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EVALUATION OF SYSTEMATIC UNCERTAINTIES CAUSED BY RADIATIVE CORRECTIONS IN EXPERIMENTS ON DEEP INELASTIC ν_ℓN-SCATTERING

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Оценка систематических неточностей, обусловленных радиационными поправками, в опытах по глубоконеупругому $\nu_\ell N$ -рассеянию

В рамках простой кварк-партонной модели сильного взаимодействия и теории Вайнберга-Салама получены компактные формулы для радиационной поправки к процессам глубоконеупругого $\nu_\ell(\bar{\nu}_\ell)N$ -рассеяния нейтрино на нуклонах, индуцированным заряженными токами. Показано, что радиационная поправка достигает 20-30% величины, характерной для глубоконеупругого ℓN -рассеяния. Полученные результаты значительно отличаются от имеющихся в литературе оценок рассматриваемого эффекта.

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Evaluation of Systematic Uncertainties Caused by Radiative Corrections in Experiments on Deep Inelastic ν_ℓ N -Scattering

Basing on the simple quark-parton model of strong interaction and on the Weinberg-Salam theory we derive compact formulae for the radiative correction to the charged current induced deep inelastic scattering of neutrinos from nucleons. The radiative correction is found to be around 20-30%, i.e., the value typical of deep inelastic ℓN -scattering. The results obtained are rather different from the presently available estimations of the considered effect.

The investigations has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

It is well known that experimental data on deep inelastic $e(\mu)N$ -scattering must be corrected for electromagnetic effects [1,2], because of the large magnitude of the electromagnetic corrections (EC) and high precision of measurements [3,4]. Provided the EC are taken into account correctly, one can interpret the data in terms of hadron structure functions and draw some conclusions about the behaviour of strong interactions (e.g., scaling violation).

Until recently the analysis of data on deep inelastic neutrino-nucleon scattering has not yet included the radiative corrections [5,6]. This was justified in the early days of neutrino physics when it was specified by poor statistics and low energy of neutrino beams. At present, however, experiments on deep inelastic $\nu(\bar{\nu})N$ -scattering are specified by rather high statistics (ten thousand of events), and a neutrino beam energy reaches 200 GeV [5,6]. Present experiments yield more precise results (5÷10%) for which the radiative corrections should not be neglected. A priopi there are no grounds at all to consider the corrections to be smaller than the attained precision of measurements. The neglect of these corrections was caused by difficulties in performing a reliable theoretical calculation. Let us discuss this point in more detail: When calculating the EC to deep inelastic scattering of charged leptons on nucleons one can separate a gauge-invariant set of diagrams in which an extra real (or virtual) photon couples

to the lepton line (corrections to the lepton current). This part of diagrams may be calculated in a model-independent manner as these contain the known hadron current. Calculations show that the EC to the lepton current are rather larger than those due to diagrams in which the photon couples to the hadron lines. Therefore the calculation of the EC to deep inelastic ℓN -scattering is reliable. In neutrino charged current induced reactions the electric current flows from leptons to hadrons, and therefore the gauge-invariant set includes all possible diagrams in which photons are emitted both from lepton and hadron lines. A model for strong interaction is hence obligatory in the calculations of EC to inclusive neutrino reactions, and no part of the EC can be evaluated in a model-independent manner. The model calculations of EC, however, cannot be applied to correct the data unless some of other experiments will justify the validity of the corresponding model. (Information of this kind can be gained by measuring the difference of $\ell^+ N - \ell^- N$ deep inelastic scattering [7]).
Nevertheless, the model calculations of the EC are very important indeed as these represent the only possibility to estimate the systematic uncertainties in extracting the physical information from data when the EC are not taken into account. These uncertainties should be remembered when interpreting the results of neutrino experiments.

We are aware of two works devoted to the problem we discuss here. In ref. [8] the EC have been calculated in the framework of the parton model of strong interaction and V-A four-fermion theory of weak interaction. Recent paper [9] presents numerical study of the EC based on the parton model calculation of Kiskis [8], and the computations have been done by a Monte Carlo program for a wide set of kinematic variables and different free parameters.

Calculations within the V-A theory contain divergences, and the finite result has been derived by redefining the Fermi constant

$$G' = G(1 + \frac{\alpha}{\pi} \ln \frac{\Lambda}{M_N}), \qquad (1)$$

where Λ is a cut-off parameter, M_N the nucleon mass, and α the fine structure constant. A procedure of this type is not justified by exact renormalization theory and can produce the result up to a constant. Within gauge theory unifying weak and electromagnetic interactions one can obtain an unambiguous result, and thus this part of calculation [8] should be revised.

In the present paper we calculate the EC to inclusive neutrino reactions induced by weak charged currents using the Weinberg-Salam theory [10]. In view of its recent successes [11] the calculations in electroweak sector can be considered as reliable. However (see above) when studying neutrino reactions, one cannot dispense with the model of strong interactions. As in ref. [8], we took a simple quark-parton model. The limits of applicability of the parton model for calculating the EC have been thoroughly discussed in literature [8,11]. In recent paper [7] we have considered this problem for the EC to the hadron current for deep inelastic νN - scattering. In this paper the calculation is made under the same assumptions as in ref. [7], for this reason we shall not consider this question here. Let us emphasize once more that the calculation of EC in the parton model should be considered only as a reliable estimate of the magnitude and gross features of the effect.

In Sec. 2 we present a scheme for calculating the contribution to EC from the diagrams with exchange by virtual particles W, Z, χ -bosons, leptons and γ-quanta (V-contribution). Much attention is paid to define the "semi-weak" interaction constant, g, as to remove the uncertainty of the result of Kiskis [8]. For this purpose we consider the one-loop approximation for total probality of μ -decay. Then we give a scheme for calculating the contributions to the EC from the diagrams with real photon emission (R-contribution).

