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К вопросу о распаде $\eta \to \pi^\circ \gamma\gamma$

В рамках киральной квантовой теории показано, что процесс $\eta \to \pi^\circ \gamma\gamma$ связан с процессом $\eta \to \pi^+ \pi^- \pi^\circ$ и не может идти через барионные петли подобно распаду $\eta \to \gamma\gamma$. Теоретические оценки ширины этого распада резко отличаются от полученных в настоящее время экспериментальных данных.

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Ebert D., Volkov M.K. **E2 - 12255**

On the Problem of the $\eta \to \pi^\circ \gamma\gamma$ Decay

We find that in the quantum chiral theory the decay $\eta \to \pi^\circ \gamma\gamma$ is closer related to the process $\eta \to \pi^+ \pi^- \pi^\circ$ by adding a meson loop than to the process $\eta \to \gamma\gamma$ going via a baryon loop. The theoretical estimates are in sharp disagreement with the present experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR;

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1. INTRODUCTION

The theoretical explanation of the decay $\eta \to \pi^\circ \gamma\gamma$ is considerably more difficult than the explanation of many other decays of the pseudoscalar SU_3-meson octet. Not accidentally, the number of papers dealing with this decay is therefore rather small compared with investigations of other meson decays. This is a direct result of the fact that the most popular and effective methods of low-energy physics as current algebra, different kinds of chiral models, and vector dominance models work badly in this case or lead to sharply lowered values of the decay probability [1].

In this note we analyze once more the decay $\eta \to \pi^\circ \gamma\gamma$ in the framework of the nonlinear quantum chiral field theory. This theory has been applied successfully to the description of numerous low-energy processes and properties of mesons [2]. As can be seen from fig. 1 it explains with a satisfactory accuracy all the main decay modes of the pseudoscalar meson octet. It looks therefore rather strange that as we shall see there is the single decay $\eta \to \pi^\circ \gamma\gamma$ *, where the prediction of this theory

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*A similar disagreement could occur for the decay $K_L \to \pi^\circ \gamma\gamma$.

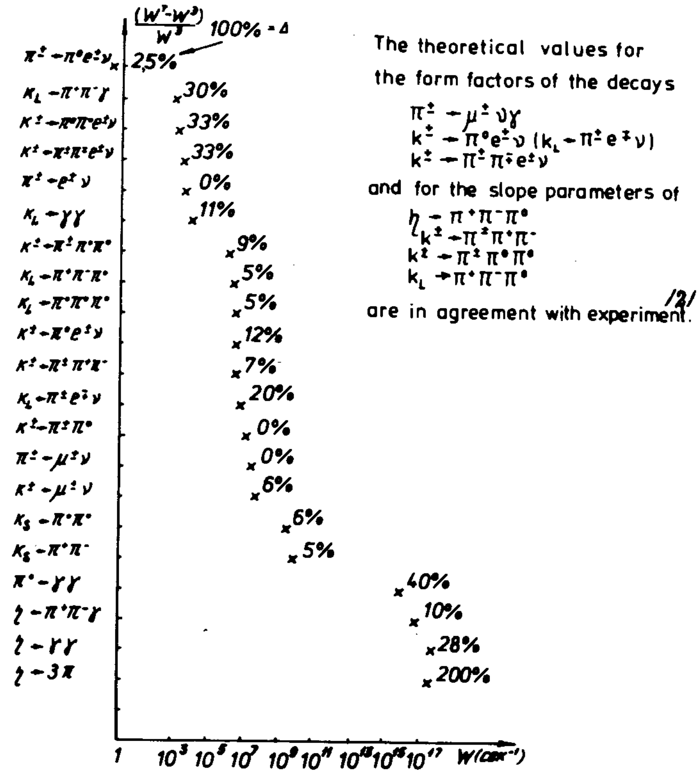

Fig. 1

diverges by three orders of magnitude from experiment. In view of this discrepancy it seems to us to be worthwile to reinspect both the theoretical estimates and the present experimental data /3/.

