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E2 - 12331

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WESS-ZUMINO MODEL AS LINEAR σ-MODEL OF SPONTANEOUSLY BROKEN CONFORMAL AND OSp(1,4)-SUPERSYMMETRIES

Submitted to ЯФ

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E2 - 12331

Модель Весса-Зумино как линейная σ-модель спонтанно нарушенных конформной и $OSp(1,4)$-суперсимметрий

Изучена структура спонтанного нарушения конформной и ортосимплектической суперсимметрий в безмассовой модели Весса-Зумино за счет классических решений фубиниевского типа. Показано, что малой группой соответствующего вакуума является градуированная подгруппа $\overset{\mathrm{I}}{OSp}(1,4)$ конформной супергруппы. Симметрия по отношению к другой $OSp(1,4)$-подгруппе ($\overset{\mathrm{II}}{OSp}(1,4)$) спонтанно нарушена до $O(2,3)$-симметрии с возникновением массивного голдстоуновского фермиона. Определено суперполевое преобразование Вейля, с его помощью действие модели переписано в терминах суперпространства $OSp(1,4)/O(1,3)$, являющегося спинорным расширением пространства анти де Ситтера. Показано, что в таком представлении спонтанно нарушенная фаза допускает стандартную σ-модельную интерпретацию. Построен $OSp(1,4)$-аналог массивной модели Весса-Зумино и изучена его вакуумная структура. Обнаружен эффект спонтанного нарушения P- и CP-четностей с константой, связанной с радиусом пространства анти де Ситтера.

Работа выполнена в Лаборатории Теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1979

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E2 - 12331

Wess-Zumino Model as Linear σ-Model of Spontaneously Broken Conformal and OSp(1,4)-Supersymmetries

The massless Wess-Zumino model is shown to exhibit the spontaneous breaking of global conformal and orthosymplectic supersymmetries on account of the Fubini-type classical solutions to the equations of motion. We study the group structure of spontaneously broken phase and analyze its particle spectrum. The little group of the ground state is found to be the graded subgroup $\overset{\mathrm{I}}{OSp}(1,4)$ of the conformal supergroup. The symmetry with respect to another OSp(1,4)-subgroup ($\overset{\mathrm{II}}{OSp}(1,4)$) is broken to O(2,3)-symmetry with emergence of massive Goldstone fermion. The superfield Weyl transformation is defined and with its help the model action is rewritten in terms of the superspace OSp(1,4)/O(1,3), spinorial extension of anti de Sitter space. In such a representation the spontaneously broken phase admits the standard σ-model interpretation. We also construct the OSp(1,4)-analog of the massive Wess-Zumino model and examine its vacuum structure. An effect of the spontaneous breaking of P- and CP-parities with the strength related to anti de Sitter radius is found.

Preprint of the Joint Institute for Nucleor Research. Dubna 1979

I. In the light of a recent progress in supergravity[1,2,3] it seems of real importance to seek and study various mechanisms of spontaneous breakdown of conformal (SU(2,2/1)) and orthosymplectic (OSp(1,4)) supersymmetries.

The nonlinear realizations of these supersymmetries have been considered in[4,5]. It is interesting to construct corresponding linear σ-models. As a first step, it is natural to explore in detail the global case. The regularities found may essentially clarify the situation in the local case which can be achieved by introducing interactions with gauge fields of supergravity.

In the present paper we show that even in the simplest linear superconformal-invariant theory, the massless Wess-Zumino model[6], conformal and OSp(1,4)-supersymmetries are spontaneously broken on account of x-dependent classical solutions to the equations of motion. These solutions[7] are similar to those revealed by Fubini[8] in the massless φ^4-theory. They break the Poincaré-symmetry but display invariance with respect to the group of motions of anti de Sitter space O(2,3). Among other solutions of the Wess-Zumino model O(2,3)-solutions are on a distinct status due to their Lorentz invariance and vanishing of the (improved) energy-momentum tensor $T_{\mu\nu}$ on them[1)]. For this reason, they pretend to describe the ground state of the spontaneously broken phase of the model.

In Sec. 2, following our previous paper[9], we study the superconformal properties of vacua associated with O(2,3)-solutions. The full little group of a fixed vacuum is shown to be a graded subgroup $\overset{\mathrm{I}}{OSp}(1,4)$ of the conformal supergroup with O(2,3) as the even subgroup. The invariance with respect to another OSp(1,4) ($\overset{\mathrm{II}}{OSp}(1.4)$ in the notation of ref.[9]) which has the

1) All other nontrivial solutions of the massless φ^4-theory (and of the massless Wess-Zumino model) have $T_{00} > 0$.

same O(2,3)-subgroup and whose odd generator is given by an orthogonal combination of superconformal spinor charges is broken to O(2,3). Conformal symmetry is also broken to O(2,3)-symmetry, as in the massless φ^4-theory /8/. Besides, chiral (γ_5-) invariance is broken.

In Sec. 3 we examine the particle content of spontaneously broken phase by transforming the model action to the manifestly O(2,3)-invariant form in which it is represented in terms of fields given on anti de Sitter space O(2,3)/O(1,3). We define a superfield analog of Weyl transformation and with its help demonstrate that the action in O(2,3)-representation automatically possesses the manifest $\overset{\mathrm{I}}{\mathrm{OSp}}(1,4)$-symmetry. In O(2,3)- (and OSp(1,4)-) invariant formalism O(2,3)-solutions reduce to constants minimizing the relevant potential. Thus, the massless Wess-Zumino model can be interpreted as the simplest linear σ-model of spontaneously broken conformal and OSp(1,4)-supersymmetries (an analogous interpretation of the massless φ^4-theory as a linear σ-model of conformal symmetry has been given by Fubini /8/). Each component of the initial multiplet is proved to be the Goldstone field with respect to a certain spontaneously broken superconformal generator. In particular, the spinor component has the meaning of Goldstino accompanying the spontaneous breakdown of $\overset{\mathrm{II}}{\mathrm{OSp}}(1,4)$-symmetry. Upon separating vacuum expectation values of boson components it acquires a "mass" which is twice the inverse radius of anti de Sitter space, in agreement with the general result obtained by Zumino /4/ within the nonlinear realization of OSp(1.4).

The second, closely related subject of the present paper is the construction and examination of the OSp(1,4)-analog of the massive Wess-Zumino model (Sec. 4). This theory reveals a rather complicated vacuum structure which includes, along with OSp(1,4)-invariant vacua, those realizing the spontaneous breakdown of OSp(1,4) to O(2,3). The most interesting phenomenon is the presence of two OSp(1,4)-invariant vacua giving rise to the spontaneous violation of discrete P- and CP-symmetries. As in the many-body problem, regimes with a different symmetry of the ground state go into each other with changing an extra ordering parameter the role of which is played by the anti de Sitter radius. When the latter tends to infinity, the fine structure of vacua disappears and there remains one fully symmetric vacuum of the usual massive Wess-Zumino model.