Sec. 3 lists and discusses numerical results. The correction we have found differs essentially in magnitude and behaviour from that one obtained by Kiskis [8]. In this context we present our results in a form suitable for their chacking and reproducing.

2. INCLUSIVE CROSS SECTION OF THE PROCESS IN ORDER $g^4\alpha$

The calculation of the inclusive cross sections $d\Sigma(E,x,y)$ for reaction

$$\nu_\ell(\bar{\nu}_\ell)N \to \ell(\bar{\ell}) + \text{hadrons} \qquad (2)$$

to order $g^4\alpha$ will be done by the following scheme: Find the one-loop approximation of the amplitude of the process

$$\nu_\ell + q_1 \to \ell + q_2, \qquad (3)$$

where q_1 and q_2 are point particles with arbitrary masses and charges $Q_1=f_1e$ and $Q_2=f_2e$ so that $|f_1-f_2|=1$. Define the "semi-weak" interaction constant up to one-loop corrections, i.e., calculate the one-loop corrections to the relation

$$\frac{g^2}{8M_W^2}=\frac{G_F}{\sqrt{2}}, \qquad (4)$$

with the Fermi constant G_F and W-boson mass M_W, and derive the finite expression for the differential cross section $d\sigma_V$ of process (3) in order $g^4\alpha$. Calculate the contribution to the cross section (2) from the diagrams with real photon emission ($d\sigma_R$)

$$\nu + q_1 \to \ell + q_2 + \gamma. \qquad (5)$$

In the sum $d\sigma_V + d\sigma_R$ replace the initial parton momentum by ξP, where P is the target nucleon momentum. Multiply the cross section by the distribution function $f_i(\xi)$ of i-th kind of partons over momenta, integrate over ξ and sum over all types of parton involved in reactions. At the final step we average the cross section over proton and neutron since neutrino experiments are typically performed with the heavy targets having almost the same number of neutrons and protons.

1. V-Contribution

The contribution to the amplitude (3) from the virtual particle exchange diagrams in the one-loop approximation has been calculated in our paper [13]. The calculation was done in the unitary gauge by the dimensional regularization method. Renormalization was made through counterterm method [19].

The amplitude has been calculated in the approximation

$$m_i^2 \ll I \ll M_W^2, \tag{6}$$

where I is an invariant of the amplitude, $m_i=(m, m_1, m_2)$ is a mass of intial or final particles (m, the lepton mass; m_1 and m_2, masses of q_1 and q_2 particles). The l.h.s. of the inequality is the necessary condition for validity of the parton model. The r.h.s. is not necessary, however, it essentially simplifies final formulae. It is equivalent to the inequality

$$E_\nu \ll \frac{M_W^2}{2M_N} \simeq 3400 \text{ GeV} \tag{7}$$

(E_ν is the lab. neutrino energy) which holds for energies of current neutrino experiments.

Among diagrams 1 to 12 the most difficult to calculation are those of type 11, 12 with exchange by W and γ quanta. Their contribution B_{WA} to the amplitude (3) in approximation (6) can be obtained from general formulae (5.21) - (5.30) of paper [14] for the VA-exchange diagrams

$$\begin{aligned} B_{WA} = \frac{e^2}{16\pi^2} C_{M_0^W}(g)\cdot\{ & -2(1+a_W)C_\alpha \otimes O_\alpha \cdot P - \\ & -2O_\alpha \otimes O_\beta\left(\delta_{\alpha\beta}+\frac{q_\alpha q_\beta}{M^2}\right)\cdot[|f_1|\cdot X\cdot\mu(-k_2;p_1)-|f_2|\cdot S'\cdot\mu(-k_2;p_2)]P_{IR}+ \\ & + O_\alpha \otimes O_\alpha \cdot(|f_2|\cdot B_2^F+|f_1|\cdot B_1^F)+2\hat{p}_1(1+\gamma_5)\otimes\hat{k}_1(1+\gamma_5)\cdot|f_1|\cdot X^{-1}\}. \end{aligned} \tag{8}$$

Here

$$R = 1 - \frac{e^2}{g^2}, \alpha_W = \frac{q^2}{M_W^2}, X = -2p_1 \cdot k_2, S = -2p_1 \cdot k_1, S' = -2p_2 \cdot k_2,$$

with q^2 transfer momentum squared

$$\mu(k_1;k_2) = \int_0^1 \frac{dy}{[k_1 y + k_2(1-y)]^2}, \qquad (9)$$

$$P = P_{IR} = \frac{1}{n-4} + \frac{1}{2}C + \ln\frac{M_W}{\eta 2\sqrt{\pi}}$$

are pole terms representing the ultraviolet and infrared divergences (η is an arbitrary parameter with mass dimensionality)

$$B_1^F = b_1(X) + 1 + \frac{\pi^2}{3} + 4\ln\frac{M_W^2}{m \cdot m_1} + 2\ln\frac{m \cdot m_1}{X}\ln\frac{q^2}{M_W^2} - \frac{1}{2}\ln\frac{Xm^2}{q^4}\ln\frac{X}{m^2} - \frac{1}{2}\ln\frac{Xm_1^2}{q^4}\ln\frac{X}{m_1^2},$$

$$B_2^F = b_2 + \frac{4}{3}\pi^2 + 4\ln\frac{M_W^2}{mm_2} + 2\ln\frac{mm_2}{S}\ln\frac{q^2}{M_W^2} - \frac{1}{2}\ln\frac{Sm^2}{q^4}\ln\frac{S}{m^2} - \frac{1}{2}\ln\frac{Sm_2^2}{q^4}\ln\frac{S}{m_2^2}, \qquad (10)$$

where

$$b_1(X) = \frac{3}{2} + 3\ln\frac{X}{M_W^2}, \quad b_2 = 5.$$

In (8) $C_{M_0^W}(g)$ is given by formulae (2.3) of paper [13] and symbol $\otimes$ means direct product of matrices. In this notation, the Born amplitude (diagram I) has the form

$$M_0^W(g) = C_{M_0^W}(g)\bar{u}(k_2)O_\alpha u(k_1)\bar{u}(p_2)O_\alpha u(p_1) => C_{M_0^W}(g)O_\alpha \otimes O_\alpha, \qquad (11)$$

with

$$O_\alpha = \gamma_\alpha(1 + \gamma_5).$$

In the approximation (6) the Born amplitude is factorized from all diagrams 2 to 12 except the last term of (8).

