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О глубоконеупругом рассеянии лептонов на ядрах

В низшем порядке по электромагнитному взаимодействию рассмотрено глубоконеупругое рассеяние заряженных лептонов на ядрах. Получены выражения для соответствующих дифференциальных сечений, когда в конечном состоянии на совпадение регистрируются рассеянный лептон и фрагмент начального ядра. Структурные функции проанализированы с помощью принципа автомодельности. Проведено рассмотрение этих функций в рамках формализма "светового" фронта" для многочастичных систем. Высказано предположение о масштабной инвариантности структурных функций по отношению к переменной ξ, являющейся некоторой сложной безразмерной комбинацией кинематических инвариантов. Указана простая связь этой переменной с импульсными характеристиками входящих в ядро нуклонов.

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On the Deep Inelastic Lepton-Nucleus Scattering

Deep inelastic scattering of charged leptons on nuclei is considered in the lowest order in electromagnetic interaction. Expressions for the corresponding differential cross sections are obtained provided the scattered lepton and the fragment of the initial nucleus are detected in coincidence. Structure functions are analysed by means of the automodelity principle. These functions are considered in the framework of the "light front" formalism for many-body systems. A hypothesis is put forward on the scale invariance of structure functions with respect to the ξ-variable, which is some complicated dimensionless combination of kinematic invariants. A simple relation of this variable to the momenta of the nucleons inside the initial nucleus is pointed out.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Scaling property of structure functions of the deep inelastic lepton-hadron interaction which has been observed expementally /1/ led to the development of the new model and field-theoretic approaches which would explain these experimental regularities. The modern ideas on lepton-hadron interactions are based mainly on the parton model /2/ and its modifications, on the automodelity principle /3/ and on the proof of existence of automodel asymptotics in quantum field theory /4/. We note for completeness that in the experiments with beams of high energy electrons (see, e.g., /5/ and references therein) the scale invariance (automodelity) property in the Bjorken variable $x^B = -q^2/2\nu$ was verified in the range $1 (GeV/c)^2 \leq q^2 \leq 15 (GeV/c)^2$ and ν up to $\sim 20\ (GeV)^2$.

However, the recent experiments with high energy electrons, muons /6/ and neutrions /7/ led to interesting results: the scale invariance in Bjorken variable is broken. The breaking effect is of an order of 30-40%*. Among the attempts to explain this effects the search for new automodel variables in the framework of the parton model (see, e.g., review paper /10/ and references therein) and calculations of deep inelastic processes in the framework of quantum chromodynamics /11/ should be noticed. Without going into the advantages and shortcomings of these attempts, we note that the study of deep inelastic lepton-nucleus scattering can shed some light in the solution of some problems appeared. Besides the possibility which these processes offer in the study of nuclear structure, they can model

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* Interest to this problem increases in connection with future experiments with high energy muon beams /8,9/.

in some sense the processes of lepton-hafron interactions (see, e.g., ref. [12,13]). Analogies in hadronic and nuclear interactions become very obvious in the framework of many-body [14] quasipotential dynamics [15] in the "light front" variables [14,16,17].

We proceed now to the description of some results on lepton-nucleus interactions which have been obtained by means of the modified automodelity principle and by means of the formalism of papers [14,17].

2. Consider the process of deep inelastic scattering of the electron (muon) with 4-momentum k on the nucleus A with 4-momentum P_A, when the final lepton with 4-momentum k' and spectator fragment $(A-1)$ of nucleus with 4-momentum P^{sp}_{A-1} are detected in coincidence.

We do not distinguish here between the protons and neutrons and consider nucleons as scalar particles. The consideration of spins and isotopic spins of nucleons requires special analysis and we hope to turn back to study these problems elsewhere.

