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Описание формфактора релятивистской двухчастичной системы в ковариантной гамильтоновой формулировке квантовой теории поля

На основе гамильтоновой формулировки квантовой теории поля развит трехмерный релятивистский формализм для описания формфакторов составных двухчастичных систем.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1979

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Form Factor of Relativistic Two-Particle System and Covariant Hamiltonian Formulation of Quantum Field Theory

Based on the Hamiltonian formulation of quantum field theory proposed by Kadyshevsky the three-dimensional relativistic approach is developed for describing the form factors of composite systems.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1979

1. INTRODUCTION

The idea of quark structure of hadrons increasingly penetrates the modern physics of elementary particles. A lot of important results have been obtained within the nonrelativistic quark model which is essential at the early stage of consideration of many problems (e.g., charmonium spectrum).

The relativistic quark models describing form factors, magnetic moments of composite particles typically make use of four-dimensional covariant equations of the type of Dirac[1] or Bethe-Salpeter equations[2].

However, the four-dimensional generalization of the nonrelativistic theory meets with failure in describing the system of particles. For instance two-particle wave function of the Bethe-Salpeter equation (compared to the nonrelativistic one) contains an extra dependence on relative time that makes its probabilistic interpretation a bit trickier.

A consistent elimination of the relative-time dependence of the wave function (WF) of a two-relativistic-particle system is achieved in the Logunov-Tavkhelidze quasipotential approach[3] in which the equations for WF are three-dimensional (like the Schrődinger equation in momentum space).

Our aim is to construct the three-dimensional formalism for describing form factors of two-particle bound systems. It should be noted that in that construction of relativistic theory it would be desirable to keep close analogy with nonrelativistic theory. This would allow one to "relativize" many results obtained in the nonrelativistic quark model.

The description of form factors of composite systems on the basis of the Logunov-Tavkhelidze relativistic quasipotential equation was considered in several papers[4,5]. In this paper we use the variant of the three-dimensional approach based on the covariant Hamiltonian formulation of quantum field theory proposed by Kadyshevsky[6,7]. The equations obtained in that approach in momentum space may be transformed to the form of a direct geometrical generalization of the relevant nonrelativistic equations (see[8,9]).

The paper is organized as follows: In Sec. 2 we discuss the main features of the diagram technique appearing in the covariant Hamiltonian formulation of field theory. In Sec. 3 we derive the three-dimensional relativistic equation for the vertex function and find its connection with that for the quasipotential wave function. In Sec. 4 we obtain the expressions for the form factor of the system through equal time two-particle (quasipotential) wave functions both in momentum and relativistic configurational representation. An explicit expression for the form factor is found for the case of two-particle interaction through the Coulomb potential.

2. THE DIAGRAM TECHNIQUE OF THE HAMILTON FORMULATION OF QUANTUM FIELD THEORY

The main difference of the Kadyshevsky diagram technique[6] from Feynman's technique is as follows: in Feynman's technique all the momenta of particles in the intermediate state are off the mass shell (virtual particles), but at each vertex the conservation law of energy-momenta holds; and on the contrary, in the Kadyshevsky approach the momenta of all particles are on the mass shell*, but

*Just this circumstance ensures the three-dimensional character of integrations in the momentum space.

each vertex contains extra incoming and outgoing lines of quasi-particles-spurious transporting additional 4-momenta $\lambda\tau$ and $\lambda\tau'$* (see Fig. 1).

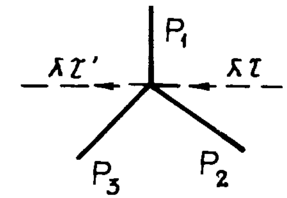

Fig. 1

The S-matrix can be represented according to[6] as follows

$$S=1+iR=T\exp\{-\int H(x)d^4x\}, \qquad (2.1)$$

where $H(x)$ is the Hamiltonian density in the expansion

$$R=\sum_{n=1}^{\infty}R_n$$

$$R_n=(-1)^n i^{n-1}\int\theta(x_{10}-x_{20})\cdot\theta(x_{20}-x_{30})\dots\theta(x_{n-10}-x_{n0}) \qquad (2.2)$$

$$\times H(x_1)H(x_2)\dots H(x_n)d^4x_1 d^4x_2\dots d^4x_n .$$

The θ-functions in (2.2) can be given the manifestly covariant form[6] because for time-like intervals (where they are important) $(x_i-x_{i+1})^2>0$ the following equality always holds *:

$$\theta(x_{i0}-x_{i+10})=\theta(\lambda(x_i-x_{i+1})), \qquad (2.3)$$

* λ^μ is the unit time-like vector, $\lambda_0^2-\vec{\lambda}^2=1$.[6,7]

with
$\lambda_0 > 0, \quad \lambda^2 = 1.$

Using the integral representation of θ-function

$$\theta(\lambda x) = \frac{1}{2\pi i} \int \frac{e^{i\tau(\lambda x)}}{\tau - i\epsilon} d\tau \qquad (2.4)$$

and passing to the Fourier transform of the Hamiltonian density

$$\mathcal{H}(p) = \int e^{-ipx} \mathcal{H}(x) d x$$

we get

$$R_n = (-1)^n \int \mathcal{H}(-\lambda_1 \tau_1) \frac{d\tau_1}{2\pi(\tau_1 - i\epsilon)} \mathcal{H}(\lambda \tau_1 - \lambda \tau_2) \frac{d\tau_2}{2\pi(\tau_2 - i\epsilon)} \cdots \qquad (2.5)$$

$$\cdots \frac{d\tau_{n-1}}{2\pi(\tau_{n-1} - i\epsilon)} \mathcal{H}(\lambda \quad \tau_{n-1}).$$

Like in[6], formula (2.5) can be obtained by iteration of the integral equation

$$R(\lambda\tau) = -\mathcal{H}(\lambda\tau) - \int \mathcal{H}(\lambda\tau - \lambda\tau') \frac{d\tau'}{2\pi(\tau' - i\epsilon)} R(\lambda\tau') \qquad (2.6)$$

provided that

$$R = \sum_{n=1}^{\infty} R_n = R(0). \qquad (2.7)$$

Based on eq. (2.6) one can formulate the diagram technique rules following the prescription of ref.[6].

