpha_{\sigma_n}$; ii) infrared regime, $A/D=0$ when some α -parameters are infinite; iii) pinch regime: $A/D=0$ at nonzero but finite α -parameters.

In the momentum representation only the first regime corresponds to integration over the region where the QCD running coupling constant is small. So, only when the first regime dominates, one may hope to justify the use of the QCD perturbation theory.

It can be shown /4/ that the SD regime is the only way to get $A/D=0$ for $T(q,p)$ in the region $|\omega|<1$.

The simultaneous vanishing of the α -parameters is conveniently described by the scaling

$$\lambda(V) = \alpha_{\sigma_1} + \dots + \alpha_{\sigma_n}; \quad \alpha_{\sigma_i} = \lambda(V)\beta_{\sigma_i}, \quad \sum_i \beta_{\sigma_i} = 1, \qquad (9)$$

where V is a subgraph composed by lines $\sigma_1, \dots, \sigma_n$. At small λ one can write

$$Q^2 A/D = Q^2\left[\lambda(V)\, a(\beta)/d(\beta) + O(\lambda^2)\right], \qquad (10)$$

and the main contribution in the large- Q^2 limit is given by the region $\lambda(V) \sim 1/Q^2$. In other words, all momenta k_i corresponding to lines of the subgraph V are highly virtual $k_i^2 \sim Q^2$. This allows one to use the dimensional counting to estimate the large - Q^2 contribution of a particular subgraph V . If all coupling constants are dimensionless, then

$$T^{(V)}_{SD}(q,p) \sim Q^{4-\sum d_i + \sum s_i}, \qquad (11)$$

where the summation goes over external lines of the subgraph V. The factor $Q^{4-\sum d_i}$ is the dimension of the subgraph V , whereas $Q^{\sum s_i}$ is due to the fact that a vector line ($s_i = 1$) adds the factor p_μ which can combine with some q_μ-factor to give $(qp) \sim Q^2$, hence p_μ must be estimated as Q . The same factor p_μ may be added by two spinor lines (each having $s_i = 1/2$) through the $(m + p_\mu \gamma^\mu)$ factor. We recall that, by definition, twist t_i is dimension (in mass units) minus spin: $t_i = d_i - s_i$. To make use of eq. (11), one must note that fields with $s_i = 0, 1/2$ have $t_i = 1$. That means the leading contribution is due to subgraphs with minimal number of external lines related to these fields. Vector fields have zero twist, and the number of external vector lines does not affect the large- Q^2 behaviour.

There exists a simple rule which facilitates the search of subgraphs responsible for the SD-regime. By assumption $A/D=0$ at $\lambda(V)=0$, i.e, at $\alpha_{\sigma_1} = \dots = \alpha_{\sigma_n} = 0$. The vanishing of the α_σ-parameter means topologically the contraction of the corresponding line σ into point. Hence the subgraph V must possess the property that after contraction of V into point the diagram is independent of large momentum invariants (proportional to Q^2). Subgraphs possessing this property are called usually Q^2-subgraphs.

For deep inelastic scattering Q^2-subgraphs are those containing the photon vertices. This corresponds to the well-known fact that the large- Q^2 behaviour of $T(q,p)$ is controlled by short distances between the photon vertices. Let us concentrate first

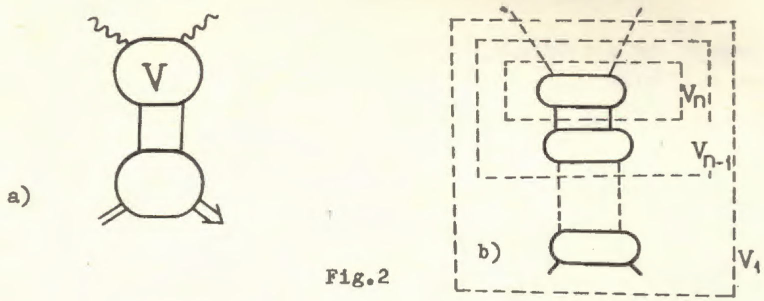

Fig.2

on a theory without vector particles, say, on the Yukawa γ_5-theory. In this case the Q^2-subgraphs responsible for a leading contribution are those having 4 external lines (see fig. 2a). For any given diagram these subgraphs can be ordered as in fig. 2b

$$V_1 \supset V_2 \supset \ldots \supset V_n . \qquad (12)$$

Each leading contribution may be characterized then by the largest Q^2-subgraph V_i for which $\lambda(V_i)$ is small, i.e., the leading contribution is given by the sum (fig. 3)

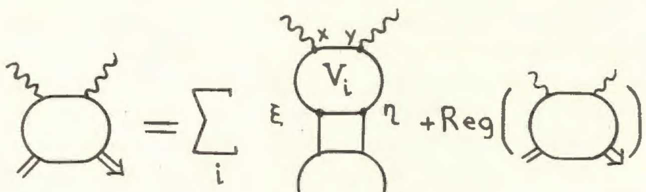

Fig.3

$$T = T^{(lead)} + R \;\; ; \;\; T^{(lead)} = \sum_{i=1}^{n} T_{SD}^{(V_i)} , \qquad (13)$$

where $T_{SD}^{(V_i)}$ corresponds to integration over the region in the α-space where $\lambda(V_i)$ is small but all λ's - related to larger leading Q^2-subgraphs $V_{i-1},\ldots,V_1$ are large. To be quantitative we define that λ is small if $\lambda < 1/\mu^2$ and large otherwise, where μ is an arbitrary scale of order Q. Then $T_{SD}^{(V_i)}$ is due to integration over the region in the α-space where $\lambda(V_i)<1/\mu^2$, but $\lambda(V_j \setminus V_i) > 1/\mu^2$ for all $j<i$. In this sense the subgraph V_i

corresponds to short distances whereas the subgraph $V_1 \setminus V_i$ outside V_i - to long ones.

The remaining term R corresponds to integration over the region where all $\lambda(V_i)$ are large $i=1,\ldots,n$ and gives only a non-leading contribution.

As we have discussed above, the question of a primary importance is whether $T^{(lead)}$ factorizes into short-distance and long-distance parts. The factorization is most easily seen in the coordinate representation where eq. (13) can be written as (see fig. 3)

$$T_{(n)}(x,y;p) = \sum_i \iint d^4\xi\, d^4\eta\; C(V_i; x,y;\xi,\eta;\mu^2)\cdot f(V_1\setminus V_i; \xi,\eta; p; \mu^2) + R_{(n)}(x,y;p) , \qquad (14)$$

where the function $C(\ldots\mu^2)$ has an infrared regularization specified by the parameter μ (e.g., a cut-off $\lambda(V_i)<1/\mu^2$ in the α-representation), whereas the function $f(..\mu^2)$ is regularized in the ultraviolet region (e.g. $\lambda(V_j \setminus V_i) > 1/\mu^2$ for $j<i$).

Note, that both subgraphs V_1 and $V_1\setminus V_i$ do not diverge as a whole in the UV-region, since they have 4 external lines with 2 spinor ones among them. Hence the standard R-operation factorizes $R(V_1) = R(V_i)R(V_1\setminus V_i)$ and does not affect the structure of eq. (14).

