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OSp(1, 4) -суперполя в киральном представлении

Изучены свойства OSp(1,4)-суперполей в расщепленной /киральной/ параметризации суперпространства OSp(1,4)/O(1,3). Найдена связь между действительным и киральным базисами в суперпространстве и построены ковариантные производные и инвариантные меры интегрирования в киральных базисах. Детально исследована групповая структура киральных представлений OSp(1, 4). Построены простейшие OSp(1, 4) -инвариантные модели: OSp(1, 4) -аналог модели Весса-Зумино и OSp(1,4) -расширение калибровочных теорий. Обсуждается связь описанного формализма с супергравитацией.

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OSp(1,4)-Superfields in Chiral Representation

The properties of OSp(1,4) -superfields in splitting (chiral) parametrization of the superspace OSp(1,4)/O(1,3) are studied. We find the connection between the real and chiral bases in superspace and construct covariant derivatives and invariant integration measures in chiral bases. The group structure of chiral representations of OSp(1, 4) is examined in detail. The simplest linear OSp(1,4) -invariant models are presented: the OSp(1,4) -analog of the Wess-Zumino model and OSp(1, 4) -extension of the Yang-Mills theory. We discuss also the relation of the described formalism to supergravity.

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I. The present paper is the continuation of the paper [1] where the OSp(1,4)-superfield formalism in the symmetrically parametrized superspace has been described. Here we pass to the non-symmetric, splitting parametrization of the superspace OSp(1,4)/O(1,3). The basic property of this parametrization is that the right- and left-handed components of corresponding Grassmann coordinate enter into superfield transformation rules in essentially different ways. Just as in the usual supersymmetry, the splitting bases appear in many cases more convenient and advantageous than the symmetric one.

The paper is planned as follows. In Sec . 2, we describe in detail the connection between symmetric and splitting parametrizations of superspace and construct covariant superfields derivatives and invariant integration measures for splitting parametrization. In Sec . 3 the simplest linear OSp(1,4)-invariant models are set up: the OSp(1,4)-analog of the Wess-Zumino model and OSp(1,4)-extension of the Yang-Mills theory. Relation to supergravity is briefly explained at the end of Sec . 2 and in Sec .4. Appendix A contains explicit expressions for OSp(1,4)-generators in the non-symmetric parametrization. In App. B we construct the complete set of representations of OSp(1,4) induced by the Lorentz supergroups (see Sec . 2) in invariant spaces of the Lorentz group.

In what follows, the paper[1] will be cited as I. Respectively, formulas from I will be referred to as (I.n), n being the number of given formula in I. Notations are the same as in I.

2. To shed more light on the results of paper I and, especially, to clarify the meaning of the restrictions (I.$35_{\pm}$) (which seem, at first sight, quite mysterious) we adopt here a more general point of view on the relation between the real superspace and chiral ones. Namely, we pursue to show that the variable change (I.41) and its right-handed analog naturally emerge within the

coordinate transformations to the following new complex parametrizations of cosets OSp(1,4)/O(1,3):

$$G(x,\theta)\to G^{+}(x^L,\theta^L,\eta^R)=OSp(1,4)/S_-\cdot S_-/O(1,3)=g(x^L)e^{i\bar\theta^L Q_+}e^{i\bar\eta^R Q_-} \qquad (1_+)$$

and

$$G(x,\theta)\to G^{-}(x^R,\theta^R,\eta^L)=OSp(1,4)/S_+\cdot S_+/O(1,3)=g(x^R)e^{i\bar\theta^R Q_-}e^{i\bar\eta^L Q_+} \qquad (1_-)$$

Here $G(x,\theta)$ is an element of the coset OSp(1,4)/O(1,3) in the symmetric parametrization (I.16), $Q_\pm=\frac{1\pm i\gamma_5}{2}Q$ are the left- and right-handed components of spinor OSp(1,4)-generator which form, together with the Lorentz generators $M_{\mu\nu}$, two complex conjugated graded subalgebras of OSp(1,4), $S_\pm \propto (M_{\mu\nu}, Q_\pm)$:

$$[M_{\mu\nu},Q_\pm]=-\frac{1}{2}\sigma_{\mu\nu}Q_\pm\ ,\quad \{Q_\pm,\bar Q_\pm\}=\frac{m}{2}\sigma^{\mu\nu}\frac{1\pm i\gamma_5}{2}M_{\mu\nu} \qquad (2a)$$

$$\{Q_\pm,\bar Q_\mp\}=\gamma^\mu\frac{1\mp i\gamma_5}{2}R_\mu \qquad (2b)$$

(we call them, resp., left- and right-handed Lorentz superalgebras).

Cumbersome calculations utilizing the Baker-Hausdorf formula, structure relations (I.1) and (2), and the Grassmann nature of coordinates θ_α indicate that $G(x,\theta)$ admits a representation in the two equivalent forms

$$G(x,\theta)=\begin{cases} G^{+}[x^L(x,\theta),\theta^L(x,\theta),\eta^R(x,\theta)]\exp\{-\frac{im^2}{4}\bar\theta\gamma^\rho\gamma_5\theta x^\nu M_{\nu\rho}\} \\ G^{-}[x^R(x,\theta),\theta^R(x,\theta),\eta^L(x,\theta)]\exp\{\frac{im^2}{4}\bar\theta\gamma^\rho\gamma_5\theta x^\nu M_{\nu\rho}\}\end{cases} \qquad (3_\pm)$$

$x^L_\mu(x,\theta),\theta^L_\alpha(x,\theta)$ and $x^R_\mu(x,\theta),\theta^R_\alpha(x,\theta)$ being just the functions given by relations (I.41) and by those arising from (I.41) after the change $\theta_+\leftrightarrow\theta_-$ while η^R and η^L are expressed as

$$\eta^R(x,\theta)=\theta_-+\frac{3}{8}im^2\bar\theta\theta\cdot x\gamma\theta_+=(1+\frac{m}{2}\bar\theta\theta)\theta^R \qquad (4)$$

$$\eta^L(x,\theta)=\theta_++\frac{3}{8}im^2\bar\theta\theta\cdot x\gamma\theta_-=(1+\frac{m}{2}\bar\theta\theta)\theta^L .$$

In what follows, the bases x^L,θ^L,η^R and x^R,θ^R,η^L associated with coset parametrizations (1_+) and (1_-) will be referred to as left- and right-handed, respectively.