3. EQUATIONS FOR THE VERTEX FUNCTION

Consider the R-matrix element obeying eq. (2.6), $\langle k_1, k_2 | R(\lambda\tau) | \mathcal{P}, M_B, J=0 \rangle$. Define the vertex function $\Gamma_{\mathcal{P}}(k_1, k_2; \lambda\tau)$ by

$$\langle k_1, k_2 | R(\lambda\tau) | \mathcal{P}, M_B, J=0 \rangle = (2\pi)^4 \delta^{(4)}(\mathcal{P} - k_1 - k_2 - \lambda\tau) \times \qquad (3.1)$$

$$\times \frac{1}{\sqrt{2k_{10}; 2k_{20} \cdot 2\mathcal{P}_0}} \times \Gamma_{\mathcal{P}}(k_1, k_2; \lambda\tau)$$

Then, using the operator equation (2.6) with fixed λ and making calculations in analogy with those in[7] in deriving the two-particle equation for the scattering amplitude, one may derive the equation for the vertex function (an analog of eq. (3.27) in[7]) which is equivalent to the Edwards equation in the Feynman-Dayson formulation of quantum field theory (we consider spinless particles):

$$\Gamma_{\mathcal{P}}(k_1, k_2; \lambda\tau) = \frac{1}{(2\pi)^3} \int d\tau' d^4k_1' d^4k_2' \cdot \Delta^{(+)}(k_1', m_1) \cdot \Delta^{(+)}(k_2', m_2) \times$$

$$\times \frac{1}{\tau' - i\epsilon} \cdot V(k_1, k_2; \lambda\tau | k_1', k_2'; \lambda\tau') \cdot \Gamma_{\mathcal{P}}(k_1', k_2'; \lambda\tau) \times \qquad (3.2)$$

$$\times \delta^{(4)}(\mathcal{P} - k_1' - k_2' - \lambda\tau'),$$

$$\Delta^{(+)}(k, m) = \theta(k_0) \cdot \delta(k^2 - m^2),$$

where m_i is the mass of an i-th particle. In what follows we shall take the 4-vector λ^μ as*

$$\lambda_\mu = \frac{\mathcal{P}_\mu}{\sqrt{\mathcal{P}^2}} = \frac{\mathcal{P}_\mu}{M}; \qquad (3.3)$$

* The choice of the 4-velocity vector $\lambda_\mu = \mathcal{P}_\mu / M$ of a composite particle as λ_μ is for the reasons of convenience and simplicity of subsequent calculations. In ref.[10,11] vector λ_μ was taken belonging to the light cone: $\lambda^2 = \lambda_0^2 - \vec{\lambda}^2 = 0$.

With the help of the spurious diagram technique eq. (3.2) is represented in Fig. 2. The composite particle has the momentum $\mathcal{P}_\mu$, spin J=0, solid lines are the constituents of the composite

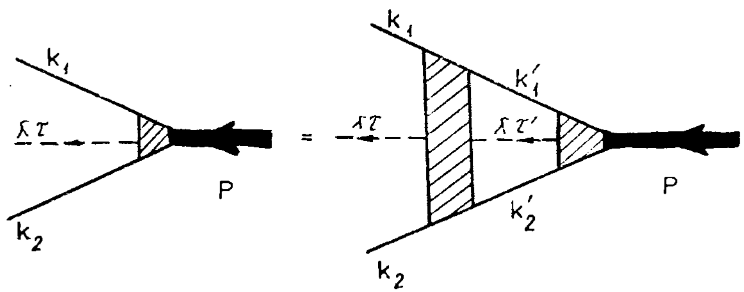

Fig. 2

particle, transferring the momenta k_1, k_2 (k'_1, k'_2), dotted lines are quasiparticles-spurions. The trapeziform block of diagrams in Fig. 3 corresponds, by construction of eq. (3.2) (also (3.27) and (4.14) in ref.[7]), to the sum of diagrams irreducible in the sense of one-spurion and two-particle cuttings. According to ref.[7], a block of this type will be considered as a quasipotential.

Putting the masses of constituents equal $m_1 = m_2 = m$, we rewrite (3.2) in the form ($\lambda_\mu = \mathcal{P}_\mu / M$)

$$\Gamma_{\mathcal{P}}(k_1, k_2; \lambda\tau) = \frac{1}{(2\pi)^3 \cdot 4} \cdot \int d\tau' \cdot \frac{d^3\vec{k}'_1}{\sqrt{m^2 + \vec{k}'^2_1}} \cdot \frac{d^3\vec{k}'_2}{\sqrt{m^2 + \vec{k}'^2_2}} \cdot \frac{1}{\tau' - i\epsilon} \times \tag{3.4}$$

$$\times V(k_1, k_2; \lambda\tau | k'_1, k'_2; \lambda\tau') \cdot \Gamma(k'_1, k'_2; \lambda\tau') \cdot \delta^{(4)}[(1 - \frac{\tau'}{M})\mathcal{P} - k'_1 - k'_2].$$

Next we use the property of the invariance under the Lorentz transformation L of the integration measure $d\Omega_k$ on the mass hyperboloid

$$k_0^2 - \vec{k}^2 = m^2 \tag{3.5}$$

$$d\Omega_k = \frac{d^3\vec{k}}{\sqrt{1 + \vec{k}^2/m^2}} = \frac{d^3\vec{k}'}{\sqrt{1 + \vec{k}'^2/m^2}}, \tag{3.6}$$

where $k' = Lk$.