In this note we intend to clarify the question which of the following two processes one should connect the decay $\eta\to\pi^0\gamma\gamma$ with, either: with the decay $\eta\to\gamma\gamma$ by adding a strong (pion) vertex, or with the decay $\eta\to\pi^0\pi^+\pi^-$, where charged pions subsequently annihilate each other and emit a photon pair? In the first case one would expect the $\eta\to\pi^0\gamma\gamma$ decay rate to be by one order of magnitude smaller than the $\eta\to\gamma\gamma$ rate in agreement with the present experimental data. In the second case the $\eta\to\pi^0\gamma\gamma$ rate should be smaller by a factor of a^2 than the rate of the decay $\eta\to\pi^0\pi^+\pi^-$, i.e., it is expected to have an order of magnitude of 10^{-2} eV instead of the value 26 eV quoted in ref. /3/. Our analysis shows that in the chiral quantum field theory the second possibility is realized, i.e., the decay rate of the process $\eta\to\pi^0\gamma\gamma$ gets its nonvanishing contributions only from meson loops but not from baryon loops.

As is shown in the following section, the insertion of an additional pion vertex in the triangle baryon loop diagram describing the process $\eta\to\gamma\gamma$ leads to a complete cancellation of all baryon loops. The only nonvanishing contributions arise from the meson loops which are investigated in the third section and yield a decay rate in agreement with the above rough estimate ($1\cdot 10^{-2}$ eV). A short discussion of the obtained results is contained in the conclusions.

2. BARYON LOOPS

The Lagrangian of the strong meson-baryon interactions appearing in the process $\eta\to\pi^0\gamma\gamma$ is given by (see, e.g. ref. /2/)*

$$\mathcal{L}^{(1)} = ig\pi^0[\bar{p}\gamma_5 p + (2a-1)\bar{\Xi}^-\gamma_5\Xi^- + 2(1-a)(\bar{\Sigma}^+\gamma_5\Sigma^+ - \bar{\Sigma}^-\gamma_5\Sigma^-)] +$$
$$+ i\frac{g}{\sqrt{3}}\eta[(3-4a)\bar{p}\gamma_5 p - (3-2a)\bar{\Xi}^-\gamma_5\Xi^- + 2a(\bar{\Sigma}^+\gamma_5\Sigma^+ + \bar{\Sigma}^-\gamma_5\Sigma^-)] + \qquad (1)$$
$$+ \frac{g^2}{\sqrt{3}M}\pi^0\eta[(3-4a)\bar{p}p + (3-8a+4a^2)\bar{\Xi}^-\Xi^- + 4a(1-a)(\bar{\Sigma}^+\Sigma^+ - \bar{\Sigma}^-\Sigma^-)],$$

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*The Lagrangian (1) may further contain terms quadratic in the meson fields with derivative couplings. As the corresponding triangle baryon loops vanish by the Furry theorem these terms of the Lagrangian are here omitted.

where g is the coupling constant of the strong interactions, $g^2/4\pi \approx 14.7$, and $a \approx 2/3$ is the usual SU_3-mixing parameter of the F- and D-couplings. The above Lagrangian leads to three types of diagrams represented in fig. 2. In the

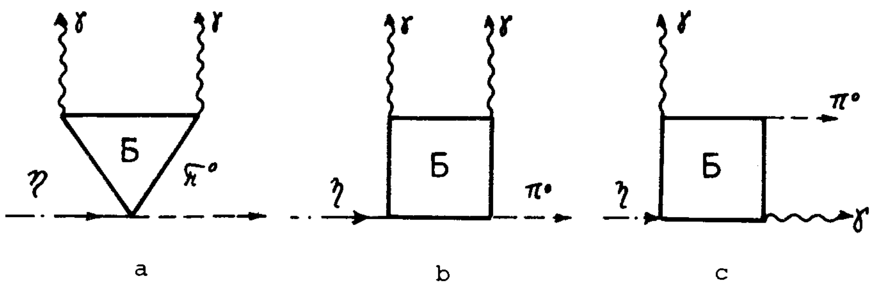

Fig. 2

following we shall prove that the triangle diagrams cancel the contributions of the box diagrams.