It is clear now that x^L_μ,θ^L_α and x^R_μ,θ^R_α are nothing but coordinates of the homogeneous spaces $OSp(1,4)/S_-$ and $OSp(1,4)S_+$. The invariance of these superspaces with respect to the realizations of OSp(1,4) as left multiplications of elements G^+ and G^- is now evident and follows directly from the structure of G^+, G^-. It may be verified, in particular, that under the

multiplication of G^+ by $G_0=e^{i\bar\epsilon Q}$ coordinates x^L_μ and θ^L_α transform according to the law (I.42). The coordinates η^R and η^L supplement the invariant subspaces x^L,θ^L and x^R,θ^R to the whole superspace OSp(1,4)/O(1,3) and have the clear meaning: they label "points" of purely Grassmannian coset spaces $S_-/O(1,3)$ and $S_+/O(1,3)$. Under the action of OSp(1,4) variables η^R,η^L transform according to the left realizations of supergroups S_-, S_+ on these cosets, with constant parameters if an OSp(1,4)-transformation belongs to S_- or S_+ and, otherwise, with parameters dependent on x^L,θ^L or x^R,θ^R, respectively. To illustrate this point we trace in detail how OSp(1,4) operates, say on elements G^+:

$$G_0G^+(x^L,\theta^L,\eta^R)=g(x^{L'})e^{i\bar Q_+\theta^{L'}}S^0_-(G_0,x^L,\theta^L)e^{i\bar Q_-\eta^R}=$$
$$=g(x^{L'})e^{i\bar Q_+\theta^{L'}}e^{i\bar Q_-\eta^{R'}}e^{\frac{i}{2}\tilde w_+^{\mu\nu}(G_0,x^L,\theta^L,\eta^R)M_{\mu\nu}}\equiv G^+(x^{L'},\theta^{L'},\eta^{R'})e^{\frac{i}{2}\tilde W_+^{\mu\nu}(G_0,x^L,\theta^L,\eta^R)M_{\mu\nu}} \qquad (5)$$

Here S^0_- is an element of the right-handed Lorentz supergroup. If $G_0\in S_-$, then $S^0_-(G_0,x^L,\theta^L)=\mathrm{const}=G_0$ and $\tilde W_+^{\mu\nu}=\tilde W_+^{\mu\nu}(G_0,\eta^R)$.

Further in this section we shall have to do only with the parametrization (1_+) keeping in mind that the transition to the basis associated with (1_-) can be performed at any stage by means of trivial interchanges $L\leftrightarrow R$, $\frac{1+i\gamma_5}{2}\leftrightarrow\frac{1-i\gamma_5}{2}$.

Transformation properties of superfields in the left-handed basis are defined in entire analogy to (I.19):

$$\tilde\Phi_k(x^L,\theta^L,\eta^R)\overset{G_0}{\to}\tilde\Phi'_k(x^{L'},\theta^{L'},\eta^{R'})=\left(\exp\{\tfrac{i}{2}\tilde W_+^{\mu\nu}(G_0,x^L,\theta^L,\eta^R)J_{\mu\nu}\}\right)_{k\ell}\tilde\Phi_\ell(x^L,\theta^L,\eta^R), \qquad (6)$$

where $J_{\mu\nu}$ are, as in I, matrices of generators of the Lorentz group. The transformation law (6) corresponds to a successive inducing: transformations belonging to S_- are induced by the subgroup O(1,3) as a little group (under the action of S_- superfields transform as in O(1,3) but with parameters which are in general functions of η^R and constant parameters of S_-) and the remaining OSp(1,4)-transformations are induced, in turn, by supergroup S_- (the constant group parameters in the S_- transformation rule are replaced by suitable functions both of x^L,θ^L and new group parameters determined from the composition law (5)). The relation of superfields $\tilde\Phi_k(x^L,\theta^L,\eta^R)$ to those in the real basis $\Phi_k(x,\theta)$ (see paper I) is **not** so simple as in the usual supersymmetry:

$$\tilde\Phi_k[x^L(x,\theta),\theta^L(x,\theta),\eta^R(x,\theta)]=\left(\exp\{-\tfrac{im^2}{4}\bar\theta\gamma^\rho\gamma_5\theta x^\nu J_{\nu\rho}\}\right)_{k\ell}\Phi_\ell(x,\theta) . \qquad (7)$$

Here x^L, θ^L, η^R are assumed to be expressed in terms of x, θ through (I.41) and (4). Note the presence of the matrix Lorentz factor in (7). It reflects the fact that the coset representatives $G(x,\theta)$ and $G^{+}(x^L, \theta^L, \eta^R)$, as seen from the relation (3_+), are not identical but differ by the Lorentz rotation $\exp\{-\frac{im^2}{4}\bar{\theta}\gamma^{\rho}\gamma_5\theta x^{\nu} M_{\nu\rho}\}$. One can be convinced that the interwining property of this matrix:

$$\exp\{\tfrac{i}{2}\widetilde{W}_{+}^{\mu\nu}(G_0, x^L, \theta^L, \eta^R) J_{\mu\nu}\} \exp\{-\tfrac{im^2}{4}\bar{\theta}\gamma^{\rho}\gamma_5\theta x^{\nu} J_{\nu\rho}\} =$$

$$= \exp\{-\tfrac{im^2}{4}\bar{\theta}'\gamma^{\rho}\gamma_5\theta' x^{\nu\prime} J_{\nu\rho}\} \exp\{\tfrac{i}{2} W^{\mu\nu}(G_0, x, \theta) J_{\mu\nu}\}$$

(where $W^{\mu\nu}(G_0, x, \theta)$ is defined by formula (I.18)) guarantees for superfields $\widetilde{\Phi}_K$ defined by (7) the transformation rule (6). The connection (7) is, of course, invertible because x_{μ} and θ_{α} can always be expressed in terms of $x^L_{\mu}, \theta^L_{\alpha}, \eta^R_{\alpha}$ upon inverting eqs. (I.41) and (4).

Covariant derivatives of superfields in the left-handed basis can be found either starting from their expressions (I.26), (I.27) in the symmetric basis and then performing the transformation (7) or directly, with the help of relevant Cartan forms (defined by the decomposition of $(G^{+})^{-1} d\, G^{+}$ in OSp(1,4)-generators). Without going into details of derivation, we quote the result:

$$\hat{\mathcal{D}}^R_{+} = (1 + \tfrac{m}{4}\bar{\eta}^R\eta^R)\frac{\partial}{\partial\bar{\eta}^R} - \tfrac{m}{4}(\sigma^{\mu\nu}\eta^R) J_{\mu\nu} \tag{8}$$

$$\hat{\mathcal{D}}^L_{+} = (1 + \tfrac{m}{2}\bar{\eta}^R\eta^R)\Big[(1 + \tfrac{m}{4}\bar{\theta}^L\theta^L)\frac{\partial}{\partial\bar{\theta}^L} - \tfrac{m}{4}(\sigma^{\mu\nu}\theta^L) J_{\mu\nu}\Big] - i(\gamma^{\nu}\eta^R)\hat{\nabla}^L_{+\nu} \tag{9}$$

$$\begin{aligned}\hat{\nabla}^L_{+\nu} = \tfrac{im}{2}\Big[&(1 + \tfrac{m}{4}\bar{\eta}^R\eta^R)(1 + \tfrac{m}{4}\bar{\theta}^L\theta^L)(\bar{\eta}^R\gamma_{\nu} - m x^L_{\rho}\bar{\theta}^L\sigma^{\rho}{}_{\nu})\frac{\partial}{\partial\bar{\theta}^L} + \\ &+ (1 - \tfrac{m}{4}\bar{\eta}^R\eta^R)(1 - \tfrac{m}{4}\bar{\theta}^L\theta^L)(\bar{\theta}^L\gamma_{\nu} - m x^L_{\rho}\bar{\eta}^R\sigma^{\rho}{}_{\nu})\frac{\partial}{\partial\bar{\eta}^R}\Big] + \\ &+ (1 + \tfrac{m}{4}\bar{\eta}^R\eta^R)(1 - \tfrac{m}{4}\bar{\theta}^L\theta^L)\big[a^{-1}(x^L)\partial^L_{\nu} - i m^2 x^L_{\rho} J^{\rho}{}_{\nu}\big].\end{aligned} \tag{10}$$