The group of motion of surface (3.5), which is a model of the Lobachevsky space[8,9], is the Lorentz group. Consequently, vector $k' = Lk$, resulting from k by a certain Lorentz transformation L, also belongs to the Lobachevsky space*. If the L is taken to be the pure Lorentz transformation (boost) Λ_λ^{-1}, corresponding to the 4-vector of velocity λ_μ: $\Lambda_\lambda^{-1}\mathcal{P} = (M, \vec{0})$, then we get

$$d\Omega_k = d\Omega_{\Delta_{k,m\lambda}}, \tag{3.7}$$

where $\Delta_{k,m\lambda}$ is a vector of the Lobachevsky space[5,8,9,12] $(\Delta_{k,m\lambda})^\mu = (\Lambda_\lambda^{-1} k)^\mu$:

$$\vec{\Delta}_{k,m\lambda} = \overrightarrow{\Lambda_\lambda^{-1} k} = \vec{k}(-)m\vec{\lambda} = \vec{k} - \vec{\lambda}\,[k_0 - \frac{\vec{k}\vec{\lambda}}{1 + \lambda_0}], \tag{3.8}$$

$$\Delta^0_{k,m\lambda} = (\Lambda_\lambda^{-1} k)^0 = k_\mu \lambda^\mu = k_0\lambda_0 - \vec{k}\vec{\lambda}.$$

The invariance of δ-function in (3.4) allows the following its representation:

$$\delta^{(4)}[(1 - \frac{\tau'}{M})\mathcal{P} - k'_1 - k'_2] = \delta^{(4)}[(1 - \frac{\tau'}{M})\Lambda_\lambda^{-1}\mathcal{P} - \Lambda_\lambda^{-1}(k'_1 + k'_2)] =$$

$$= \delta[M - \tau' - \Delta^0_{k'_1,m\lambda} - \Delta^0_{k'_2,m\lambda}] \cdot \delta^{(3)}(\vec{\Delta}_{k'_1,m\lambda} + \vec{\Delta}_{k'_2,m\lambda}). \tag{3.9}$$

Allowing for (3.7) and (3.9) eq. (3.4) is transformed to the form

$$\Gamma_{\mathcal{P}}(k_1, k_2; \lambda\tau) = \frac{1}{(2\pi)^3 \cdot 4} \cdot \int \frac{d^{(3)}\vec{\Delta}_{k'_1,m\lambda}}{\Delta^0_{k'_1,m\lambda}} \times \tag{3.10}$$

$$\times \frac{1}{\Delta^0_{k'_1,m\lambda}[M - 2\Delta^0_{k'_1,m\lambda} - i\epsilon]} V(k_1, k_2; \lambda\tau | k'_1, k'_2; \lambda\tau) \cdot \Gamma_{\mathcal{P}}(k'_1, k'_2; \lambda\tau')$$

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* For details see refs.[8,9].

where r' according to (3.9) is defined by

$$r' = M - 2\Delta^{\circ}_{k'_1, m\lambda} \qquad (3.11)$$

and k'_1 and k'_2 in the r.h.s. of (3.10) are connected with the parameter of integration $\Delta_{k'_1, m\lambda}$:

$$k'_1 = \Lambda_\lambda \cdot \Delta_{k'_1, m\lambda} = \Delta_{k'_1, m\lambda} (+) m\lambda$$
$$(3.12)$$
$$k'_2 = \Lambda_\lambda \cdot \Delta_{k'_2, m\lambda} = \Lambda_\lambda \cdot (-\Delta_{k'_1, m\lambda}) = (-\Delta_{k'_1, m\lambda})(+) m\lambda .$$

At this step, the following note should be made. The parameter r' given by (3.11) is an invariant. This follows from the invariance of $\Delta^{\circ}_{k'_1, m\lambda}$ under the Lorentz transformations that is seen directly from (3.8). The invariance of $\Delta^{\circ}_{k'_1, m\lambda}$ can also be established from the fact that the vector $\vec{\Delta}_{k,p}$ (k and p are some four-vectors, $k^2 = p^2 = m^2$) under the Lorentz transformations suffers only the three-dimensional, Wigner rotation [5]. To see this, let $k' = Lk$, $p' = Lp$; consider $\Delta_{k', p'}$:

$$\vec{\Delta}_{k',p'} = (\overrightarrow{\Lambda^{-1}_{p'}\ k'}) = (\overrightarrow{\Lambda^{-1}_{L_p} \cdot Lk}) = (\overrightarrow{\Lambda^{-1}_{L_p} \cdot L \cdot (\Lambda_p \cdot \Delta_{k,p})}) = \qquad (3.13)$$

$$= V(L, p') \cdot \vec{\Delta}_{k,p}$$

where $V(L,p')$ is the Wigner rotation matrix defined by the relation $V(L,p) = L^{-1}_p L \cdot \Lambda_{L^{-1}p}$. The invariance of r' can be made more manifest by rewriting (3.11) as:

$$r' = M - \sqrt{s_{k'}} , \qquad \text{where } s_{k'} = (k'_1 + k'_2)^2 . \qquad (3.14)$$