If the electron interacts with one nucleon of the nucleus only the cross section of this process in one photon exchange approximation is of the form

$$\frac{d\sigma(\ell A\to \ell'(A-1)\,X)}{dq^2 d\nu_{\ell A} d\nu_{sp}\, d\kappa} \sim \left(\frac{4\pi\alpha}{q^2}\right)^2 \frac{(\nu^2_{\ell A}-m^2_A q^2)^{-1/2}}{\lambda(s_{\ell A}, m^2_\ell, m^2_A)} \times \{-2(q^2+2m^2_\ell)\tilde{W}^{\ell A}_1(q^2,\nu_{\ell A},\nu_{sp},\kappa)+$$

$$+[(s_{\ell A}-\nu_{\ell A}-m^2_A-m^2_\ell)^2+q^2(m^2_A-\frac{\nu^2_{\ell A}}{q^2})]\tilde{W}^{\ell A}_2(q^2,\nu_{\ell A},\nu_{sp},\kappa)\}. \qquad (1)$$

Kinematic invariants in (1) are defined in the following manner

$$q^2 = (k-k')^2, \quad \nu_{\ell A} = (P_A q), \quad \nu_{sp} = (P^{sp}_{A-1} q)$$

$$\kappa = (P_A P^{sp}_{A-1}), \quad s_{\ell A} = (k+P_A)^2.$$

$$\frac{d\vec{P}^{sp}_{A-1}}{E^{sp}_{A-1}} = \frac{d\nu_{sp}\, d\kappa\, d\phi_{sp}}{(\nu^2_{\ell A}-m^2_A q^2)^{1/2}}. \qquad (2)$$

m_A is the nucleus mass, m_ℓ is the lepton mass, $\lambda(x,y,z) = (x-y-z)^2 - 4yz$.

Note, that before the more precise definition we consider the process in an arbitrary Lorentz frame in which the initial lepton and nucleus collide along the z-axis. Integration over the azimuthal angle of the spectator fragment gives that from the tensor

$$\hat{W}^{\ell A}_{\mu\nu} = (-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2})\hat{W}^{\ell A}_1 + (P_{A,\mu}-\frac{\nu_{\ell A}}{q^2}q_\mu)(P_{A,\nu}-\frac{\nu_{\ell A}}{q^2}q_\nu)\hat{W}^{\ell A}_2 +$$

$$+(P^{sp}_{A-1,\mu}-\frac{\nu_{sp}}{q^2}q_\mu)(P^{sp}_{A-1,\nu}-\frac{\nu_{sp}}{q^2}q_\nu)\hat{W}^{\ell A}_3 + \qquad (3)$$

$$+\frac{1}{2}[(P_{A,\mu}-\frac{\nu_{\ell A}}{q^2}q_\mu)(P^{sp}_{A-1,\nu}-\frac{\nu_{sp}}{q^2}q_\nu)+(P_{A,\nu}-\frac{\nu_{\ell A}}{q^2}q_\nu)(P^{sp}_{A-1,\mu}-\frac{\nu_{sp}}{q^2}q_\mu)]\hat{W}^{\ell A}_4$$

which defines the nuclear (hadronic) part of the diagram of process considered, contribution to the cross section is given by two structure functions $\tilde{W}^{\ell A}_1$ and $\tilde{W}^{\ell A}_2$ only. $\tilde{W}^{\ell A}_{\mu\nu}$ is related to the initial tensor

$$\hat{W}^{\ell A}_{\mu\nu} = \sum_N (2\pi)^4 \delta^{(4)}(P_A + q - P^{sp}_{A-1} - P_N) \langle P_A | J_\nu(0) | N, P^{sp}_{A-1}\rangle \langle P^{sp}_{A-1}, N | J_\mu(0) | P_A\rangle \qquad (4)$$

in the following way

$$\int_0^{2\pi} d\phi_{sp}\, \hat{W}^{\ell A}_{\mu\nu} = \int_0^{2\pi} d\phi_{sp}\, d\nu_{sp}\, d\kappa\, \delta(\nu_{sp}-(P^{sp}_{A-1}q))\,\delta(\kappa-(P_A P^{sp}_{A-1}))\,\hat{W}^{\ell A}_{\mu\nu} = \tilde{W}^{\ell A}_{\mu\nu}. \qquad (5)$$

(For the details of the corresponding kinematics see, e.g. [18], where the process of deep inelastic lepton-hadron scattering $\ell N \to \ell' h X$ is considered, when in the final state lepton ℓ' and hadron h are detected in coincidence).