2. The invariant action of the massless Wess-Zumino model in the standard superfield notation is

$$S=\int d^4x\, d^4\theta \Big\{\Phi_+(x,\theta_+)\exp\big(\tfrac{1}{2}\bar\theta\,\partial\!\!\!/\,\gamma_5\theta\big)\,\Phi_-(x,\theta_-) + \\ + \frac{\sqrt{2}}{3}g\big[\delta^{\Gamma}(\theta_-)\Phi_+^3(x,\theta_+)+\delta^{\Gamma}(\theta_+)\,\Phi_-^3(x,\theta_-)\big]\Big\}, \tag{1}$$

where $\Phi_\pm(x,\theta_\pm)=A_\pm(x)+\bar\theta_\pm\Psi_\pm(x)+\frac{1}{2}\bar\theta_\pm\theta_\pm F_\pm(x)$ are two conjugated chiral superfields, $\delta^{\Gamma}(\theta_\pm)=\frac{1}{2}\bar\theta_\pm\theta_\pm$, $\theta_\pm=\frac{1}{2}(1\pm i\gamma_5)\theta$, $\bar\theta_\pm=\bar\theta\,\frac{1\pm i\gamma_5}{2}$ and θ is the Majorana spinor coordinate[2]. After integration over $d^4\theta$, transition to the real components A, B, F, G and Majorana spinor Ψ by the formulae

$$A_\pm=\frac{1}{\sqrt{2}}(A\pm iB),\quad F_\pm=\frac{1}{\sqrt{2}}(F\pm iG),\quad \Psi_\pm=\frac{1}{2}(1\pm i\gamma_5)\Psi$$

and elimination of the auxiliary fields F, G by their equations of motion

$$F=-g(A^2-B^2)\ ,\quad G=2gAB \tag{2}$$

the action (1) takes the form

$$S=\int d^4x\Big\{\frac{1}{2}\big[(\partial A)^2+(\partial B)^2+i\bar\Psi\,\partial\!\!\!/\,\Psi\big]-\frac{g^2}{2}(A^2+B^2)^2-g\bar\Psi(A-B\gamma_5)\Psi\Big\}\ . \tag{3}$$

The maximal invariance group of the action is the conformal supergroup /6/ with respect to which the components of superfields $\Phi_\pm(x,\theta_\pm)$ form left- and right-handed scalar multiplets of weight 1/2 /6/. Under odd superconformal transformations $\Phi_\pm(x,\theta_\pm)$ transform according to

$$\delta\Phi_\pm(x,\theta_\pm)=\bar\beta\Big[\frac{\partial}{\partial\bar\theta_\pm}+i\,\partial\!\!\!/\,\theta_\pm\Big]\Phi_\pm(x,\theta_\pm)-\frac{i}{2}\Big[\bar\theta_\pm\,\partial\!\!\!/\,\beta-\frac{1}{2}(\bar\theta_\pm\theta_\pm)\frac{\partial}{\partial\theta_\pm}\,\partial\!\!\!/\,\beta\Big]\Phi_\pm(x,\theta_\pm), \tag{4}$$

where

$$\beta=\alpha_1-i(x\gamma)\alpha_2, \tag{5}$$

and α_1, and α_2 are spinor parameters of supertranslations (generator S_α) and proper superconformal transformations (generator T_α).

In addition to the fully symmetric vacuum $(A=B=F=G=\Psi=0)$ the model under consideration has vacua with smaller invariance groups corresponding to the phase with spontaneously broken superconformal symmetry. Like in standard linear σ-models, symmetries of these vacua respect symmetries of the related anomalous vacuum expectation values of fields. The latter are found from the condition that the action have an extremum on them.

2) Notations are the same as in ref. /9/.

In other words, they should be solutions to the equations of motion. It is natural to demand that the ground state of the spontaneously broken phase preserve Lorentz invariance as well as P- and CP-parities. Then, the permissible vacuum structure of the massless Wess-Zumino model is determined by classical solutions for the field $A(x)$ (and $F(x)$) at $B=G=\Psi=0$, i.e., in fact, by solutions of the massless φ^4-theory. The equations of motion of such a theory are known to have no nontrivial constant solutions[8] (the trivial one $A=0$ corresponds to the symmetric phase). There exist, however, x-dependent Lorentz-invariant solutions which break the Poincaré symmetry but are instead invariant under another subgroup of the conformal group, the anti de Sitter group O(2,3). It is an extension of the Lorentz group $O(1,3) \propto M_{\mu\nu}$ by the vector generator $R_\mu = \frac{1}{2}(P_\mu - m^2 K_\mu)$ $([R_\mu, R_\nu] = -i m^2 M_{\mu\nu})$ P_μ and K_μ being the generators of 4-translations and conformal boosts, m a scale parameter ($[m]=L^{-1}$). The O(2,3)-invariant solution for $A(x)$ and that one suggested by it for $F(x)$ are of the form[7,8]

$$A_0(x) = \frac{m}{g}\,\frac{2}{1+m^2x^2} \equiv \frac{m}{g}\,\alpha(x)\,,$$

$$F_0(x) = -\frac{m^2}{g}\,\frac{4}{(1+m^2x^2)^2} = -\frac{m^2}{g}\,\alpha^2(x)\,. \qquad (6)$$

It is convenient to combine (6) into the superfunctions

$$\Phi^0_\pm(x,\theta_\pm) = \frac{m}{\sqrt{2}g}\,\alpha(x)\left[1 - \frac{m}{2}\,\alpha(x)\,\bar\theta_\pm\theta_\pm\right] \equiv \frac{m}{\sqrt{2}g}\, f_0(x,\theta_\pm) \qquad (7)$$

which can be interpreted as the expectation values of superfields $\Phi_\pm(x,\theta_\pm)$ over the vacuum of the spontaneously broken phase $|\bar{0}\rangle$:

$$\Phi^0_\pm(x,\theta_\pm) = \langle \bar{0}|\,\Phi_\pm(x,\theta_\pm)|\bar{0}\rangle\,.$$

It is seen that m measures the strength of the spontaneous breaking of superconformal and conformal symmetries.