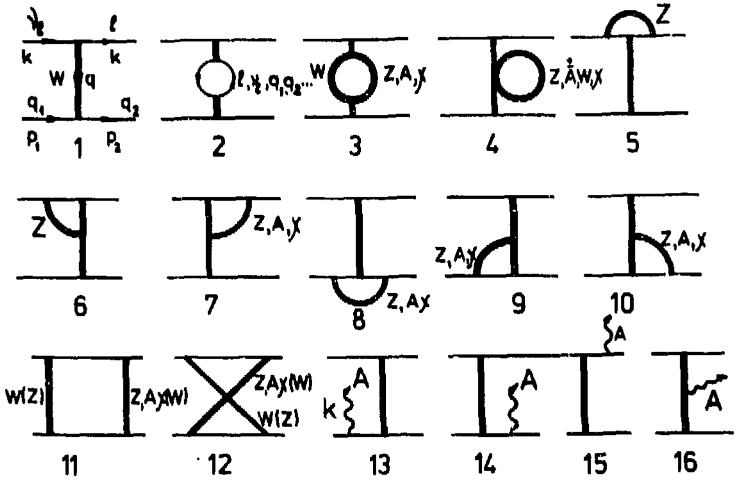

Fig.1. Diagrams for the reaction (3) in order $g_F^4\alpha$.

Calculating the interference of this term with the Born amplitude

$$\mathrm{Re}\{[\hat{p}_1(1+\gamma_5)\otimes\hat{k}_1(1+\gamma_5)]\cdot(O_\alpha\otimes O_\alpha)^*\} = -\frac{1}{2}X|O_\alpha\otimes O_\alpha|^2 \qquad (12)$$

we can factorize the amplitude $M_0^W(g)$ from the sum of all diagrams 1 to 12. In the approximation (6) we get

$$M_{1-12} = M_0^W(g)\cdot(1+\frac{\alpha}{4\pi}F+\frac{\alpha}{2\pi}\cdot\delta\cdot P_{IR}), \qquad (13)$$

with

$$\delta = 1+f_1^2+f_2^2-2p_1\cdot k_2|f_1|\mu(-k_2;-p_1)+2p_2\cdot k_2|f_2|\mu(-k_2;p_2)- \qquad (14)$$

$$-2p_1\cdot p_2|f_1|\cdot|f_2|\mu(-p_1;p_2).$$

This term represents the genuine infrared divergences which cancel out by the contribution of real photon emission

$$F = F_{V\chi} + 2G_{IR} + F_e . \tag{15}$$

Notation in (15) is as follows: $F_{V\chi}$ is a constant originating from the diagrams with exchange by $W^{\pm}$, Z, and γ-bosons; paper [18] provides the following expression

$$(1 - R)F_{V\chi} = W(0) - W(-1) + 2G - \frac{3}{4R} + \frac{35}{4} - 5R + 2R\,V_2(0) + \tag{16}$$

$$+ (-\frac{1}{2} + R - 3R^2)\,\frac{\ln R}{1 - R};$$

G_{IR} is a pole contribution representing infrared divergences which result from renormalization on the mass shell of W-boson, the explicit form of G_{IR} being given by formula (3.19) of ref. [15]; F_e is a contribution of the diagrams with virtual γ-quantum exchange; using (7) to (9) and (11) we obtain

$$F_e^F = |f_1| + |f_2| \cdot [\frac{3}{2R} - \frac{3}{2} - \frac{\pi^2}{3} + 2(1 - \frac{1}{4}\ln\frac{m_1^2}{M_W^2})\ln\frac{m_1^2}{M_W^2} +$$

$$+ 2(1 - \frac{1}{4}\ln\frac{m_2^2}{M_W^2})\ln\frac{m_2^2}{M_W^2} - 3\ln\alpha_W + \ln^2\alpha_W] + \tag{17}$$

$$+ f_1^2 \cdot (\frac{3}{4R} - \frac{11}{4} + \frac{3}{2}\ln\frac{m_1^2}{M_W^2}) + f_2^2 \cdot (\frac{3}{4R} - \frac{11}{4} + \frac{3}{2}\ln\frac{m_2^2}{M_W^2}) +$$

$$+ \frac{3}{2}\ln\frac{m^2}{M_W^2} + |f_1| \cdot (B_1^F - 1) + |f_2| \cdot B_2^F .$$

Now we will show how to remove the remaining divergences, G_{IR}. In the amplitude (13) to (17) the semi-weak interaction constant is not yet fixed. The relation (4) holds only in the tree approximation, while the constant g should be defined up to one-loop corrections. For this aim it is necessary to calculate some observable dependent on g up to one-loop corrections and to constrain it by comparing with the measured value.