Let us write here the virtual photon absorption cross section on nucleus when the spectator fragment $(A-1)$ is detected in the final state

$$\frac{d\sigma(\gamma A\to (A-1)X)}{d\vec{P}^{sp}_{A-1}/E^{sp}_{A-1}} \sim \frac{4\pi^2\alpha}{\nu_{\ell A}+q^2/2}\,\epsilon^*_\mu \epsilon_\nu \hat{W}^{\ell A}_{\mu\nu}. \qquad (6)$$

ϵ_μ is the virtual photon polarization 4-vector which satisfies the gauge condition $\epsilon_\mu q_\mu = 0$. In the reference frame, where $q = (q_0, 0, 0, q_3)$, the polarization vector ϵ_μ may be chosen in the form

$$\epsilon_\mu^{T,\pm} = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0) \quad \text{- transversal polarization}$$

$$\epsilon_\mu^{L} = \frac{1}{\sqrt{-q^2}}(q_3, 0, 0, q_0) \quad \text{- longitudinal polarization}$$

Integrating the cross section (6) over the azimuthal angle of the spectator, we obtain the following expression for the absorption cross section of transversally and longitudinally polarized photons

$$\left\langle \frac{d\sigma^T(\gamma A \to (A-1)X)}{d\vec{P}^{sp}_{A-1}/E^{sp}_{A-1}} \right\rangle \sim \frac{4\pi^2\alpha}{\nu_{\ell A} + q^2/2} \tilde{W}_1^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) \tag{7a}$$

$$\left\langle \frac{d\sigma^L(\gamma A \to (A-1)X)}{d\vec{P}^{sp}_{A-1}/E^{sp}_{A-1}} \right\rangle \sim \frac{4\pi^2\alpha}{\nu_{\ell A} + q^2/2}\Big[-\tilde{W}_1^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) + \Big(m_A^2 - \frac{\nu_{\ell A}^2}{q^2}\Big) \times$$
$$\times \tilde{W}_2^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa)\Big]. \tag{7b}$$

Here and below the symbol $<\ >$ denotes that integration over the spectator azimuthal angle is performed in the corresponding cross section.

When $q^2 \to 0$ a relation between the cross sections of the processes $\ell A \to \ell'(A-1)X$ and $\gamma A \to (A-1)X$ can be obtained. Let us write for this purpose the cross section (1) in the laboratory frame

$$\frac{d\sigma(\ell A \to \ell'(A-1)X)}{dq^2 d\nu_{\ell A} d\nu_{sp} d\kappa} \sim \Big(\frac{4\pi\alpha}{q^2}\Big)^2 \frac{(\nu_{\ell A}^2 - m_A^2 q^2)^{-1/2}}{m_A^2} \cdot \frac{E_{k'}}{E_k} \times$$
$$\times \Big[2\sin^2\frac{\theta}{2}\tilde{W}_1^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) + m_A^2 \cos^2\frac{\theta}{2}\tilde{W}_2^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa)\Big]. \tag{8}$$

Here θ is the lepton scattering angle, E_k and $E_{k'}$ are energies in the initial and final states, respectively.