Transformation properties of vacuum $|\bar{0}\rangle$ with respect to the conformal supergroup are determined by the transformation properties of the vacuum superfield (7). Now we proceed to describe them in brief, **following** our paper[9] devoted to the analysis of superconformal properties of solutions (6). The invariance of vacuum $|\bar{0}\rangle$ under O(2,3)-transformations is expressed by the relations

$$M_{\mu\nu}\,\Phi^0_\pm(x,\theta_\pm) = 0\,, \qquad R_\mu\,\Phi^0_\pm(x,\theta_\pm) = 0. \qquad (8)$$

Each of odd generators S_α and T_α in itself yields no zero

when acting on $\Phi^0_\pm(x,\theta_\pm)$ that can easily be verified by substituting $\Phi^0_\pm(x,\theta_\pm)$ for $\Phi_\pm(x,\theta_\pm)$ into the superconformal transformation law (4). However, there holds the relation

$$(S - mT)\,\Phi^0_\pm(x,\theta_\pm) = 0 \qquad (9)$$

indicating that the spinor generator $\sim (S - mT)$ should also be included into the little group of vacuum $|\bar{0}\rangle$. It corresponds to the special choice of function β in the law (4):

$$\beta = \frac{1}{\sqrt{2}}(1 + imx\gamma)\,\beta_{\mathrm{I}}\,, \qquad (10)$$

where β_{I} is a constant spinor parameter connected with the normalized generator $Q_{\mathrm{I}} = \frac{1}{\sqrt{2}}(S - mT)$. The generator Q_{I} enlarges the algebra of group O(2,3) to the algebra of the graded group $\mathrm{OSp}^{\mathrm{I}}(1,4) \propto (M_{\mu\nu}, R_\mu, Q_{\mathrm{I}})$ which is thus the full little group of $|\bar{0}\rangle$.

All the remaining independent generators of the conformal supergroup are not zero on $\Phi^0_\pm(x,\theta_\pm)$ and hence are associated with spontaneously broken symmetries. It is convenient to choose these generators so that they belong to the coset space $SU(2,2/1)/\mathrm{OSp}^{\mathrm{I}}(1,4)$:

$$D\,,\ \Pi_5\,,\ G_\mu = \frac{1}{2}(P_\mu + m^2 K_\mu)\,,\ Q_{\mathrm{II}} = \frac{1}{\sqrt{2}}(S + mT). \qquad (11)$$

Here D and Π_5 are the generators of dilatations and chiral (γ_5-) transformations, resp.

We see that in the bose-sector of the present model (as in the O(2,3)-sector of the massless φ^4-theory[8]) there comes out broken scale invariance and invariance under the fixed combination of translations and proper conformal transformations (generator G_μ). Chiral invariance is also broken. The spontaneously broken component of odd superconformal transformations is represented by the generator Q_{II} corresponding to the choice $\beta = \frac{1}{\sqrt{2}}(1 - imx\gamma)\,\beta_{\mathrm{II}}$ in the law (4). This generator, like Q_{I}, enlarges the algebra O(2,3) to the orthosymplectic superalgebra, $\mathrm{OSp}^{\mathrm{II}}(1,4)$ (a closure of $\mathrm{OSp}^{\mathrm{I}}(1,4)$, $\mathrm{OSp}^{\mathrm{II}}(1,4)$ coincides with the conformal superalgebra[9]). Thus, the breakdown of superconformal symmetry in the massless Wess-Zumino model proceeds not via the subgroup of usual supersymmetry but via the subgroup $\mathrm{OSp}^{\mathrm{II}}(1,4)$.

Transformations with generators from the set (11), being applied to vacuum $|\bar{0}\rangle$, produce orbits of equivalent vacua, in the same way as in ordinary σ-models. The relevant field expectation values are classical solutions rotated with respect to (6) by the same transformations (they may involve bosonic as well as Grassmann fermionic parameters /7,9/). Theories built upon such vacua are equivalent to each other in virtue of the superconformal invariance of the action. In what follows, without loss of generality, we shall proceed from the solutions (6) and vacuum $|\overline{0}\rangle$ which is unambiguously fixed by the conditions (8), (9) and the requirement of P- and CP-conservation /9/.

To conclude this Section, we note that vacuum $|\overline{0}\rangle$, like O(2,3)-vacua of the massless φ^4-theory, from the energetic point of view is not distinguished in comparison with the fully symmetric vacuum $(A=B=\Psi=F=G=0)$: on solutions (6) the improved energy-momentum tensor $T_{\mu\nu}$ vanishes (as well as the spin-vector current and, correspondingly, the supercurrent). This is the essential difference between the present model (and the massless φ^4-theory) and standard σ-models of internal symmetries where, as a rule, vacua of the spontaneously broken phase already at the classical level possess the lower energy as compared to the symmetric vacuum. Perhaps, the situation will alter after allowing for the radiative corrections[3]. In what follows, we confine our consideration to the classical level reserving for the future the analysis of the question under which conditions $|\overline{0}\rangle$ dominates over the symmetric vacuum.

3. Now we turn to determining the physical spectrum of the spontaneously broken phase. As usual, this implies the transition to fields with zero vacuum expectation values. However, upon a direct subtraction of anomalous values (6) from the initial fields the potential part of the rearranged Lagrangian would get explicit dependence on X_μ. For a better correspondence with usual σ-models it is more convenient to reconstruct before the action to the form in which it displays the manifest O(2,3)-invariance. By this procedure, solutions (6) reduce to constants and all the coordinate dependence of the corresponding Lagrangian turns out to be concentrated in kinetic terms of fields where it enters through the metric of anti de Sitter space O(2,3)/O(1,3) playing in O(2,3)-formalism the role of a background space /8,11/.

The most straightforward way to arrive at the O(2,3)-representation is to apply the relevant Weyl transformation /11/ directly to fields in the component action (3), by analogy with the procedure employed, say, in the massless φ^4-theory /8/. Nevertheless, we prefer to proceed from the action in the superfield form (1) having for the object to demonstrate that Weyl transformation for the physical components is in fact a part of a more general superfield transformation describing the transition to the manifestly OSp(1,4)-invariant formulation of the massless Wess-Zumino model.

Remind before the meaning of Weyl transformations and also what is understood by Weyl covariance of conformal-invariant theories.

Any conformal-invariant theory can be represented not only by the standard, manifestly Poincaré-invariant Lagrangian but equally by Lagrangians manifestly invariant with respect to other subgroups of the conformal group isomorphic to groups of motions of conformally flat spaces (a Riemannian space is said to be conformally flat if its metric differs from $\eta_{\mu\nu}=(1,-1,-1,-1)$ merely by a local factor /11/; such spaces have isomorphic conformal groups and comprise both Minkowski and anti de Sitter spaces). This property is called Weyl covariance. Weyl transformations (see e.g. /11/) connect equivalent sets of fields representing a given conformal-invariant theory in different conformally flat spaces. The concrete form of Weyl transformation to a certain conformally flat space can easily be found from purely group-theoretical considerations: its structure, up to an unessential scale factor, is fixed by the requirement that the relevant subgroup of the conformal group be realized with zero weight on the Weyl transformed fields. For instance, the transformation from Poincaré- covariant fields $\Lambda_K(X)$ to O(2,3)-covariant fields $\widetilde{\Lambda}_K(X)$ can be determined from the condition of absence of weight terms in the generator $R_\mu=\frac{1}{2}(P_\mu-m^2K_\mu)$ when the latter is applied to $\widetilde{\Lambda}_K(X)$. The result is:

$$\widetilde{\Lambda}_K(x)=\left[\rho\,\alpha(x)\right]^{-d_\Lambda}\Lambda_K(x)\,. \tag{12}$$

Here d_Λ is the dimensionality of $\Lambda_K(x)$ (in mass units) and the function $\alpha(x)$ is the same as in (6). Without loss of generality, the scale multiplier ρ will be set one from now on.