If the subgraph V_i has quark external lines, then one must use the Fierz identity

$$\delta^{\alpha}_{\alpha'}\,\delta^{\beta'}_{\beta} = \sum_{a=S,V,T,A,P} (\Gamma_a)^{\alpha}_{\beta}\,(\Gamma^a)^{\beta'}_{\alpha'} \qquad (15)$$

to factorize the spinor structure into V_i - and $V_1\setminus V_i$-parts.

Summing over all relevant diagrams one obtains

$$T(x,y;p) = \int d^4\xi\, d^4\eta \sum_k C_k(x,y;\xi,\eta;\mu^2)\, f_k(\xi,\eta;p;\mu^2) + R(x,y;p), \qquad (16)$$

where k numerates different intermediate 2-particle states, and C_k, f_k are the appropriately regularized Green functions

$$C_k(x,y;\xi,\eta;\mu^2) = \mathrm{Reg}^{IR}_{\mu^2} \langle 0| S^+ T(J(x)J(y)j_k(\xi)j_k(\eta)S)|0\rangle \qquad (17)$$

$$f_k(\xi,\eta;p;\mu^2) = \mathrm{Reg}^{UV}_{\mu^2} \langle p| S^+ T(\varphi_k(\xi)\varphi_k(\eta)S)|p\rangle ,$$

φ_k being the Lagrangian fields and j_k - the corresponding currents. Note that C_k is an amputated Green function, i.e., its legs are added to the f_k -function.

The product $\varphi_k(\xi)\varphi_k(\eta)$ with account of the UV-regularization procedure is a bilocal operator

$$\vartheta_k(\xi,\eta;\mu^2) = \mathrm{Reg}^{UV}_{d\mu^2}\left(\varphi_k(\xi)\,\varphi_k(\eta)\right) \tag{18}$$

which is well-defined in the following sense. If one expands $\varphi(\xi)\varphi(\eta)$ into the Taylor series

$$\varphi_k(\xi)\varphi_k(\eta) = \sum_n \frac{(\xi-\eta)^{\mu_1}\ldots(\xi-\eta)^{\mu_n}}{n!}\,\varphi_k\left(\tfrac{\xi+\eta}{2}\right)\overleftrightarrow{\partial}_{\mu_1}\ldots\overleftrightarrow{\partial}_{\mu_n}\varphi_k\left(\tfrac{\xi+\eta}{2}\right), \tag{19}$$

then there appear new vertices $\varphi\overleftrightarrow{\partial}^n\varphi$ producing divergences which are not removed by the ordinary R -operation, and one must define an additional renormalization recipe for these operators to obtain a meaningful expression. The $\mathrm{Reg}^{UV}_{d\mu^2}$ — prescription provides in fact such a recipe, because it means that $\lambda(v)>1/\mu^2$ just for those subgraphs which give rise to the UV -divergences related to the new vertices.

Note that eq. (16) is nothing else but the operator product expansion on the light cone:

$$T\left(J(x)J(y)\right) = \sum_k \int d^4\xi\, d^4\eta\; \vartheta_k(\xi,\eta;\mu^2)\,C_k(x-y,\xi,\eta;\mu^2) + R(x,y). \tag{20}$$

To obtain the standard expansion of $T(q,p)$ over the matrix elements of local operators, one must use eq. (19) and then re-expand the $\varphi\overleftrightarrow{\partial}^n\varphi$ -operators over the traceless symmetric ones, i.e., over operators having definite Lorentz spin and twist, and then integrate in eq. (16) over ξ,η . Only the operators having lowest twist give a leading contribution whereas the higher twist operators give $O(1/Q^2)$ contributions which must be added to R .

The lowest-twist contribution may be written in the form suggested by the parton model. Parton densities $f(\beta,\mu^2)$ are defined by

$$\langle P|\varphi_k\{\partial_{\mu_1}\ldots\partial_{\mu_n}\}\varphi_k|P\rangle = \{P_{\mu_1}\ldots P_{\mu_n}\}\int_0^1 \frac{d\beta}{\beta}\,\beta^n f_k(\beta,\mu^2), \tag{21}$$

where $\{\ \}$ denotes traceless symmetric part of a tensor. Substituting eqs. (19), (21) into eq. (16) we obtain

$$T(q,p) = \int_0^1 \frac{d\beta}{\beta}\sum_k t_k(\beta p,q;\mu^2)\,f_k(\beta,\mu^2) + R(q,p), \tag{22}$$

where $t(\beta p,q;\mu^2)$ is the IR -regularized amplitude constructed according to ordinary rules of perturbation theory

$$t_k(\beta p,q;\mu^2) = \int d^4\xi\, d^4\eta\, d^4x\; e^{i\beta p(\xi-\eta)+iqx}\,C_k(x,\xi,\eta;\mu^2). \tag{23}$$

One may worry about the convergence of the integral in eq. (22) at $\beta=0$, because the parton densities are known to behave like $1/\beta$ for $\beta\to 0$. But a more careful analysis (cf., e.g., /2/) shows that $t(\beta p,q;\mu^2)$ behaves like β^2 as $\beta\to 0$:

$$t(\beta p,q) = \sum_{n=2}^{\infty}\beta^n\,\frac{q^{\mu_1}\ldots q^{\mu_n}}{Q^{2n}}\,\{P_{\mu_1}\ldots P_{\mu_n}\}\,E_n(Q^2/\mu^2,g) \tag{24}$$

so really there are no problems with the "wee" partons, and the hard scattering formula eq. (22) works just for the forward Compton amplitude, not only for its discontinuity, i.e., the structure functions of deep inelastic scattering

$$W(q,p) = \int_{1/\omega}^1 \frac{d\beta}{\beta}\sum_k w_k(\beta p,q;\mu^2)\,f_k(\beta;\mu^2) + R(q,p), \tag{25}$$

where the integration over β is bounded away from zero by spectral properties of w_k which is nonzero only if $2\beta(pq)\geq Q^2$, i.e., for $\beta\geq 1/\omega$.

In a gauge theory a leading Q^2 -subgraph may possess an arbitrary number of external vector lines, i.e., one must sum over the gluons taking part in parton subprocess (fig. 4a). Every gluon line adds the field $A^a_\mu(z)$ into the matrix element related to the f -function and modifies some propagator $S^c(x_\alpha - x_\beta)$ related to the C -function

$$S^c(x_\alpha - x_\beta) \to g\int d^4z\; S^c(x_\alpha - z)\,\gamma^\mu \tau_a\, S^c(z - x_\beta), \tag{26}$$

where τ_a is a matrix of the gauge group in the fundamental (quark) representation. The sum over gluon lines inserted into the (x_α,x_β)-line (fig. 4b) gives $\mathcal{S}^c$ - the propagator of a spinor particle in an external gluonic field, i.e., the perturbative solution to the equation