Note also that for arbitrary vector λ the parameter r' is defined as follows:

$$r' = \lambda(\mathcal{P} - k'_1) - \sqrt{[\lambda \cdot (\mathcal{P} - k'_1)]^2 + 2(\mathcal{P} \cdot k'_1) - M^2} . \qquad (3.15)$$

The vertex function $\Gamma_{\mathcal{P}}(k_1, k_2; \lambda r)$ is represented by a four-leg diagram, but with two external momenta collinear in virtue of our choice $\lambda_\mu = \frac{\mathcal{P}_\mu}{M}$. For the bound system of two spinless particles moving with the zero relative orbital momentum, J=0, (see, (3.1)) $\Gamma_{\mathcal{P}}(k_1, k_2; \lambda r)$ may depend only on the Lorentz scalars

$$\mathcal{P}^2 = M^2 , \quad k_1^2 = k_2^2 = m^2 , \quad (\lambda r)^2 = r^2 ;$$
$$(3.16)$$
$$k_1 \cdot k_2 ; \quad \mathcal{P}k_1 = M(\lambda k_1) , \quad \mathcal{P}k_2 = M(\lambda k_2) , \quad \lambda \cdot \mathcal{P} .$$

However, as all the momenta are on the mass shells and $\lambda_\mu = \frac{\mathcal{P}_\mu}{M}$, and $\lambda\mathcal{P} = \sqrt{M^2}$, only four parameters, r , $k_1 \cdot k_2$, $\mathcal{P} \cdot k_1$, $\mathcal{P} \cdot k_2$, are essential*. From the collinearity of λ_μ and $\mathcal{P}_\mu$ and conservation law $\mathcal{P} - \lambda r - k_1 - k_2 = 0$ one may easily derive three independent relations for these four parameters:

$$M - r = \sqrt{(k_1 + k_2)^2} = \sqrt{2(m^2 + k_1 k_2)}$$
$$(3.17)$$
$$\mathcal{P} \cdot k_1 = \mathcal{P} \cdot k_2 ; \quad \mathcal{P} \cdot k_1 = M\sqrt{\frac{m^2 + k_1 \cdot k_2}{2}} .$$

So, the vertex function $\Gamma_{\mathcal{P}}(k_1, k_2; \lambda r)$ for $\vec{\lambda} \parallel \vec{\mathcal{P}}$ depends only on one scalar parameter. That is the main difference (and certain advantage) with respect to the parametrization of wave functions of relativistic systems through the light-cone vector $\lambda^2 = \lambda_0^2 - \vec{\lambda}^2 = 0$ considered in ref. [10] what results in [10] in the dependence of a wave function on an additional variables.

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*We will however keep, for analogy with quantum mechanics, the dependence of the wave function on parameter $\sqrt{\mathcal{P}^2} = M$, eigenvalues of the Hamiltonian in the quasipotential equation (3.10) (see also (3.22)). In the nonrelativistic limit $\Psi_M(r) \to \Psi_{E_q}(r)$ [5,8,9] .

As an independent variables we choose $\mathcal{P}\cdot k_1 = = M\Delta^{\circ}_{k_1,m\lambda_{\mathcal{P}}} \equiv M\Delta^{\circ}_{k_1,\frac{m}{M}\mathcal{P}}$ and introduce the notation

$$\Gamma_{\mathcal{P}}(k_1,k_2;\lambda\tau) \equiv \Gamma_{M,J=0}(\Delta^{c}_{k_1,m\lambda}), \tag{3.18}$$

$$V(\vec{\Delta}_{k_1,m\lambda};\vec{\Delta}_{k_1',m\lambda};\mathcal{P}^2) = (4m^2)^{-1}V(k_1,k_2;\lambda\tau|k_1',k_2';\lambda\tau') \tag{3.19}$$

(factor $(4m^2)^{-1}$ is taken for the purpose of the more direct analogy with the nonrelativistic formalism) so that for $J=0$ the vertex function $\Gamma_{M,J=0}(\Delta^{\circ})$ will obey the following equation

$$\Gamma_{M,J=0}(\Delta^{\circ}_{k,m\lambda}) = \frac{1}{(2\pi)^3}\int \frac{d^3\vec{\Delta}_{k',m\lambda}}{m^{-1}\Delta^{\circ}_{k',m\lambda}}\cdot\frac{V(\vec{\Delta}_{k,m\lambda};\vec{\Delta}_{k',m\lambda};\mathcal{P}^2)\cdot\Gamma_{M,J=0}(\Delta^{\circ}_{k',m\lambda})}{m^{-1}\Delta^{\circ}_{k',m\lambda}\cdot[M-2\Delta^{\circ}_{k',m\lambda}-i\epsilon]}. \tag{3.20}$$

It is not difficult to establish, up to a factor, the connection between the vertex function and the quasipotential wave function considered in [7,8]. Defining

$$\Psi_{M,J=0}(\Delta^{\circ}_{k,m\lambda}) = \frac{\Gamma_{M,J=0}(\Delta^{\circ}_{k,m\lambda})}{2^{3/2}\cdot\Delta^{\circ}_{k,m\lambda}\cdot[M-2\Delta^{\circ}_{k,m\lambda}]} \tag{3.21}$$

from (3.20) we derive the equation for the wave function in terms of invariant variables*:

$$\frac{\Delta^{\circ}_{k,m\lambda}}{m}(M-2\Delta^{\circ}_{k,m\lambda})\Psi_{M,J=0}(\Delta^{\circ}_{k,m\lambda}) = \tag{3.22}$$

$$= \frac{1}{(2\pi)^3}\int d\Omega_{\Delta_{k',m\lambda}}\cdot V(\vec{\Delta}_{k,m\lambda};\vec{\Delta}_{k',m\lambda};\mathcal{P}^2)\Psi_{M,J=0}(\Delta^{\circ}_{k',m\lambda})$$

found earlier in ref. [7] with other parametrizing variables (3.15).