This procedure is easily realized applying to μ-decay. Indeed, only two modes of μ-decay, $\mu \to e\nu\nu$ and $\mu \to e\nu\nu\gamma$, are observed, i.e., the measured inverse total lifetime should be equated to the calculated sum of probabilities

$$\frac{1}{\tau_\mu} = W_1(\mu \to e\nu\nu) + W(\mu \to e\nu\nu\gamma), \tag{18}$$

where W_1 contains the one-loop corrections to μ-decay. For the sum (18) we got

$$\frac{g^4 \cdot m_\mu^5}{3\cdot 2^{11}\cdot \pi^3 M_W^4} \cdot [1 + \frac{\alpha}{2\pi}(F_{V\chi} - 2G_{IR} + F_{e\mu})] \tag{19}$$

with

$$F_{e\mu} = \frac{3}{4R} + 9 - \pi^2 , \tag{20}$$

and m_μ muon mass.

Defining the physical constant, g_F by the equation

$$\frac{g_F^2 \cdot m_\mu^5}{3\cdot 2^{11}\cdot \pi^3 M_W^4} = \frac{1}{\tau_\mu}, \tag{21}$$

and inserting the recent data [16] on m_μ, τ_μ and proton mass M_p, we obtain

$$G_F = \sqrt{2}\cdot \frac{g_F^2}{8M_W^2} = 10^{-5} \cdot \frac{1.0245}{M_p^2} . \tag{22}$$

It seems convenient to use just this value of the constant in Born amplitudes since the relations (19) and (21), (22) represent the simplest way to constrain the product of two infinite unobservables, g and G_{IR}, by the observable, $g_F(\tau_\mu)$. One, of course, may use another definition of g_F in terms of τ_μ, e.g., involving the corrections calculated within the four-fermion theory [17] which differs from (22) by 0.2%. This would shift the numerical results listed below by 0.4%. Therefore, one should clearly realize what is the constant used in Born amplitudes*.

Writing $M_0^W(g)$ in (13) in terms of g_F to order $g_F^4 \alpha$, we get

$$M_{1-12} = M_0^W(g_F) \cdot [1 + \frac{\alpha}{4\pi} \cdot (F_e - F_{e\mu}) + \frac{\alpha}{2\pi} \cdot \delta \cdot P_{IR}]. \qquad (23)$$

Thus, the divergent contribution G_{IR} does not enter into final expressions if all their parameters are written in terms of observables.

Note that in the amplitude (23) contributions of the heavy boson exchange diagrams, $F_{V\chi}$, also cancel. This results from the approximation (6) (r.h.s. inequality) and from similarity of the process (3) and μ-decay: up to difference in electric charges these are different channels of the same reaction. Because of the cancellation of $F_{V\chi}$ the resulting correction appears to be independent of some parameters of the Weinberg-Salam theory ($\sin^2\theta_W$ and M_χ). And what is more, this cancellation means that the correction to processes (3) does not depend on the gauge model unifying weak and electromagnetic interactions. A similar statement

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* Relations like (19) were derived also in papers [18]. We repeated those calculations within our scheme. Paper [19] suggests another method for fixing g through considering the total probability $W_1(W \to \mu\nu_\mu) + W(W \to \mu\nu_\mu \gamma)$, however, there are no experimental data on those decays, therefore we have fixed g via the relation (18).

is valid for μ-decay [20], too. Due to the above noted similarity of two processes it is not surprising that it is also valid for the process (3). So, the radiative correction is given just by the electromagnetic sector, therefore in what follows we shall use the notion of the EC.

From the relation (23) we obtain the contribution of diagrams 1 to 12 to the cross section of process (3)

$$d^2\sigma_V^{\nu,\bar\nu} = d^2\sigma_0 \cdot T_0^{\nu,\bar\nu} \cdot \frac{\alpha}{\pi}\left(\frac{1}{2}\cdot F^{\nu,\bar\nu} + \delta\cdot P_{IR}\right), \tag{24}$$

where

$$d^2\sigma_0^{\nu,\bar\nu} = K\cdot dXdY\cdot\delta[(p_1+k_1-k_2)^2-m_2^2], \tag{25}$$

$$F^{\nu,\bar\nu} = F_e^{\nu,\bar\nu} - F_{e\mu} \tag{26}$$

with

$$Y = -(k_1-k_2)^2,\quad T_0^{\nu} = 1,\quad T_0^{\bar\nu} = \left(\frac{X}{S}\right)^2,\quad K = \frac{1}{S_N}\sigma_0,$$

$$\sigma_0 = \frac{g_F^4\cdot S_N}{2^5\pi M_W^4},\quad S_N = -2P\cdot k_1,$$

and P, the initial nucleon momentum. The form factor F_e^{ν} is given by formula (17). To find $F_e^{\bar\nu}$, one should make the change $b_2 \to b_1(S)$ and $b_1(X)\to b_2$ in formula (10).

The V-contribution to the process (2) from the i-th parton scattering is obtained from formulae (24) to (26) by changing p_1 to ξP, multiplying by the distribution function $f_i(\xi)$, and integrating over ξ. In usual scaling variables x and y

$$x = \frac{Y}{2M_N\nu},\quad y = \frac{\nu}{k_{10}},\quad \nu = k_{10}-k_{20},$$

we get

$$\left(\frac{d^2\Sigma_V}{dx\,dy}\right)_i^{\nu,\bar\nu} = \left(\frac{d^2\Sigma_0}{dx\,dy}\right)_i^{\nu,\bar\nu} \cdot \frac{\alpha}{\pi}\left(\frac{1}{2}F^{\nu,\bar\nu} + \delta\cdot P_{IR}\right)\Big|_{\xi=x} \qquad (27)$$

with

$$\left(\frac{d^2\Sigma_0}{dx\,dy}\right)_i^{\nu,\bar\nu} = \sigma_0 \cdot T_0^{\nu,\bar\nu}\cdot x\cdot f_i(x). \qquad (28)$$