In the contraction limit expressions (8)-(10) become covariant derivatives of the usual supersymmetry in the left-handed chiral basis. The form of the derivative $\hat{\mathcal{D}}^R_{+}$ is so simple because it is nothing else than the covariant derivative of the realization of S_- on the coset space $S_-/O(1,3) = e^{i\bar{Q}_-\eta^R}$. Its covariance with respect to the whole supergroup OSp(1,4) follows from the above-mentioned fact that the general transformation law (6) may be obtained from the transformation law for superfields $\widetilde{\Phi}_K(x^L, \theta^L, \eta^R)$ in supergroup S_- (it corresponds to the choice of $\widetilde{W}_{+}^{\mu\nu}(G_0 \in S_-, \eta^R)$) by changing constant parameters of S_- to certain functions of coordinates x^L and θ^L (these functions are still constants relative to η^R-differentiation).

Now it becomes clear what is the origin of the constraints $(I.35_+)$. The condition $(I.34_+)$ in the splitting parametrization takes the form:

$$(\hat{\mathcal{D}}^R_{+}\widetilde{\Phi}(x^L, \theta^L, \eta^R))_K = 0\,. \tag{11_+}$$

Expanding (11_+) in powers of $\eta^R (\eta^R\eta^R\eta^R = 0)$ it can be observed that for all superfields on which the covariant derivative (8) has nonzero matrix part equation (11_+) permits only the trivial solutions $\widetilde{\Phi}_K = 0$. If matrix part is zero equation (11_+) goes over to the condition:

$$\frac{\partial}{\partial\bar{\eta}^R}\widetilde{\Phi}_K(x^L, \theta^L, \eta^R) = 0 \tag{12}$$

which simply means the absence of η^R-dependence. The only class of the Lorentz group representations on which the second term of (8) vanishes is the class $(I.35_+)$. For scalar superfields the matrix part is absent at all, while for superfields with indices it is nullified due to the presence of projectors $\frac{1+i\gamma_5}{2}$ in the generators of representations $(I.35_+)$ (and the algebraic property $(\frac{1-i\gamma_5}{2}\sigma^{\mu\nu})_{\alpha}{}^{\beta}(\frac{1+i\gamma_5}{2}\sigma_{\mu\nu})_{\gamma}{}^{\delta} = 0$). The constraints $(I.35_-)$ have the analogous interpretation (in the basis (1_-)).

The restrictions $(I.35_+)$ can also be understood based on pure group-theoretical considerations. So far as x^L, θ^L and x^R, θ^R are coordinates of the homogeneous spaces $OSp(1,4)/S_-$ and $OSp(1,4)/S_+$, the superfields dependent only on x^L, θ^L or on x^R, θ^R should transform in OSp(1,4) according to representations induced by the corresponding little groups, S_- or S_+. In other words, they should form, in external indices, linear multiplets of these supergroups. In Appendix B we list all linear representations of S_- and S_+ realized in spaces of finite-dimensional representations of the Lorentz group. As follows from our analysis, some Lorentz irreducible multiplet is an invariant space of the supergroup $S_-(S_+)$ only provided it transforms in the Lorentz group according to one of the representations $(I.35_+)((I.35_-))$ (the generator Q_- (Q_+) is zero on such a multiplet). In all other cases several different Lorentz multiplets are needed for composing an irreducible linear multiplet of S_- or S_+. In the superfield language, this effectively comes out as appearance of dependence on $\eta^R(\eta^L)$.

It is instructive to compare the situation with that one taking place in the usual supersymmetry, chiral representations of which are induced in complex invariant spaces of contracted versions of supergroups S_-, S_+ viz. $S_-(m=0)$, $S_+(m=0)$. In the contraction limit, the r.h.s. of (2a) vanishes so that one may set $Q_-(m=0)$ ($Q_+(m=0)$) zero on any given Lorentz multiplet without conflict with the structure relations of $S_-(m=0)$ ($S_+(m=0)$) (this multiplet should be complex, otherwise one arrives at the trivial result $Q(m=0)=0$). Thus, any irreducible complex multiplet of the Lorentz group can be taken as an invariant space of the supergroup $S_-(m=0)$ ($S_+(m=0)$) and hence as a carrier of a certain representation of the whole supergroup realized on cosets over $S_-(m=0)$ or $S_+(m=0)$. This is the reason why in the usual supersymmetry chiral superfields with an arbitrary external Lorentz index are permissible.

Note that the purely chiral representations of the conformal superalgebra are also restricted to the classes $(I.35_+)$ /2/. This seems natural in the light of the property of the conformal superalgebra to be a closure of two superalgebras OSp(1,4) generated by orthogonal combinations of superconformal spinor charges and having the common O(2,3)-subalgebra /3/. Actually, OSp(1,4) plays the crucial role in forming linear representations of the conformal superalgebra: in the forthcoming paper we show that all they can be induced by a simple procedure in invariant spaces of irreducible representations of OSp(1,4).

Now we obtain the OSp(1,4)-invariant integration measures in left- and right-handed chiral superspaces. Performing the variable change (I.41) and (4) in the measure (I.31) we get the invariant measure of the superspace OSp(1,4)/O(1,3) in the parametrization (1_+):

$$\mathcal{D}\mathcal{M} = d^4x^L d^2\theta^L d^2\eta^R \alpha^4(x^L)\left(1+\tfrac{3}{2}m\bar\theta^L\theta^L\right)\left(1+\tfrac{m}{2}\bar\eta^R\eta^R\right) \qquad (13)$$

(this measure could also be obtained straightforwardly as outer product of the relevant Cartan forms). The invariant measure in the subspace x^L, θ^L may now be found by integrating (13) over $1/m\, d^2\eta^R$:

$$\mathcal{D}\mathcal{M}^L = d^4x^L d^2\theta^L \alpha^4(x^L)\left(1+\tfrac{3}{2}m\bar\theta^L\theta^L\right). \qquad (14)$$

The latter coincides with the measure we have derived earlier /8/. The measure in the superspace x^R, θ^R follows from (14)

through involution:

$$\mathcal{D}\mathcal{M}^R = d^4x^R d^2\theta^R \alpha^4(x^R)\left(1+\tfrac{3}{2}m\bar\theta^R\theta^R\right). \qquad (15)$$

We conclude this Section with several comments concerning the relation of the group structure presented here to the fundamental chiral structure of supergravity revealed recently by Ogievetsky and Sokatchev /4/.

