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E2 - 12363

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* Dnepropetrovsk State University.

Иванов Е.А., Сорин А.С. E2 - 12363

Суперполевой подход к суперсимметрии в пространстве анти де Ситтера

Развит суперполевой подход к глобальной суперсимметрии в пространстве анти де Ситтера. Установлены общие трансформационные законы для OSp(1,4) -суперполей и построены все основные элементы OSp(1,4) -ковариантного формализма в симметричном базисе, такие, как ковариантные производные, инвариантная мера интегрирования в суперпространстве OSp(1,4)/O(1,3) и т.д. Исследуются вопросы приводимости OSp(1,4) -суперполей и найдена реализация OSp(1,4) в левом и правом киральных суперпространствах.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1979

Ivanov E.A., Sorin A.S. E2 - 12363

Superfield Approach to Anti de Sitter Supersymmetry

A self-contained superfield approach to global supersymmetry in anti de Sitter space (OSp(1,4)) is developed. General transformation laws for OSp(1,4) -superfields are established, and all basic elements of the OSp(1,4)-covariant formalism in the real basis, such as covariant superfield derivatives, invariant integration measure over the superspace OSp(1,4)/O(1,3), etc., are explicitly given. We analyse also the reducibility questions and find realizations of OSp(1,4) in the left- and right-handed chiral superspaces.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1979

1. Currently a good deal of attention is being paid to the orthosymplectic supergroup OSp(1,4), the minimal extension of the group O(2,3) ($\sim$Sp(4)) by Majorana spinor generators /1-8/.

There exist obvious indications that this supergroup has strong relevance to the dynamics of supersymmetric theories. For instance, Deser and Zumino /2/ have argued that spontaneously broken supergravity should be constructed as a theory of the spontaneously broken local OSp(1,4)-symmetry (OSp(N,4) for the O(N)-extended supergravity). With such a construction, it becomes possible to remove the unwanted cosmological term arising due to the super-Higgs effect (through cancellation with a similar term coming from the pure gauge supergravity Lagrangian) and simultaneously to adjust the reasonable order of the mass splitting between bosons and fermions. An analogous approach to spontaneously broken supergravity was developed on the basis of the vierbein formulation of OSp(1,4)-symmetry /4-6/.

An additional evidence in favour of significance of OSp(1,4) is associated with its role as a subgroup in the Wess-Zumino /9/ conformal supergroup. In fact, the conformal supergroup is a closure of two its different graded subgroups OSp(1,4) with the common O(2,3)-subgroup /7/. We have shown in paper /7/ that one of these OSp(1,4) is the stability group of classical instanton-like solutions of the simplest superconformal-invariant theory, the massless Wess-Zumino model, i.e., it plays there the same role as the group O(2,3) in the massless φ^4 -theory /10/. The other OSp(1,4) is spontaneously broken on these solutions to O(2,3)-symmetry. By analogy, the OSp(1,4)-structure of spontaneously broken supergravity may be thought to emerge due to a similar mechanism /7,8/. Note also that the Euclidean analog of OSp(1,4), the extension of the group O(5) by Dirac spinor generators, may happen to be the stability group of generalized bosonic-fermionic instantons in Euclidean supersymmetric gauge theories (like O(5) in the usual Yang-Mills case[11]).

Taking into account all this, it seems of real importance to construct and analyze theories with global OSp(1,4)-invariance, i.e., supersymmetric theories in anti de Sitter space $\sim O(2,3)/O(1,3)$. The first nontrivial theory of this type, non-linear realization of the OSp(1,4)-symmetry, has recently been considered by Zumino [3]. He has found in particular, that the relevant Goldstone fermion possesses the mass, which is twice the inverse radius of anti de Sitter space. This result was reproduced in another context in our paper[8] where the ordinary massless Wess-Zumino model has been revealed to be the simplest linear σ-model of spontaneously broken conformal and OSp(1,4)-supersymmetries. In the same paper, we have constructed the OSp(1,4)-analog of the massive Wess-Zumino model and studied its vacuum structure. However, the methods we have used to obtain the corresponding Lagrangians were, to a great extent, heuristic. It is desirable to have general algorithms for constructing models with linear realization of the OSp(1,4) analogous to those employed in the usual supersymmetry.

The most adequate and elegant formulation of conventional linear supersymmetric theories is achieved with the use of the superfield concept[12]. The present and subsequent[13] papers are devoted to the description of a consistent superfield approach to OSp(1,4)-supersymmetry.

The supergroup OSp(1,4) can naturally be realized in the superspace $\sim OSp(1,4)/O(1,3)$, the spinorial extension of anti de Sitter space $\sim O(2,3)/O(1,3)$. For the first time, such a realization has been considered by Keck[1]. He has studied the transformation properties of a general scalar OSp(1,4)superfield and performed its reduction to irreducible pieces. But it remained unclear how to construct OSp(1,4)-invariants from superfields and hence how to set up nontrivial Lagrangian densities. We give explicitly all elements relevant to the construction of OSp(1,4)-invariant Lagrangians of arbitrary structure: covariant derivatives of superfields with any external Lorentz index, invariant measures of integration over superspace, etc.

The paper is organized as follows. In Sections 2,3 we describe the OSp(1,4)-covariant superfield techniques in the symmetrical-parametrized superspace proceeding from general theory of group realizations in homogeneous spaces. In Section 4, we study the problem of reduction of general OSp(1,4)-superfields and find realizations of OSp(1,4) in the left- and right-handed chiral superspaces.

2. The structure relations of the superalgebra OSp(1,4) can be taken as[1,3] 1)

$$[M_{\mu\nu}, M_{\lambda\rho}] = i(\eta_{\mu\rho}M_{\nu\lambda} + \eta_{\nu\lambda}M_{\mu\rho} - \eta_{\mu\lambda}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\lambda})$$
$$[M_{\mu\nu}, R_\lambda] = i(\eta_{\nu\lambda}R_\mu - \eta_{\mu\lambda}R_\nu), \quad [R_\mu, R_\nu] = -i m^2 M_{\mu\nu} \tag{1a}$$
$$[M_{\mu\nu}, Q] = -\frac{1}{2}\sigma_{\mu\nu}Q, \quad [R_\mu, Q] = -\frac{m}{2}\gamma_\mu Q$$
$$\{Q, \bar{Q}\} = \gamma_\mu R^\mu + \frac{m}{2}\sigma^{\mu\nu}M_{\mu\nu}. \tag{1b}$$

Even generators R_μ and $M_{\mu\nu}$ form the algebra of the group O(2,3), $M_{\mu\nu}$ being the Lorentz subgroup generators. The odd generator Q has transformation properties of the O(2,3)-spinor and obeys the Majorana condition $Q = C\bar{Q}^T$. We have introduced explicitly into (1) the dimensional parameter of contraction m ($[m] = L^{-1}$) to have at each step clear correspondence with the standard supersymmetry whose algebra results from (1) in the limit $m \to 0$.