2. R-Contribution

To calculate the contribution to EC from the process (5), diagrams 13 to 16, one should integrate the cross section of (5), $d\sigma_R$, over the whole phase space of photons since in the final state of (5) only lepton characteristics are measured. Following our procedure we separate the infrared-divergent part, $d\sigma_R^{IR}$, out of the cross section $d\sigma_R$:

$$d\sigma_R = d\sigma_R^{IR} + d\sigma_R - d\sigma_R^{IR} = d\sigma_R^{IR} + d\sigma_R^{F}. \qquad (29)$$

where

$$\frac{d^2\sigma_R^{IR}}{dX\,dY} = \frac{e^2}{(2\pi)^3}\int\frac{d^3k}{2k_0}\cdot 4F_{IR}\,\frac{d^2\sigma_0}{dX\,dY}\cdot T^{\nu,\bar\nu}\,,$$

$$T^{\nu} = 1 + \frac{m_1^2 - m_2^2 - m^2}{S}\,,\quad T^{\bar\nu} = \frac{X(X + m_2^2 - m_1^2 - m^2)}{S^2}\,, \qquad (30)$$

and F_{IR} is given by formula (4.8) of paper [13].

All cumbersome calculations (spuring, subtraction of d^3k, reduction of similar terms, and d^3k-integration) were performed by the analytic calculation system SCHOONSCHIP [22]. Integration was made by substitution of obtained expressions. Table of the substitutions and final formulae for $(d^2\Sigma_R^F)_i^{\nu,\bar\nu}$ are given in the Appendix.

Now we present the scheme for calculating $(d^2\Sigma_R^{IR})_i^{p,\bar p}$, the infrared part of the contribution to the cross section (2) produced by bremsstrahlung (5) at scattering on the i-th parton. In formula (30) we make the following change of variables

$$X = \xi X_N,\quad S = \xi \cdot S_N,\quad m_1 = \xi \cdot M_N, \tag{31}$$

where $X_N = S_N(1-y)$, then average (30) over the parton momentum distribution, and obtain

$$\left(\frac{d^2\Sigma_R^{IR}}{dx\,dy}\right)_i^{p,\bar p} = \sigma_0 \cdot S_N \cdot y \int d\xi\, T^{p,\bar p}(\xi)\cdot \xi f_i(\xi)\,\delta(p_2^2+m_2^2)\frac{e^2}{(2\pi)^3}\int \frac{d^3k}{2k_0}\cdot 4F_{IR}. \tag{32}$$

To (32) we add and subtract

$$\left(\frac{d^2\Sigma_0}{dx\,dy}\right)_i^{p,\bar p}\cdot \frac{\alpha}{\pi}\cdot \delta^{IR}, \tag{33}$$

where

$$\delta^{IR} = (2\pi)^2 \cdot S_N \cdot y \cdot \int_{\bar x}^{1} d\xi\,\delta(p_2^2+m_2^2)\frac{1}{(2\pi)^3}\int \frac{d^3k}{2k_0}\,4\cdot F_{IR}. \tag{34}$$

In (34) $\bar x$ is a root of the equation

$$V(\xi) = (S_N - X_N)\xi - Y + \xi^2 \cdot m_1^2 - m_2^2 = 0. \tag{35}$$

We calculate the infrared-free difference (32)-(33) straightforward in the approximation (6) and get

$$\left(\frac{d^2\Sigma_R^{TR\,\nu,\bar\nu}}{dxdy}\right)_i = \left(\frac{d^2\Sigma_R^{IR\,\nu,\bar\nu}}{dxdy}\right)_i + \left(\frac{d^2\Sigma_0^{\nu,\bar\nu}}{dxdy}\right)_i \cdot \frac{\alpha}{\pi}\delta^{IR} + \sigma_0 \cdot T_0^{\nu,\bar\nu} \cdot \frac{\alpha}{\pi} \cdot R(f_i), \tag{36}$$

where

$$R(f_i) = \int_x^1 \frac{\xi f_i(\xi) - x f_i(x)}{\xi - x} \cdot I(\xi) d\xi, \tag{37}$$

$$I(\xi) = -1 + |f_1| \ln\frac{\xi^2 X_N^2}{m^2 m_1^2} + |f_2| \frac{S_N \xi}{X_N \xi + Y} \ln\frac{(X_N \xi + Y)^2}{m^2 r}$$
$$- f_1^2 - \frac{m_2^2}{r} \cdot f_2^2 - |f_1| \cdot |f_2| \frac{Y}{\xi S_X^N} \ln\frac{(\xi S_X^N)^2}{m_1^2 r}, \tag{38}$$

with

$$S_X^N = S_N - X_N, \quad r = v + m_2^2, \quad m_1 = \xi M_N, \quad v = S_X^N \xi - Y.$$

To calculate the integral (34), we split it into two parts. The first part is integrated over k_0 from 0 to $\bar{k}$, where $\bar{k}$ is an infinitesimal, the second from $\bar{k}$ to the kinematic maximum $k_0^{max}(\xi)$. The first integral will be denoted by δ_1^{IR}, the second by δ_2. In δ_2 integration over d^3k is carried out with the use of Table A1, thereby we obtain

$$\delta_2 = \int_x^1 \frac{S_N y}{v} \cdot I(\xi) d\xi. \tag{39}$$

The integral (39) is calculated by making the identity transformation

$$I(\xi) = I(\xi) - I(x) + I(x), \tag{40}$$

followed by straightforward calculations

$$\delta_2 = I(x)\ln\frac{v_{max}}{2\bar{k}m_2} + |f_2|\cdot[-2\Phi[-r_x(1-y)]-\Phi(1)-\frac{1}{2}\ln^2\frac{v_{max}}{m_2^2} +$$

$$+\frac{1}{r_y}[\ln(\frac{1}{x}-y\cdot r_x)\ln(\frac{S_N}{m^2}yr_y(r_y+x))-\Phi(1)+\Phi(\frac{1}{x}-yr_x)]] + \tag{41}$$

$$+f_2^2\ln\frac{v_{max}}{m_2^2} + |f_1\cdot|f_2|\cdot[2\cdot\Phi(1)+\frac{1}{2}\ln^2\frac{v_{max}}{m_2^2} - \ln x\cdot\ln\frac{S_x^N}{M_N^2} -$$

$$-\Phi(x) + \frac{1}{2}\ln^2 x].$$

where $v_{max} = v|_{\xi=1}$, $r_x = 1/x-1$, $r_y = 1/y-1$ and $\Phi(x)$ is the Spence function.