It is natural to expect that the notion of Weyl covariance generalizes to superconformal-invariant theories. In other words,

3) The fact that the symmetric, conformal-invariant phase of the massless φ^4-theory is unstable against radiative corrections has been pointed out in /10/. Perhaps, this means that it would be more correct to construct quantum theory from the beginning upon the O(2,3)-invariant vacuum.

they should admit equivalent representations in any superspace having the same dimensionality as the usual superspace and including one or another conformally flat space as the maximal even subspace. The manifest invariance group of a given formulation of a theory is expected to be that graded subgroup of the conformal supergroup which is a suitable spinor extension of the group of motions of corresponding even subspace. We have as yet no general rigorous proof of the above statements but are able to confirm them in the particular case we are interested in.

To find a superfield extension of (12), $\Phi_\pm(x,\theta_\pm) \to \widetilde{\Phi}_\pm(x,\theta_\pm)$, transforming the action (1) to the representation in the superspace $OSp(1,4)/O(1,3) \supset O(2,3)/O(1,3)$, one may require that generators of a certain subgroup $OSp(1,4)$ of the conformal supergroup, say $\overset{I}{OSp}(1,4)$, contain no weight terms when realized on $\widetilde{\Phi}_\pm(x,\theta_\pm)$. It is not hard to be convinced that for removing the weight factor $-\frac{i}{2\sqrt{2}}\bar{\theta}_\pm \not{\partial}(1+imx\gamma)\beta_I$ from the transformation with the generator $Q_I = \frac{1}{\sqrt{2}}(S-mT)$ ($\beta = \frac{1}{\sqrt{2}}(1+imx\gamma)\beta_I$ in the transformation rule (4)) it is sufficient to multiply superfields $\Phi_\pm(x,\theta_\pm)$ by superfunctions $f_0^{-1}(x,\theta_\pm)$, the inverse of (7). The weight connected with O(2,3)-translations is taken away by the variable change $\theta_\pm \to \frac{1}{\sqrt{a(x)}}\theta_\pm$. As a result, we have

$$\widetilde{\Phi}_\pm(x,\theta_\pm) = f_0^{-1}\left(x,\frac{1}{\sqrt{a}}\theta_\pm\right)\Phi_\pm\left(x,\frac{1}{\sqrt{a}}\theta_\pm\right) = a^{-1}(x)\left(1+\frac{m}{2}\bar{\theta}_\pm\theta_\pm\right)\Phi_\pm\left(x,\frac{1}{\sqrt{a}}\theta_\pm\right) \quad (13a)$$

$$\Phi_\pm(x,\theta_\pm) = f_0(x,\theta_\pm)\widetilde{\Phi}_\pm(x,\sqrt{a}\,\theta_\pm) = a(x)\left(1-\frac{m}{2}a(x)\bar{\theta}_\pm\theta_\pm\right)\widetilde{\Phi}_\pm(x,\sqrt{a}\,\theta_\pm) \quad (13b)$$

or, in components:

$$\widetilde{A}_\pm(x) = a^{-1}(x)\,A_\pm(x)\,, \quad \widetilde{\Psi}_\pm(x) = a^{-3/2}(x)\,\Psi_\pm(x)$$
$$\widetilde{F}_\pm(x) = a^{-2}(x)\,F_\pm(x) + a^{-1}(x)\,m\,A_\pm(x)\,. \quad (14)$$

For the physical components, as promised, the mapping (14) reduces to the transformation (12). A nontrivial novel structure of this mapping manifests itself in relation between the auxiliary components: $\widetilde{F}_\pm(x)$ appears a fixed combination of O(2,3)-covariant fields $a^{-2}(x)\,F_\pm(x)$ and $a^{-1}(x)\,A_\pm(x)$.

The odd $\overset{I}{OSp}(1,4)$-transformations induced for $\widetilde{\Phi}_\pm(x,\theta_\pm)$ by the law (4) (via the connection (13a)) are as follows:

$$\delta_{Q_I}\widetilde{\Phi}_\pm(x,\theta_\pm) = \bar{\beta}\sqrt{a}\left[1+\frac{m}{2}\bar{\theta}_\pm\theta_\pm\left(1+\frac{3}{2}imx\gamma\right)\right]\frac{\partial}{\partial\bar{\theta}_\pm}\widetilde{\Phi}_\pm(x,\theta_\pm) + \frac{i}{\sqrt{a}}\bar{\beta}\gamma^\mu\theta_\pm\partial_\mu\widetilde{\Phi}_\pm(x,\theta_\pm) \quad (15)$$

$$\delta_{Q_I}\widetilde{A}_\pm = \sqrt{a}\,\bar{\beta}\widetilde{\Psi}_\pm$$
$$\delta_{Q_I}\widetilde{\Psi}_\pm = \sqrt{a}\,\frac{1\pm i\gamma_5}{2}\left(-\frac{i}{a}\gamma^\mu\partial_\mu\widetilde{A}_\pm + \widetilde{F}_\pm\right)\beta \quad (16)$$
$$\delta_{Q_I}\widetilde{F}_\pm = \sqrt{a}\,\bar{\beta}\left[-\frac{i}{a}\gamma^\mu\partial_\mu\widetilde{\Psi}_\pm + m\left(1+\frac{3}{2}imx\gamma\right)\widetilde{\Psi}_\pm\right]$$

where as before $\beta = \frac{1}{\sqrt{2}}(1+imx\gamma)\beta_I$. Under O(2,3)-translations $\widetilde{\Phi}_\pm(x,\theta_\pm)$ transform as

$$\delta_R\widetilde{\Phi}_\pm(x,\theta_\pm) = -\frac{im^2}{2}\lambda^\mu x^\nu\bar{\theta}_\pm\sigma_{\mu\nu}\frac{\partial}{\partial\bar{\theta}_\pm}\widetilde{\Phi}_\pm(x,\theta_\pm) + \frac{1}{2}\left[\lambda^\mu + 2m^2(\lambda x)x^\mu - m^2x^2\lambda^\mu\right]\partial_\mu\widetilde{\Phi}_\pm(x,\theta_\pm)\,, \quad (17)$$

λ^μ being the infinitesimal transformation parameter. Thus, superfields $\widetilde{\Phi}_\pm(x,\theta_\pm)$ transform in $\overset{I}{OSp}(1,4)$ exclusively due to nonlinear shifts of their arguments x_μ, $\theta_\pm$ and, hence, they are $OSp(1,4)$-scalars. Correspondingly, the fields $\widetilde{A}_\pm(x)$, $\widetilde{\Psi}_\pm(x)$, $\widetilde{F}_\pm(x)$ form conjugated left- and right-handed scalar OSp(1,4)-multiplets. Under contraction $m \to 0$ the laws (15)-(17) go into the transformation rules of scalar multiplets of the usual supersymmetry. The fact that OSp(1,4) has scalar multiplets has been established earlier by Keck/12/ but without indicating their connection with the realization of OSp(1,4) in the left- and right-handed chiral superspaces. Our results show that this connection is quite analogous to that one in the usual supersymmetry.