Since O(2,3) is the group of motions of anti de Sitter space [14,15] its spinorial extension OSp(1,4) determines the simplest possible supersymmetry in this space. Like its Minkowski counterpart, OSp(1,4)-symmetry admits the natural representation in a superspace x_μ, θ_α but with anti de Sitter space as the even subspace[1,3]. Clearly, to construct the OSp(1,4)-covariant formalism in superspace one needs, first of all, basic relations of the O(2,3)-covariant formalism in anti de Sitter space.

Anti de Sitter space is a space of constant negative curvature homeomorphic to the homogeneous (coset) space O(2,3)/O(1,3). Due to the latter fact we can take advantage of general constructive methods of group realizations in homogeneous spaces[16-19].

One-to-one correspondence between the coset space O(2,3)/O(1,3) and anti de Sitter space means that coordinates of the

1) Our conventions on metric and γ-matrices coincide with those of Salam and Strathdee[12]. Indices μ, ν, ρ, λ refer to Lorentz vectors and $\alpha, \beta, \gamma, \delta$ to spinors. Summation over repeated indices is meant everywhere.

latter can be identified with parameters of left cosets of group O(2,3) over its subgroup O(1,3). To different parametrizations of cosets there correspond equivalent systems of curvilinear coordinates in anti de Sitter space. Of common use is the exponential parametrization

$$\exp\{iz^\mu R_\mu\} L,$$

where L is the set of the Lorentz group elements. However, we find it more convenient to deal with the coordinates x_μ related to z_μ by

$$x_\mu = z_\mu \frac{\mathrm{tg}\frac{mz}{2}}{mz}, \quad g(x) \stackrel{def}{=} \exp\{iz^\mu(x) R_\mu\} \tag{2}$$

with $z = \sqrt{z^\mu z_\mu}$ [2]. This choice is advantageous as it diagonalizes the metric of anti de Sitter space and makes most simple covariant derivatives of fields. Note that x_μ are stereographic projections of the Cartesian coordinates on a four-dimensional hypersphere with radius $1/m$ in a five-dimensional pseudo-Euclidean space with metric $(\eta_{\mu\nu}, 1)$ [3].

The group O(2,3) can be realized in the space O(2,3)/O(1,3) as left multiplications of cosets:

$$g(x) \xrightarrow{g_0 = \exp\{i\lambda^\mu R_\mu + \frac{i}{2}\lambda^{\mu\nu} M_{\mu\nu}\} \in O(2,3)} g_0 g(x) = g(x')\, e^{\frac{i}{2} u^{\mu\nu}(g_0, x) M_{\mu\nu}} \tag{3}$$

Shifts with $g_0 \in$ O(1,3) induce on x_μ usual Lorentz transformations which form the little (stability or structure) subgroup of realization (3). Shifts with $g_0 = \exp\{i\lambda^\mu R_\mu\}$ result in nonlinear transformations

$$\delta_R x_\mu = \frac{1}{2}(\lambda_\mu + 2m^2(\lambda x) x_\mu - m^2 x^2 \lambda_\mu) \tag{4}$$

Transformation properties of Lorentz irreducible fields $\varphi_\kappa(x)$ with respect to realization (3) can be naturally defined following the induced representations method:

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[2] A similar parametrization of cosets O(2,3)/O(1,3) has been used also by Gürsey and Marchildon /6/. However, their treatment of O(2,3)(and OSp(1,4)) essentially differs from ours. Together with MacDowell and Mansouri /4/ and Chamseddine /5/ they regard these groups as purely gauge i.e.,as acting in some internal tangent space. In such a treatmént parameters of cosets are fields over usual Minkowski space-time which is not affected by group transformatinns at all.

[3] These coordinates also determine a particular parametrization of the coset space O(2,3)/O(1,3). Their explicit relatinn to z_μ was given in refs. /1,3/.

$$\varphi'_\kappa(x') = (\exp\{\tfrac{i}{2} u^{\mu\nu}(g_0, x) J_{\mu\nu}\})_{\kappa\ell}\, \varphi_\ell(x), \tag{5}$$

where $(J_{\mu\nu})_{\kappa\ell}$ are matrices representing generators of the little group O(1,3) on fields $\varphi_\kappa(x)$. For infinitesimal O(2,3)-translations (4) functions $u^{\mu\nu}(\lambda, x)$ are given by:

$$u^{\mu\nu}(\lambda, x) = m^2(\lambda^\mu x^\nu - \lambda^\nu x^\mu) = \frac{1}{2}(\partial^\mu \delta_R x^\nu - \partial^\nu \delta_R x^\mu). \tag{6}$$

At this point, one essential remark is to be made. Insofar as the subgroup O(1,3) of O(2,3) is identified with the physical Lorentz group the law (5) is the most general O(2,3)-transformation law for fields defined over anti de Sitter space (up to a change of coordinates). For instance, given some linear O(2,3) multiplet $\Phi(x)$ such that $\Phi(x) \xrightarrow{O(2,3)} \Phi'(x') = \bar{g}\,\Phi(x)$ where $\bar{g}$ is an appropriate matrix representation of O(2,3), it can be decomposed into the direct sum of Lorentz irreducible fields with transformation properties (5) by means of the equivalence replacement $\Phi(x) \to \tilde{\Phi}(x) = \exp\{-iz^\mu(x)\bar{R}_\mu\}\Phi(x)$, where $\bar{R}_\mu$ are matrices of generators R_μ in the representation $\bar{g}$. This phenomenon is a particular manifestation of the relationship between linear and nonlinear group realizations /17/. As an important example, we write down explicitly the equivalency transformation by which some O(2,3)-spinor $\Psi^S_\alpha(x)$ $(\bar{R}_\mu = \frac{m}{2}\gamma_\mu)$ is expressed in terms of components $\Psi_\alpha(x)$ comprising a Lorentz spinor:

$$\Psi^S_\alpha(x) = (\exp\{\tfrac{im}{2} z^\mu(x)\gamma_\mu\})_\alpha{}^\beta\, \Psi_\beta(x) = \sqrt{\frac{\alpha(x)}{2}}(1 + imx^\mu\gamma_\mu)_\alpha{}^\beta\, \Psi_\beta(x) \equiv \Lambda_\alpha^\beta(x)\, \Psi_\beta(x) \tag{7}$$

with

$$\alpha(x) = \frac{2}{1 + m^2 x^2}. \tag{8}$$

The transformation (7) has been used earlier in papers /3,20/ however without explanation of its group meaning.