It is not hard to see that the realization of OSp(1,4) in the symmetric basis considered in I can be embedded into a very general supergroup consisting both of arbitrary translations of x, θ and gauge Lorentz rotations of external superfield indices. However, the group really relevant to supergravity is in fact much smaller: it is given by a direct product of general coordinate groups in conjugated chiral superspaces x^L, θ^L and x^R, θ^R /4/. This group is realized so that all local Lorentz rotations appear in the theory not independently but turn out to be induced by transformations of superspace coordinates (like Lorentz rotations in OSp(1,4)-transformation rules (I.19), (6)). The real part of x^L and $x^R=(x^L)^*$ is identified in the Ogievetsky-Sokatchev approach with the usual space-time coordinate while the imaginary part is postulated to be a function of the remaining variables. The latter is the axial superfield, the fundamental object, in terms of which all geometrical characteristics of superspace (supercurvature, supertorsion, etc.) and all transformation laws can be expressed. It is interesting to compare our results deduced following a quite different procedure with this general picture.

Our starting point will be the observation that the realizations of OSp(1,4) given by the rule (I.42) and its conjugate lie in the above-mentioned general coordinate groups (just as chiral realizations of usual supersymmetry). With this in mind, we may, step by step, establish the correspondence with the Ogievetsky-Sokatchev formalism. Like them we might choose coordinates (x^L, θ^L, θ^R), (x^R, θ^R, θ^L) to represent left- and right-handed splitting bases instead of using for this purpose coordinate sets (x^L, θ^L, η^R), (x^R, θ^R, η^L). Indeed, eq.(4) indicates that η^R, η^L are canonically related to θ^R, θ^L. Likewise, we might associate with the symmetric basis the coordinates $\tilde{x}_\mu = \frac{1}{2}(x^L_\mu + x^R_\mu)$, $\tilde\theta = \theta^L + \theta^R$ instead of x_μ, θ_α. The explicit form of the equivalence mapping between these two sets of variables can immediately be found with the help of the relations (I.41) and their right-handed counterparts:

$$\tilde{x}_\mu = [1 + \frac{m^2}{16\alpha(x)}(\bar\theta\theta)^2]x_\mu$$
$$\tilde\theta = \theta - \frac{m}{2}\bar\theta\theta(1 + \frac{3}{4}imx\gamma)\theta. \tag{16}$$

In terms of $\tilde{x}_\mu, \tilde\theta_\alpha$ the transitions to the shifted bases are extremely simple and have almost the same form as in the usual supersymmetry

$$\begin{pmatrix} x^L_\mu \\ x^R_\mu \end{pmatrix} = \begin{pmatrix} \tilde{x}_\mu - \frac{1}{4}\bar{\tilde\theta}\gamma_\mu\gamma_5\tilde\theta\alpha^{-1}(\tilde{x}) \\ \tilde{x}_\mu + \frac{1}{4}\bar{\tilde\theta}\gamma_\mu\gamma_5\tilde\theta\alpha^{-1}(\tilde{x}) \end{pmatrix}, \quad \begin{pmatrix} \theta^L \\ \theta^R \end{pmatrix} = \begin{pmatrix} \tilde\theta_+ \\ \tilde\theta_- \end{pmatrix} \tag{17}$$

So, only the shift of the boson coordinate is essential and unremovable whereas shifts of Grassmann variables can be absorbed into the equivalence redefinition of coordinates. This agrees with the basic concepts of Ogievetsky and Sokatchev[1]. It is clear now what plays the role of "axial superfield" for the superspace OSp(1,4)/O(1,3). Namely, this is $\frac{1}{2i}(x^L_\mu - x^R_\mu) = \frac{i}{4}\bar{\tilde\theta}\gamma_\mu\gamma_5\tilde\theta\alpha^{-1}(\tilde{x}) = \frac{i}{4}\bar\theta\gamma_\mu\gamma_5\theta\alpha^{-1}(x)$. It should be expected that all OSp(1,4)-covariant objects and OSp(1,4)-transformation rules found in the present paper admit reexpression in terms of this fundamental geometrical characteristic (and its derivatives).

3. We construct now OSp(1,4)-analogs of the simplest theories with global Poincare supersymmetry: Wess-Zumino model/6/ and supersymmetric Yang-Mills theory/7/.

3a. The OSp(1,4)-invariant superfield action for the self-interacting scalar OSp(1,4)-multiplet with the overall mass M and dimensionless coupling constant g can be written as

$$S = S_K + S_M + S_g = \int \mathcal{D}m\, \Phi_+(x,\theta)\Phi_-(x,\theta) + \frac{1}{2}M[\int\mathcal{D}m^L\, T^{+2}(x^L,\theta^L) + \int\mathcal{D}m^R\, T^{-2}(x^R,\theta^R)] + \frac{\sqrt2}{3}g[\int\mathcal{D}m^L\, T^{+3}(x^L,\theta^L) + \int\mathcal{D}m^R\, T^{-3}(x^R,\theta^R)]. \tag{18}$$

The invariant integration measures $\mathcal{D}m$, $\mathcal{D}m^L$ and $\mathcal{D}m^R$ are defined by formulas (I.31), (14) and (15). Chiral superfields in the symmetric basis, Φ_+ and $\Phi_- = (\Phi_+)^*$ are related to "truncated" superfields T^+ and $T^- = (T^+)^*$ as in (I.37$_\pm$).

1) We suspect that coordinates x_μ, θ_α (which are both shifted when transitions to chiral bases are performed) correspond to the Siegel's formulation of superspace supergravity /5/. This formulation seems to be basically equivalent to the Ogievetsky-Sokatchev approach.

Using the rules for changing variables in Grassmann integrals and the connection (I.41) together with its conjugate we have checked that the second and third pieces in (18) do not change their form under replacements $x^L_\mu, x^R_\mu \to x_\mu$, $\theta^L \to \theta_+$, $\theta^R \to \theta_-$. Then, integrating (18) over $d\theta$, going to real components A, B, F, G:

$$A_\pm = \frac{1}{\sqrt2}(A \pm iB), \quad F_\pm = \frac{1}{\sqrt2}(F \pm iG)$$

and finally eliminating the auxiliary fields F and G by their equations of motion

$$F = -(m+M)A - g(A^2 - B^2), \quad G = (M-m)B + 2gAB \tag{19}$$

we are left with the action which includes only the physical components $\Psi(x), A(x), B(x)$ and is expressed solely in terms of anti de Sitter space:

$$S = \int \mathcal{D}m_s \{\frac{1}{2}(\nabla^\mu A\nabla_\mu A + \nabla^\mu B\nabla_\mu B + i\bar\Psi\gamma^\mu\nabla_\mu\Psi) - V_M(A,B,\Psi)\}. \tag{20}$$

Here ∇_μ and $\mathcal{D}m_s$ are, resp., O(2,3)-covariant derivative and O(2,3)-invariant integration measure for anti de Sitter space given by (I.12) and (I.15) (in fact, the matrix part of $\nabla_\mu\Psi$ makes no contribution to (20) because of the Majorana spinor property $\bar\Psi(x)\gamma_\mu\Psi(x) = 0$). The potential $V_M(A,B,\Psi)$ has the form

$$V_M(A,B,\Psi) = \frac{1}{2}(M+m)(M-2m)A^2 + \frac{1}{2}(M-m)(M+2m)B^2 + \frac{1}{2}M\bar\Psi\Psi + \frac{g^2}{2}(A^2+B^2)^2 + gMA(A^2+B^2) + g\bar\Psi(A - B\gamma_5)\Psi. \tag{21}$$

Expressions (20) and (21) coincide with those we have obtained earlier by a different method/8/. In the contraction limit they reduce to the standard Wess-Zumino/6/ action. For more details see/8/ where the vacuum structure associated with the potential (21) was studied.