Now we are ready to obtain the OSp(1,4)-invariant representation of the action (1). Upon the substitution (13b) and the subsequent variable change $\theta_\pm \to \frac{1}{\sqrt{a}}\theta_\pm$ the potential and kinetic parts of the action take the form:

$$S_V = \frac{\sqrt{2}}{3}g\int d^4x\,d^4\theta\left[\mathcal{M}_+(x,\theta_+)\delta^{\Gamma}(\theta_-)\widetilde{\Phi}_+^3(x,\theta_+) + \mathcal{M}_-(x,\theta_-)\delta^{\Gamma}(\theta_+)\widetilde{\Phi}_-^3(x,\theta_-)\right] \quad (18)$$

$$S_K = \int d^4x\,d^4\theta\,\mathcal{M}(x,\theta)\,e^{-\frac{1}{4}\bar{\theta}\gamma^\mu\gamma_5\theta\hat{\nabla}_\mu}\widetilde{\Phi}_+(x,\theta_+)\,e^{\frac{1}{4}\bar{\theta}\gamma^\rho\gamma_5\theta\hat{\nabla}_\rho}\widetilde{\Phi}_-(x,\theta_-)\,, \quad (19)$$

where

$$\mathcal{M}_\pm(x,\theta_\pm) = a^4(x)\left(1-\frac{3}{2}m\,\bar{\theta}_\pm\theta_\pm\right) \quad (20)$$

$$\mathcal{M}(x,\theta) = a^4(x)\left[1-\frac{3}{2}m\,\bar{\theta}\theta + \frac{3}{8}m^2(\bar{\theta}\theta)^2\right] \quad (21)$$

$$\hat{\nabla}_\mu = a^{-1}(x)\,\partial_\mu - \frac{im}{2}\left[\bar{\theta}(\gamma_\mu + m x^\rho\sigma_{\rho\mu})\right]\frac{\partial}{\partial\bar{\theta}}\,. \quad (22)$$

Without going into details (the complete derivation will be given in a separate paper devoted to the superfield formulation of

OSp(1,4)-symmetry) we indicate only that $d^4x\, d^2\theta_\pm\, \mathcal{M}_\pm(x,\theta_\pm)$ are the OSp(1,4)-invariant integration measures over conjugated chiral superspaces (their invariance with respect to the transformations (15), (17) can be checked straightforwardly, by making use of general rules for changing variables in Grassmann integrals/13/), the objects $\exp\{\mp\frac{1}{4}\bar\theta\gamma^\rho\gamma_5\theta\hat\nabla_\rho\}\widetilde\Phi_\pm(x,\theta_\pm)$ are nothing but the chiral OSp(1,4)-superfields in the real basis, $\hat\nabla_\rho$ and $d^4x\, d^4\theta\, \mathcal{M}(x,\theta)$ have, resp., the meaning of the vector covariant derivative and invariant integration measure in this basis. Thus, the action of the usual massless Wess-Zumino model is identical to the action of its direct OSp(1,4)-analog, in the same way-as the action of the massless φ^4-theory is identical to that of its counterpart in anti de Sitter space/8/. Point out once more that this remarkable fact should be traced to the generalized Weyl covariance of the action (1) caused by its superconformal invariance.

Integrating over $d^4\theta$ and eliminating the auxiliary fields $\widetilde F(x)$, $\widetilde G(x)$ by their equations of motion

$$\begin{aligned}\widetilde F(x) &= m\widetilde A(x) - g\,(\widetilde A^2(x) - \widetilde B^2(x))\\ \widetilde G(x) &= m\widetilde B(x) + 2g\widetilde A(x)\widetilde B(x)\end{aligned} \tag{23}$$

one reduces the action S=(18)+(19) to the form

$$S=\int d^4x\, a^4(x)\Big[\frac{1}{2}\eta^{\mu\nu}(\nabla_\mu\widetilde A\nabla_\nu\widetilde A+\nabla_\mu\widetilde B\nabla_\nu\widetilde B+i\bar{\widetilde\Psi}\gamma_\mu\nabla_\nu\widetilde\Psi)+m^2(\widetilde A^2+\widetilde B^2)- -\frac{g^2}{2}(\widetilde A^2+\widetilde B^2)^2-g\bar{\widetilde\Psi}(A-B\gamma_5)\widetilde\Psi\Big] \tag{24}$$

where $\nabla_\mu = a^{-1}(x)\partial_\mu+\dots$ is the O(2,3)-covariant derivative, dots stand for the matrix part which is not essential for our purposes (it does not contribute to (24) because of scalarity of fields $\widetilde A, \widetilde B$ and the Majorana nature of $\widetilde\Psi$). Taking into account that the quantity $g_{\mu\nu}\stackrel{\text{def}}{=}a^2\eta_{\mu\nu}$ can be interpreted as the metric tensor of anti de Sitter space with radius $\tau=m^{-1}$/11/ $(g^{-1}_{\mu\nu}\equiv g^{\mu\nu}=a^{-2}(x)\eta^{\mu\nu}$, $a(x)\eta_{\mu\nu}$ and $a^{-1}(x)\eta^{\mu\nu}$ being appropriate direct and inverse vierbeins) and also that $a^4(x)=\sqrt{-\|g_{\mu\nu}\|}$, $m^2=-\frac{1}{12}R$ where R is the scalar curvature for the metric $g_{\mu\nu}(x)$, one recognizes (24) as the standard conformal-invariant action for massless fields in curved background. Note that the representation (24) and the equations (23) might be attained directly by applying Weyl transformation (12) to the fields A, B, Ψ in (3) and (2). However, when doing so, it is difficult to make out the manifest OSp(1,4)-invariance of the action (24).