Let us define the covariant differentials and derivatives of fields $\varphi_\kappa(x)$. This can be done quite simply using the method of Cartan differential forms. In our case, these forms are found from the decomposition:

$$\begin{aligned} g^{-1}(x) dg(x) &= i\mu_S^\nu(x, dx) R_\nu + \tfrac{i}{2} \nu_S^{\mu\rho}(x, dx) M_{\mu\rho} \equiv \\ &\equiv i\alpha(x) dx^\nu R_\nu - im^2 \alpha(x) x^\mu dx^\rho M_{\mu\rho}. \end{aligned} \tag{9}$$

The form $\mu^\nu_S = \alpha(x) dx^\nu$ transforms under shifts (3) homogeneously as the Lorentz 4-vector, with parameters $u^{\mu\nu}(g_0, x)$ and thus is the covariant differential of the coordinate x^ν. The inhomogeneously transforming form $\nu_s^{\mu\rho} = -m^2 a(x)(x^\mu dx^\rho - x^\rho dx^\mu)$ (the Lorentz connection) enters into the covariant differentials of fields $\varphi_k(x)$:

$$\nabla \varphi_k(x) = d\varphi_k(x) + \frac{i}{2} \nu_s^{\mu\rho}(x,dx)(J_{\mu\rho})_{k\ell}\, \varphi_\ell(x) \tag{10}$$

which transform as $\varphi_k(x)$ themselves. The covariant derivatives $\nabla_\rho \varphi_k(x)$ are naturally defined as coefficients of the expansion of $\nabla\varphi_k(x)$ in forms μ_s^ρ :

$$\nabla \varphi_k(x) = \mu_s^\rho(x,dx)\, \nabla_\rho \varphi_k(x) \tag{11}$$

whence

$$\nabla_\rho \varphi_k(x) = a^{-1}(x)\, \partial_\rho \varphi_k(x) - i m^2 x^\mu (J_{\mu\rho})_{k\ell}\, \varphi_\ell(x). \tag{12}$$

It is worth noting the useful formula [4)]

$$[\nabla_\rho, \nabla_\lambda] = -i m^2 J_{\rho\lambda} \tag{13}$$

which can be deduced either directly or using the following general method. One should evaluate the commutator of two independent covariant differentials (10), extract from both sides of the obtained identity independent products of forms μ_s^ρ with taking into account the Maurer-Cartan structure equations for μ_s^ρ , $\nu_s^{\rho\mu}$, and, finally, identify coefficients of these products (the structure equations for anti de Sitter space are readily extracted from more general ones for the superspace OSp(1,4)/O(1,3) which are given in the next Section).

Obtained formulae allow us to construct all objects relevant to the geometry of anti de Sitter space. The contraction of two forms μ_s^ν gives the invariant interval:

$$ds^2 = \mu_s^\nu \mu_{s\nu} = g^s_{\mu\nu}(x)\, dx^\mu dx^\nu = a^2(x)\, \eta_{\mu\nu}\, dx^\mu dx^\nu \tag{14}$$

and the outer product of four forms μ_s^ν, the invariant volume element:

$$\mathcal{D}m_s = \mu_s^{\nu_1} \wedge \mu_s^{\nu_2} \wedge \mu_s^{\nu_3} \wedge \mu_s^{\nu_4} = d^4x \sqrt{-\| g^s_{\mu\nu} \|} = d^4x\, a^4(x)\,. \tag{15}$$

One immediately observes that $g^s_{\mu\nu}(x) = a^2(x)\, \eta_{\mu\nu}$ plays the role of anti de Sitter metric, $a(x)\, \eta_{\mu\nu}$ being an appropriate vierbein. The curvature tensor can be defined now in the standard fashion. As a matter of fact, its components are given already by eq. (13): $R^{\rho\lambda}_{\mu\nu} = -m^2(\delta^\rho_\mu \delta^\lambda_\nu - \delta^\lambda_\mu \delta^\rho_\nu)$, taking into account that (12) and (13) are

4) Hereafter we mean that the matrix part of each covariant derivative in operators of the type $\nabla_\rho \nabla_\mu \ldots \nabla_\lambda$ acts on all free Lorentz indices to the right of it, including vector indices of covariant derivatives.

particular cases of well known general covariance formulae (corresponding to the above special choice of vierbein).

To conclude, having expressions for covariant derivatives of fields $\varphi_k(x)$ and for the O(2,3)-invariant volume element (which is the measure of integration over the space O(2,3)/O(1,3)) we can construct O(2,3)-invariant Lagrangian densities of any desirable structure in $\varphi_k(x)$. Also, the problem of reduction of $\varphi_k(x)$ to O(2,3)-irreducible pieces can be solved (by representing two Casimir operators of O(2,3) in terms of covariant derivatives (12) and proceeding further like Grensing/21/ in his analysis of O(1,4)-de Sitter fields).

As $m \to 0$, all the expressions obtained reduce to their trivial Minkowski analogs (within our definition of anti de Sitter coordinate, the complete correspondence with Minkowski space arises upon rescaling $x_\mu \to \frac{1}{2} x_\mu$).

3. To develop the covariant superfield technique for OSp(1,4)-symmetry, we shall follow, as before, general recipes of the theory of group realizations in coset spaces and represent OSp(1,4) by left shifts in the superspace OSp(1,4)/O(1,3). For cosets of OSp(1,4) over group O(1,3) we take the parametrization

$$G(x,\theta) = O(2,3)/O(1,3) \cdot OSp(1,4)/O(2,3) = g(x)\, \exp\{i(1 - \tfrac{m}{3}\bar\theta\theta)\bar\theta Q\}\,, \tag{16}$$

where θ_α are the Grassmann coordinates associated with generator Q_α and comprising a Majorana spinor. The parametrization (16) differs from that adopted by Keck/1/ and Zumino/3/, besides the different choice of the coordinate system in space O(2,3)//O(1,3) also by the opposite arrangement of even and odd factors. We adhere to this sequence in order that under the left shifts belonging to subgroup O(2,3) coordinate θ_α transform according to the induced representation law (5), i.e., like a Lorentz spinor. Within this choice, different θ -monomials do not mix under O(2,3)-transformations, and as a consequence, components of OSp(1,4) superfields have the uniform transformation properties (5) in all their Lorentz indices. At the same time, with making use of the parametrization by Keck/1/ and Zumino/3/, the spinor coordinate (denoted here by θ^s) behaves like a O(2,3)-spinor, i.e., transforms under O(2,3)-translations as $\delta\theta^s = \frac{im}{2} \lambda^\rho \gamma_\rho \theta^s$. For this reason, components of corresponding superfields, in indices associated with θ -monomials, form linear multiplets of O(2,3) what seems to us less convenient because of the lack both of uniformity and explicit correspondence with the ordinary supersymmetry. The connection of our coordinates x_μ, θ_α with those of Keck[1] and Zumino[3] is given by eq. (2) and by the formula of the type (7)

$$\theta^S_\alpha = \Lambda_\alpha{}^\beta(x)\,\theta_\beta\,(1-\tfrac{m}{3}\bar\theta\theta)\,. \tag{17}$$