3b. OSp(1,4)-invariant gauge theories are constructed analogously to the usual supersymmetric ones.

We prefer to work in the left-handed basis (1$_+$). Let I_i be matrices of generators of some group of internal symmetry G. Introduce the real Lie algebra valued gauge superfield:

$$V(x^L,\theta^L,\eta^R) = V^*(x^L,\theta^L,\eta^R) = V^i(x^L,\theta^L,\eta^R)I_i \tag{22}$$

and postulate for it the following law of local G-transformations:

$$e^{2gV(x^L,\theta^L,\eta^R)} = e^{-i\Lambda^+(x^L,\theta^L,\eta^R)}e^{2gV(x^L,\theta^L,\eta^R)}e^{i\Lambda(x^L,\theta^L)}, \tag{23}$$

where g is a constant and Λ, Λ^{+} are two conjugated superfunctions with values in the same algebra and subject to the conditions:

$$\hat{\mathcal{D}}^{R}_{+}\Lambda = \hat{\mathcal{D}}^{L}_{+}\Lambda^{+} = 0 \tag{24}$$

($\hat{\mathcal{D}}^{R}_{+}$ and $\hat{\mathcal{D}}^{L}_{+}$ are defined by formulae (8) and (9)). Then the left handed (with respect to the Lorentz index) spinor superfield $e^{-2gV}\hat{\mathcal{D}}^{L}_{+}e^{2gV}$ transforms in G as

$$e^{-2gV}\hat{\mathcal{D}}^{L}_{+}e^{2gV} \to e^{-i\Lambda}e^{-2gV}\hat{\mathcal{D}}^{L}_{+}e^{2gV}e^{i\Lambda} + e^{-i\Lambda}\hat{\mathcal{D}}^{L}_{+}e^{i\Lambda} . \tag{25}$$

A complication in comparison with the usual supersymmetry arises only at the stage of construction of covariant superfield strengths. The operator $\bar{\hat{\mathcal{D}}}^{R}_{+}\hat{\mathcal{D}}^{R}_{+}$ being applied to the superfield (25), does not produce the covariant quantity because in the OSp(1,4)-case it does not annihilate the inhomogeneous term in (25). The correct generalization of the standard procedure implies the use of the projection operator

$$\Pi^{(1/2,0)}_{-} = -\frac{1}{2m}(\bar{\hat{\mathcal{D}}}^{R}_{+}\hat{\mathcal{D}}^{R}_{+} - 2m) \tag{26}$$

which singles out of the left-handed spinor OSp(1,4)-superfield the pure chiral part dependent only on x^{L} and θ^{L} (see Appendix B). As is shown in Appendix B, such a superfield divides in general into two pieces each closed under the action of OSp(1,4). One of them is a pure chiral superfield, the other a non-chiral superfield, F- and A- components in the expansion of which in η^{R} are related as $F_{\alpha}(x^{L},\theta^{L}) = -m A_{\alpha}(x^{L},\theta^{L})$ (formulae (B.15), (B.14)). With making use of the explicit form (9) for the covariant derivative $\hat{\mathcal{D}}^{L}_{+}$ it can be established that the inhomogeneous term in (25) is just a superfield of the second type. Therefore the action of the projection operator (26) on that term gives zero[2]. As a result, the left-handed chiral spinor superfield:

$$W_{+\alpha} = -\left[2m\Pi^{(1/2,0)}_{-}e^{-2gV}\hat{\mathcal{D}}^{L}_{+}e^{2gV}\right]_{\alpha} = (\bar{\hat{\mathcal{D}}}^{R}_{+}\hat{\mathcal{D}}^{R}_{+} - 2m)_{\alpha}^{\ \beta}e^{-2gV}\hat{\mathcal{D}}^{L}_{+\beta}e^{2gV} \tag{27}$$

transforms in G homogeneously

$$W_{+\alpha} \to e^{-i\Lambda}W_{+\alpha}e^{i\Lambda} \tag{28}$$

and can serve as the covariant strength. In the limit $m \to 0$ it becomes the covariant superfield strength of the conventional supersymmetric Yang-Mills theory.

The invariant action for the gauge superfield is set up in the standard manner

2) This fact can be verified straightforwardly, using the algebra of covariant derivatives (I.28).

$$S = \frac{1}{g^{2}}\int \mathcal{D}\mathcal{M}^{L}\,\mathrm{Tr}(\overline{W}_{+}W_{+}) + h.c. \tag{29}$$

and in the Wess-Zumino gauge has the form

$$S = \int d^{4}x\, a^{4}(x)\,\mathrm{Tr}\left[-\frac{1}{4}\widetilde{F}_{\mu\nu}\widetilde{F}^{\mu\nu} + \frac{i}{2}\bar{\lambda}\gamma^{\mu}\widetilde{\mathcal{D}}_{\mu}\lambda + \frac{1}{2}D^{2}\right], \tag{30}$$

where

$$\widetilde{F}_{\mu\nu} = \nabla_{\mu}v_{\nu} - \nabla_{\nu}v_{\mu} + ig[v_{\mu}, v_{\nu}] \tag{31}$$

$$\widetilde{\mathcal{D}}_{\mu}\lambda = \nabla_{\mu}\lambda + ig[v_{\mu},\lambda] . \tag{32}$$

The action (30) describes the Yang-Mills field v^{i}_{μ} in anti de Sitter space minimally coupled to the massless Majorana spinor λ^{i}_{α} belonging to the regular representation of the group G . Transformations of OSp(1,4)-symmetry in the Wess-Zumino gauge are as follows

$$\delta v_{\mu} = i\bar{\beta}\gamma_{\mu}\lambda \ , \quad \delta D = i\bar{\beta}\gamma^{\mu}\gamma_{5}\widetilde{\mathcal{D}}_{\mu}\lambda$$
$$\delta\lambda = -\frac{i}{2}\sigma^{\mu\nu}\beta\widetilde{F}_{\mu\nu} + \gamma_{5}\beta D , \tag{33}$$

where $\beta = (1 - i m x^{\mu}\gamma_{\mu})\sqrt{\frac{a(x)}{2}}\,\epsilon \equiv \Lambda^{-1}(x)\epsilon$.