Taking the limit $q^2 \to 0$ $(\sin^2\frac{\theta}{2} \to 0)$ in this expression we obtain

$$\frac{d\sigma(\ell A \to \ell'(A-1)X)}{dq^2 d\nu_{\ell A} d\nu_{sp} d\kappa}\Big|_{q^2\to 0} \sim \frac{\alpha^2}{\nu_{\ell A}} \frac{E_{k'}}{E_k} \lim_{q^2\to 0}\Big[\frac{\tilde{W}_2^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa)}{q^4}\Big]. \tag{9}$$

On the other hand the absorption cross section of the longitudinally polarized photons should vanish when $q^2 \to 0$. This leads to the following relation

$$\lim_{q^2\to 0} \tilde{W}_1^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) = \lim_{q^2\to 0}\Big[\frac{\nu_{\ell A}^2}{q^2}\tilde{W}_2^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa)\Big]. \tag{10}$$

Cross section of transversally polarized photons when $q^2 \to 0$ is the total photoabsorption cross section and according to (7a) takes the form

$$\frac{d\sigma(\gamma A \to (A-1)X)}{d\nu_{sp} d\kappa}\Big|_{q^2\to 0} \sim \frac{4\pi^2\alpha}{\nu_{\ell A}^2} \lim_{q^2\to 0}\big[\tilde{W}_1^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa)\big]. \tag{11}$$

From the formulas (9), (10), (11) we obtain the following relation between the cross sections

$$\frac{d\sigma(\ell A \to \ell'(A-1)X)}{dq^2 d\nu_{\ell A} d\nu_{sp} d\kappa}\Big|_{q^2\to 0} \sim \frac{\alpha}{\nu_{\ell A}} \frac{E_{k'}}{E_k} \frac{1}{q^2} \frac{d\sigma(\gamma A \to (A-1)X)}{d\nu_{sp} d\kappa}. \tag{12}$$

3. Let us carry out now a dimensional analysis of structure functions $\tilde{W}_1^{\ell A}$ and $\tilde{W}_2^{\ell A}$ and apply the automodelity principle in the form for electromagnetic and weak interactions /3/ to the kinematical invariants q^2, $\nu_{\ell A}$, ν_{sp}, which contain the lepton momenta, and in the form for strong interactions /19/ to the invariant κ, which contains the hadron momenta only. We obtain that if the momenta undergo the scale transformations,

the structure functions $\tilde{W}_1^{\ell A}$ and $\tilde{W}_2^{\ell A}$ are transformed in the following way

$$\tilde{W}_1^{\ell A}(\lambda^2 q^2, \lambda^2 \nu_{\ell A}, \lambda^2 \nu_{sp}, \kappa) = \lambda^{-2} \tilde{W}_1^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa), \tag{13a}$$

$$\tilde{W}_2^{\ell A}(\lambda^2 q^2, \lambda^2 \nu_{\ell A}, \lambda^2 \nu_{sp}, \kappa) = \lambda^{-4} \tilde{W}_2^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa). \tag{13b}$$

It follows from here that in the Bjorken limit ($s_{\ell A}, \nu_{\ell A}, q^2 \gg m^2$, $q^2/\nu_{\ell A}$ fixed) in the target fragmentation region (κ - finite, $\nu_{sp} \to \infty$ with $\nu_{sp}/\nu_{\ell A}$ fixed) the structure functions behave as follows

$$\lim m_A^2 \tilde{W}_1^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) = f_1\left(\frac{q^2}{\nu_{\ell A}}, \frac{\nu_{sp}}{\nu_{\ell A}}, \kappa\right), \tag{14a}$$

$$\lim m_A^2 \nu_{\ell A} \tilde{W}_2^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) = f_2\left(\frac{q^2}{\nu_{\ell A}}, \frac{\nu_{sp}}{\nu_{\ell A}}, \kappa\right): \tag{14b}$$

Writing the kinematic invariants used in the center-of-mass frame we obtain

$$\lim m_A^2 \tilde{W}_1^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) = F_1(x_{\ell A}^B, x_{\ell A}^F, \vec{P}^{sp}_{A-1,\perp}), \tag{15a}$$

$$\lim m_A^2 \nu_{\ell A} \tilde{W}_2^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) = F_2(x_{\ell A}^B, x_{\ell A}^F, \vec{P}^{sp}_{A-1,\perp}), \tag{15b}$$

where

$$x_{\ell A}^B = -q^2/2\nu_{\ell A}, \tag{16a}$$

$$x_{\ell A}^F = 2P^{sp}_{A-1,z}/\sqrt{s_{\ell A}}. \tag{16b}$$

Similar conclusions can be obtained for structure functions of deep inelastic lepton-hadron interactions $\ell N \to \ell' hX$ substituting the lepton-nucleus kinematic variables by the corresponding lepton-hadron variables.