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* In a different way this equation was deduced in [5,12].

4. RELATIVISTIC FORM FACTOR OF COMPOSITE SYSTEM

Consider first the simplest case with the following Hamiltonian density:

$$\mathcal{H}(x) = -z_1\cdot\phi_1^{+}(x)\phi_1(x)A(x) - z_2\phi_2^{+}(x)\phi_2(x)A(x). \tag{4.1}$$

All the fields in (4.1) are spinless. Earlier, in paper [5], following the results of ref. [4] the form factor of a composite system was defined as a matrix element of the current local operator between bound states in terms of the quasi-potential wave functions satisfying the Kadyhsevsky equation (3.22). In contrast to the four-dimensional Bethe-Salpeter approach in which the wave function depends on two times, in the quasi-potential formalism the composite system is described in terms of one time variable, proper time of the system $\tau = \mathcal{P}/M(x_1+x_2)$ [4]. It is clear that before and after the photon absorption the two-particle system has different momenta $\mathcal{P}$ and $\mathcal{P}'$ and, correspondingly, different proper times

$$\tau = \lambda\cdot X,\ \tau' = \lambda'\cdot X \quad (X = x_1+x_2,\ \lambda = \mathcal{P}/M,\ \lambda' = \mathcal{P}'/M). \tag{4.2}$$

In the quasi-potential approach, based on equating proper times, the wave function obeys equation (3.22), written in the same variables as in [5], and, hence, for the same quasipotentials it coincides with (3.21). Then, to the expression for the current matrix element found earlier in [5] we may associate the invariant expression

$$\langle\mathcal{P}'|J(0)|\mathcal{P}\rangle = z_1\frac{1}{(2\pi)^3}\int d\tau\, d\tau'\, d^4p\, d^4k\, d^4k'\cdot\Delta^{(+)}(p,m)\times$$

$$\times\Gamma^{(+)}_{\mathcal{P}'}(p,k';\lambda'\tau')\cdot\frac{1}{(\tau'+i\epsilon)(\tau-i\epsilon)}\cdot\Gamma_{\mathcal{P}}(p,k;\lambda\tau)\times \tag{4.3}$$

$$\times\Delta^{(+)}(k,m)\cdot\Delta^{(+)}(k',m)\cdot\delta^{(4)}(\mathcal{P}-p-k-\lambda\tau)\cdot\delta^{(4)}(\mathcal{P}'-p-k'-\lambda'\tau') + (1\to 2).$$

which graphically is defined by the diagram in Fig.3*.

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* Let us emphasize that because of the different proper times of a system before and after interaction the diagram in Fig. 3 differs from those arising in the Kadyshevsky formalism for S-matrix with fixed λ .

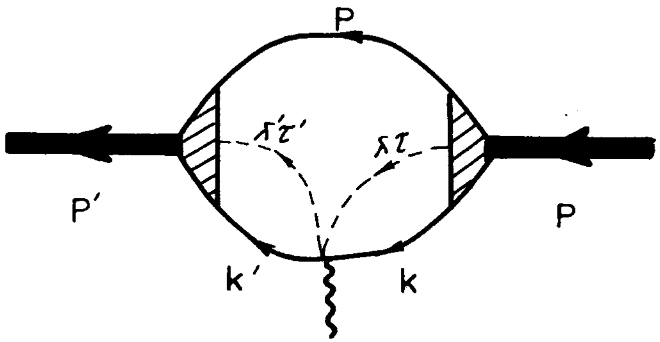

Fig. 3

Calculations analogous to those of the previous section yield the following invariant expression for the current between states with J=0:

$$\langle \mathcal{P}' | J(0) | \mathcal{P} \rangle = \frac{z_1}{(4\pi)^3} \int \frac{d^3 \vec{p}}{\sqrt{m^2+\vec{p}^{\,2}}} \times \tag{4.4}$$

$$\times \frac{\Gamma^+_{M,J=0}(\Lambda^\circ_{p,m\lambda'})}{\Delta^\circ_{p,m\lambda'}[M-2\Delta^\circ_{p,m\lambda'}]} \cdot \frac{\Gamma_{M,J=0}(\Lambda^\circ_{p,m\lambda})}{\Delta^\circ_{p,m\lambda}[M-2\Lambda^\circ_{p,m\lambda}]} + (1 \to 2).$$

Due to the scalar nature of the current we can define (in our case J=0)

$$\langle \mathcal{P}' | J(0) | \mathcal{P} \rangle = F(q^2), \tag{4.5}$$

with

$$q^2 = (\mathcal{P}' - \mathcal{P})^2 . \tag{4.6}$$

In terms of the quasipotential wave function given by (3.21) we obtain the following expression for the invariant form factor

$$F(q^2) = \frac{z_1+z_2}{(2\pi)^3} \int \frac{d^3\vec{p}}{\sqrt{m^2+\vec{p}^{\,2}}} \cdot \Psi^+_{M,J=0}(\Delta^\circ_{p,m\lambda'}) \cdot \Psi_{M,J=0}(\Delta^\circ_{p,m\lambda}). \tag{4.7}$$

For simplicity we consider the elastic form factor, i.e., $\sqrt{\mathcal{P}^2} = \sqrt{\mathcal{P}'^2} = M$.