We emphasize that the action (30) in itself produces no new consequences since upon the Weyl transformation:

$$v_{\mu}(x), \lambda(x), D(x) \to v'_{\mu}(x) = a(x)v_{\mu}(x),\ \lambda'(x) = a^{3/2}(x)\lambda(x),\ D'(x) = a^{2}(x)D(x) \tag{34}$$

it reduces to the ordinary action of the corresponding supersymmetric Yang-Mills theory in Minkowski space. This is because the action (30) is conformally invariant and, hence, Weyl-covariant. Moreover, the component transformations (34) can be extended to the Weyl superfield transformation which takes the action (29) into the superfield action of the related gauge theory in the usual superspace (for brevity, we do not give it explicitly; it is similar to Weyl transformation of chiral superfields defined in /8/). This reflects the generalized Weyl covariance of supersymmetric Yang-Mills theories caused by their superconformal invariance.

A nontrivial novel theory may be set up, e.g., by coupling the gauge superfield to massive scalar OSp(1,4)-multiplet belonging to a unitary representation of group G (such interactions are introduced in the same way as in the usual supersymmetry). Models of this type possess no superconformal invariance (and Weyl covariance) and therefore allow no transition to Minkowski space. We intend to explore them in the future.

For completness, we finally quote general expressions for component Lagrangian densities belonging to scalar and vector OSp(1,4)-multiplets:

$$\mathcal{L}_{I}(x) = a^4(x)[F(x) + 3m\, A(x)] \tag{35}$$

$$\mathcal{L}_{II}(x) = a^4(x)[D(x) + 12m^2 A(x) + 12m\, F(x)] \tag{36}$$

. $\mathcal{L}_{I}$ and $\mathcal{L}_{II}$ have the positive parity and change by a divergence under OSp(1,4)-transformations.

4. In the present and preceding[1] papers we have described the superfield approach to supersymmetry in anti de Sitter space and constructed the simple linear globally OSp(1,4)-invariant models. We hope that the methods elaborated here can be generalized to accomodate the case of OSp(N,4)-supersymmetry. The construction and study of models with global OSp(N,4)-symmetry is an interesting and urgent task since such models describe the "flat" limit of O(N)-extended supergravity a self-contained superfield formulation of which is as yet unknown.

Our approach is based on the consistent application of group-theoretical methods. However, as has been already explained partly in the text, all the relevant relations could in principle be obtained by a limiting procedure from more general local theories. For instance, the basic elements of O(2,3)-formalism constructed in paper[1] can be deduced in a different manner, by noting that the space O(2,3)/O(1,3) is a particular solution of Einstein's equations with the negative cosmological term and substituting the relevant background metric into general relativity formulae. Likewise, the superspace OSp(1,4)/O(1,3) (as well as OSp(1,4)/S_- and OSp(1,4)S_+) is expected to be a particular solution of equations of superfield supergravity[4,5,9]. Therefore, all relations of OSp(1,4)-covariant formalism should result from general formulae of a self-contained superfield supergravity on inserting the corresponding particular values of gauge superfields. To verify this, we might proceed, say, from the explicit form of OSp(1,4)-solution for the Ogievetsky-Sokatchev axial superfield found in Sec. 2. But, as a closed formulation of OSp(1,4)-supergravity (as well as of conformal one) in terms of the axial superfield (an invariant action, superfield equations of motion, etc.) is not constructed for the time being, it is simpler to work within component supergravity[10] which seems to be equivalent to a certain gauge of the complete superfield theory. A particular set of gauge fields of component supergravity related to the superspace OSp(1,4)/O(1,3) includes (in our notation) O(2,3)- de Sitter solution for the vierbein $e^a_\mu = \delta^a_\mu\, a(x)$ and the constant solution for the auxiliary field $S = 3\sqrt{2}\, m$ (all other gauge fields, ψ_μ, A_μ, P take zero values). Inserting this set, e.g., in the general transformation law for a scalar multiplet of local supersymmetry[11] with the restriction to OSp(1,4)-transformations only, one recovers the transformation law (I.43). Analogously, starting from general couplings of a scalar multiplet with supergravity[12], it is possible to regain the action considered in Sec. 3. The advantage of the approach we have developed is that it allows one to deduce all the relations of OSp(1,4)-supersymmetry on the basis of OSp(1,4)-superalgebra (I.1) alone and provides a deep understanding of the group structure of this important second global limit of local supersymmetry. Moreover, it may serve as a useful guide in constructing a closed superfield formulation of OSp(1,4)-supergravity.

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Appendix A. OSp(1,4)-generators in nonsymmetric parametrization.

Taking into account that the left- and right-handed splitting bases (1_+), (1_-) are related through involution, it is sufficient to know expressions for generators in one of them, say in the basis (1_+):

$$M_{\mu\nu} = i(x^L_\mu \partial^L_\nu - x^L_\nu \partial^L_\mu) + \frac{1}{2}\bar\theta^L \sigma_{\mu\nu}\frac{\partial}{\partial\bar\theta^L} + M^{S-}_{\mu\nu} \tag{A.1}$$

$$Q = -a^{-1}(x^L)\Lambda(x^L)\gamma^\mu\theta^L\partial^L_\mu + i\Lambda(x^L)\left[1 - \frac{m}{2}\bar\theta^L\theta^L\left(1 - \frac{3}{2}im\, x^L_\mu\gamma^\mu\right)\right]\frac{\partial}{\partial\bar\theta^L} + \tag{A.2}$$
$$+\left(1 + \frac{m}{2}\bar\theta^L\theta^L\right)\Lambda(x^L)Q^{S-} + \frac{im}{4}\Lambda(x^L)(\sigma_{\mu\nu} + 4m\, x^L_\mu\gamma_\nu)\theta^L M^{S-\mu\nu}$$

$$R_\mu = i\left(\frac{1 - m^2 x^{L2}}{2}\delta^\nu_\mu + m^2 x^L_\mu x^{L\nu}\right)\partial^L_\nu + \frac{m^2}{2}x^L_\nu\bar\theta^L\sigma_\mu{}^\nu\frac{\partial}{\partial\bar\theta^L} + m^2 x^{L\nu}M^{S-}_{\mu\nu}. \tag{A.3}$$

Here

$$M^{S-}_{\mu\nu} = \frac{1}{2}\bar\eta^R\sigma_{\mu\nu}\frac{\partial}{\partial\bar\eta^R} + J_{\mu\nu} \tag{A.4}$$

$$Q^{S-} = i\left(1 - \frac{m}{2}\bar\eta^R\eta^R\right)\frac{\partial}{\partial\bar\eta^R} + \frac{im}{4}\sigma^{\mu\nu}\eta^R J_{\mu\nu} \tag{A.5}$$

are generators of supergroup S_- realized in the coset space $S_-/O(1,3)$.

Appendix B. Irreducible spaces of supergroups S_- and S_+.