To calculate δ_1^{IR} we integrate it over ξ with the δ-function in that system in which the photon emission is isotropic ($\vec{Q}=\vec{k}_1+\vec{p}_1-\vec{k}_2=0$). With $\bar{k}$ infinitesimal, we arrive at the integral

$$\delta_1^{IR} = (2\pi)^2\cdot\int_0^{\bar{k}}\frac{d^3k}{(2\pi)^3\cdot 2k_0}\cdot(\frac{k_2}{k_2\cdot k} - |f_1|\frac{p_1}{p_1\cdot k} - |f_2|\frac{p_2}{p_2\cdot k})^2\Big|_{v=0} \tag{42}$$

and calculate this divergence by the method of dimensional regularization in a manner of paper [20]. After rather cumbersome calculations we find

$$\delta_1^{IR} = -\delta\cdot P_{IR} + \delta_1^F ,$$

$$\delta_1^F = I(x)\ln\frac{2k}{M_W} + \ln\frac{S_0}{m\cdot m_2} - |f_2|\cdot[\Phi(1)+\ln^2\frac{S_0}{m\cdot m_2}] +$$

$$+ |f_1|\cdot[\ln\frac{S_0}{m_2^2}\ln\frac{m\cdot m_1}{X_0} + \frac{1}{2}\ln^2\frac{X_0}{Y} - \frac{1}{4}\ln^2\frac{m^2}{S_0} - \frac{1}{4}\ln^2\frac{m_1^2 S_0}{Y^2} - \Phi(1)] +$$

$$+ f_1^2\cdot\ln\frac{Y}{m_1\cdot m_2} + f_2^2 + |f_1|\cdot|f_2|\cdot[\Phi(1) + \ln^2\frac{Y}{m_1\cdot m_2}], \qquad (43)$$

with

$$S_0 = xS_N, \quad X_0 = x\cdot X_N, \quad m_1 = xM_N .$$

3. Derivation of Final Formulae for EC

The $g_F^4\alpha$ -order contribution to the cross section of reaction (2) from the scattering processes (3) and (5) on the i-th parton is as follows

$$\left(\frac{d^2\Sigma}{dxdy}\right)_i^{\nu,\bar\nu} = \left(\frac{d^2\Sigma_0}{dxdy}\right)_i^{\nu,\bar\nu} \cdot \frac{\alpha}{\pi} \cdot \delta_F^{\nu,\bar\nu} + \left(\frac{d^2\Sigma_R^{TR}}{dxdy}\right)_i^{\nu,\bar\nu} + \left(\frac{d^2\Sigma_R^F}{dxdy}\right)_i^{\nu,\bar\nu}, \qquad (44)$$

where $\delta_F = \delta_1^F + \delta_2 + \frac{1}{2}F$ and $(d^2\Sigma_R^F)_i^{\nu,\bar\nu}$ are given in the Appendix.

To obtain the final expressions for total cross sections of processes (2) in order $g_F^4\alpha$, , one should sum (44) over all partons and antipartons in the nucleon. In this summation, we take into account that the formülae for scattering of neutrino on quarks are the same as for $\bar\nu\bar{Q}$ -scattering, and those for $\nu\bar{Q}$ -scattering are the same as for $\bar\nu Q$ -scattering. To simplify the derivation of the numerical result, we use the three-quark model and ignore the s-quark contribution, i.e., terms with $\sin^2\theta_C$. Based on the general formula (44), the EC can be estimated within the model with arbitrary number of quarks, however, it is evident that the contribution to radiative processes from heavy quarks is suppressed due to the large mass. In this approximation, the contribution to (2) comes only from u-, d-, $\bar{u}$- and $\bar{d}$-quarks, therefore it is natural to assume the equal masses of all final quarks.

Define the EC as the ratio

$$\delta^{\nu,\bar\nu}(E,x,y) = \frac{d^2\Sigma_1^{\nu,\bar\nu}}{dx\,dy} \Big/ \frac{d^2\Sigma_0^{\nu,\bar\nu}}{dx\,dy}, \qquad (45)$$

where $d^2\Sigma_0^{\nu,\bar\nu}$ and $d^2\Sigma_1^{\nu,\bar\nu}$ are the cross sections of processes (2), resp., in order g_F^4 and $g_F^4\alpha$ summed over all partons involved in the reaction and averaged over the proton and neutron. Inserting (44) into (45) we get

$$\delta^{\nu}(E,x,y) = \frac{xQ(x)\delta_F^{\nu} + R(Q) + S^{\nu}(Q) + (1-y)^2 x\bar Q(x)\delta_F^{\bar\nu} + (1-y)^2 R(\bar Q) + S^{\bar\nu}(\bar Q)}{x\left[Q(x) + (1-y)^2 \bar Q(x)\right]}, \qquad (46)$$

where (notation of ref. [23])

$$Q(x) = u_v(x) + d_v(x) + 2s(x), \quad \bar Q(x) = 2s(x)$$

functions $S^{\nu,\bar\nu}$ are given in the Appendix and

$$\delta^{\bar\nu}(E,x,y) = \frac{(1-y)^2 xQ(x)\delta_F^{\bar\nu} + (1-y)^2 R(Q) + S^{\bar\nu}(Q) + x\bar Q(x)\delta_F^{\nu} + R(\bar Q) + S^{\nu}(\bar Q)}{x\left[(1-y)^2 Q(x) + \bar Q(x)\right]}. \qquad (47)$$

III. DISCUSSION

In Figure 2 we show the δ-factor for deep inelastic scattering of the muon and electron neutrino and antineutrino on the isoscalar target at energy 100 GeV. When drawing the figures, we did not approach the kinematic boundaries where the l.h.s. inequality (6) may be violated.