As the momenta of all particles participating in the process $\eta \to \pi^\circ\gamma\gamma$ are small in comparision with the baryon masses the integration over the loop momenta is performed by neglecting all but quadratic dependences on the external momenta. For the triangle diagrams we then obtained the contribution

$$T^{(1)} = i\,\frac{3-6a+2a^2}{3\sqrt{3}}\left(\frac{4\pi e g}{M}\right)^2 \epsilon_1^\mu \epsilon_2^\nu \,[g^{\mu\nu} q_1 q_2 - q_1^\nu q_2^\mu]. \qquad (2)$$

Here q_1, q_2, ϵ_1^μ and ϵ_2^ν are the momenta and polarization four-vectors of the two emitted photons, M is an averaged mass of the baryon octet and e the electric charge ($e^2/4\pi = \alpha = 1/137$).

The box diagrams of figs. 2b, c, yield the following expression

$$T^{(2)} = -i\,\frac{3-6a+2a^2}{6\sqrt{3}}\left(\frac{4\pi e g}{M}\right)^2 \epsilon_1^\mu \epsilon_2^\nu [g^{\mu\nu} q_1 q_2 - q_1^\nu q_2^\mu + 2(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) - \frac{(m_\pi^2+m_\eta^2)}{2} g^{\mu\nu} + i\,\frac{3M^2}{\pi^2} g^{\mu\nu} (R(p_1) + R(p_2))], \qquad (3)$$

$$T^{(3)} = -i\,\frac{3-6a+2a^2}{6\sqrt{3}}\left(\frac{4\pi e g}{M}\right)^2 [g^{\mu\nu} q_1 q_2 - q_1^\nu q_2^\mu - 2(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) + \frac{(m_\pi^2+m_\eta^2)}{2} g^{\mu\nu} - i\,\frac{3M^2}{\pi^2} g^{\mu\nu} (R(p_1) + R(p_2))], \qquad (4)$$

where p_1 and p_2 are the momenta of the η meson and pion, respectively, and R(p) denotes the logarithmically divergent integral

$$R(p) = \int \frac{d^4 k}{(M^2 - k^2)(M^2 - (k-p)^2)}\;. \qquad (5)$$

It can now easily be seen that the baryon diagrams shown in fig. 2 cancel each other so that the baryon loop contribution to the amplitude is zero.

To get an impression of the order of magnitude of the baryon loops, let us suppose that there would be no cancellations. From the triangle diagrams which yield the maximum contribution we obtaine the estimate

$$\Gamma^{(\Delta)} = \frac{3}{4\pi} m_\eta \alpha^2 (3-6a+2a^2)^2 \left(\frac{m_\pi g}{3\pi M}\right)^4 \int_1^b dx \sqrt{x^2-1}\,(b-x)^2 \approx 0.05 \text{ eV}, \qquad (6)$$

where $b = \dfrac{m_\pi^2 + m_\eta^2}{2 m_\pi m_\eta}$. From this it follows that even if there were no cancellation of baryon loops, the disagreement with the experimental data would nevertheless amount to two orders of magnitude.

Concluding this section we show that an analogous cancellation mechanism works for the singlet part of the η meson. To see this, it is sufficient to quote the Lagrangian describing the interaction of the η° meson with the charged baryons

$$\mathcal{L}^{(2)} = ig'\eta^\circ[\bar{p}\gamma_5 p + \bar{\Sigma}^+\gamma_5\Sigma^+ + \bar{\Sigma}^-\gamma_5\Sigma^- + \bar{\Xi}^-\gamma_5\Xi^-] +$$
$$+ \frac{gg'}{M}\pi^\circ\eta^\circ[\bar{p}p + (2a-1)\bar{\Xi}^-\Xi^- + 2(1-a)(\bar{\Sigma}^+\Sigma^+ - \bar{\Sigma}^-\Sigma^-)]. \quad (7)$$

Comparing the Lagrangian (7) and (1) we see that the relations between the weight factors of the triangle and box diagrams are analogous to those of the octet part of the η meson. This way, $\eta^0-\eta^8$ mixing does not influence our conclusion about the cancellation of the baryon loop contributions. The decay $\eta\to\pi^\circ\gamma\gamma$ is therefore not connected with the decay $\eta\to\gamma\gamma$.