It is interesting to study the structure functions $\tilde{W}_1^{\ell N}$ and $\tilde{W}_2^{\ell N}$ of lepton-hadron interaction $\ell N \to \ell' hX$ in the framework of general principles of quantum field theory [4,20].

4. We shall study in this section a relation between the structure functions $\tilde{W}_1^{\ell A}$ and $\tilde{W}_2^{\ell A}$ and the usual structure

functions $W_1^{\ell N}$ and $W_2^{\ell N}$ of deep inelastic lepton-hadron interaction. Describe for this purpose the initial nucleus and spectator fragment in terms of many-body [14,17] quasipotential wave functions $\Phi([x_i, P_\perp^{(i)}])$ in the "light front" variables [14,16,17]. The cross section of the process $\ell A \to \ell'(A-1)X$ in such an approach takes the form

$$\frac{d\sigma(\ell A \to \ell'(A-1)X)}{dq^2\, d\nu_{\ell A}\, d\nu_{sp}\, d\kappa} \sim \frac{\lambda(s_{\ell N}, m_\ell^2, m_N^2)}{\lambda(s_{\ell A}, m_\ell^2, m_A^2)} (\nu_{\ell A}^2 - m_A^2 q^2)^{-1/2} \times$$
$$\times \left| \frac{I(X^{sp}, \vec{P}^{sp}_{A-1,\perp})}{1 - X^{sp}} \right|^2 \frac{d\sigma(\ell N \to \ell' X)}{dq^2\, d\nu_{\ell N}}. \tag{17}$$

Here $\frac{d\sigma(\ell N \to \ell' X)}{dq^2\, d\nu_{\ell N}}$ is the cross section of deep inelastic lepton-hadron interaction, m_N is the nucleon mass. The kinematic invariants

$$s_{\ell N} = (k + p_N)^2, \quad \nu_{\ell N} = p_N q \tag{18}$$

of lepton-hadron interaction are related to the invariants of lepton-nucleus interaction in the following manner :

$$s_{\ell N} = s_{\ell A} + m_{A-1}^2 - 2(kP^{sp}_{A-1}) - 2\kappa, \tag{19a}$$

$$\nu_{\ell N} = \nu_{\ell A} - \nu_{sp}, \tag{19b}$$

p_N in formula (18) is the 4-momentum of the nucleon from nucleus A which interacted with lepton, $I(X^{sp}, \vec{P}^{sp}_{A-1,\perp})$ is the overlap integral of the initial nucleus and the fragment-nucleus wave functions. (See in this connection [21], where X^{sp} however, is defined in a slightly different way).

$$I(X^{sp}, \vec{P}^{sp}_{A-1,\perp}) = \int_0^1 \prod_{i=1}^{A-1} \frac{dx_i^{(A-1)'}}{x_i^{(A-1)'}} \delta\left(1 - \sum_{i=1}^{A-1} x_i^{(A-1)'}\right) \int \prod_{i=1}^{A-1} d\vec{p}_\perp^{(i)'} \times$$
$$\times \delta\left(\vec{P}^{sp}_{A-1,\perp} - \sum_{i=1}^{A-1} \vec{p}_\perp^{(i)'}\right) \Phi^+_{P^{sp}_{A-1}}([x_i^{(A-1)'}, \vec{p}_\perp^{(i)'}]) \Phi_{P_A}([x_i^{(A)}, \vec{p}_\perp^{(i)}]). \tag{20}$$