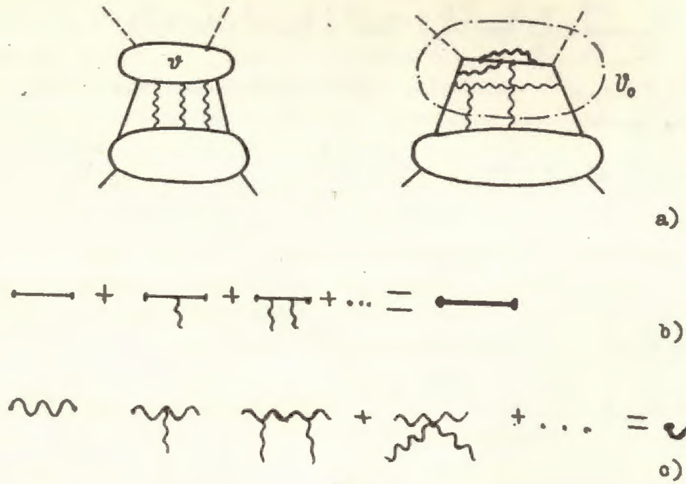

Fig.4

$$(i\,\partial/\partial x_\alpha^\mu + g\hat{A}_\mu)\gamma^\mu S^c(x_\alpha, x_\beta) = -\delta^4(x_\alpha - x_\beta), \tag{27}$$

where $(\hat{A}_\mu)_{AB} = A_\mu^a(\tau_a)_{AB}$. The solution of eq. (27) can be written as

$$S^c_{AB}(x_\alpha, x_\beta; A) = \hat{E}_{AB}(x_\alpha, x_\beta; A)\left\{S^c(x_\alpha - x_\beta) + R(x_\alpha, x_\beta)\right\}. \tag{28}$$

We use the short-hand notation

$$\hat{E}(x, y; A) = P\exp\left(ig\int_y^x \hat{A}_\mu(z)\,dz^\mu\right), \tag{29}$$

where P means that the integral is path-ordered along the integration contour which is the straight line connecting x and y. The function $R(x,y;G)$ satisfies the equation

$$i\gamma^\mu \frac{\partial}{\partial x^\mu} R(x,y) + g\gamma^\mu\left[\int_0^1 t\,dt\, G_{\mu\nu}(y + t(x-y))\right]\cdot (R(x,y) + S^c(x-y)) = 0 \tag{30}$$

whence it follows that R depends on the gluon field only through the field strength $G_{\mu\nu}$. Any operator of the $OG\ldots G$-type has twist higher than that of O, because $G_{\mu\nu}$ is antisymmetric in μ, ν. Hence, $R(x,y;G)$ is responsible only for power corrections of $(1/Q^2)^N$-type and will be ignored hereafter.

In a non-Abelian gauge theory gluon lines may be inserted also into the gluon and ghost propagators, and then

$$\delta_{ab} D^c(x_\alpha - x_\beta) \to \tilde{E}_{ab}(x_\alpha, x_\beta; A)\left\{D^c(x_\alpha - x_\beta) + O(G)\right\}, \tag{31}$$

where $\tilde{E}$ is defined by eq.(29), but one should take there $\tilde{A}_\mu \equiv A_\mu^a \sigma_a$ rather than $\hat{A}_\mu$, σ_a being a matrix of the gauge group in the gluonic (adjoint) representation.

To unite the exponentials corresponding to neighbouring lines, one must commute the exponential with τ- or σ-matrices

$$\tilde{E}_{ab}(x,y;A)(\sigma_c)_{bd} = (\sigma_f)_{ab}\tilde{E}_{bd}(x,y;A)\tilde{E}_{fc}(x,y;A) \tag{32}$$
$$\hat{E}_{AB}(x,y;A)(\tau_a)_{BC} = (\tau_b)_{AB}\hat{E}_{BC}(x,y;A)\tilde{E}_{ba}(x,y;A).$$

Note the additional $\tilde{E}$-factors in the r.h.s. of eqs. (32).

Using eqs. (32) one can easily verify that for a subgraph with quark external lines all exponentials resulting from commutations are cancelled by those entering into the modified proparators (31), whereas the remaining $\hat{E}$ -factors sum up into $\hat{E}(\xi,\eta)$. As a result, there appears the gauge-invariant bilocal operator

$$O_\nu(\xi,\eta;A;\mu^2) = \mathrm{Reg}^{UV}_{\mu^2}\left(\bar{\psi}(\xi)\gamma_\nu \hat{E}(\xi,\eta;A)\,\psi(\eta)\right) \tag{33}$$

which can be expanded into the Taylor series eq.(19) over the gauge-invariant local operators

$$\bar{\psi}\gamma_{\mu_1}\hat{D}_{\mu_2}\ldots\hat{D}_{\mu_n}\psi, \tag{34}$$

where $\hat{D}_\mu = \partial_\mu - ig\hat{A}_\mu$ is the covariant derivative acting on the quark field. These operators can be related to gauge-invariant quark $f_a(\beta)$ and antiquark $f_{\bar{a}}(\beta)$ densities

$$\frac{i^{n-1}}{2}\langle P|\bar{\psi}_a\{\gamma_{\mu_1}\hat{D}_{\mu_2}\ldots\hat{D}_{\mu_n}\}\psi_a|P\rangle = \tag{35}$$
$$= \{P_{\mu_1}\ldots P_{\mu_n}\}\int_0^1 d\beta/\beta\left[f_a(\beta,\mu^2) + (-1)^n f_a(\beta,\mu^2)\right].$$

This justifies the use of a modified QCD parton model for deep inelastic scattering.

3. Parton Model and Hadronic Form Factors

Unlike deep inelastic scattering there exist in literature at least two different views (both inspired by the parton model) on the mechanism responsible for the large-Q^2 behaviour of hadronic EM form factors.

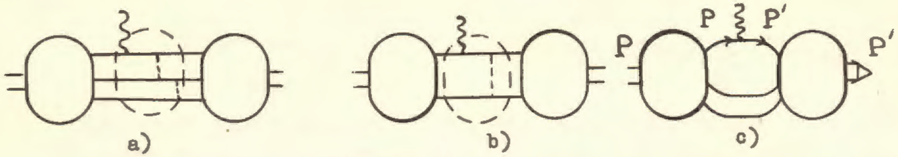

Fig.5

The first one is the hard rescattering picture of Brodsky and Farrar[10], in which it is assumed that large momentum transfer absorbed by a valence quark is distributed among other valence quarks by a short-distance rescattering process (fig. 5a,b). In the infinite momentum frame (IMF) all valence quarks in initial and final states are assumed to carry finite fractions of the hadron momentum. As a result, the momenta corresponding to the subprocess lines are all of order Q . The dimensional analysis (cf. eq.(11)) then gives the quark counting rules (QCR)[10,11]

$$F_H(Q) \sim (Q^2)^{1-n_H}, \tag{36}$$

where n_H is the number of valence quarks inside the hadron H .

Another mechanism proposed by Feynman[12] implies that the large- Q^2 behaviour of a form factor is dominated by a configuration in which the valence quark that absorbes the high momentum transfer takes the whole hadron momentum. All other quarks are wee and can be associated with the hadron both in initial and final states (fig. 5c). In this case short distances are clearly irrelevant.