For further transition to the relativistic configurational representation introduced in [8,9], let us rewrite (4.7) in a somewhat different form. To this end, we write allowing for (3.14)

$$\vec{\Delta}_{p,m\lambda'} = \overrightarrow{\Lambda^{-1}_{\mathcal{P}'} \cdot p} = \overrightarrow{\Lambda^{-1}_{\mathcal{P}'} \cdot \Lambda_{\mathcal{P}} \cdot \Delta_{p,m\lambda}} = V(\Lambda_{\mathcal{P}}, \mathcal{P}') \cdot \overrightarrow{\Lambda^{-1}_{\Delta_{\mathcal{P}',\mathcal{P}}} \cdot \Delta_{p,m\lambda}}, \tag{4.8}$$

where

$$\Delta_{\mathcal{P}',\mathcal{P}} \equiv \Lambda^{-1}_{\mathcal{P}} \mathcal{P}' = \mathcal{P}'(-)\mathcal{P}. \tag{4.9}$$

The 4-momentum transfer squared, $q^2 = t = (\mathcal{P}' - \mathcal{P})^2$, is expressed in terms of the momentum transfer in the Lobachevsky space (4.9) as follows [8,9]

$$t = (\mathcal{P}' - \mathcal{P})^2 = 2M^2 - 2M\sqrt{M^2 + \vec{\Delta}^2_{\mathcal{P}',\mathcal{P}}} = 2M(M - \Delta^\circ_{\mathcal{P}',\mathcal{P}}). \tag{4.10}$$

Therefore, form factor $F(t) = F(q^2)$ can be regarded as a function of invariant variable $\vec{\Delta}^2_{\mathcal{P}',\mathcal{P}}$. With (4.8), the form factor (4.7) may be represented as a convolution of wave functions in the Lobachevsky space

$$F(\vec{\Delta}^2_{\mathcal{P}',\mathcal{P}}) = \frac{z_1+z_2}{(2\pi)^3} \int \frac{d^3\vec{\Delta}_{p,m\lambda}}{\sqrt{m^2+\vec{\Delta}^2_{p,m\lambda}}} \cdot \Psi^+_{M,J=0} \times \tag{4.11}$$

$$\times ([\vec{\Delta}_{p,m\lambda}(-)\frac{m}{M}\vec{\Delta}_{\mathcal{P}',\mathcal{P}}]^\circ) \cdot \Psi_{M,J=0}(\Delta^\circ_{p,m\lambda}).$$

Now let us find the form factor in the relativistic configurational representation (RCR) (see discussion in [13]) expanding, after [8,9], the wave functions over the functions which form complete and orthogonal system on the mass hyperboloid:

$$\xi(\vec{\Delta}_{p,m\lambda};\vec{r}) = \left(\frac{\Delta^{\circ}_{p,m\lambda} - \vec{\Delta}_{p,m\lambda}\cdot\vec{n}}{m}\right)^{-1-irm} \qquad (4.12)$$

$\vec{r} = r\vec{n}$; $\vec{n}^2 = 1$; $0 < r < \infty$.

These functions have been obtained in [14] and realize the principle series of unitary irreducible representations of the Lorentz group[15].

For local quasipotential $V(\vec{\Delta}_{k,m\lambda}(-)\vec{\Delta}_{k',m\lambda};\mathcal{P}^2)$ eq. (3.22) in RCR takes the form[19]:

$$\frac{\hat{H}_0}{m}(M - 2\hat{H}_0)\Psi_M(\vec{r}) = V(\vec{r},\mathcal{P}^2)\Psi_M(\vec{r}), \qquad (4.13)$$

where the free Hamiltonian operator satisfying the condition

$$\hat{H}_0\xi(\vec{\Delta},\vec{r}) = \Delta_0\xi(\vec{\Delta},\vec{r}) \qquad (4.14)$$

is a finite-difference operator[8,9]

$$\hat{H}_0 = m\,\mathrm{ch}\left(\frac{i}{m}\frac{\partial}{\partial r}\right) + \frac{i}{r}\,\mathrm{sh}\left(\frac{i}{m}\frac{\partial}{\partial r}\right) - \frac{\Delta_{\theta,\phi}}{2mr^2}\exp\left(\frac{i}{m}\frac{\partial}{\partial r}\right) \qquad (4.15)$$

($\Delta_{\theta,\phi}$ is the Laplacian on sphere).

Using the "addition" theorem[8,14] for the "plane waves" (4.12) (see footnote on page 18) we express the form factor in terms of the quasipotential wave functions in RCR:

$$F(\vec{\Delta}^2_{\mathcal{P}'\mathcal{P}}) = (z_1 + z_2)\int d^3\vec{r}\,\xi^*\left(\frac{m}{M}\vec{\Delta}_{\mathcal{P}'\mathcal{P}},\vec{r}\right)|\Psi_{M,J=0}(\vec{r})|^2 \qquad (4.16)$$

For the S-state of a composite system, we consider here, (4.16) yields

$$F(\vec{\Delta}^2_{\mathcal{P}'\mathcal{P}}) = (z_1 + z_2)\frac{y}{\mathrm{sh}\,y}\cdot 4\pi\int_0^\infty r^2 dr\cdot\frac{\sin rmy}{rmy}\cdot|\Psi_{M,J=0}(r)|^2, \qquad (4.17)$$

where $y = \mathrm{Arch}\frac{\Delta^{\circ}_{\mathcal{P}'\mathcal{P}}}{M} = \mathrm{Arch}\left(1 - \frac{t}{2M^2}\right)$ is the so-called rapidity corresponding to the transfer momentum t.