3. PION LOOPS

There is the alternative possibility to relate the process $\eta\to\pi^\circ\gamma\gamma$ to the process $\eta\to\pi^\circ\pi^+\pi^-$ via a meson loop. In the framework of the chiral theory considered here this is the unique possibility to get a nonvanishing decay rate for the process $\eta\to\pi^\circ\gamma\gamma$.

It is worth mentioning that in the case of low-energy pion-pion interactions (excluding baryon loops) the quantum chiral theory admits a perturbative expansion in powers of two small expansion parameters

$$\left(\frac{m_\pi}{4\pi F}\right)^2 \approx 0.01 \qquad \text{and} \qquad \left[\frac{q^2}{(4\pi F)^2}\right] \ll 1,$$

where q^2 is the squared energy-momentum of the interacting mesons which is always essentially

smaller than $(4\pi F)^2 \approx 1.4\ \mathrm{GeV}^2$. Thus, the lowest order contributions provide us certainly with the main information on the process under consideration.

Let us now quote those parts of the Lagrangian which are necessary for the description of the process $\eta\to\pi^\circ\gamma\gamma$ via meson loops*. The electromagnetic interaction of the pions is described by the expression

$$\mathcal{L}^{(A)} = e^2 :A_\mu^2\,\pi^+\pi^-: + ie A_\mu :(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+): . \quad (8)$$

The four-pion part of the chiral invariant Lagrangian reads

$$\mathcal{L}_1^{(\pi)} = \frac{1}{6F^2} :[(\vec{\pi}\,\partial_\mu\vec{\pi})^2 - \vec{\pi}^2(\partial_\mu\vec{\pi})^2]: . \quad (9)$$

Finally, the relevant part of the symmetry-breaking term has the form [4]

$$\mathcal{L}_2^{(\pi)} = \frac{m_\pi^2}{\sqrt{3}} :\pi^\circ\eta\left(1-\frac{\vec{\pi}^2}{6F^2}\right): + \frac{m_\pi^2}{12F^2} :\vec{\pi}^2\eta^2: + \frac{m_\pi^2}{24F^2} :(\vec{\pi}^2)^2: . \quad (10)$$

Here F is the decay constant of the $\pi^\pm$-meson ($F\approx 95$ MeV). For the following it is convenient to divide the meson fields in eqs. (9), (10) into neutral "external" (π°) and charged "internal" fields (ϕ,ϕ^*). This yields

$$\bar{\mathcal{L}}_1^{(\pi)} = \frac{1}{3F^2} :[\pi^\circ\partial_\mu\pi^\circ\partial_\mu(\phi^*\phi) - \pi^{\circ 2}\partial_\mu\phi^*\partial_\mu\phi - \phi^*\phi(\partial_\mu\pi^\circ)^2]: , \quad (9')$$

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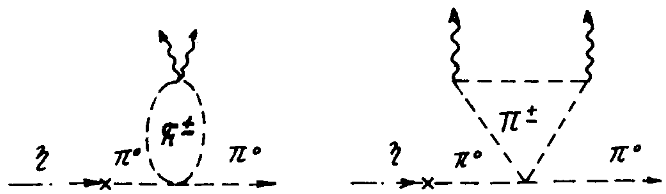
*The mass terms which are added to the chiral invariant part of the meson Lagrangian and break the group $SU(3)\times SU(3)$ and $SU(2)\times SU(2)$ play here an important role. As they led in the case of the decay $\eta\to\pi^\circ\pi^+\pi^-$ to an enlarged decay rate, we would expect that our estimate for the process $\eta\to\pi^\circ\gamma\gamma$ should be enlarged, too. We choose the scheme of symmetry breaking given in ref. [4].

$$\bar{\mathfrak{L}}_2^{(\pi)} = \frac{m_\pi^2}{\sqrt{3}} :\pi^\circ\eta\left(1-\frac{\phi^*\phi}{3F^2}\right): + \frac{m_\pi^2}{6F^2}:\eta^2\phi^*\phi: + \frac{m_\pi^2}{6F^2}:\pi^{\circ 2}\phi^*\phi: \qquad (10')$$