The Feynman mechanism works only if the amplitude for a single quark to carry the whole hadron momentum is large enough. The two pictures exclude one another, hence if the Feynman mechanism dominates, then the hard rescattering picture must break down. To get a feeling about the interrelation between the two mechanisms, let us assume that the BF-diagram (fig.5a,b) can be written in the form suggested by the parton model

$$F(Q) \sim \int \varphi^*(y, k_\perp') \varphi(x, k_\perp)\, dx\, dy\, d^2k_\perp d^2k_\perp' V(Q^2(1-x)(1-y)+(k_\perp-k_\perp')^2), \tag{37}$$

where φ^*, φ are the wave functions describing the dissipation of the hadron into its constituents and $V((k-k')^2)$ describes interaction between the struck quark and spectators (fig. 6). We will assume also that the wave function falls off rapidly with growing transverse momentum $k_\perp$, i.e.,one may replace $(k_\perp-k_\perp')^2$ by a constant parameter $M^2 \simeq 2\langle k_\perp^2\rangle$. Then, if we take $V(t) \sim t^{-\alpha}$

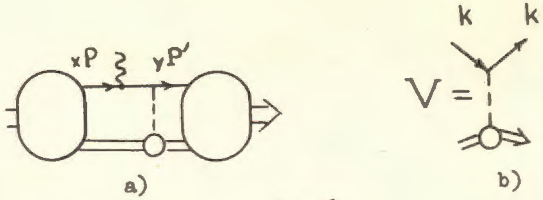

Fig.6

$$F(Q) \sim \int dx\, dy\, \varphi^*(y)\, \varphi(x)\, [Q^2(1-x)(1-y)+M^2]^{-\alpha}. \tag{38}$$

From eq.(38) it is clear that one may nelgect M^2 only if the integral

$$\int^1 dx\, (1-x)^{-\alpha}\, \varphi(x) \tag{39}$$

converges. Otherwise the hard rescattering picture does not work. According to eq.(36) $\alpha = n_H - 1$ for theories with dimensionless coupling constants like QCD or Yukawa γ_5 -theory, and short distances dominate if $\varphi(x)$ behaves at $x \to 1$ like $(1-x)^\beta$ with $\beta > n_H - 2$. In the reversed situation, i.e.,when $\beta < \alpha - 1$ the large- Q^2 behaviour is controlled by the $x \to 1$ behaviour of the wave function

$$F(Q) \sim (Q^2)^{-\beta-1}. \tag{40}$$

In the old-fashioned parton model it was assumed that $V(t)$ very rapidly vanishes as $t \to \infty$; $\alpha \sim \infty$, and the Feynman mechanism always dominates. However, the QCR can hold in this picture as well, if $\beta = n_H - 2$, i.e.,$\varphi_{(\pi)}(x) \sim const$, $\varphi_{(p)}(x) \sim (1-x)$, etc.

Our goal in the following sections is to analyze within a field-theoretical framework whether it is possible to justify the hard rescattering picture in the simplest case $n=2$.

4. Bound State Form Factors in Perturbation Theory

It is impossible to see bound states in any finite order of perturbation theory. Hence we must consider a full amplitude given by sum over all orders. To investigate the EM form factor of pion treated as the QCD bound state of quark and antiquark, we start with the full amplitude $T_5(p_1, p_2, p_1', p_2', q)$ describing the process $q\bar{q}\gamma \to q\bar{q}$ (fig. 7). Of course, we must take such a $q\bar{q}$ -combination which has nonzero projection onto the pion state $|P\rangle$:

$$\langle 0|\varphi_q \varphi_{\bar{q}}|P\rangle \neq 0.$$

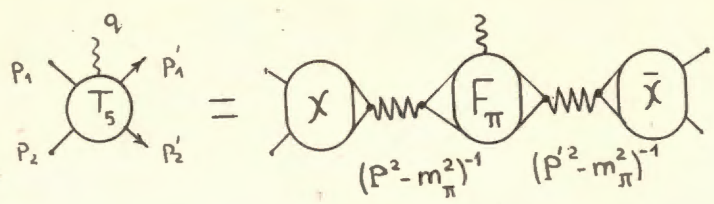

Fig.7

In this case the auxiliary amplitude T_5 has two poles/13/ related to pion bound states (fig. 7)

$$T_5(p_1, p_2, p_1', p_2', q) = i^2 \frac{\chi_{P'}(p_1' - p_2')\, F_\pi(q)\, \chi^*_P(p_1 - p_2)}{(P^2 - m_\pi^2)(P'^2 - m_\pi^2)}, \tag{41}$$

where $P = p_1 + p_2$, $P' = p_1' + p_2'$; $F_\pi(q)$ is the pion form factor

$$\langle P' | J^\mu(q) | P \rangle = (2\pi)^4 \delta^4(q - P' + P), \tag{42}$$

and χ, χ^* are the Bethe-Salpeter wave functions, which characterize the magnitude of the projection of the chosen set of fundamental fields $(\varphi_q, \varphi_{\bar q})$ onto the pion state $|P\rangle$.

In the next section we will show that in any finite order of perturbation theory the large-Q^2 behaviour of T_5 is given by an expression where long- and short-distance contributions factorize (see fig. 8)

$$T_5(p_1, p_2, p_1', p_2', q) = \int d^4\varkappa\, d^4\lambda\, d^4\varkappa'\, d^4\lambda'\, f^*(p_1', p_2'; \varkappa', \lambda', \mu^2) \cdot$$
$$C(\varkappa, \lambda, \varkappa', \lambda'; q; \mu^2)\, f(p_1, p_2; \varkappa, \lambda; \mu^2) + O(1/Q^4), \tag{43}$$

where f^*, f are related to long distances and C- to short ones. As usual, $1/\mu$ is the boundary between short and long distances.

If we make an assumption that the asymptotical behaviour of the full amplitude T_5 is given by the sum of those of all relevant diagrams, then eq.(43) is valid for the full amplitude T_5, where f and f^* are now given by the Green functions analogous to those given by eqs.(17). In particular,

$$f(p_1, p_2; \varkappa, \lambda; \mu^2) = \mathrm{Reg}^{UV}_{\mu^2} \langle 0 | S^+ T(\varphi_q(\varkappa)\varphi_{\bar q}(\lambda)\tilde\varphi_q(p_1)\tilde\varphi_{\bar q}(p_2) S) | 0 \rangle \tag{44}$$

The functions f, f^* must also possess poles corresponding to the pion bound states (fig. 9a)

$$f(p_1, p_2; \varkappa, \lambda; \mu^2) = i \frac{\chi_P(p_1 - p_2)}{P^2 - m_\pi^2} \langle 0 | \mathcal{O}(\varkappa, \lambda; \mu^2) | P \rangle. \tag{45}$$

Comparing eqs.(41) and (43), (45) (see also figs. (7) and (8))

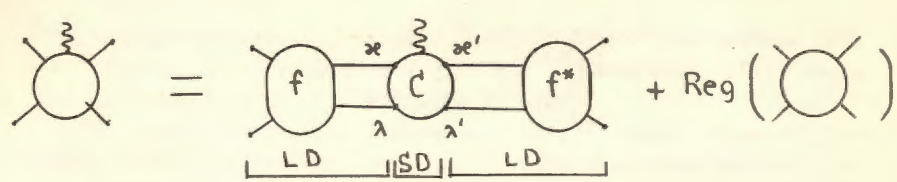

Fig.8

we conclude that in perturbation theory (see fig. 9b)

$$F_\pi(q) = \int d^4\varkappa\, d^4\lambda\, d^4\varkappa'\, d^4\lambda'\, \langle P' | \mathcal{O}(\varkappa', \lambda'; \mu^2) | 0 \rangle \cdot$$
$$\cdot\, C(q, \varkappa, \lambda, \varkappa', \lambda'; \mu^2) \langle 0 | \mathcal{O}(\varkappa, \lambda; \mu^2) | P \rangle + O(1/Q^4). \tag{46}$$

Expanding the bilocal operators into the Taylor series (eq.(19)) and introducing the parton wave function/7/

$$(2i)^n \langle 0 | \mathrm{Reg}^{UV}_{\mu^2} \varphi(0) \{ \overleftrightarrow{\partial}_{\mu_1} \ldots \overleftrightarrow{\partial}_{\mu_n} \} \varphi(0) | P \rangle = \{ P_{\mu_1} \ldots P_{\mu_n} \} \int_{-1}^{1} \xi^n \varphi(\xi, \mu^2) d\xi \tag{47}$$

gives the hard scattering formula

$$F_\pi(q) = \int_{-1}^{1} d\xi \int_{-1}^{1} d\eta\, \varphi^*(\eta, \mu^2)\, C\left(\frac{1+\xi}{2} P, \frac{1-\xi}{2} P, \frac{1+\eta}{2} P', \frac{1-\eta}{2} P', q, \mu^2\right) \varphi(\xi, \mu^2). \tag{48}$$

The parton wave function $\varphi(\xi)$ describes the dissociation of the pion into quark and an antiquark with momenta $\frac{1+\xi}{2} P$ and $\frac{1-\xi}{2} P$, respectively.