Return to the analysis of structure of the spontaneously broken phase. As has been anticipated at the beginning of this Section, the classical solutions (6) in terms of O(2,3)-covariant fields reduce to constants:

$$\widetilde A_0=\frac{m}{g}\,,\quad \widetilde F_0=\widetilde B_0=\widetilde G_0=\widetilde\Psi_0=0 \tag{25}$$

$$\langle\bar 0|\widetilde\Phi_\pm(x,\theta_\pm)|\bar 0\rangle=\langle\bar 0|\widetilde\Phi_\pm(0,0)|\bar 0\rangle=\frac{m}{\sqrt 2\, g}\,. \tag{26}$$

Being X-independent, these solutions can be obtained directly from the condition of extremum of the potential in the action (24):

$$V(\widetilde A,\widetilde B,\widetilde\Psi)=-m^2(\widetilde A^2+\widetilde B^2)+\frac{g^2}{2}(\widetilde A^2+\widetilde B^2)^2+g\bar{\widetilde\Psi}(\widetilde A-\widetilde B\gamma_5)\widetilde\Psi \tag{27}$$

It is remarkable that they supply just the minimum to the potential, i.e. play the role analogous to constant solutions in linear σ-models of internal symmetries. Indeed, due to the wrong sign of the "mass" term the bosonic part of (27) strongly resembles the usual Higgs potential and attains minima on the circle of radius $\left|\frac{m}{g}\right|$ in $\widetilde A$-$\widetilde B$ plane:

$$\widetilde A^2+\widetilde B^2=\frac{m^2}{g^2}\,. \tag{28}$$

The general solution of this equation and the related solution for the auxiliary components can be written as

$$\widetilde A_\lambda=\frac{m}{g}\cos\lambda\,,\quad \widetilde B_\lambda=\frac{m}{g}\sin\lambda$$

$$\widetilde F_\lambda=\frac{m^2}{g}(\cos\lambda-\cos 2\lambda)\,,\ \widetilde G_\lambda=\frac{m^2}{g}(\sin\lambda+\sin 2\lambda)\,, \tag{29}$$

where λ is an arbitrary parameter which reflects a degeneracy with respect to chiral transformations. In virtue of chiral invariance, solutions with different λ should be treated on equal footing and without loss of generality one may choose the solution (25) corresponding to $\lambda=0$.

On passing to the field with zero vacuum expectation value $\widetilde A'=\widetilde A-\frac{m}{g}$, the potential (27) rearranges to the form:

$$V=-\frac{m^4}{2g^2}+2m^2\widetilde A'^2+m\bar{\widetilde\Psi}\widetilde\Psi+2mg\widetilde A'(\widetilde A'^2+\widetilde B^2)+\frac{g^2}{2}(\widetilde A'^2+\widetilde B^2)^2+ +g\bar{\widetilde\Psi}(\widetilde A'-\widetilde B\gamma_5)\widetilde\Psi\,. \tag{30}$$

Thus, the spectrum of the spontaneously broken phase consists of the massless pseudoscalar field $\widetilde B$, the scalar field $\widetilde A'$ with mass $2m$, and the Majorana spinor $\widetilde\Psi$ of the same mass4) all defined on anti de Sitter space.

4) For fields over anti de Sitter space the "mass" is a rather ambiguous concept. We define bare mass parameters of the Lagrangian in the conventional manner, via terms quadratic in fields.

Let us clarify the status of these fields with respect to superconformal transformations. As expected, the little group $\overset{I}{OSp}(1,4)$ is realized on them homogeneously. At the same time, generators from the set (11) give rise to inhomogeneous transformations. We begin by considering the action of the $\overset{II}{OSp}(1,4)$-generator Q_{II}. After a little labour we find:

$$\delta_{Q_{II}}\widetilde{\Psi}_{\pm}=\sqrt{a}\,\frac{1\pm i\gamma_5}{2}\left(-\frac{i}{a}\gamma^{\mu}\partial_{\mu}\widetilde{A}_{\pm}+\widetilde{F}_{\pm}-2m\widetilde{A}_{\pm}\right)\frac{1}{\sqrt{2}}\left(1-imx\gamma\right)\beta_{II}\,. \quad (31)$$

Extracting from $\widetilde{A}_{\pm}$ the vacuum value, we observe that Q_{II}-transformation of $\widetilde{\Psi}(x)$ starts with a constant:

$$\delta_{Q_{II}}\widetilde{\Psi}=-\sqrt{2}\,\frac{m^2}{g}\beta_{II}+i\sqrt{2}\,\frac{m^3}{g}(x\gamma)\beta_{II}+O(x^2,\widetilde{A}'_{\pm})\,. \quad (32)$$

Hence, $\widetilde{\Psi}$ is the Goldstone fermion (Goldstino) accompanying the spontaneous breaking of $\overset{II}{OSp}(1,4)$-symmetry. The fact that it possesses the mass term $m\overline{\widetilde{\Psi}}\widetilde{\Psi}$, unusual for Goldstone fields, agrees with the general result obtained by Zumino /4/ in the framework of the nonlinear realization.

It is interesting to trace in detail how the OSp(1,4)-structure arises in the Wess-Zumino model. When deriving the representation (24), we proceeded from the superfield formulation manifestly invariant with respect to $\overset{I}{OSp}(1,4)$. However, we would come to the same result choosing the subgroup $\overset{II}{OSp}(1,4)$ to begin with. This is clear already from the fact that (24) is an even function of parameter m. In other words, the action (24) simultaneously describes the linear realizations of two different OSp(1,4)-supersymmetries, $\overset{I}{OSp}(1,4)$ and $\overset{II}{OSp}(1,4)$. So far as the superconformal symmetry is unbroken, these OSp(1,4) are on entirely equal status. After allowing for the solution (25) the degeneracy is removed: $\overset{I}{OSp}(1,4)$ takes the role of the stability subgroup of (25) while $\overset{II}{OSp}(1,4)$ gets broken to O(2,3). If one chooses the solution with $\lambda=\pi$ the situation is reversed. More generally, the solution with a fixed λ is stable under the subgroup $\overset{(\lambda)}{OSp}(1,4)=e^{i\lambda\Pi_5}\overset{I}{OSp}(1,4)\,e^{-i\lambda\Pi_5}$ and breaks the symmetry with respect to $\overset{(\lambda+\pi)}{OSp}(1,4)=e^{i\lambda\Pi_5}\overset{II}{OSp}(1,4)\,e^{-i\lambda\Pi_5}$ (all $\overset{(\lambda)}{OSp}(1,4)$ have the common even subgroup $O(2,3)\propto(M_{\mu\nu},R_{\lambda})$ /9/).