The most essential contributions to the decay rate of $\eta\to\pi^\circ\gamma\gamma$ arise from the loop diagrams



a b

Fig. 3

shown in figs. 3-5*. The first sort of diagrams is related to the transition $\eta\to\pi^\circ$, where π° then goes over into π^+,π^- and π° with the subsequent annihilation of the charged pions into two photons (cf. fig. 3a, b). The corresponding amplitude is given by

$$T_\pi^{(1)} = -\frac{\epsilon_1^\mu\epsilon_2^\nu}{3\sqrt{3}}\left(\frac{2m_\pi e}{F}\right)^2 \frac{(3q_1q_2 - m_\pi^2 - m_\eta^2/2)}{m_\eta^2 - m_\pi^2} J_{\mu\nu}(q_1,q_2), \qquad (11)$$

* There exist additional diagrams with kaon loops. As these contributions are small in comparision with the pion loops, they have been omitted.

Note that the Lagrangians (10), (1o′) contain a mixing term $\pi^\circ\eta$ which after diagonalizing the mass matrix leads to a small shift in the masses of the π° and η mesons. As the angle of the $\pi^\circ\eta$ mixing is, however, small, we neglect these mass shifts here and work with the nondiagonalized expressions. The loop expressions obtained from a diagonalized Lagrangian are quoted in the Appendix.

where

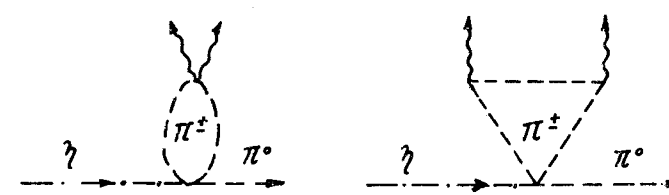
$$J_{\mu\nu}(q_1,q_2) = \int \frac{d^4k}{[m_\pi^2-(k+q_1)^2][m_\pi^2-(k-q_2)^2]}\left[g^{\mu\nu} + \frac{4k^\mu k^\nu}{m_\pi^2-k^2}\right] =$$

$$= i\pi^2\frac{(g^{\mu\nu}q_1q_2 - q_1^\nu q_2^\mu)}{q_1q_2}\left\{\frac{2m_\pi^2}{q_1q_2}\left[\operatorname{arctg}\left(\frac{2m_\pi^2}{q_1q_2}-1\right)^{-1/2}\right]^2 - 1\right\}. \qquad (12)$$

In calculating $T_\pi^{(1)}$ we have omitted gauge-noninvariant terms of the form

$$\int\frac{d^4k}{(k^2-m^2)}\left[g^{\mu\nu}+\frac{4k^\mu k^\nu}{m^2-(k+q_1)^2}\right],\ \int\frac{d^4k}{(k^2-m^2)}\left[g^{\mu\nu}+\frac{4k^\mu k^\nu}{m^2-(k-q_2)^2}\right]. \qquad (13)$$

The second sort of pion loop diagrams is related to the direct transition of the η meson into 3 pions (cf. eqs. (10), (10′) and fig. 4a,b)



a b

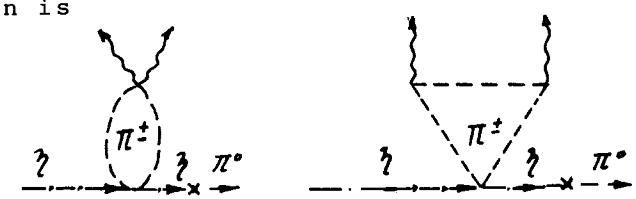
Fig. 4

They yield the following contribution

$$T_\pi^{(2)} = -\frac{\epsilon_1^\mu\epsilon_2^\nu}{3\sqrt{3}}\left(\frac{2m_\pi e}{F}\right)^2\frac{1}{2}J_{\mu\nu}(q_1,q_2). \qquad (14)$$

Finally, there is a third type of loop diagrams which is related to the process $\eta\to\pi^+\pi^-\eta$ and

$\eta \to \pi^{\circ}$ (cf. fig. 5a, b). The respective contribution is



a b

Fig. 5

$$T_{\pi}^{(3)} = \frac{\epsilon_1^{\mu}\epsilon_2^{\nu}}{3\sqrt{3}}\left(\frac{2m_{\pi}e}{F}\right)^2 \frac{m_{\pi}^2}{2(m_{\eta}^2 - m_{\pi}^2)} J_{\mu\nu}(q_1, q_2). \qquad (15)$$