A very important observation is that the BS wave functions (present in eq.(41)) which depend on a particular set of fundamental fields have disappeared in eq.(46). That means we can start

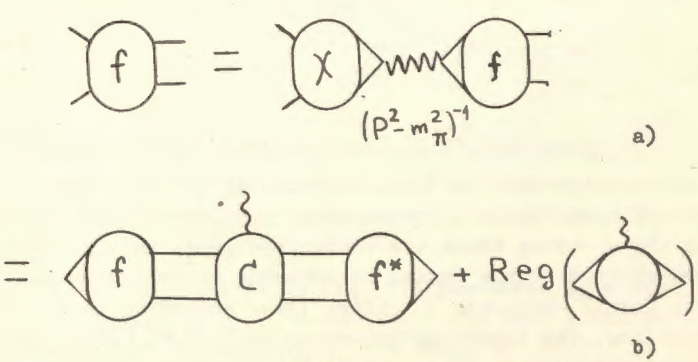

Fig.9

with another set $K(\varphi_q, \varphi_{\bar q}, \varphi_g)$ which contains an arbitrary number of gluon field operators φ_g, the only requirement on K being $\langle 0 | K | P \rangle \neq 0$. This is essential for QCD, because, as it was argued by Suura/14/, if one assumes quark confinement, then the simplest combination $\bar\psi \gamma_5 \psi$ has zero projection onto the pion bound state. Whether this is true or not, we prefer to use in place of $\bar\psi\gamma_5\psi$ a more safe combination $j\, j_5$ of two colour-singlet currents $j = \bar\psi \psi$, $j_5 = \bar\psi \gamma_5 \psi$. In the coordinate representation this corresponds to the change from $\bar\psi(a_1)\gamma_5\psi(a_2)$ to a gauge-invariant superposition of quark and gluon fields (fig. 10)

$$K(\psi, \bar\psi, A) = \bar\psi(a_1)\gamma_5\, \mathcal{S}^c(a_1, a_2; A)\, \psi(a_2), \tag{49}$$

where $\mathcal{S}^c(a_1,a_2;A)$ is given by eq.(28). Note, that up to $O(G)$ terms and the numerical factor $S^c(a_1 - a_2)$ the combination $K(\psi,\bar\psi,A)$ coincides with the combination $\tilde K$

$$\tilde K(\psi, \bar\psi, A) = \bar\psi(a_1)\gamma_5 \left(P\exp ig \int_{a_2}^{a_1} \hat A_\mu(z) dz^\mu\right)\psi(a_2) \tag{50}$$

which was argued in ref./14/ to be a right one to be used as a pion interpolating field.

We emphasize that to justify eq.(46), it is sufficient to prove the factorization (eq.(43)) for an arbitrary set $K(\psi,\bar\psi,A)$ having nonzero projection onto $|P\rangle$. In particular, we may choose the same infrared cut-off both for quark and antiquark related

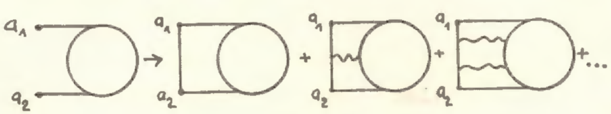

Fig.10

to the same pion state, i.e., take $p_1^2 = p_2^2$; $p_1'^2 = p_2'^2$ to avoid the factors like $\ln p_1^2/p_2^2$ which are irrelevant to our problem.

5. Factorization of the Auxiliary Green Function

As discussed in Sec. 2, to study the asymptotical behaviour of a particular Feynman amplitude, one should consider regions in the α-space where the factor $A(\alpha)/D(\alpha)$ (see eq.(7)) vanishes. The short-distance regime corresponds to vanishing A/D when $\lambda(V)=0$ for some Q^2-subgraph V (fig. 11a). According to dimensional counting, the resulting behaviour is $T_\mu \sim (P_\mu + P'_\mu)/Q^2$, which is realized when spinor (i.e., quark) lines add up the $P_\nu, P_{\nu'}$-factors.

This corresponds in our case to the axial projection in the Fierz identity eq.(15). Other projections correspond either to higher twist operators or to those having vanishing matrix elements, e.g.,

$$\langle 0 | \bar\psi \gamma_\mu \partial \ldots \partial \psi | P \rangle = 0$$

by parity conservation.

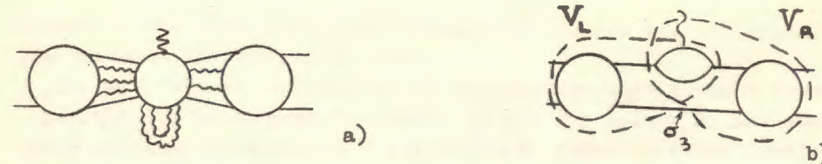

Fig.11

It can be shown also/8/ that the configuration fig. 11b, which works in some simple field theories ($\varphi^3_{(4)}$, $\varphi^3_{(6)}$) gives no leading contribution in theories with spin-1/2 quarks. More precisely, it can be shown that if V_L gives $O(1/Q^2)$ contribution, then V_R gives the $O(1/Q^4)$ one.

As a result, it is possible to order the Q^2-subgraphs just like for deep inelastic scattering (with an obvious modification) and to write T_5 as in eq.(13). Then one should prove that the remainder (for which $\lambda(V) > 1/\mu^2$ for any leading Q^2-subgraph V) gives only nonleading contribution. However, T_5 has a more complicated topology than that of $T(q,p)$, and there appear additional possibilities to get $A/D = 0$ due to infrared regime when some α-parameters $\alpha_{\sigma_1}, \ldots, \alpha_{\sigma_n}$ are infinite. The corresponding lines $\sigma_1, \ldots, \sigma_n$ are usually called soft.

This possibility is based on the fact that $D(\alpha)$ contains all α-parameters of a given, one-particle irreducible (1PI) diagram whereas it is possible that some α-parameters do not enter into the function A. Then $A/D = 0$ if $\alpha_{\sigma_j} = \infty$. The limit $\alpha_\sigma \to \infty$ means topologically the removal of the σ-line from the diagram. This gives a simple rule (analogous to that for Q^2-subgraphs) for finding a set $\{\sigma_i\} \equiv S$ of soft lines: after removing the lines $\{\sigma_i\}$ the diagram must be independent of large momentum variables (but remains connected).