The Goldstone fields may be assigned not only to Q_{II} but also to all of the remaining generators from the set (11). The field $\widetilde{B}(x)$ transforms inhomogeneously under the action of the generator Π_5 and therefore is the Goldstonion associated with the spontaneous breaking of chiral symmetry. Its "masslessness" can be traced to the fact that chiral symmetry is purely internal for

which reason the standard arguments of Goldstone's theorem apply. The component $\widetilde{A}'(x)$ plays the double role. With respect to the generator Π_5 it is the Higgs field while with respect to D the Higgs and simultaneously Goldstone field (dilaton) for its infinitesimal scale transformation begins with a constant. The "mass" term $2m^2\widetilde{A}'^2$ has the meaning analogous to that of $m\overline{\widetilde{\Psi}}\widetilde{\Psi}$. In both cases, for invariance of the action under corresponding spontaneously broken transformations (i.e., dilatations and $\overset{II}{OSp}(1,4)$-supertranslations) it is necessary that the Goldstone field kinetic terms occur in the combination with these unconventional mass terms (because of an explicit dependence of the transformations on x_{μ}). As to the generator $G_{\mu}=\frac{1}{2}(P_{\mu}+m^2K_{\mu})$, there is no independent Goldstone field for it among components of the initial multiplet. However, taking notice of the fact that G_{μ}-transformation of the field $\widetilde{A}'(x)$ begins with a term $\sim(t^{\mu}x_{\mu})$, where t_{μ} is an appropriate group parameter, it is clear that a missing Goldstonion is imitated by the gradient $\partial_{\mu}\widetilde{A}'$ whose G_{μ}-transformation includes a purely constant term $\sim t_{\mu}$ [5].

To summarize, we have shown that the ordinary massless Wess-Zumino model is the linear σ-model simultaneously of two spontaneously broken supersymmetries, SU(2,2/1) and $\overset{II}{OSp}(1,4)$, realized, respectively, in homogeneous spaces SU(2,2/1)/$\overset{I}{OSp}(1,4)$ and $\overset{II}{OSp}(1,4)$/O(2,3). Their breakdown is induced by $\overset{I}{OSp}(1,4)$-invariant classical solutions to the equations of motion. These solutions display the coordinate dependence when considered in Minkowski space but reduce to constants in anti de Sitter space. The regime of spontaneous breaking is stable: tachyons do not appear.

4. Having the expressions for the OSp(1,4)-invariant integration measures in chiral superspaces ($20_{\pm}$) we are in a position to construct OSp(1,4)-symmetric models with scalar potentials of an arbitrary structure in superfields $\widetilde{\Phi}_{\pm}(x,\theta_{\pm})$. All such models, except the special case of the massless $\widetilde{\Phi}^3$ theory considered above, contain dimensional constants (beyond m). For this reason they possess no superconformal symmetry and, as a result, are not equivalent to any kind of supersymmetric theories in Minkowski space. The simplest model is set up by adding to the action (18)+(19) the OSp(1,4)-invariant mass term:

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[5] An analogous phenomenon in nonlinear realizations is known as the inverse Higgs phenomenon /14/.

$$S_M = M\int d^4x\, d^4\theta \left[\mathcal{M}_+(x,\theta_+)\delta^{\Gamma}(\theta_-)\tilde{\Phi}_+^2(x,\theta_+) + \mathcal{M}_-(x,\theta_-)\delta^{\Gamma}(\theta_+)\tilde{\Phi}_-^2(x,\theta_-)\right] . \qquad (33)$$

The theory thus constructed is the OSp(1,4)-analog of the usual massive Wess-Zumino model[6b] and provides a nontrivial example of linear globally supersymmetric theory in curved space-time. After eliminating the auxiliary fields $\tilde{F}, \tilde{G}$ through their equations of motion

$$\begin{aligned} \tilde{F} &= (m-2M)\tilde{A} - g(\tilde{A}^2 - \tilde{B}^2) \\ \tilde{G} &= (m+2M)\tilde{B} + 2g\tilde{A}\tilde{B} \end{aligned} \qquad (34)$$

we find that the kinetic part of the Lagrangian density in the action $S_K + S_V + S_M$ expressed in terms of the physical components $\tilde{A}, \tilde{B}, \tilde{\Psi}$ coincides with the corresponding part of the density of the action (24). The potential part is now given by the expression:

$$\begin{aligned} V_M = {} & (M+m)(2M-m)\tilde{A}^2 + (M-m)(2M+m)\tilde{B}^2 + M\bar{\tilde{\Psi}}\tilde{\Psi} + \frac{g^2}{2}(\tilde{A}^2+\tilde{B}^2)^2 + \\ & + 2gM\tilde{A}(\tilde{A}^2+\tilde{B}^2) + g\bar{\tilde{\Psi}}(\tilde{A} - \tilde{B}\gamma_5)\tilde{\Psi} . \end{aligned} \qquad (35)$$

The potential (35) in contrast to (27) is not symmetric under the change $m \to -m$. This reflects noninvariance of the present model with respect to the supergroup $\overset{II}{OSp}(1,4)$ (and thereby to the conformal supergroup, the closure of $\overset{I}{OSp}(1,4)$ and $\overset{II}{OSp}(1,4)$).

As is well known, the conventional massive Wess-Zumino model exhibits no nontrivial vacuum structure[15]. In the present case, the situation is quite different. Depending on a relation between the parameters M and m, the potential (35) attains minima on four different sets of constant solutions to the equations of motion ($m>0$ fixed):

a) $|M| \geqslant m$

$$\langle\tilde{A}\rangle_0 = \langle\tilde{B}\rangle_0 = \langle\tilde{\Psi}\rangle_0 = \langle\tilde{F}\rangle_0 = \langle\tilde{G}\rangle_0 = 0 , \quad \langle T_{\mu\nu}\rangle_0 = 0 \qquad (36)$$

b) $M \geqslant 2m$ or $M \leqslant 0$

$$\langle\tilde{A}\rangle_0 = \frac{m-2M}{g}, \langle\tilde{B}\rangle_0 = \langle\tilde{\Psi}\rangle_0 = \langle\tilde{F}\rangle_0 = \langle\tilde{G}\rangle_0 = 0, \langle T_{\mu\nu}\rangle_0 = g_{\mu\nu} mM \frac{(m-2M)^2}{g^2} \qquad (37)$$

c) $-m \leqslant M \leqslant 0$ or $m \leqslant M \leqslant 2m$

$$\langle\tilde{A}\rangle_0 = -\frac{m+M}{g}, \langle\tilde{F}\rangle_0 = \frac{(m+M)(M-2m)}{g}, \langle\tilde{B}\rangle_0 = \langle\tilde{\Psi}\rangle_0 = \langle\tilde{G}\rangle_0 = 0, \langle T_{\mu\nu}\rangle_0 = g_{\mu\nu} M^2 \frac{(M+m)^2}{2g^2} \qquad (38)$$

d) $0 \leqslant M \leqslant m$

$$\langle\tilde{A}\rangle_0 = -\frac{m+2M}{2g}, \langle\tilde{B}\rangle_0 = \pm\frac{\sqrt{(m+2M)(3m-2M)}}{2g}, \langle\tilde{F}\rangle_0 = \langle\tilde{G}\rangle_0 = \langle\tilde{\Psi}\rangle_0 = 0$$

$$\langle T_{\mu\nu}\rangle_0 = -g_{\mu\nu} mM \frac{(m+2M)(m-2M)}{2g^2} \qquad (39)$$