An additional canonical redefinition of the Grassmann coordinate with the help of θ -dependent factor $(1-\frac{m}{3}\bar\theta\theta)$ in (16) is made to simplify subsequent formulae (it does not alter O(2,3)-properties of θ_α).

Transformation properties of the superspace OSp(1,4)/O(1,3) and defined over it superfields $\Phi_K(x,\theta)$ (K is the Lorentz index) with respect to an arbitrary OSp(1,4)-transformation are specified by the formulae (compare with (3) and (5)):

$$G(x,\theta) \to G_0 G(x,\theta) = G(x',\theta')\exp\{\tfrac{i}{2} W^{\mu\nu}(G_0,x,\theta)\, M_{\mu\nu}\} \tag{18}$$

$$\Phi_K(x,\theta) \to \Phi'_K(x',\theta') = \left(\exp\{\tfrac{i}{2} W^{\mu\nu}(G_0,x,\theta)\, \mathcal{J}_{\mu\nu}\}\right)_{K\ell}\, \Phi_\ell(x,\theta) \tag{19}$$

with $(\mathcal{J}_{\mu\nu})_{K\ell}$ again being the matrix realization of generators $M_{\mu\nu}$.

Clearly, for $G_0 = g_0 \in O(2,3)$ transformations (18) and (19) reduce to (3) and (5). In particular, $W^{\mu\nu}(g_0,x,\theta) = u^{\mu\nu}(g_0,x)$, and generators $M_{\mu\nu}$ and R_λ in realization on superfields $\Phi_K(x,\theta)$ are simply

$$\begin{aligned} M_{\mu\nu} &= i(x_\mu\partial_\nu - x_\nu\partial_\mu) + \tfrac{1}{2}\bar\theta\sigma_{\mu\nu}\frac{\partial}{\partial\bar\theta} + \mathcal{J}_{\mu\nu} \\ R_\lambda &= i\left(\frac{1-m^2x^2}{2}\delta^\nu_\lambda + m^2 x_\lambda x^\nu\right)\partial_\nu + \frac{m^2}{2}x^\nu\bar\theta\sigma_{\lambda\nu}\frac{\partial}{\partial\bar\theta} - m^2 x^\nu \mathcal{J}_{\nu\lambda}\,. \end{aligned} \tag{20}$$

Much more involved is the structure of odd OSp(1,4)-transformations generated by shifts with $G_0 = e^{i\bar\epsilon Q}$. Making use of the commutation relations (1) and formulae of Zumino[3], through rather cumbersome calculations we have found that generators Q_α realized on superfields $\Phi_K(x,\theta)$ are of the form:

$$\begin{aligned} Q_\alpha = (1-\tfrac{m}{4}\bar\theta\theta)\Lambda_\alpha{}^\beta(x)\Big\{ i\Big[(1+\tfrac{m}{4}\bar\theta\theta)\delta_\beta{}^\gamma + m\theta_\beta\bar\theta^\gamma - \tfrac{m}{4}(\gamma^\nu\theta)_\beta(\bar\theta(\gamma_\nu - \\ - m x^\rho\sigma_{\rho\nu}))^\gamma\Big]\frac{\partial}{\partial\bar\theta^\gamma} - \tfrac{1}{2}a^{-1}(x)(\gamma^\nu\theta)_\beta\partial_\nu + i\tfrac{m}{4}\big[(2m\gamma_\mu x_\nu + \sigma_{\nu\mu})\theta\big]_\beta \mathcal{J}^{\nu\mu}\Big\}\,, \end{aligned} \tag{21}$$

where matrix $\Lambda_\alpha{}^\beta(x)$ and function $a(x)$ are introduced by eqs. (7) and (8). Differential parts of generators (20) and (21) transform arguments x_μ and θ_α , while matrix ones change the superfield form via the Lorentz-rotation with x_μ- and θ_α -dependent parameters. In the contraction limit, (20) and (21) convert into the generators of usual supersymmetry (with additional rescaling $x_\mu \to \frac{1}{2}x_\mu$).

We now turn to making up covariants of the realization given by transformation laws (18), (19). The relevant Cartan forms are introduced by the identity which is a direct extension of the decomposition (9):

$$G^{-1}(x,\theta)\,dG(x,\theta) = i\bar\tau(x,\theta,dx,d\theta)Q + i\mu^\nu(x,\theta,dx,d\theta)R_\nu + \tfrac{i}{2}\nu^{\rho\lambda}(x,\theta,dx,d\theta)M_{\rho\lambda}\,. \tag{22}$$

The forms μ^ν, $\bar\tau^\alpha$ are easily verified to transform under shifts (18) according to the general law (19), independently of each other. So they have the meaning of covariant differentials of coordinates x_ν, θ_α . The inhomogeneously transforming form $\nu^{\rho\lambda}$ is nothing but the connection over the Lorentz group (which is the structure group of the present realization). This form determines the covariant differentials of superfields:

$$\mathcal{D}\Phi_K(x,\theta) = d\Phi_K(x,\theta) + \tfrac{i}{2}\nu^{\rho\lambda}(\mathcal{J}_{\rho\lambda})_{K\ell}\Phi_\ell(x,\theta)\,. \tag{23}$$

Inhomogeneity of its transformation is just such as to compensate the noncovariant term which arises from $d\Phi_K(x,\theta)$ when $\Phi_K(x,\theta)$ undergoes the transformation (19). As a result, $\mathcal{D}\Phi_K(x,\theta)$ transforms like $\Phi_K(x,\theta)$ itself.