For example, the σ_3-line in fig. 11b possesses this property, i.e., $A(\alpha)/D(\alpha) \sim 1/\alpha_3$ as $\alpha_3 \to \infty$. The main contribution is due to integration over $\mathcal{I}_3 \sim Q^2/p^2$, i.e., over $\alpha_3 \sim Q^2/p^4$. In the

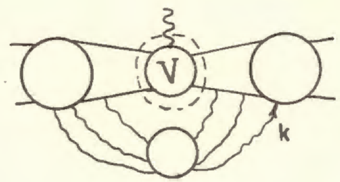

Fig.12

momentum space this corresponds to integration over $k \sim p^2/Q \to 0$, hence the IR regime for the σ_3 -line corresponds to the Feynman process. However, using the "inversed" dimensional counting prescription

$$k_\mu \sim (p_\mu + p'_\mu)\frac{p^2}{Q^2} \;;\; (pk) \sim (pp')\frac{p^2}{Q^2} \sim p^2 \;;\; k^2 \sim \frac{p^4}{Q^2} \tag{51}$$

(which is equivalent to a more careful α -representation analysis) gives that in QCD the IR-integration over α_3 gives only $O(1/Q^4)$ contribution.

Another possibility is to combine the short-distance and the infrared regimes, i.e., to take $\lambda(V) = 0$; $\alpha_\sigma = \infty$, $\sigma \in S$ (fig.12). Physically, this corresponds to a short-distance subprocess accompanied by soft-gluon exchanges between initial and final states. In the α -representation this corresponds to the following structure of the exponential factor

$$\exp i \frac{Q^2}{p^2}\frac{A(\bar\alpha)}{D(\bar\alpha)} = \exp i \frac{Q^2}{p^2}\left[\left(\frac{1}{\tilde\alpha_{\sigma_1}} + \ldots + \frac{1}{\tilde\alpha_{\sigma_n}}\right)\varphi_1(\bar\alpha) + \lambda(V)\cdot \varphi_2(\alpha)\right], \tag{52}$$

i.e., the dominant contribution is due to integration over $\tilde\lambda(V) \sim p^2/Q^2$, $\tilde\alpha_{\sigma_j} \sim Q^2/p^2$ (or over $\lambda(V) \sim 1/Q^2$, $\alpha_{\sigma_j} \sim Q^2/p^4$). Note, that V must be Q^2 -subgraph for a diagram with all soft lines removed, and not for the initial diagram.

To obtain the large- Q^2 contribution, we integrate first over the small-$\lambda(V)$ region. By dimensional counting estimate the resulting behaviour is $1/Q^2$. Then we must consider the reduced diagram (fig. 12) with the subgraph V contracted into point. In all gauge theories (in covariant gauges) on graph by graph level the reduced diagrams behave like Q^0 , and the resulting contribution is the leading one. There appear however numerous cancellations between different graphs due to gauge invariance. As a result, all terms responsible for the Q^0- factor drop out if all external li-

nes of the diagram are colour-singlets. This is analogous to a familiar QED result that light by light scattering is free from IR singularities.

The correspondence between the IR singularities and the $O(1)$ contribution from the soft region is straightforward. Both are caused by the fact that in the small- k region the corresponding momentum integral looks like

$$\int_0 \frac{d^4k}{k^2}\, T(p,p',k) \;;\; T \sim \frac{\{(pp'),(pk),(p'k),k^2\}}{[2(pk)+O(p^2)][2(p'k)+O(p^2)]} \tag{53}$$

whence it follows both that there appears logarithmic IR-divergence for $p^2 = p'^2 = 0$ and that integration in the region $k \sim p^2/Q$ gives $O(1)$ contribution for nonzero p^2, p'^2 .

If one takes a gauge-invariant sum of diagrams then the (pp')-term in eq.(53) disappears,

$$\lim_{\varkappa \to 0} \varkappa^2 T(p,p',\varkappa k)\Big|_{p^2=p'^2=0} = 0 , \tag{54}$$

the resulting integral converges and the IR-region gives $O(1/Q^2)$ contribution. Eq.(54) can be proved in QCD /15/ using the axial gauge Ward identity /16/ for diagrams with colour-singlet external lines. The cancellation however is a gauge-invariant phenomenon since both the full T_5 -function and the pure short-distance term (see below) are the gauge-invariant quantities.

Hence the combined UV-IR regime also gives a non-leading contribution into T_5 . The cancellation is due to colour neutrality of the pion, because it is impossible for a coloured particle to start with T_5 having only colour-singlet external lines. Physically, this cancellation is due to the fact that a soft gluon has large wave length and "feels" only the total colour of a system. So, it decouples from colour-singlet states.

To complete our study of factorization, we must consider the short-distance-dominated configurations fig. (11a). Just like in Sec. 2, the summation over gluon lines inserted into propagators related to parton subprocess results in a modification given by eqs.(28), (31). However in this case there appears also an additional possibility to insert the gluon lines going out of, say, the initial pion state into external quark or gluon lines related to the final state pion, and vice versa (fig. 13a).

One must fix first the gluon lines related to (say) the final state (B-lines) and sum over gluon lines related to the initial state (A-lines). For a spinor external line the sum over A-lines gives (fig. 13b)

$$\psi(\xi) \to \Psi(\xi) = \psi(\xi) + g\int d^4z\, S^c(\xi - z)\gamma^\mu \hat{A}_\mu(z)\psi(z) + \ldots \qquad (55)$$

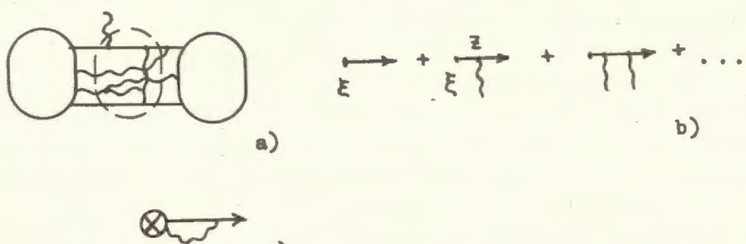

Fig.13

However, the integral in eq.(55) is IR-divergent unless $A(z)\psi(z)$ vanishes rapidly than $1/z$ in the limit $z \to \infty$. For the $\psi(z)$ field we assume here the translation-invariant estimate $\psi(z) \sim const$, whereas $A_\mu(z)$ must be estimated in perturbation theory as $A_\mu \sim 1/z$ for large z, in accordance with the large-z behaviour of the propagator

$$\langle 0|T(A_\mu(z)A_\nu(z'))|0\rangle \sim g_{\mu\nu}/(z-z')^2 . \qquad (56)$$

Hence $\psi A \sim 1/z$ and the integral in eq.(55) diverges logarithmically.

The convergence can be improved if we require that the boundary condition $\Psi(0) = \psi(0)$ is fulfilled. Then

$$\Psi(\xi) = \psi(\xi) + g\int d^4z\,(S^c(\xi - z) - S^c(-z))\gamma_\mu \hat{A}_\mu(z)\psi(z) + \ldots \qquad (57)$$

and the integrals converge at large z.

