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OF DIRECT MODEL-INDEPENDENT  
VERIFICATION OF QCD PREDICTIONS  
IN DEEP-INELASTIC  $\mu$  N SCATTERING

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Об одном способе прямой модельно-независимой проверки предсказаний квантовой хромодинамики в глубоконеупругом  $\mu N$  рассеянии

Предложена простая процедура проверки предсказаний квантовой хромодинамики, использующая лишь экспериментально определяемые моменты структурных функций глубоконеупругого  $\mu N$ -рассеяния.

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On a New Procedure of Direct Model-Independent Verification of QCD Predictions in Deep-Inelastic  $\mu N$  Scattering

A simple procedure of the verification of QCD predictions is suggested. This procedure deals only with experimentally defined moments of deep-inelastic  $\mu N$  scattering structure functions.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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Recently scaling violations predicted by Quantum Chromodynamics (QCD)<sup>/1/</sup> have been compared with the data on  $\mu N$  (at Fermi-lab<sup>/2/</sup>) and  $\nu N$  (CERN<sup>/3/</sup>) deep inelastic scattering.

QCD does not allow us to determine the form of the structure functions because of the divergence of Fourier transform of two current operators product. For this reason a number of different techniques have been developed for reconstructing the structure functions from their moments<sup>/4,5/</sup>. But it turns out that the mathematically rigorous reconstruction of the structure functions at definite points is impossible without extra assumptions<sup>/6/</sup>.

Thus, the most direct way for verifying the QCD is the comparison of the moments of structure functions with experiment<sup>/7/</sup>. In the case of  $\nu N$  deep inelastic scattering there exists a possibility (for checking QCD) by comparing it with the data on  $\alpha F_3(x, Q^2)$  structure functions whose moments are defined by the nonsinglet operators contribution only<sup>/3/</sup>. The  $\mu N$  scattering is more complicated because the relevant moments (see, e.g.<sup>/5/</sup>)

$$M(n, Q^2) = M_{N\bar{N}}(n, Q^2) \exp[-\lambda_{N\bar{N}}(n) \cdot S] + \quad (1) \\ + M_+(n, Q^2) \exp[-\lambda_+(n) \cdot S] + M_-(n, Q^2) \exp[-\lambda_-(n) \cdot S]$$

$$S = \ln \left[ \ln(Q^2/\Lambda^2) / \ln(Q_0^2/\Lambda^2) \right]$$

contain in addition to the nonsinglet part  $M_{NS}$  two extra  $M_{\pm}$  moments, that appear due to the contribution of twist-2 singlet operators occurring in the short-distance expansion of the product of two electromagnetic currents. The coefficients  $\lambda_{NS}$  and  $\lambda_{\pm}$  are given explicitly by the theory. They are defined by the anomalous dimensions of Wilson operators. For the theory with three colours and four flavours they equal:  $\lambda_{NS}(n) = 0.42667, 0.83733$  and  $1.08038$  for  $n = 2, 4$  and  $6$  respectively, and analogously  $\lambda_{+}(n) = 0.74667, 1.85234$  and  $2.46039$ ;  $\lambda_{-}(n) = 0.0, 0.81699$  and  $1.07427$ .

Formula (1) can also be expressed in terms of the quark and gluon momentum distributions<sup>18/</sup>

$$M(n, Q^2) = M(n, Q_0^2) \exp[-\lambda_{NS}(n) \cdot S] - Y_n(s) \cdot A_0(n, Q_0^2) + X_n(s) \cdot \sum_i A_i(n, Q_0^2). \quad (2)$$

The explicit form of  $Y_n(s)$  and  $X_n(s)$  functions can be determined in the theory. Equation (2) can, in principle, be used for checking QCD. Unfortunately, eq.(2) includes the unknown gluon and quark momentum distributions  $A_0(n, Q_0^2)$  and  $A_i(n, Q_0^2)$  ( $i = u, d, s, c, \dots$ ). Thus, one is forced to apply to certain assumptions about these moments. The choice of these assumptions influences substantially the value of the coupling-strength parameter  $\Lambda$ , found by fitting the experimentally defined moments (see<sup>18/</sup>).

In<sup>12/</sup> formula (1) was used in analysing  $\mu N$  data. The moments  $M_{NS}(n, Q_0^2)$  and  $M_{\pm}(n, Q_0^2)$  were expressed through combinations of the gluon and quark moments, which together with  $\Lambda$  were considered as free fitted parameters (thus on the whole 12 parameters for  $n = 2, 4, 6$ ).