Explicitly, the forms $\bar\tau^\beta$, μ^ν, $\nu^{\rho\lambda}$ are as follows:

$$\begin{aligned} \bar\tau^\beta &= (1-\tfrac{m}{2}\bar\theta\theta)\Big\{d\bar\theta^\alpha\big[[1+\tfrac{9}{32}m^2(\bar\theta\theta)^2]\delta_\alpha^\beta - \tfrac{m}{2}\theta_\alpha\bar\theta^\beta\big] - \tfrac{im}{2}a(x)\,dx_\mu\big[\bar\theta(\gamma^\mu - m x_\nu\sigma^{\nu\mu})\big]^\beta\Big\} \\ \mu^\nu &= a(x)\,dx^\nu + \tfrac{i}{2}(1-\tfrac{m}{4}\bar\theta\theta)\,\bar\tau\gamma^\nu\theta \\ \nu^{\rho\lambda} &= \tfrac{im}{2}(1-\tfrac{m}{4}\bar\theta\theta)\,\bar\tau\big[\sigma^{\rho\lambda} + m(x^\rho\gamma^\lambda - x^\lambda\gamma^\rho)\big]\theta - m^2(x^\rho\mu^\lambda - x^\lambda\mu^\rho)\,. \end{aligned} \tag{24}$$

In constructing the covariant derivatives of superfields we shall do in close parallel with the pure anti de Sitter case. Namely, we extract from $\mathcal{D}\Phi_K(x,\theta)$ (23) the covariant differentials of coordinates x_ν, θ_α, i.e., forms μ^ν, $\bar\tau^\alpha$ and identify with covariant derivatives, respectively vector and spinor, coefficients of these forms:

$$\mathcal{D}\Phi_K(x,\theta) = \mu^\nu\hat\nabla_\nu\Phi_K(x,\theta) + \bar\tau^\alpha\hat{\mathcal{D}}_\alpha\Phi_K(x,\theta)\,. \tag{25}$$

The objects thus defined are manifestly OSp(1,4)-covariant by construction. With the explicit expressions for Cartan forms (24) definition (25) implies:

$$\hat\nabla_\nu = a^{-1}(x)\partial_\nu + \tfrac{im}{2}\bar\theta(\gamma_\nu - m x^\mu\sigma_{\mu\nu})\frac{\partial}{\partial\bar\theta} - i m^2 x^\mu \mathcal{J}_{\mu\nu} \tag{26}$$

$$\hat{\mathcal{D}}_\alpha = (1-\tfrac{m}{4}\bar\theta\theta)\Big\{\Big[(1+\tfrac{3}{4}m\bar\theta\theta)\delta_\alpha^\beta + \tfrac{m}{2}\theta_\alpha\bar\theta^\beta\Big]\frac{\partial}{\partial\bar\theta^\beta} - \tfrac{i}{2}(\gamma^\mu\theta)_\alpha\hat\nabla_\mu - \tfrac{m}{4}(\sigma^{\mu\nu}\theta)_\alpha\mathcal{J}_{\mu\nu}\Big\} \tag{27}$$

As $m \to 0$, these operators contract into the usual vector and spinor covariant derivatives of "flat" supersymmetry. Let us point out that dx_μ and ∂_μ enter into forms (24) and covariant derivatives (26) and (27) only through their O(2,3)-counterparts μ^μ_S , ∇_μ thus ensuring correct O(2,3)-properties for (24), (26) and (27). Note also that the pieces of forms (24) independent of x_μ , dx_μ coincide with Cartan forms for the realization of OSp(1,4) in the purely Grassmannian coset space OSp(1,4)/O(2,3).

Though the structure of covariant derivatives (26) and (27) is rather complicated their commutator algebra turns out, beyond expectation, almost as simple as in the case of usual supersymmetry:

$$
\begin{aligned}
&[\hat{\nabla}_\mu, \hat{\nabla}_\nu] = -i m^2 J_{\mu\nu} \\
&\{\hat{\mathcal{D}}_\alpha, \hat{\mathcal{D}}_\beta\} = -\frac{m}{2}(\sigma^{\mu\nu}C)_{\alpha\beta} J_{\mu\nu} + \frac{1}{i}(\gamma^\nu C)_{\alpha\beta}\hat{\nabla}_\nu \qquad (28) \\
&[\hat{\nabla}_\mu, \hat{\mathcal{D}}_\alpha] = \frac{m}{2i}(\gamma_\mu \hat{\mathcal{D}})_\alpha
\end{aligned}
$$

multiplication of covariant derivatives being understood here in the sense explained in the footnote to formula (13). To learn what are these (anti) commutators we have taken advantage of the general method mentioned after eq. (13) (the straightforward computation of them is also possible, but it involves a lot of tedious labour). In application to the present case, that method consists in evaluating an antisymmetrized second-order covariant differential of $\Phi_K(x,\theta)$ and equating afterwards coefficients of independent products of forms μ^ν, $\overline{\tau}^\alpha$ in both sides of the resulting identity. This procedure essentially exploits the Maurer-Cartan structure equations for the superspace OSp(1,4)/O(1,3):

$$
\begin{aligned}
&\mathcal{D}_2\overline{\tau}(d_1) - \mathcal{D}_1\overline{\tau}(d_2) + \frac{m}{2i}[\overline{\tau}(d_1)\gamma^\nu\mu_\nu(d_2) - \overline{\tau}(d_2)\gamma^\nu\mu_\nu(d_1)] = 0 \qquad (29) \\
&\mathcal{D}_2\mu_\nu(d_1) - \mathcal{D}_1\mu_\nu(d_2) - i\,\overline{\tau}(d_1)\gamma_\nu\tau(d_2) = 0 \\
&\mathcal{D}_2\nu^{\rho\lambda}(d_1) - \mathcal{D}_1\nu^{\rho\lambda}(d_2) - m^2[\mu^\rho(d_1)\mu^\lambda(d_2) - \mu^\rho(d_2)\mu^\lambda(d_1)] - im\,\overline{\tau}(d_1)\sigma^{\rho\lambda}\tau(d_2) = 0
\end{aligned}
$$

which are derived in Appendix.