The function $\Psi(\xi)$ which is a solution of the Dirac equation

$$i\gamma^\mu(\partial_\mu - ig\hat{A}_\mu)\Psi = 0 \qquad (58)$$

can be written as

$$\Psi(\xi) = \hat{E}(0,\xi)\{\psi(\xi) + O(G,\psi)\} . \qquad (59)$$

One can, of course, choose another boundary condition. For instance, the function

$$\Psi(\xi, z_0) = P\,(\hat{E}(z_0, 0)\,\Psi(\xi)) \qquad (60)$$

also satisfies eq.(58). The point z_0 can be taken arbitrary far away from the region where the short-distance subprocess takes place. This means the sum of gluon insertions into an external line is an IR-sensitive quantity. The parameter z_0 works here like an IR cut-off. One can interpret this so that the field $A_\mu(z)$ is switched off outside the region of $|z_0|$ radius. This IR-regularization can be introduced from the very beginning to give a meaning to eq.(55). But our goal was to show also that the whole IR-sensitivity can be absorbed by the phase factor $\hat{E}(z_0, \xi)$ whereas the $O(G,\psi)$ terms in $\Psi(\xi, z_0)$ (eq.(60)) are z_0-independent, i.e., IR-insensitive.

The matrix elements of the $\Psi(\xi, z_0)$-operator have double-logarithmic dependence on the splitting parameter μ; e.g., the 1-loop diagram (fig.13c) behaves like $g^2 \ln(\mu|z_0|)\ \ln p^2/\mu^2$. The phase factor $\hat{E}(z_0, \xi)$ signalizes that the double logarithmic terms $g^2\ln^2 Q^2/p^2$ are present on the graph by graph level.

It is natural to take the same z_0 for all external lines of the Q^2-subgraph V related to the initial (or final) state. Then the antiquark field $\bar{\psi}$ has just the opposite factor $\hat{E}(0, z_0)$, and the product $\bar{\psi}\psi$ is IR-insensitive. If one takes into account also that insertions into the gluon external line give

$$B_\mu(\xi) \to \mathcal{B}_\mu(\xi) = \tilde{E}(z_0, \xi; A)\{B_\mu(\xi) + O(G(A), B)\} \qquad (61)$$

and uses the commutation formulas (32), then all the z_0-dependence responsible for double-logarithmic terms is cancelled and the only effect of summation over gluon lines is the appearance of the gauge-invariant bilocal operators $\mathcal{O}_\nu(\lambda, \varkappa; A)$, $\mathcal{O}_{\nu'}(\lambda', \varkappa'; B)$:

$$\mathcal{O}_\nu(\lambda, \varkappa; A) = \bar{\psi}(\lambda)\gamma_5\gamma_\nu \hat{E}(\lambda, \varkappa; A)\,\psi(\varkappa) \qquad (62)$$

in eq.(46). They can be expanded into the Taylor series over the gauge-invariant local operators. Defining

$$(2i)^n\langle 0|\bar{\psi}\gamma_5\{\gamma_\mu \overset{\leftrightarrow}{\hat{D}}_{\mu_1}\ldots \overset{\leftrightarrow}{\hat{D}}_{\mu_n}\}\psi|P\rangle = \{P_\mu P_{\mu_1}\ldots P_{\mu_n}\}\frac{1+(-1)^n}{2} f_n(\mu^2) \qquad (63)$$

we introduce the gauge-invariant wave functions $\varphi(\xi, \mu^2)$ (cf. eq.(47))

$$\int_{-1}^{1}\varphi(\xi, \mu^2)\,d\xi = f_n(\mu^2)\,\frac{1+(-1)^n}{2} . \qquad (64)$$

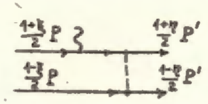

Fig.14

Matrix elements of operators with an odd number of derivatives vanish. As a result, the wave function $\varphi(\xi)$ is symmetric $\varphi(\xi)=\varphi(-\xi)$.

Contributions due to higher-twist operators have additional factors $(M/Q)^{t_i-2}$ as compared to the twist-2 term and can be ignored in the large- Q^2 limit.

6. Large-Q^2 Behaviour of the Pion EM Form Factor

Having established the desired factorization properties of the auxiliary Green function we may proceed to exploiot calculations of the short-distance amplitude $C(\dots)$ (eq.(48)) in perturbative QCD. Taking $\mu=Q$ and using the Born approximation for $C(\dots)$ (see fig. 14)

$$C_{BORN} = \frac{2g^2}{(1-\xi)(1-\eta)Q^2}\cdot\frac{C_F}{N_c} \qquad (65)$$

(where $C_F=4/3$ and $N_c=3$ is the number of colours), and using the symmetry relation $\varphi(\xi)=\varphi(-\xi)$ we obtain /7/

$$F_\pi(q)=\frac{8\pi\alpha_s(Q)}{Q^2}\cdot\frac{C_F}{N_c}\left(\int_{-1}^{1}\frac{\varphi(\xi,Q^2)}{1-\xi^2}d\xi\right)^2. \qquad (66)$$

Just like in Sec. 3 this formula is meanigful only if the wave function vanishes for $\xi^2=1$. However, eq.(66) <u>must</u> be valid in perturbation theory, where it was derived. Really, it can be shown /8/ that $\varphi(\xi)\sim 1-\xi^2$ for $\xi^2\sim 1$ in each finite (nontrivial) order of perturbation theory. In principle, it is, of course, possible that the full wave function $\varphi(\xi)$ given by a sum over all orders does not vanish at $\xi^2=1$, i.e., our assumption that summation over all orders commutes with the $Q^2\to\infty$ limit is wrong. In this case essentially nonperturbative methods should be invented.

The situation, however, is not so unhappy. In what follows we will give the arguments that the wave function related to a

bound state must vanish at $\xi^2=1$. To see this we investigate the Q^2 -dependence of the wave function.

The study of Q^2 -dependence of the wave function defined by eqs.(63),(64) is an independent problem which can be investigated by calculating the anomalous dimensions of matrix elements (63). It is straightforward to obtain that in the basis chosen (eq.(63)) the anomalous dimensions form a triangular matrix /7,8/

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right)f_n(\mu^2,g)=\sum_{k=0}^{n} Z_{nk}(g)\,f_k(\mu^2,g) \qquad (68)$$

which is diagonal only in a conformal basis, i.e., for operators

$$K_{\mu\mu_1\dots\mu_n}=\bar\psi\gamma_5\left\{\gamma_\mu\partial_+^n C_n^{3/2}(2\overleftrightarrow{D}/\partial_+)_{\mu_1\dots\mu_n}\right\}\psi, \qquad (69)$$

where $C_n^{3/2}(x)$ is the Gegenbauer polynomial, $\partial_+=\vec\partial+\overleftarrow{\partial}$, $2\overleftrightarrow{D}=\vec D-\overleftarrow{D}$, $\partial_+^n(D/\partial_+)^k\equiv\partial_+^{n-k}D^k$. As a result, in QCD

$$k_n(Q^2)\equiv\int_{-1}^{1}\varphi(\xi,Q^2)\,C_n^{3/2}(\xi)\,d\xi=k_n(Q_0^2)\left(\frac{\ln Q_0^2/\Lambda^2}{\ln Q^2/\Lambda^2}\right)^{\gamma_n/B}, \qquad (70)$$

where γ_n is the standard nonsinglet anomalous dimension

$$\gamma_n=C_F\left(1-\frac{2}{(n+1)(n+2)}+4\sum_{j=2}^{n+1}1/j\right) \qquad (71)$$

and $B=11-2N_f/3$.