As follows from formulae of Sec.II, the δ-factor depends not only on kinematic variables (E, x, y) but also on the masses of interacting particles, the W-boson mass, and the shape of the parton distribution.

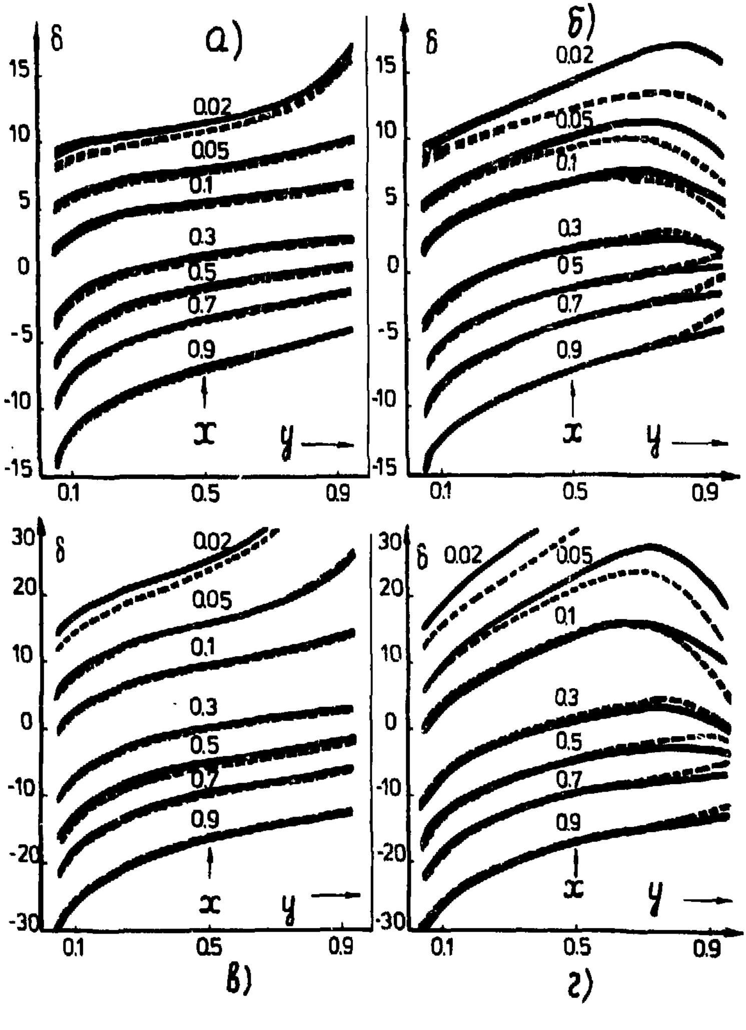

Fig.2. Electromagnetic corrections, δ, at a neutrino energy of 100 GeV calculated with parton distributions from paper [25] (solid lines) and [23] (dotted lines) for processes a) ν_μ , b) $\bar{\nu}_\mu$, c) ν_e , d) $\bar{\nu}_e$ -scattering.

Let us discuss the choice of the latter parameters. The masses of charged leptons were taken from experiment [16], the neutrino masses were put zero. The W-boson mass was calculated within the Weinberg-Salam theory from the experimental value of $\sin^2\theta_W$. For definiteness, we took the interval

$$0.15 \leq \sin^2\theta_W \leq 0.40 \qquad (48)$$

which covers the recent experimental data on $\sin^2\theta_W$. The interval (48) constrains the M_W by

$$100\ \text{GeV} \geq M_W \geq 60\ \text{GeV}. \qquad (49)$$

For the initial quark mass, requiring the kinematic relations for processes (2) and (3) to be equal, we derive $m_1 = xM_N$, (ξM_N). The final quark mass was considered as a constant free parameter varied within the limits

$$0.1\ \text{GeV} \leq m_2 \leq 1.0\ \text{GeV}. \qquad (50)$$

For parton spectra we used the parametrizations of papers [23] and [25].

The mass variation in the limits (49), (50), and the dependence of δ on the parton distributions give rise to a spectrum of δ values at fixed E, x and y. It appears that the spectrum due to the mass variation does not drop out of the width of solid lines (see Figures 2). The sensitivity to the choice of distribution functions is much more manifest, especially at small x. So, for $0.1 \leq x \leq 0.9$ and $0.1 \leq y \leq 0.9$ the model we consider provides rather definite predictions for the correction, beyond these limits either the model is not valid or the correction is very sensitive to the parameters.

We studied also the E-dependence of δ at fixed x and y. It appears to be weak ($\Delta\delta < 5\%$) within the total width of neutrino spectra for beams produced at FNAL and SPS accelerators, i.e., the correction can be considered as a constant for the whole neutrino spectrum and calculated, e.g., for the energy of intensity peak.