From eqs. (11), (14), (15) we obtain the following total expression for the amplitude of the decay $\eta \to \pi^{\circ}\gamma\gamma$.

$$T_{\eta\to\pi^{\circ}\gamma\gamma} = -\frac{\epsilon_1^{\mu}\epsilon_2^{\nu}}{\sqrt{3}}\left(\frac{2m_{\pi}e}{F}\right)^2 \frac{(q_1 q_2 - \frac{2}{3}m_{\pi}^2)}{(m_{\eta}^2 - m_{\pi}^2)} J_{\mu\nu}(q_1, q_2). \qquad (16)$$

The total decay rate of the decay $\eta \to \pi^{\circ}\gamma\gamma$ obtained from eq. (16) reads

$$\Gamma_{\eta\to\pi^{\circ}\gamma\gamma} = \frac{m_{\pi}}{3\pi\Delta}\left(\frac{m_{\pi}}{\pi F}\right)^4 \left(\frac{\alpha}{2\Delta(\Delta^2-1)}\right)^2 \int_0^{(\frac{\Delta-1}{2})^2} dy\left(y-\frac{1}{6}\right)^2 \sqrt{\left[\left(\frac{\Delta+1}{2}\right)^2 - y\right]\left[\left(\frac{\Delta-1}{2}\right)^2 - y\right]}\, f(y), \qquad (17)$$

where $\Delta = m_{\eta}/m_{\pi}$ and the function $f(y)$ is given by (cf. eq. (12))

$$f(y) = \begin{cases} \left[\dfrac{\mathrm{arctg}^2\sqrt{\dfrac{y}{1-y}}}{y} - 1\right]^2 & (y \le 1) \\ (16y)^{-1}\left[(\ln^2 z + 4y - \pi^2)^2 + (2\pi \ln z)^2\right] & (y \ge 1) \end{cases} \qquad (18)$$

$$z = \frac{\sqrt{y} + \sqrt{y-1}}{\sqrt{y} - \sqrt{y-1}}.$$

The numerical estimate of the integral appearing in eq. (17) yields ~ 16. Inserting this number and the values of the remaining parameters into eq. (17) we obtain the following value of the rate of the decay $\eta \to \pi^{\circ}\gamma\gamma$

$$\Gamma_{\eta\to\pi^{\circ}\gamma\gamma} \approx 1 \cdot 10^{-2}\ \mathrm{eV}. \qquad (19)$$

The value (19) differs by three orders of magnitude from the experimental data quoted in the Rosenfeld table /3/.

4. CONCLUSIONS

As has already been found in previous works /1/ the theoretical estimates for the decay rate of the process $\eta \to \pi^{\circ}\gamma\gamma$ sharply disagree with the experimental data if one assumes that this process goes as the decay $\eta \to \gamma\gamma$ via baryon loops. In particular, our investigations show that in the framework of the quantum chiral field theory all baryon loop diagrams cancel each other completely. Thus, the process $\eta \to \pi^{\circ}\gamma\gamma$ should be related to the decay $\eta \to 3\pi$ rather than to the decay $\eta \to 2\gamma$. As we have shown, there arises a nonvanishing contribution to the decay $\eta \to \pi^{\circ}\gamma\gamma$ from the meson loops which is, however, also too small for explaining the present experimental data.

It can easily be understood that we cannot expect to get large contributions from meson loops because the process $\eta \to \pi^{\circ}\pi^{+}\pi^{-} \to \pi^{\circ}\gamma\gamma$ is related to two small effects: 1) the relevant part of the Lagrangian arises from small terms breaking the chiral symmetry, 2) the above process is directly related to the Compton effect on neutral mesons which is small because the polarization of neutral mesons is small /5/.