In Sec. 2 we have remarked that linear representations of the supergroup OSp(1,4) in the parametrizations (1_+), (1_-) can be regarded as induced in invariant spaces of its supersubgroups S_-, S_+, respectively (with O(1,3) as the structure group). In other words, given, e.g., a space closed under the action of S_- [3]:

$$\Phi_\kappa(\eta^R) = P_\kappa + \bar{\eta}^R N_\kappa + \bar{\eta}^R \eta^R B_\kappa \qquad (B.1)$$

(κ being the external Lorentz index), one immediately arrives at the space invariant under the whole supergroup OSp(1,4) (and, hence, carrying its some linear representation) simply replacing constant coefficients in (B.1) by functions over the homogeneous space $OSp(1,4)/S_-$:

$$P_\kappa, N_\kappa, B_\kappa \to P_\kappa(x^L, \theta^L), N_\kappa(x^L, \theta^L), B_\kappa(x^L, \theta^L) \,. \qquad (B.2)$$

So, the problem of implementing all inequivalent linear representations of OSp(1,4) in the left-handed basis (1_+) reduces to finding all inequivalent irreducible invariant spaces of supergroup S_- of the type (B.1) (the analogous statement, with the change $S_- \to S_+$, holds also for the right-handed basis). Henceforth, we restrict our consideration to the case of S_- keeping in mind that invariant spaces of S_+ can be obtained from those of S_- by involution (just as S_+ itself from S_-).

To obtain all the irreducible S_--invariant spaces of type (B.1) it is sufficient to find the spectrum of two Casimir operators of S_-:

$$\hat{C}_1 = M^{S-}_{\mu\nu} M^{S-\mu\nu} - \frac{1}{m} \bar{Q}^{S-} Q^{S-}, \quad \hat{C}_2 = M^{S-}_{\mu\nu} M^{S-\mu\nu} + \frac{i}{2} \varepsilon^{\mu\nu\rho\lambda} M^{S-}_{\mu\nu} M^{S-}_{\rho\lambda}, \qquad (B.3)$$

where $M^{S-}_{\mu\nu}$, Q^{S-} -are given by (A.4) and (A.5). Inserting in (B.3) the explicit forms for generators and using (8) we get:

$$(\hat{C}_1)_\kappa{}^\ell = \frac{1}{m} (\bar{\hat{\mathcal{D}}}{}^R_+ \hat{\mathcal{D}}^R_+)_\kappa{}^\ell + (\hat{K}_1)_\kappa{}^\ell, \quad (\hat{C}_2)_\kappa{}^\ell = (\hat{K}_1 + \hat{K}_2)_\kappa{}^\ell \,, \qquad (B.4)$$

where $(\bar{\hat{\mathcal{D}}}{}^R_+ \hat{\mathcal{D}}^R_+)_\kappa{}^\ell = (\bar{\hat{\mathcal{D}}}{}^{R\beta}_+)_{\beta\kappa}{}^{\gamma m} (\hat{\mathcal{D}}^R_{+\gamma})_m{}^\ell$ and $\hat{K}_1$, $\hat{K}_2$ are purely matrix parts of the Casimir operators of the Lorentz group;

$$\hat{K}_1 = J_{\mu\nu} J^{\mu\nu} \,, \quad \hat{K}_2 = \frac{i}{2} \varepsilon^{\mu\nu\rho\lambda} J_{\mu\nu} J_{\rho\lambda} \,. \qquad (B.5)$$

We suppose that external indices of superfields (B.1) are rotated by the finite-dimensional irreducible representations of the Lorentz group, $D^{(p,q)}$, where p and q are positive integer and

3) We apply here terms "invariant space" and "superfield" on equal footing.

half-integer. For such representations the spectrum of operators (B.5) has the form:

$$\hat{K}_1^{(p,q)} = 4p(p+1) + 4q(q+1), \quad \hat{K}_2^{(p,q)} = 4p(p+1) - 4q(q+1) \,. \qquad (B.6)$$

Now, the use of the simple identity for the covariant derivative $\hat{\mathcal{D}}^R_+$:

$$(\bar{\hat{\mathcal{D}}}{}^R_+ \hat{\mathcal{D}}^R_+)^2 = \frac{m^2}{2} (\hat{K}_1 - \hat{K}_2) + 2m \bar{\hat{\mathcal{D}}}{}^R_+ \hat{\mathcal{D}}^R_+$$

yields the spectrum of operator $\hat{C}_1 - \hat{K}_1$:

$$(\hat{C}_1 - \hat{K}_1)^{(p,q)}_{(\pm)} = 1 \pm (1 + 2q) \,. \qquad (B.7)$$

Finally, the total spectrum of operators $\hat{C}_1$ and $\hat{C}_2$ on superfields (B.1) is given by:

$$\hat{C}^{(p,q)}_{1\,(\pm)} = 4p(p+1) + 4q(q+1) + 1 \pm (1+2q), \quad \hat{C}_2^{(p,q)} = 8p(p+1) \,. \qquad (B.8)$$

It is seen that the eigenvalue of $\hat{C}_2$ is uniquely determined by the superfield external index, κ. At the same time, to each fixed q there correspond two different eigenvalues of $\hat{C}_1$ distinguished by signes (+,-). Consequently, each superfield (B.1) with the fixed Lorentz index (i.e. p and q fixed) contains two irreducible inequivalent subspaces of supergroup S_-. The normalized projection operators which single out these subspaces are constructed in a standard manner:

$$\Pi^{(p,q)}_\pm = \frac{\hat{C}_1 - \hat{C}^{(p,q)}_{1\,(\mp)}}{\hat{C}^{(p,q)}_{1\,(\pm)} - \hat{C}^{(p,q)}_{1\,(\mp)}} = \pm \frac{\bar{\hat{\mathcal{D}}}{}^R_+ \hat{\mathcal{D}}^R_+ - [1 \mp (1+2q)]m}{2m(1+2q)} \qquad (B.9)$$

$$(\Pi^{(p,q)}_+ + \Pi^{(p,q)}_- = 1, \ \Pi^{(p,q)}_+ \Pi^{(p,q)}_- = \Pi^{(p,q)}_- \Pi^{(p,q)}_+ = 0) \,.$$

With the help of these operators irreducible parts of a superfield $\Phi_\kappa(\eta^R)$ are expressed as follows

$$\tilde{\Phi}^\pm_\kappa(\eta^R) = (\Pi_\pm \Phi(\eta^R))_\kappa = P^\pm_\kappa + \bar{\eta}^R (Y^\pm N)_\kappa - \frac{m}{4} \bar{\eta}^R \eta^R [1 \pm (1+2q)] P^\pm_\kappa \,, \qquad (B.10)$$

where

$$P^\pm_\kappa = \mp \frac{m[1 \mp (1+2q)] P_\kappa + 4B_\kappa}{2m(1+2q)} \qquad (B.11)$$

and $Y^\pm$ are the operators projecting out of $N_{\alpha\kappa}$ Lorentz irreducible pieces:

$$Y^\pm = \pm \frac{1 - i\gamma_5}{2} \frac{1}{2(1+2q)} \left[1 \pm (1+2q) + \frac{1}{2} \sigma^{\mu\nu} J_{\mu\nu} \right] . \qquad (B.12)$$