Formulae (4.16) and (4.17) are the relativistic generalization of the expression for the nonrelativistic form factor in r-representation. They were found in a different way in ref.[16].

Now, let us analyze the vector photon coupling case. For the electromagnetic current matrix element we get following (4.2)-(4.7)*:

$$\langle\mathcal{P}'|J_\mu(0)|\mathcal{P}\rangle = (z_1 + z_2)\frac{2}{(2\pi)^3}\int\frac{d^3\vec{p}}{\sqrt{m^2+\vec{p}^2}}\cdot\Psi^+_{M,J=0}(\Delta^{\circ}_{p,m\lambda'})\times \qquad (4.18)$$

$$\times\left[\frac{\mathcal{P}'\cdot\Delta^{\circ}_{p,m\lambda'} + \mathcal{P}\cdot\Delta^{\circ}_{p,m\lambda}}{M} - p\right]_\mu\cdot\Psi_{M,J=0}(\Delta^{\circ}_{p,m\lambda}).$$

The invariant form factor F(t) will be defined by

$$\langle\mathcal{P}'|J_\mu(0)|\mathcal{P}\rangle = (\mathcal{P}' + \mathcal{P})_\mu\cdot F(t). \qquad (4.19)$$

In virtue of the relation

$$(\mathcal{P}' + \mathcal{P})_\mu\cdot\left(\frac{\mathcal{P}'\cdot\Delta^{\circ}_{p,m\lambda'} + \mathcal{P}\cdot\Delta^{\circ}_{p,m\lambda}}{M} - p\right)^\mu =$$

$$= \frac{\mathcal{P}'\cdot\mathcal{P}}{M}(\Delta^{\circ}_{p,m\lambda'} + \Delta^{\circ}_{p,m\lambda}), \qquad (4.20)$$

$$2(\mathcal{P}'\cdot\mathcal{P}) = 2M^2 - t, \quad (\mathcal{P}' + \mathcal{P})^2 = 4M^2 - t,$$

the current (4.18) yields

$$F(t) = \frac{2M^2 - t}{M(4M^2 - t)}\cdot\frac{2(z_1 + z_2)}{(2\pi)^3}\cdot\int\frac{d^3\vec{p}}{\sqrt{m^2+\vec{p}^2}}\times \qquad (4.21)$$

$$\times\Psi^+_{M,J=0}(\Delta^{\circ}_{p,m\lambda'})\cdot(\Delta^{\circ}_{p,m\lambda'} + \Delta^{\circ}_{p,m\lambda})\cdot\Psi_{M,J=0}(\Delta^{\circ}_{p,m\lambda}).$$

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* For real wave functions the current (4.18) obeys the condition of transversality $(\mathcal{P}' - \mathcal{P})^\mu\langle\mathcal{P}'|J_\mu|\mathcal{P}\rangle = 0$.

Then, using the method resulting in (4.11) we obtain

$$F(t)\equiv F(\vec{\Delta}^2_{\mathcal{P}'\mathcal{P}})=\frac{4M^2-2t}{M(4M^2-t)}\cdot\frac{z_1+z_2}{(2\pi)^3}\cdot\int\frac{d^3\vec{\Delta}_{p,m\lambda}}{\sqrt{m^2+\vec{\Delta}^2_{p,m\lambda}}}\times \tag{4.22}$$

$$\times\Psi^+_{M,J=0}([\vec{\Delta}_{p,m\lambda}(-)\frac{m}{M}\vec{\Delta}_{\mathcal{P}'\mathcal{P}}]^{\circ})\cdot[(\vec{\Delta}_{p,m\lambda}(-)\frac{m}{M}\vec{\Delta}_{\mathcal{P}'\mathcal{P}})^{\circ}+\Delta^{\circ}_{p,m\lambda}]\cdot\Psi_{M,J=0}(\Delta^{\circ}_{p,m\lambda}).$$

Now we will transform (4.22) into RCR. To this end we represent the integral (4.22) in terms of the quasipotential wave functions in RCR

$$\int\frac{d^3\vec{\Delta}_{p,m\lambda}}{\Delta^{\circ}_{p,m\lambda}}\cdot d^3\vec{r}_1\cdot d^3\vec{r}_2\cdot\xi(\vec{\Delta}_{p,m\lambda}(-)\frac{m}{M}\vec{\Delta}_{\mathcal{P}'\mathcal{P}};\vec{r}_1)\cdot\xi^*(\vec{\Delta}_{p,m\lambda};\vec{r}_2)\times \tag{4.23}$$

$$\times[(\vec{\Delta}_{p,m\lambda}(-)\frac{m}{M}\vec{\Delta}_{\mathcal{P}'\mathcal{P}})^{\circ}+\Delta^{\circ}_{p,m\lambda}]\cdot\Psi^+_{M,J=0}(\vec{r}_1)\cdot\Psi_{M,J=0}(\vec{r}_2).$$

With the relation (4.14), (4.23) can be rewritten in the following form

$$\int\frac{d^3\vec{\Delta}_{p,m\lambda}}{\Delta^{\circ}_{p,m\lambda}}\cdot d^3\vec{r}_1\cdot d^3\vec{r}_2\cdot\{(\hat{H}_{0,r_1}+\hat{H}^*_{0,r_2})\times \tag{4.24}$$

$$\times\xi(\Delta_{p,m\lambda}(-)\frac{m}{M}\vec{\Delta}_{\mathcal{P}'\mathcal{P}};\vec{r}_1)\xi^*(\vec{\Delta}_{p,m\lambda},\vec{r}_2)\}\cdot\Psi^+_{M,J=0}(\vec{r}_1)\Psi_{M,J=0}(\vec{r}_2).$$