There remain the baryon loops. In order to obtain here finite contributions comparable with the present data, one should allow for large deviations from the chiral symmetry. This possibility seems, however, to be unjustified in view of the successful description of a large number of other processes within this theory. Note also that in distinction to the decay $\eta \to \gamma\gamma$, there arise no anomalies in the process $\eta \to \pi^{\circ}\gamma\gamma$ because the relevant integrals are convergent. What can we conclude from all this? As the decay $\eta \to \pi^{\circ}\gamma\gamma$ represents the only case among a large number of decays of the meson octet (cf. fig. 1) where a sharp disagreement of the predictions of the chiral theory with data is found, it seems to us to be useful to check once more the experimental situation. If one nevertheless wants to believe in the data, there arises the question whether there exist other kinds of dominating decay mechanisms which may explain the decay $\eta \to \pi^{\circ}\gamma\gamma$.

APPENDIX

In this appendix we shall diagonalize the Lagrangian (10′) by introducing rotated π° and η-fields

$$\pi^{\circ} = \cos\alpha\, \pi^{\circ\prime} + \sin\alpha\, \eta', \qquad \eta = -\sin \pi^{\circ\prime} + \cos\alpha\, \eta'. \tag{A.1}$$

If we choose α equal to

$$\mathrm{ctg}\, 2\alpha = \frac{\sqrt{3}}{2}\,\frac{(m_{\pi^+}^2 - m_\eta^2)}{m_{\pi^+}^2} \approx -12.5, \quad \alpha \approx -2{,}3^{\circ} \tag{A.2}$$

the $\pi^{\circ}\eta$ term in (10′) vanishes and the part of the Lagrangian which is relevant for the decay $\eta \to \pi^{\circ}\pi^{+}\pi^{-}$ takes the form

$$\mathcal{L}_{\mathrm{eff}} = \frac{\sin 2\alpha}{3F^2} :[\tfrac{1}{2}(\pi^{\circ\prime}\partial_\mu \eta' + \eta'\partial_\mu \pi^{\circ\prime})\partial_\mu(\phi^*\phi) - \pi^{\circ\prime}\eta'\partial_\mu\phi^*\partial_\mu\phi - \partial_\mu\pi^{\circ\prime}\partial_\mu\eta'\phi^*\phi]: - \cos 2\alpha\,\left(\frac{m_\pi^2}{3\sqrt{3}\,F^2}\right):\pi^{\circ\prime}\eta'\phi^*\phi: \tag{A.3}$$

Instead of the 3 types of diagrams shown in figs. 3-5 there remain only diagrams of the type 4a, b which yield the contribution

$$\bar{T}_{\eta\to\pi^{\circ}\gamma\gamma} = -\epsilon_1^{\mu}\epsilon_2^{\nu}\left(\frac{e}{F}\right)^2\Big\{\cos 2\alpha\left(\frac{2m_\pi^2}{3\sqrt{3}}\right) - \sin 2\alpha\left(2q_1q_2 - m_\pi^2 - \frac{m_\eta^2}{3}\right)\Big\}J_{\mu\nu}(q_1,q_2). \tag{A.4}$$

Let us rewrite this amplitude as

$$T_{\eta\to\pi^{\circ}\gamma\gamma} = -\epsilon_1^{\mu}\epsilon_2^{\nu}\left(\frac{e m_\pi}{F}\right)^2\Big\{a\frac{q_1q_2}{m_\pi^2} + b\Big\}J_{\mu\nu}(q_1,q_2) \tag{A.5}$$

and compare its coefficients a and b with the analogous coefficients obtained from eqs. (16). We have

$$(16)\quad a = \frac{4}{\sqrt{3}}\,\frac{m_\pi^2}{m_\eta^2 - m_\pi^2} = 0.15;\quad b = -\frac{8m_\pi^2}{3\sqrt{3}(m_\eta^2 - m_\pi^2)} = -0.1;$$

$$(A.4)\quad a = -2\sin 2\alpha = 0.16;\quad b = \frac{2\cos 2\alpha}{3\sqrt{3}} + \frac{m_\eta^2 + 3m_\pi^2}{3m_\pi^2}\sin 2\alpha \approx -0.1. \tag{A.6}$$

The amplitude (A.5) leads therefore to the same decay rate for the process $\eta \to \pi^{\circ}\gamma\gamma$ as the amplitude calculated from the nondiagonalized Lagrangian.

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