It follows from (28) that, in contrast to the case of ordinary ("flat") supersymmetry, OSp(1,4)-covariant derivatives $\hat{\nabla}_\mu$, $\hat{\mathcal{D}}_\alpha$ do not generate the algebra isomorphic to the initial one (1). Indeed, relations (28), like their O(2,3)-counterparts (13) contain only tangent space pieces of Lorentz generators. Besides, relative signs between different structures in the right-hand side of (28) are somewhat distinguished from those appearing in (1). Strictly speaking, relations (28) (equally as (13)) should not at all be referred to as introducing any algebra in its conventional meaning because (anti) commutators between covariant derivatives are defined in (28) quite differently from those between infinitesimal group generators[5] (see the footnote to eq. (13)). Rather, eqs. (28) have to be regarded as an equivalent form of the structure equations (29). If, nevertheless, one attempts to treat (26), (27) as generators of certain superfield transformations and begins to commute (anticommute) them in the usual way, one immediately observes that they do not form any closed superalgebra (even together with OSp(1,4) generators (20), (21)) and it is not clear to which more extensive (finite-dimensional) superalgebra they could pertain. Note that (anti) commutators (in the usual sense) of $\hat{\nabla}_\mu$, $\hat{\mathcal{D}}_\alpha$ with OSp(1,4)-generators are again the same objects but Lorentz rotated according to the general rule (19).
For instance:

$$
\{\hat{\mathcal{D}}_\alpha, Q_\beta\} = \frac{1}{4}\Big[\frac{\partial}{\partial\overline{\epsilon}^\beta}W^{\mu\nu}(\epsilon,x,\theta)\Big]_{\epsilon=0}(\sigma_{\mu\nu})_\alpha{}^\gamma\hat{\mathcal{D}}_\gamma \; (=0 \text{ as } m\to 0) \,.
$$

This way, OSp(1,4)-covariance of $\hat{\nabla}_\mu$, $\hat{\mathcal{D}}_\alpha$ displays itself at the commutator level. Recall that the covariant spinor derivative of usual supersymmetry commutes with the 4-translation generator P_μ and anticommutes with the spinor generator. It can be defined not only through the Cartan forms method but also alternatively, as the generator of right supertranslations. In the OSp(1,4)-case, only the first approach appears constructive because the right action of OSp(1,4) on cosets OSp(1,4)/O(1,3) does not commute with the left one/1/. Covariant derivatives (26), (27) cannot certainly be identified with generators of right OSp(1,4)-transformations since, as pointed out above, their "algebra" does not close with respect to operation of usual (anti) commuting.

Having the explicit expressions (24) for the Cartan forms we may readily construct all geometrical characteristics of the superspace OSp(1,4)/O(1,3). As in the case of conventional superspace, there exist three independent invariant "intervals":

[5] In usual supersymmetry, due to the absence of vierbein parts in corresponding covariant derivatives, both definitions completely coincide. This is the reason why these derivatives form the true superalgebra (it is reproduced, of course, from (28) in the contraction limit).

$\mu^\nu\mu_\nu$, $\bar\tau\tau$, $\bar\tau\gamma_5\tau$. The components of the curvature and torsion supertensors (in the tangent space representation) are extracted most easily from relations (28): they are given, respectively, by coefficients of the matrix $\frac{i}{2}\,\mathcal{J}_{\mu\nu}$ and covariant derivatives in the r.h.s. of (28). In obvious notation:

$R^{\lambda\rho}_{\mu\nu} = -m^2(\delta^\lambda_\mu\delta^\rho_\nu - \delta^\rho_\mu\delta^\lambda_\nu)$, $R^{\lambda\rho}_{\alpha\beta} = im(\sigma^{\lambda\rho}C)_{\alpha\beta}$, $T^\mu_{\alpha\beta} = -i(\gamma^\mu C)_{\alpha\beta}$,
$T^\beta_{\mu\alpha} = -T^\beta_{\alpha\mu} = -\frac{i}{2}m(\gamma_\mu)_\alpha{}^\beta$, all other components being zero. The same results follow from the consideration of the structure equations (29). As is expected, in the limit $m\to 0$ there survives only the supertorsion component $T^\nu_{\alpha\beta}$ associated with the superspace of ordinary supersymmetry.

To conclude this Section, we calculate the OSp(1,4)-invariant measure of integration over superspace OSp(1,4)/O(1,3) (the invariant volume element) the knowledge of which is important for constructing OSp(1,4)-invariant Lagrangians. It is defined in a standard manner/22/ through superdeterminant (Berezinian) of the supervierbein E:

$$\mathrm{Ber}\,E = \det(A - BD^{-1}C)\det D^{-1},$$

where the supermatrix E and matrices A, B, C, D are introduced by:

$$(\mu^\nu, \bar\tau^\alpha) = (dx^\rho, d\bar\theta^\rho)\begin{pmatrix} A_\rho^\nu(x,\theta) & B_\rho^\alpha(x,\theta) \\ C_\beta^\nu(x,\theta) & D_\beta^\alpha(x,\theta)\end{pmatrix} \equiv (dx, d\bar\theta)\,E(x,\theta)\,. \qquad (30)$$

Substituting for A, B, C, D their explicit expressions which are straightforwardly extracted from formulae (24), we find:

$$\mathcal{D}m \equiv d^4x\,d^4\theta\,\mathrm{Ber}\,E = d^4x\,d^4\theta\,a^4(x)\left[1 + \tfrac{3}{2}m\bar\theta\theta + \tfrac{3}{8}m^2(\bar\theta\theta)^2\right]. \qquad (31)$$

The invariance of measure (31) with respect to the transformations of coordinates x_μ and θ_α generated by (20) and (21) can be verified directly, by using the rules for changing variables in the Grassmann integrals/23/. Its factorization has the clear meaning: the factor $d^4x\,a^4(x)$ is the O(2,3)-invariant measure of integration over the space O(2,3)/O(1,3), and the remaining part coincides with the integration measure for the superspace OSp(1,4)/O(2,3) invariant with respect to the left action of OSp(1,4) in that superspace. Note that the full invariant volume of the space OSp(1,4)/ /O(2,3) obtained by integration over the measure $d^4\theta\,[1 + \frac{3}{2}m\bar\theta\theta + \frac{3}{8}m^2(\bar\theta\theta)^2]$ is $3m^2 > 0$ what is to be compared with the case of the coset space of the standard supergroup over the Poincare group the volume of which is zero.

4. Let us study the problem of reduction of general OSp(1,4)-superfield

$$\Phi_\kappa(x,\theta) = A_\kappa(x) + \bar\theta\,\Psi_\kappa(x) + \tfrac{1}{4}\bar\theta\theta\,F_\kappa(x) + \tfrac{1}{4}\bar\theta\gamma_5\theta\,G_\kappa(x) + \tfrac{1}{4}\bar\theta i\gamma_\mu\gamma_5\theta\,A^\mu_\kappa(x) + \tfrac{1}{4}\bar\theta\theta\,\bar\theta\,\chi_\kappa(x) + \tfrac{1}{32}(\bar\theta\theta)^2 D_\kappa(x) \qquad (32)$$

with the standard OSp(1,4)-supertranslation law

$$\delta\Phi_\kappa(x,\theta) = i(\bar\epsilon Q\,\Phi(x,\theta))_\kappa\,, \qquad (33)$$

where ϵ is an anticommuting constant spinor parameter, and Q_α is given by (21).