The Gegenbauer moments are easily inverted

$$\varphi(\xi,Q^2)=(1-\xi^2)\sum_{n=0}^{\infty}\frac{1+(-1)^n}{2}\,\frac{n+3/2}{(n+1)(n+2)}\,k_n(Q^2)\,C_n^{3/2}(\xi), \qquad (72)$$

where

$$k_n(Q^2)=k_n(Q_0^2)\left(\ln(Q_0^2/\Lambda^2)/\ln(Q^2/\Lambda^2)\right)^{\gamma_n/B}. \qquad (73)$$

Each term in the sum (72) behaves at $\xi^2\sim 1$ as $1-\xi^2$, because for $1-\xi^2\ll 1$

$$\frac{n+3/2}{(n+1)(n+2)}(1-\xi^2)\,C_n^{3/2}(\xi)\sim(1-\xi^2)^{1/2}J_1\left((n+\tfrac{3}{2})\sqrt{1-\xi^2}\right)\sim 1-\xi^2, \qquad (74)$$

where J_1 is the Bessel function. However, if the coefficients $k_n(Q^2)$ do not vanish rapidly enough as $n\to\infty$, then the infinite sum of higher harmonics radically changes the behaviour of $\varphi(\xi)$ at $\xi^2\sim 1$. Really, if $k_n\sim n^\alpha$, then in the region $n\gg 1/\sqrt{1-\xi^2}$ using the well-known asymptotic relation

$$J_1(x) \sim \cos x / \sqrt{x} \quad , x \gg 1 \tag{75}$$

we find

$$\sum_{N_0}^{\infty} n^{\alpha} \sqrt{1-\xi^2}\, J_1\left((n+\tfrac{3}{2})\sqrt{1-\xi^2}\right) \sim (1-\xi^2)^{-\alpha/2}, \tag{76}$$

where $N_0 = A/\sqrt{1-\xi^2}$; $A \gg 1$. As a result,

$$\varphi(\xi) \sim a(1-\xi^2) + b(1-\xi^2)^{-\alpha/2}, \tag{77}$$

where the first term is given by the sum over $n < N_0$. It dominates if $\alpha < -2$.

What one may expect is that for a bound state wave function $\alpha \leq 0$, because otherwise the wave function is infinite at $\xi^2 = 1$, and such a pathological behaviour must be rejected. It can be shown that in QCD the constant behaviour (i.e., $\alpha = 0$) at particular Q_0^2 must be rejected also, because it induces the pathological behaviour for $Q^2 < Q_0^2$. This follows from the fact that

$$\gamma_n \sim 4 C_F \ln n \tag{78}$$

for large n . Really, substituting eqs.(78) into eq.(73) yields

$$k_n(Q^2) \simeq k_n(Q_0^2)\, n^{\alpha(Q^2)} \; ; \; \alpha(Q^2) = \frac{4C_F}{B}\left[\ln\ln\frac{Q_0^2}{\Lambda^2} - \ln\ln\frac{Q^2}{\Lambda^2}\right]. \tag{79}$$

So, if $\alpha(Q_0^2) = 0$, then $\alpha(Q^2) > 0$ for $Q^2 < Q_0^2$. Note, that if $k_n(Q_0^2) = n^{-\ell}$, ℓ being some positive number, then pathological behaviour appears for $Q^2 < \tilde{Q}^2(\ell)$, where

$$\tilde{Q}^2(\ell) = \Lambda^2 \left(\frac{Q_0^2}{\Lambda^2}\right)^{\exp(-\ell B/4 C_F)} . \tag{80}$$

From eq.(80) it follows that $\tilde{Q} \to \Lambda$ as $\ell \to \infty$. However, eq.(72) is valid only for Q not too close to Λ , say, for $Q \gg M$. Hence, the pathological behaviour is absent in the region where eq.(72) works only if $k_n(Q_0^2)$ vanishes more rapidly than $n^{-\ell_0}$, ℓ_0 being the number defined by $\tilde{Q}(\ell_0) = M$.

Another observation is that for very large Q^2 all terms with $n \geq 2$ die away and only the $n=0$ term remains, i.e.,

$$\lim_{\mu^2 \to \infty} \varphi(\xi, \mu^2) = \frac{3}{4} k_0 (1-\xi^2) . \tag{81}$$

The parton wave function $\varphi(\xi, Q^2)$ satisfies a very specific normalization condition

$$i P_\nu \int_{-1}^{1} \varphi(\xi, Q^2)\, d\xi = \langle 0 | \bar{\psi} \gamma_5 \gamma_\nu \psi | P \rangle = i P_\nu f_\pi \tag{82}$$

because the matrix element of the axial current is known from

$\pi \to \mu\nu$ decay: $f_\pi = 133$ MeV. That means $k_0(Q^2) = f_\pi$ for all Q^2 , since the axial current has zero anomalous dimension. As a result, the pion EM form factor (at least for very large Q^2) can be expressed in terms of fundamental constants

$$F_\pi^{(as)}(Q) = \frac{8\pi \alpha_s(Q) f_\pi^2}{Q^2} . \tag{83}$$

Interpolating this formula into the region $Q^2 \sim 1 \div 3$ GeV^2, we find that $F_\pi^{(as)}$ crosses the curve $F_\pi^{(exp)} = (1+Q^2/0.471)^{-1}$ (which reproduces the experimental data in this region) at $Q^2 \simeq 1.4$ GeV^2. For $Q^2 > 1.4$ GeV^2 the curve $F_\pi^{(as)}$ goes lower, mainly due to decrease of the coupling constant $\alpha_s(Q)$. Anyway, the asymptotical formula (83) predicts a magnitude of the right order for $F_\pi(Q)$ in the region $Q^2 \gtrsim 1$ GeV^2, and this indicates that a better agreement can be achieved by using a wave function that differs from $\varphi(\xi, \infty)$, and also by taking into account power corrections (which are large at moderately large Q^2) and the next-order corrections for the short-distance amplitude.

7. Conclusion

The large-Q^2 behaviour of the bound state form factors is a rather old problem. It was intensively studied in various field-theoretical models during the last 10 years/7,8,10-12,14,17-27/. To complete the paper, we summarize the new ideas which helped us to solve this problem in QCD.

First of all, the standard bound-state formalism/13/ plays a secondary role in our investigation. We propose to use an OPE-like description of the bound state structure by matrix elements of certain local gauge-invariant operators. However, such a description works only if it is established that short distances dominate the large-Q^2 behaviour of the auxiliary Green function T_5 . We prove the SD-dominance using a direct analysis of perturbation theory diagrams in the α-representation. The trickiest piece was to prove the cancellation of the leading soft contributions. However, such a cancellation was shown to be quaranteed by colour neutrality of the pion.

Once we have established the factorization of the short- and long-distance contributions for the leading power term, the further analysis parallels the classic OPE treatment of electroproduction. In particular, we use the fact that the leading term is independent of the splitting parameter μ to derive a renormalization group equation. Identifying the reduced matrix elements of the local operators with the moments of the parton wave function, we obtain the parton picture of a new type. A possible problem for this picture is that partons may be arbitrarily soft. But we show that in QCD the wave function must vanish in the soft region and, hence, produces an effective damping of the soft-parton contribution.

Thus, the short-distance parton picture provides a self-consistent description of the large-Q^2 behaviour of the pion EM form factor in QCD.

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