Using the "addition" theorem for "plane waves" *

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* The possibility of applying in (4.24) the addition theorem follows from the dependence of $\Psi_{M,J=0}(r)$ on the modulus of the "relativistic coordinate" r.

$$\int d\omega_{\vec{n}}\,\xi(\vec{\Delta}_{p,m\lambda}(-)\frac{m}{M}\vec{\Delta}_{\mathcal{P}'\mathcal{P}};\vec{r})=\int d\omega_{\vec{n}}\,\xi(\vec{\Delta}_{p,m\lambda};\vec{r})\xi^*(\frac{m}{M}\vec{\Delta}_{\mathcal{P}'\mathcal{P}};\vec{r})$$

their orthogonality and Hermiticity of the operator of free Hamiltonian, $\hat{H}_0$ /17/ (17) we get

$$F(t)\equiv F(\vec{\Delta}^2_{\mathcal{P}'\mathcal{P}})=\frac{4M^2-2t}{mM(4M^2-t)}\cdot(z_1+z_2)\times \tag{4.25}$$

$$\times 2\,\mathrm{Re}\int d^3\vec{r}\,\xi^*(\frac{m}{M}\vec{\Delta}_{\mathcal{P}'\mathcal{P}};\vec{r})\cdot\Psi^+_{M,J=0}(\vec{r})\,\hat{H}_0\,\Psi_{M,J=0}(\vec{r}).$$

Since in the considered case of s-wave the wave function depends only on the modulus of the radius-vector, r, the expression (4.25) takes the form

$$F(\vec{\Delta}^2_{\mathcal{P}'\mathcal{P}})=\frac{4M^2-2t}{mM(4M^2-t)}\cdot\frac{y}{\operatorname{sh}y}\cdot 8\pi(z_1+z_2)\times \tag{4.26}$$

$$\times\,\mathrm{Re}\int_0^{\infty}r^2dr\,\frac{\sin rmy}{rmy}\cdot\Psi^+_{M,J=0}(r)\cdot\hat{H}^{rad}_0\cdot\Psi_{M,J=0}(r),$$

where

$$y=\operatorname{Arch}(1-t/2M^2)=\operatorname{Arch}\frac{\Delta^{\circ}_{\mathcal{P}'\mathcal{P}}}{M} \tag{4.27}$$

$$\hat{H}^{rad}_0=m\operatorname{ch}(\frac{i}{m}\frac{\partial}{\partial r})+\frac{i}{r}\operatorname{sh}(\frac{i}{m}\frac{\partial}{\partial r}). \tag{4.28}$$

Consider a particular case when $V(r)$ is the attraction Coulomb field

$$V(r)=-\frac{e^2}{r}. \tag{4.29}$$

The ground state wave function which is a solution to eq. (4.13) with quasipotential (4.29) has the form /16/

$$\Psi_{M,J=0}(r)=\mathrm{const}\cdot e^{-rmx_n}\quad;\quad M=2m\cos x, \tag{4.30}$$

where x_n is defined by the condition of quantization

$$\frac{e^2}{\sin 2x_n} = n; \quad n=1,2,3.... \quad (4.31)$$

Inserting (4.30) in (4.26) yields the form factor in the form

$$F(\vec{\Delta}^2_{\mathcal{P},\mathcal{P}}) \approx \frac{\mathrm{ch}\, y}{1+\mathrm{ch}\, y} \cdot \frac{y}{\mathrm{sh}\, y} \cdot \left[\frac{\sin x_n}{4x_n^2+y^2} + \frac{2x_n \cos x_n}{(4x_n^2+y^2)}\right]. \quad (4.32)$$

For large transfer momenta $|t| \gg M^2$, $y \approx \ln(\frac{|t|}{M^2})$, and the form factor has the following asymptotic behaviour

$$F(t) \equiv F(\vec{\Delta}^2_{\mathcal{P}\mathcal{P}}) \sim \frac{1}{t \cdot \ln(|t|/M^2)}. \quad (4.33)$$

Note that in the nonrelativistic theory (where $m y = \Delta_{Eucl.} = |(\vec{p}-\vec{k})|$) with the Coulomb potential the form factor falls off by the dipole law: $F(\vec{\Delta}^2_{Euclid}) \sim \frac{1}{\vec{\Delta}^2_{Eucl.}} = \frac{1}{(\vec{p}-\vec{k})^2}$ [18] that contradicts the predictions of dimensional quark counting rules [19] whereas formula (4.33) is consistent with the predictions of dimensional analysis for two-body systems [11].

5. CONCLUSION

In this paper, we have developed the three-dimensional covariant approach for describing the form factors of relativistic systems within the covariant Hamiltonian formulation of quantum field theory proposed by V.G.Kadyshevsky. Let us summarize the main steps and the results obtained.

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* In this connection see also paper [16].

1. For describing the currents of composite systems on the basis of the Hamiltonian formalism we have applied the rules of spurion diagram technique offered in [6].

2. In the framework of the Hamilton formalism and spurion diagram technique of Kadyshevsky we derived the equation for the vertex function (3.10), (3.20) (an analog of the Edwards equation) in which momenta of all particles are on the mass shell. The link between the vertex function and quasipotential wave function is established by eq. (3.21).

3. For the Coulomb interaction we have expressed the form factor of a two-relativistic-particle system in terms of vertex and quasipotential functions, (4.4), (4.25), (4.32).

The difference from the corresponding non-relativistic expression is that the relativistic form factor possesses the asymptotic behaviour consistent with dimensional quark counting rules [19] predictions.

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