From (B.10) and (B.12) it follows that the η^R-independent invariant spaces do exist only for sign minus in (B.10). Indeed, only in this case we may nullify the coefficient for $\bar{\eta}^R \eta^R$ (by putting there $q=0$). For representations $D^{(p,0)}$ the generators $J_{\mu\nu}$ contain the projector $\frac{1+i\gamma_5}{2}$, therefore, due to the trivial algebraic property $\left(\frac{1-i\gamma_5}{2}\sigma^{\mu\nu}\right)_\alpha^{\ \beta}\left(\frac{1+i\gamma_5}{2}\sigma_{\mu\nu}\right)_\delta^{\ \gamma}=0$, the component $(Y^- N)_{\alpha K}$ is zero automatically. Analogously, for supergroup S_+ the η^L -independent invariant functions exist provided their external indices belong to representations $D^{(0,q)}$ of the Lorentz group. These results explain why OSp(1,4) has no left-handed chiral superfields with $q\neq 0$ and right-handed chiral ones with $p\neq 0$ (see Sec . 2). The projection operator which singles out the η^R -independent invariant spaces of supergroup S_- has the very simple form:

$$\Pi_-^{(p,0)}=\frac{2m-\overline{\hat{\mathcal{D}}_+^R}\hat{\mathcal{D}}_+^R}{2m}. \qquad (B.13)$$

As an example, we list here the simplest irreducible spaces of supergroup S_-.

a. Scalar superfields $(p=q=0)$.

$$\hat{C}_2^{(0,0)}=0,\begin{cases}\hat{C}_{1(+)}^{(0,0)}=2, & \tilde{\Phi}^+(\eta^R)=P^+ +\bar{\eta}^R N^+ -\frac{m}{2}\bar{\eta}^R\eta^R P^+\\ \hat{C}_{1(-)}^{(0,0)}=0, & \tilde{\Phi}^-(\eta^R)=P^-\end{cases}$$

b. Spinor superfields ($p=0$, $q=\frac{1}{2}$ or $p=\frac{1}{2}$, $q=0$).

$$\hat{C}_2^{(0,1/2)}=0,\begin{cases}\hat{C}_{1(+)}^{(0,1/2)}=6, & \tilde{\Phi}_\alpha^+(\eta^R)=P_\alpha^+ +N_{\mu\nu}^+(\sigma^{\mu\nu}\eta^R)_\alpha-\frac{3}{4}m\bar{\eta}^R\eta^R P_\alpha^+\\ \hat{C}_{1(-)}^{(0,1/2)}=2, & \tilde{\Phi}_\alpha^-(\eta^R)=P_\alpha^- +N^-\eta_\alpha^R+\frac{m}{4}\bar{\eta}^R\eta^R P_\alpha^-\end{cases}$$

$$\hat{C}_2^{(1/2,0)}=6,\begin{cases}\hat{C}_{1(+)}^{(1/2,0)}=5, & \tilde{\Phi}_\alpha'^+(\eta^R)=P_\alpha'^+ +N_\mu^+(\gamma^\mu\eta^R)_\alpha-\frac{m}{2}\bar{\eta}^R\eta^R P_\alpha'^+ \qquad (B.14)\\ \hat{C}_{1(-)}^{(1/2,0)}=3, & \tilde{\Phi}_\alpha'^-(\eta^R)=P_\alpha'^- \qquad (B.15)\end{cases}$$

c. Vector superfields ($p=\frac{1}{2}$, $q=\frac{1}{2}$).

$$\hat{C}_2^{(1/2,1/2)}=6,\begin{cases}\hat{C}_{1(+)}^{(1/2,1/2)}=9, & \tilde{\Phi}_\mu^+(\eta^R)=P_\mu^+ +\bar{N}_\nu^+(\delta_\mu^\nu-\frac{1}{4}\gamma^\nu\gamma_\mu)\eta^R-\frac{3}{4}m\bar{\eta}^R\eta^R P_\mu^+\\ \hat{C}_{1(-)}^{(1/2,1/2)}=5, & \tilde{\Phi}_\mu^-(\eta^R)=P_\mu^- +\bar{N}^-\gamma_\mu\eta^R+\frac{m}{4}\bar{\eta}^R\eta^R P_\mu^- .\end{cases}$$

The boson components of all these superfields have the structure $S+iP$, $V_\mu+iA_\mu$, $N_{\mu\nu}+iN_{\mu\nu}^*$ where $S, P, V_\mu, A_\mu, N_{\mu\nu}$ and $N_{\mu\nu}^*$ are, resp., scalar, pseudoscalar, vector, axial vector, antisymmetric tensor and its dual, all real.

Note that for p fixed (hence, $\hat{C}_2$ fixed) there always exist two different values of q to which the same eigenvalue of the Casimir operator $\hat{C}_1$ corresponds. These q are shifted by 1/2.

By the Schur lemma two such representations of the supergroup S_- are equivalent. Thus, the scalar superfield $\tilde{\Phi}^+(\eta^R)$ from the above set is equivalent to the spinor one $\tilde{\Phi}_\alpha^-(\eta^R)$ that may easily be verified by acting on $\tilde{\Phi}^+$ by the covariant derivative $\hat{\mathcal{D}}_+^R$ ($P^+\sim N^-$, $N_\alpha^+\sim P_\alpha^-$). Analogously, one may be convinced of that the spinor superfield $\tilde{\Phi}_\alpha'^+(\eta^R)$ is equivalent to the vector superfield $\tilde{\Phi}_\mu^-(\eta^R)$.

References

1. Ivanov E.A., Sorin A.S., preprint JINR E2-12363, 1979, Dubna.
2. Aneva B.L., Mikhov S.G., Stoyanov D.T., Teor.Mat.Fiz., 1977, 31, p. 177.
3. Ivanov E.A., Sorin A.S., preprint JINR E2-11610, 1978, Dubna; Teor. mat. fiz. 1979, 39, p. 172.
4. Ogievetsky V.I., Sokatchev E., Nucl.Phys., 1977, B124, p. 309; Phys.Lett., 1978, B79, p. 222; Yad. fiz., 1978, 28, p. 1633; Talk presented at the Seminar on quantum gravity, Moscow, 1978.
5. Siegel W., Gates S.J., Nucl.Phys., 1979, B147, p. 77.
6. Wess J., Zumino B., Nucl.Phys., 1974, B70, p. 39; Phys.Lett., 1974, B49, p.52.
7. Ferrara S., Zumino B., Nucl.Phys., 1974, B79, p. 413.
8. Ivanov E.A., Sorin A.S., preprint JINR, E2-12331, 1979, Dubna.
9. Wess J., Zumino B., Phys.Lett., 1977, B66, p. 361; 1978, B79, p. 394.
10. Ferrara S., Grisaru M.T., P.van Nieuwenhuizen, Nucl.Phys., 1978, B138, p. 430.
11. Ferrara S., P. van Nieuwenhuizen, Phys.Lett., 1978, B76, p. 404.
12. Cremmer E., Julia B., Scherk J., Ferrara S., Girardello L., P. van Nieuwenhuizen, Phys.Lett., 1978, B79, 231; Nucl.Phys., 1979, B147, p. 105.

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