$$x_i^{(A)} = \frac{P_{i,0} + P_{i,z}}{P_{A,0} + P_{A,z}}, \quad 0 < x_i^{(A)} < 1, \quad \sum_{i=1}^{A} x_i^{(A)} = 1 \tag{21a}$$

$$x_i^{(A-1)'} = \frac{P'_{i,0} + P'_{i,z}}{P^{sp}_{A-1,0} + P^{sp}_{A-1,z}}, \quad 0 < x_i^{(A-1)'} < 1, \quad \sum_{i=1}^{A-1} x_i^{(A-1)'} = 1 \tag{21b}$$

$$X^{sp} = \frac{P^{sp}_{A-1,0} + P^{sp}_{A-1,z}}{P_{A,0} + P_{A,z}} . \tag{21c}$$

The variables $x_i^{(A)}$, $\vec{p}_\perp^{(i)}$ are related to the integration variables $x_i^{(A-1)'}$, $\vec{p}_\perp^{(i)'}$ as follows:

$$x_i^{(A)} = X^{sp} x_i^{(A-1)'}, \quad \vec{p}_\perp^{(i)} = \vec{p}_\perp^{(i)'} \quad i = 1, 2, \ldots, A-1$$

$$x_A^{(A)} = 1 - X^{sp} \tag{22}$$

$$\vec{p}_\perp^{(A)} = -\vec{P}^{sp}_{A-1,\perp} \quad \text{in the frame, where } \vec{P}_{A,\perp} = 0 .$$

Note, that in the case of lepton-deuteron scattering the overlap integral is replaced by the deuteron relativistic wave function.

The formula analogous to (17) may be obtained for the case of photoabsorption on nucleus. It is of the form:

$$\left\langle \frac{d\sigma(\gamma A \to (A-1)X)}{d\vec{P}^{sp}_{A-1}/E^{sp}_{A-1}} \right\rangle \sim \frac{\nu_{\ell N} + q^2/2}{\nu_{\ell A} + q^2/2} \left| \frac{I(X^{sp}, \vec{P}^{sp}_{A-1,\perp})}{1 - X^{sp}} \right|^2 \sigma(\gamma N \to X) . \tag{23}$$

Here $\sigma(\gamma N \to X)$ is the total photoabsorption cross section on nucleon.

Using formulas (1), (7a), (7b), (17), (23) and the well-known expression for the cross section of deep inelastic electroproduction and photoabsorption on nucleon (see, e.g., ref. /22/) we obtain the relation between the structure functions $\tilde{W}_i^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa)$ and the structure functions $W_i^{\ell N}(q^2, \nu_{\ell N})$:

$$\tilde{W}_1^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) \sim \left| \frac{I(X^{sp}, \vec{P}^{sp}_{A-1,\perp})}{1 - X^{sp}} \right|^2 W_1^{\ell N}(q^2, \nu_{\ell N}) , \tag{24a}$$

$$\tilde{W}_2^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) \sim \left| \frac{I(X^{sp}, \vec{P}^{sp}_{A-1,\perp})}{1 - X^{sp}} \right|^2 \cdot \frac{m_N^2 - \frac{\nu_{\ell N}^2}{q^2}}{m_A^2 - \frac{\nu_{\ell A}^2}{q^2}} W_2^{\ell N}(q^2, \nu_{\ell N}) . \tag{24b}$$

Taking into account the automodel nature of structure functions $W_1^{\ell N}(q^2, \nu_{\ell N})$ and $\nu_{\ell N} W_2^{\ell N}(q^2, \nu_{\ell N})$ in the Bjorken limit, we obtain the following properties of scale invariances for the functions $\tilde{W}_i^{\ell A}$ in the target fragmentation region:

$$\lim \tilde{W}_1^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) \sim \left| \frac{I(x^F_{\ell A}, \vec{P}^{sp}_{A-1,\perp})}{1 - X^F_{\ell A}} \right|^2 \phi_1\left(\frac{x^B_{\ell A}}{1 - x^F_{\ell A}} \right) , \tag{25a}$$

$$\lim \nu_{\ell A} \tilde{W}_2^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) \sim \left| \frac{I(x^F_{\ell A}, \vec{P}^{sp}_{A-1,\perp})}{1 - x^F_{\ell A}} \right|^2 (1 - x^F_{\ell A}) \phi_2\left(\frac{x^B_{\ell A}}{1 - x^F_{\ell A}} \right) \tag{25b}$$

which agree with (15).