In the usual supersymmetry, superfields of the type (32) are known to be local-reducible/12/. The reduction is effected by imposing covariant conditions of first and higher orders in covariant derivatives/12,24/. One may attempt to proceed in the analogous way also in the OSp(1,4)-case. The simplest covariant conditions of first order in derivatives are now

$$\left(\frac{1\mp i\gamma_5}{2}\right)_\alpha{}^\beta(\hat{\mathcal{D}}_\beta\,\Phi(x,\theta))_\kappa = 0 \qquad (34_\pm)$$

which directly generalize the well known constraints isolating chiral representations in the usual supersymmetry.

Equations (34) are solved most simply when rewritten in the component form. Examining the thus obtained system of differential equations we have found that, unlike the case of "flat" supersymmetry, it possesses nontrivial solutions ($\Phi_\kappa \neq 0$) not for any superfields but only for those which are transformed by one of the following representations of the Lorentz group:[6)]

$$D^{(p,0)} \qquad \text{for condition } (34_+) \qquad (35_+)$$
$$D^{(0,q)} \qquad \text{for condition } (34_-), \qquad (35_-)$$

where $D^{(p,q)}$ are matrices of nonunitary finite-dimensional representations of the Lorentz group, p and q are positive integers and half-integers (see, e.g., paper/14/).

In superfields $\Phi_{\pm\kappa}(x,\theta)$ of the classes $(35_\pm)$ the conditions $(34_\pm)$ pick out as independent components $A_{\pm\kappa}$, $F_{\pm\kappa}$ and $\Psi_{\pm\kappa}$ (the latter, in suppressed spinor index associated with θ_α, should be either left- or right-handed depending on the lower sign) and express the remaining components in terms of the independent ones:

$$G_{\pm\kappa} = \pm iF_{\pm\kappa}\,,\quad A^\mu_{\pm\kappa} = \pm i(\nabla^\mu A_\pm)_\kappa$$
$$\chi_{\pm\kappa} = -i(\gamma^\mu\nabla_\mu\Psi_\pm)_\kappa - 2m\,\Psi_{\pm\kappa} \qquad (36_\pm)$$
$$D_{\pm\kappa} = -(\nabla^\mu\nabla_\mu A_\pm)_\kappa - 8m\,F_{\pm\kappa}\,,$$

6) The group meaning of these restrictions is explained in our paper /13/.

where ∇_μ is the O(2,3)-covariant derivative defined by (12). Note the similarity of $(36_\pm)$ to the corresponding relations of standard supersymmetry. The analogy becomes most striking after going back to the superfield notation. We have checked that the solutions $(36_\pm)$ admit the compact superfield representation:

$$\Phi_{\pm\kappa}(x,\theta)=\left(\exp\{\mp\tfrac{1}{4}\bar\theta\gamma^\rho\gamma_5\theta\hat\nabla_\rho\}\right)_{\kappa\ell}T^\pm_\ell(x,\theta_\pm) \tag{$37_\pm$}$$

$$T^\pm_\kappa(x,\theta_\pm)=A_{\pm\kappa}(x)+\bar\theta_\pm\Psi_{\pm\kappa}(x)+\tfrac{1}{2}\bar\theta_\pm\theta_\pm F_{\pm\kappa}(x), \tag{$38_\pm$}$$

where $\hat\nabla_\rho$ is the OSp(1,4)-covariant vector derivative (27) and $\theta_\pm=\frac{1\pm i\gamma_5}{2}\theta$. Relations (37) are seen to appear as a direct "covariantization" $(\partial_\rho\to\hat\nabla_\rho)$ of familiar formulae of "flat" supersymmetry which describe the transition to the symmetric basis in corresponding "truncated" chiral superfields. These formulae are just the contraction limit $(m=0)$ of (37). Without loss of generality, one may put $\Phi_{+\kappa}(x,\theta)=\Phi^*_{-\kappa}(x,\theta)$ and $T^+_\kappa(x,\theta_+)=(T^-_\kappa(x,\theta_-))^*$ where symbol $*$ means involution (complex conjugation plus reversion of the order of anticommuting factors).

Thus, we come to the conclusion that superfields from the restricted set $(35_\pm)$ possess invariant chiral subspaces. Those superfields which are transformed by the direct sum of representations $D^{(\rho,0)}\oplus D^{(0,\rho)}$ (they can be submitted to the reality condition) contain invariant subspaces of both chiralities. A simplest example is a real scalar superfield $\Phi(x,\theta)$. It contains two irreducible conjugated scalar OSp(1,4)-multiplets involving, as suggested by the field content of $(38_\pm)$, eight real independent components. This fact has been established earlier by Keck/1/ through a straightforward analysis of transformation properties of superfield components.

Further reduction of superfields (32) can be effected by imposing supplementary conditions of higher order in covariant derivatives with the structure dictated by the structure of Casimir operators of supergroup OSp(1,4). The corresponding procedure will repeat, in its main steps, that one employed in the usual case /12,24/. However, the knowledge of supplementary conditions and projection operators which single out higher representations of OSp(1,4) is, in our opinion, rather of academic interest. Based on analogy with the conventional supersymmetry, it should be expected that in OSp(1,4)-invariant theories of real interest

(OSp(1,4)-symmetric Yang-Mills theory, OSp(1,4)-supergravity, etc.) a minimal set of relevant fields will be automatically picked out on account of additional local invariances (with the elimination of a subset of auxiliary fields afterwards through equations of motion).