The aim of the present note is to draw attention to the possibility for checking QCD predictions with no use of any information irrelevant to QCD. We mean the following simple procedure.

Let us write down the relation (1) for arbitrary fixed values of the squared momenta transfer  $Q_1^2, Q_2^2, Q_3^2$  (but with the same value of  $Q_0^2$  in (1))

$$M(n, Q_1^2) = M_{NS}(n, Q_0^2) \exp[-\lambda_{NS}(n) \cdot S_1] + M_{+}(n, Q_0^2) \exp[-\lambda_{+}(n) \cdot S_1] + M_{-}(n, Q_0^2) \exp[-\lambda_{-}(n) \cdot S_1] \quad (3)$$

$$M(n, Q_2^2) = M_{NS}(n, Q_0^2) \exp[-\lambda_{NS}(n) \cdot S_2] + M_{+}(n, Q_0^2) \exp[-\lambda_{+}(n) \cdot S_2] + M_{-}(n, Q_0^2) \exp[-\lambda_{-}(n) \cdot S_2]$$

$$M(n, Q_3^2) = M_{NS}(n, Q_0^2) \exp[-\lambda_{NS}(n) \cdot S_3] + M_{+}(n, Q_0^2) \exp[-\lambda_{+}(n) \cdot S_3] + M_{-}(n, Q_0^2) \exp[-\lambda_{-}(n) \cdot S_3]$$

$$S_j = \ln [ \ln(Q_j^2/\Lambda^2) / \ln(Q_0^2/\Lambda^2) ], \quad j = 1, 2, 3.$$

Now, let us assume the moments  $M(n, Q_j^2)$  ( $j = 1, 2, 3$ ) at these points  $Q_1^2, Q_2^2, Q_3^2$  to be known. In this case the system (3) allows us to express the unknown values  $M_{NS}(n, Q_0^2)$  and  $M_{\pm}(n, Q_0^2)$  through the known moments  $M(n, Q_j^2)$ .

The substitution in (1) of  $M_{NS}(n, Q_0^2)$  and  $M_{\pm}(n, Q_0^2)$  thus defined gives us the moment  $M(n, Q^2)$  at arbitrary  $Q^2$

$$M(n, Q^2) = \frac{\Delta_{NS}(\Lambda)}{\Delta(\Lambda)} \exp[-\lambda_{NS}(n) \cdot S] + \frac{\Delta_{+}(\Lambda)}{\Delta(\Lambda)} \exp[-\lambda_{+}(n) \cdot S] + \frac{\Delta_{-}(\Lambda)}{\Delta(\Lambda)} \exp[-\lambda_{-}(n) \cdot S] \quad (4)$$

where  $\Delta$  is the determinant of the whole system (3) and  $\Delta_{NS}, \Delta_{\pm}$  the determinants corresponding to unknown  $M_{NS}(n, Q_0^2), M_{\pm}(n, Q_0^2)$ .

Thus if we know some  $n$ -th moment at three arbitrarily chosen large enough  $Q_j^2$  ( $Q_j^2 > \Lambda^2 \sim 0.25 \text{ GeV}^2$ ) we can calculate  $M(n, Q^2)$  at arbitrary  $Q^2$  without using any other information. Let us mention that at present there exists the experimental information about values of moments  $M(n, Q^2)$  at  $n = 2, 4, 6$  for the momenta transfer  $3 \leq Q^2 < 50 \text{ GeV}^2$ <sup>12/</sup>.

This allows, even now, the direct and model-independent examination of QCD.

It is important to emphasize that the procedure suggested here allows one to find the  $Q^2$  dependence of  $\sum$  quarks and gluon moments. Really, if we apply the above described procedure this time to equation (2), we can express  $M(n, Q_0^2)$ ,  $A_0(n, Q_0^2)$ , and  $\sum_j A_j(n, Q_0^2)$  through the experimentally defined moments  $M(n, Q_j^2)$  ( $j = 1, 2, 3$ ). Then due to the arbitrariness in the choice of point  $Q_0^2$  in eqs. (1) and (2), we may perform the same procedure for any large enough value of  $Q_0^2 > A^2$ .

The results of fitting by formulae (4), (1), (2) of the experimental data together with the discussion of their consequences and the role of high order  $\alpha_s$  contributions<sup>19)</sup> should be published in a subsequent paper.

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