Here $T_{\mu\nu}$ is the energy-momentum tensor defined according to the general prescriptions for curved backgrounds [16]:

$$T_{\mu\nu} = g_{\mu\nu}V_M - \frac{1}{24} g_{\mu\nu} R(\tilde{A}^2 + \tilde{B}^2) \quad + \text{derivative terms}$$

The existence domains of vacua (37)-(40) are pictured in Fig. 1

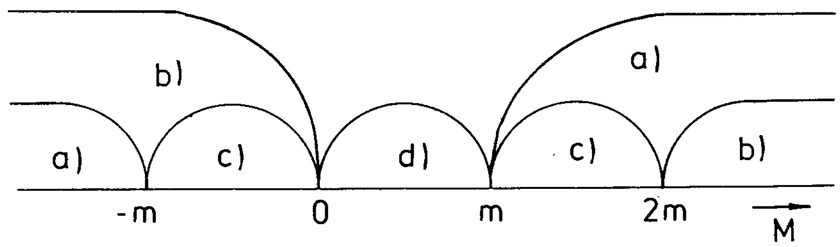

Fig.1

(the vacuum structure for the choice $m<0$ can be obtained by reversing the positive direction on axis M).

For the lack of room, we do not give explicit forms for rearranged potentials in sectors a)-d) and are forced to limit ourselves to a number of comments.

First of all, point out that the physical masses in all these sectors are real, i.e.,ghosts are absent in the particle spectrum (this is because solutions (36)-($39_\pm$) supply true minima to the potential (35)).

Vacua (36), (37) correspond to the symmetric phase. In terms of zero vacuum expectation fields the potential in sector b) looks as in a) but with $M' = m - M$ instead of M. The boson and fermion masses split already at the symmetric level, m being a splitting parameter.

Vacuum (38) gives rise to the spontaneous breaking of OSp(1,4) down to O(2,3)-symmetry as $\langle\tilde{F}\rangle_0 \neq 0$ over it. In this sector, $\tilde{\Psi}$ is the Goldstino. Its mass is $m_\Psi = 2m$ again in agreement with the general theorem of Zumino[4]. The masses of the Higgs fields $\tilde{A}', \tilde{B}$ are related to $\tilde{\Psi}$ by the simple mass formula

$$m^2_{\tilde{A}'} + m^2_{\tilde{B}} = m^2_{\tilde{\Psi}} . \qquad (41)$$

The most interesting and unexpected feature of the model under consideration is the presence of two stable OSp(1,4)-symmetric vacua ($39_\pm$) with spontaneously broken P- and CP-parities.

Within the range $0 \leq M \leq m$ there are no other stable vacua, i.e., solutions ($39_\pm$) give absolute minima to the potential (35) in this range. The constant of the P- and CP-violating interaction in the diagonalized potential appears to be a function of bare parameters g, m, M. We expect that the phenomenon of P- and CP-violation will occur also in other, more realistic OSp(1,4)-invariant models.

It is seen from Fig. 1 that with M fixed and the space radius m^{-1} varying, phases with a different symmetry of the ground state change each other. In this sense the parameter m is similar to the ordering parameter (temperature) in the many-body problem. In the case of large curvature ($m \gg M$) the phase with spontaneously broken P- and CP-parities dominates. To the small curvature there corresponds the fully symmetric phase[6]. When $M=0$, the solutions (37)-($39_\pm$) go into solutions from the set (29), with $\lambda=0$, π, $\frac{2}{3}\pi$, $\frac{4}{3}\pi$, respectively. In this limit, the violation of P- and CP-parities becomes unobservable because, due to chiral invariance, sectors b)-d) turn out to be related by equivalence field transformations.

In the limit $m \to 0$, $M \neq 0$ leading to the usual massive Wess-Zumino model there survives, as expected, the symmetric phase alone. As $m \to 0$, the fine vacuum structure locked in the interval $-m \leq M \leq 2m$ degenerates into the point $M=0$.

5. In this paper we have studied simplest σ-models with spontaneously broken global conformal and OSp(1,4)-supersymmetries. In conclusion we discuss in brief what happens in the local case, i.e., after coupling these models to gauge fields of supergravity.

To maintain global superconformal symmetry, one must couple the massless Wess-Zumino model to conformal supergravity/1/. As far as all the fields $\tilde{A}', \tilde{B}, \tilde{\Psi}$ of the spontaneously broken phase are of the Goldstone type, it is clear without explicit calculations that in the local case they can be removed from the Lagrangian by the Higgs effect. The Lagrangian in unitary gauge is expected to contain only terms of gauge fields and to show the manifest invariance only with respect to local OSp(1,4)-transformations. Just such a situation has been observed recently by Kaku and Townsend for the case of the self-interacting massless scalar multiplet minimally coupled to conformal supergravity/18/. They have shown that in the gauge $\tilde{A}=\mathrm{const}$, $\tilde{B}=0$, $\tilde{\Psi}=0$ the relevant

6) A similar situation for some open metrics has been revealed also in /17/.

Lagrangian coincides with the pure gauge Lagrangian of OSp(1,4)-supergravity/19/ (that of Poincaré supergravity plus a fixed combination of the gravitino mass term and the cosmological term). We have verified that this gauge exactly corresponds to the classical solution (25), i.e. is the usual unitary gauge if one works in terms of the Goldstone fields $\tilde{A}'$, $\tilde{B}$, $\tilde{\Psi}$ with zero vacuum expectation values.

To make the model of Sec.4 locally OSp(1,4)-invariant, one has to couple it to OSp(1,4)-supergravity. To reproduce features of the real world the resulting model should give a reasonable order of the mass splitting between bosons and fermions involved (i.e., the splitting parameter m must be $\sim$ 1 GeV), and, besides, ensures the cosmological term to be observably small (vanishing). Presumably, these requirements can both be satisfied only in sector c) corresponding to the phase with spontaneously broken OSp(1,4)-symmetry, where the Deser-Zumino/3/ mechanism of compensating cosmological terms may be operative. The detailed discussion of this possibility and also of the question how the CP-violating phase d) manifests itself in the local case will be given elsewhere.

Finally, point out that the model of Sec. 4 may bear interest irrespective of its possible relation to supergravity. Assuming that integration in the invariant action is only over a small space-time region of an order of the hadron size, m^{-1} being $\sim$ radius of this region, this model can be regarded as a supersymmetric extension of the bag model with the anti de Sitter geometry proposed in/20/.

We express our deep gratitude to V.P.Akulov, A.A.Kapustnikov, D.V.Volkov, V.Yu.Tzeitlin for useful discussions. We are especially grateful to V.I.Ogievetsky for interest in the work, kind attention, and valuable discussions.

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Received by Publishing Department
on March 23 1979.