From the existence of representations $(37_\pm)$ it follows that OSp(1,4) can be realized on "truncated" chiral superfields $T^\pm_\kappa(x,\theta_\pm)$. To find these realizations, we reduce $(37_\pm)$ in analogy with the case of usual supersymmetry to certain nonlinear shifts of superfield arguments. For the purpose, we unlink first the matrix and differential parts in the operator $\exp\{\mp\frac{1}{4}\bar\theta\gamma^\rho\gamma_5\theta\hat\nabla_\rho\}$. This can easily be done using the Baker-Hausdorf formula and the basic property of Grassmann coordinates $(\theta)^5=0$. We have

$$\begin{aligned}\exp\{\mp\tfrac{1}{4}\bar\theta\gamma^\rho\gamma_5\theta\hat\nabla_\rho\}&=\exp\{\pm\tfrac{im^2}{4}\bar\theta\gamma^\rho\gamma_5\theta x^\nu J^\pm_{\nu\rho}\}\times\\&\times\exp\{\mp\tfrac{1}{4}[\bar\theta\gamma^\rho\gamma_5\theta\alpha^{-1}(x)\partial_\rho+2im\bar\theta\theta\cdot\bar\theta\gamma_5(1+\tfrac{3}{4}imx\gamma)\tfrac{\partial}{\partial\bar\theta}]\},\end{aligned} \tag{39}$$

where $J^\pm_{\nu\rho}$ are matrices of generators of the Lorentz group in representations $(35_\pm)$. Applying further the general identity $e^{f(z)\partial_z}\varphi(z)=\varphi(e^{f(z)\partial_z}z)$ we rewrite $(37_\pm)$ as follows

$$\tilde\Phi_{\pm\kappa}(x,\theta)\equiv\left(\exp\{\mp\tfrac{im^2}{4}\bar\theta\gamma^\rho\gamma_5\theta x^\nu J^\pm_{\nu\rho}\}\right)_{\kappa\ell}\Phi_{\pm\ell}(x,\theta)=\begin{cases}T^+_\kappa(x^L,\theta^L)\\ T^-_\kappa(x^R,\theta^R)\end{cases} \tag{$40_\pm$}$$

where

$$\begin{aligned}\begin{pmatrix}x^L_\mu\\ \theta^L\end{pmatrix}&=\exp\{-\tfrac{1}{4}[\bar\theta\gamma^\rho\gamma_5\theta\alpha^{-1}(x)\partial_\rho+2im\bar\theta\theta\cdot\bar\theta\gamma_5(1+\tfrac{3}{4}imx\gamma)\tfrac{\partial}{\partial\bar\theta}]\}\begin{pmatrix}x_\mu\\ \theta_+\end{pmatrix}=\\&=\begin{pmatrix}[1+\frac{m^2}{16\alpha(x)}(\bar\theta\theta)^2]x_\mu-\frac{1}{4}\bar\theta\gamma_\mu\gamma_5\theta\alpha^{-1}(x)\\ \theta_+-\frac{m}{2}\bar\theta\theta(\theta_++\frac{3}{4}imx\gamma\theta_-)\end{pmatrix}\end{aligned} \tag{41}$$

and $x^R_\mu=(x^L_\mu)^*$, $\theta^R=C\bar\theta^{LT}$ result from (41) simply by changes $\theta_+\leftrightarrow\theta_-$. Based on these relations, and making use of the fact that the realization of OSp(1,4) on superfields $\Phi_{\pm\kappa}(x,\theta)$ (and on canonically related superfields $\tilde\Phi_{\pm\kappa}(x,\theta)$) is known we are now in a position to deduce the OSp(1,4)-transformation rules of chiral superfields $T^\pm_\kappa$. Transformation properties of their arguments x^L_μ, θ^L_α and x^R_μ, θ^R_α are uniquely determined by the properties of x_μ, θ_α owing to the explicit connection (41) (and the analogous one between x_μ, θ_α and x^R_μ, θ^R_α). As expected, these pairs of variables form invariant spaces with respect to the action of OSp(1,4). Under the Lorentz-rotations and O(2,3)-translations they behave like coordinates x_μ and θ_α. Their odd transformations are given by

$$\delta_Q x^L_\mu = i\,\alpha^{-1}(x^L)\,\bar{\epsilon}\,\Lambda(x^L)\,\gamma_\mu\,\theta^L \qquad (42)$$
$$\delta_Q \bar{\theta}^L = \bar{\epsilon}\,\Lambda(x^L)\left[1 - \frac{m}{2}\bar{\theta}^L\theta^L\left(1 - \frac{3}{2}\,i m\, x^L_\mu \gamma^\mu\right)\right]\frac{1+i\gamma_5}{2}$$

(infinitesimal variations of x^R_μ, $\bar{\theta}^R_\alpha$ are of the same form up to the replacements $L \to R$, $\frac{1+i\gamma_5}{2} \to \frac{1-i\gamma_5}{2}$). Matrix parts of OSp(1,4) -transformations of T^+_k and T^-_k can also be shown to depend, respectively,either on x^L_μ, θ^L_α or on x^R_μ, θ^R_α (at this point, it is significant that generators $J^{\pm}_{\mu\nu}$ of representations ($35_\pm$) do contain projectors $\frac{1\pm i\gamma_5}{2}$).

For illustration, we write down the transformations induced by the supershifts (42) for components of scalar chiral superfields $T^\pm(x, \theta_\pm)$:

$$\delta_Q A_\pm = \bar{\beta}\,\Psi_\pm \qquad (43)$$
$$\delta_Q \Psi_\pm = \frac{1\pm i\gamma_5}{2}\left(-i\gamma^\mu \nabla_\mu A_\pm + F_\pm\right)\beta$$
$$\delta_Q F_\pm = \bar{\beta}\left(-i\gamma^\mu \nabla_\mu \Psi_\pm - m\,\Psi_\pm\right)$$

where $\beta = \Lambda^{-1}(x)\epsilon$. The laws (43) are the same as those we have found earlier/8/. In the contraction limit they become the usual transformation laws of scalar multiplets of "flat" supersymmetry.

More detailed study of chiral realizations of OSp(1,4) will be carried out in subsequent paper/13/ where also some linear OSp(1,4)-symmetric models will be presented.

Appendix. The derivation of the structure equations.

As a first step, we differentiate the decomposition (22) and antisymmetrize independent differentials to obtain:

$$d_2(G^{-1}d_1G) - d_1(G^{-1}d_2G) = i\left[d_2\mu(d_1) - d_1\mu(d_2)\right]R + i\left[d_2\bar{\iota}(d_1) - d_1\bar{\iota}(d_2)\right]Q + i\left[d_2\nu(d_1) - d_1\nu(d_2)\right]M \qquad (A.1)$$

(for brevity we have suppressed Lorentz indices). Further, the l.h.s. of (A.1) can be written as the commutator:

$$\left[G^{-1}d_1G,\, G^{-1}d_2G\right] \qquad (A.2)$$

Inserting in the latter again the decomposition (22), making use of the (anti) commutator algebra (1) and finally comparing coefficients of OSp(1,4)-generators in both sides of (A.1), we come to the equations (29).

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Received by Publishing Department
on April 2 1979.