Note, that analysis of this type can be applied immediately to the processes of weak deep inelastic lepton-nucleus interactions.

5. Recently the problem of the so-called ξ-scaling in the lepton-hadron interactions is widely discussed (see, e.g., ref. /11/). The meaning of this phenomenon consists in the following. The structure functions $W_1^{\ell N}$ and $\nu_{\ell N} W_2^{\ell N}$ may be functions not of the well-known Bjorken variable $x^B_{\ell N} = -q^2/2\nu_{\ell N}$, but the functions of some more complicated dimensionless combination of invariants q^2 and $\nu_{\ell N}$. In the framework of parton model the variable $\xi_{\ell N}$ is related to the parton momentum in the following way /23/:

$$\xi_{\ell N} = \frac{p_+^{part}}{P_+^{hadr}} = \frac{p_0^{part} + p_z^{part}}{P_0^{hadr} + P_z^{hadr}} , \tag{26}$$

where p_μ^{part} is the 4-momentum of parton, and P_μ^{hadr} is the 4-momentum of hadron, to which this parton belongs.

As far as the variables of the type (26) enter into our consideration in a natural way (see *Section 4*), it may be supposed that the following automodel relations should hold:

$$m_A^2 \tilde{W}_1^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) = F_1 (\xi_{\ell A}^B, \xi_{\ell A}^F, \vec{P}^{sp}_{A-1,\perp}), \qquad (27a),$$

$$m_A^2 \nu_{\ell A} \tilde{W}_2^{\ell A}(q^2, \nu_{\ell A}, \nu_{sp}, \kappa) = F_2 (\xi_{\ell A}^B, \xi_{\ell A}^F, \vec{P}^{sp}_{A-1,\perp}), \qquad (27b)$$

where

$$\xi_{\ell A}^F = \frac{P^{sp}_{A-1,+}}{P_{A,+}} = \frac{P^{sp}_{A-1,0} + P^{sp}_{A-1,z}}{P_{A,0} + P_{A,z}}, \qquad (28a)$$

$$\xi_{\ell A}^B = \frac{P_{N,+}}{P_{A,+}} = \frac{P_{N,0} + P_{N,z}}{P_{A,0} + P_{A,z}} = \frac{-q^2 + m_x^2 - m_N^2 + \sqrt{(-q^2 + m_x^2 - m_N^2) - 4(\vec{P}^{sp\,2}_{A-1\perp} + m_N^2)q^2}}{2(\nu_{\ell A} + \sqrt{\nu_{\ell A}^2 - m_A^2 q^2})}, \qquad (28b)$$

m_x^2 is the missing mass squared.

The variables $\xi_{\ell A}^B$ and $\xi_{\ell A}^F$ in the asymptotic region turn into the $x_{\ell A}^B$ and $x_{\ell A}^F$, respectively.

The relation $(\xi_{\ell A}^B + \xi_{\ell A}^F = 1)$ between the $\xi_{\ell A}^B$ and $\xi_{\ell A}^F$ variables allows one to think that a number of variables in the functions F_1 and F_2 may be reduced to two. (A similar relation between the variables $x_{\ell A}^B$ and $x_{\ell A}^F$ is realized only by neglecting all the particle masses, entering the reaction). The last suggestions require however the critical verification in the experiment.

One can think that the dependence of the type (27) should hold also for the structure functions of deep inelastic lepton-hadron interactions, when the final lepton and one of the produced hadrons are detected in coincidence.

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