As is seen from Figures 2 the EC is large (especially in $\nu_e N$-scattering), consequently, it may give rise to the observed effects, for instance, to some deviations of x and y distributions from parton model predictions, to simulation of the violation of Bjorken scaling, and so on. These large values of the EC are not surprising; the correction to ℓN-scattering at an energy of 100 GeV also appears to be 20-30%. For both $\nu_\ell N$- and ℓN-scattering there is observed the similar behaviour of the correction (growth with increasing y and decreasing x) [2]. As in ℓN-scattering [26], an approximate ratio 2 : 1 also is valid for the corrections to neutrino reactions with final electrons and muons. It is interesting to note that for $y \to 0$ the EC for all the four processes, $\nu_\ell(\bar\nu_\ell)N$-, $\ell(\bar\ell)N$--scattering, coincide within high accuracy. It is just that region where the difference in mechanisms of these reactions is small. With increasing y the EC to ℓN-scattering are growing rapidly due to the one-photon exchange; the EC to νN- and $\bar\nu N$-scattering for $y \neq 0$ do not coincide because of the different helicities of neutrino and antineutrino. It is noteworthy to mention a somewhat surprising behaviour of the EC to $\bar\nu N$-scattering for $x \sim 0$ and $y \sim 1$. The decrease of the EC in this region for other processes is not observed. According to (47), in the region $y \to 1$ only the antiquarks contribute (function $\bar Q(x)$). From comparison of (46) and (47) it follows that for $y \to 1$ δ^{ν} differs from $\delta^{\bar\nu}$ only by the shape of parton spectrum. An additional calculation has shown that δ^{ν} computed for two functions $Q(x)$ and $\bar Q(x)$ differ from each other, especially at small x, where maximal contribution comes from $R(Q)$ and $S(Q)$ from which the parton distribution functions cannot be factorized. As follows from calculations, $\delta[\bar Q(x)]$ is smaller than $\delta[Q(x)]$, consequently for $y \to 1$ $\delta^{\bar\nu} < \delta^{\nu}$, i.e., the EC do not simulate y-anomalies in $\bar\nu$-reactions.

In conclusion, we compare our results with those obtained in the four-fermion theory (ref.[8]). In the recent paper[9] one can find the graphs for δ calculated with the same parameters and parton distributions as ours (Fig.2). The comparison reveals that the agreement is only in general outline. Numerically, the EC differ in magnitude by an order of the EC itself (for $x < 0.3$). For $y \to 1$ and $x \to 0$ the behaviour of δ^{ν} is also different.

Unfortunately, we cannot compare the analytic form of final formulae since in papers[8,9] it has not been obtained and numerical calculations have been done using Monte-Carlo integration, while in this paper we have derived compact final expressions which contain one-dimensional integrals of the product of the parton spectrum by the calculated functions that essentially simplifies the programming for computer.

As was mentioned in the Introduction, the calculation of the EC to the inclusive neutrino reactions seems to be very important for estimating systematic uncertainties in information gained from these reactions on properties of strong interactions. We think the study we carried out is of great interest because of an essential difference of our results from the estimate available in literature.

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APPENDIX

FORMULAE FOR FINITE PART OF BREMSSTRAHLUNG CROSS SECTION

1. Table of Integrals

$$J[\kappa] = \frac{1}{\pi} \int \delta(p_2^2 + m_2^2) \frac{d^3k}{k_0} \cdot \kappa . \qquad (A.1)$$

$$J[\frac{1}{z^2}] = \frac{1}{m^2 V},\quad J[\frac{1}{u^2}] = \frac{1}{m_1^2 V},\quad J[\frac{1}{zu}] = \frac{1}{V} L_X = \frac{1}{V\sqrt{\lambda_X}} \ln \frac{X+\sqrt{\lambda_X}}{X-\sqrt{\lambda_X}} .$$

$$J[\frac{1}{u}] = L_u = \frac{1}{\sqrt{\lambda_Y}} \ln \frac{S_X + 2m_1^2 + \sqrt{\lambda_Y}}{S_X + 2m_1^2 - \sqrt{\lambda_Y}},\quad J[\frac{1}{z}] = L_A = \frac{1}{\sqrt{A}} \ln \frac{X+Y-m^2+\sqrt{A}}{X+Y-m^2-\sqrt{A}},$$

$$J[1] = \frac{V}{T},\quad A = (X+Y-m^2)^2 - 4m^2 \cdot T,\quad \lambda_Y = S_X^2 - 4m_1^2 \cdot Y ,$$

$$\lambda_X = X^2 - 4m^2 \cdot m_1^2,\quad T = S_X - Y + m_1^2,\quad V = \frac{\pi}{2} , \tag{A.2}$$

$$u = -2p_1 \cdot k,\quad z = -2k_2 \cdot k .$$

Integrals (A.2) are defined by exact formulae, the subsequent ones (A.3) are given in the approximation (6).

$$J[z] = \frac{V \cdot v(X+Y)}{2\tau^2},\quad m_1^2 J[\frac{z}{u^2}] = \frac{X}{S_X},\quad m_1^2 \cdot J[\frac{z^2}{u^2}] = \frac{V \cdot X^2}{S_X^2},$$

$$J[\frac{z}{u}] = \frac{V}{S_X^2}(X \cdot \tilde{\rho}_u - \frac{e}{\tau}),\quad \tilde{\rho}_u = \ln \frac{S_X^2}{m_1^2 \tau},\quad e = Xv - YS,$$

$$J[\frac{z^2}{u}] = \frac{V^2}{S_X^2} \cdot [X^2 \tilde{\rho}_u + \frac{1}{2} \frac{e^2}{\tau^2} + \frac{X(YS - 2e)}{\tau}],$$

$$J[u] = \frac{V^2 (S_X + 2m_1^2)}{2\tau^2},\quad m^2 J[\frac{u}{z^2}] = \frac{X}{P_X},\quad m^2 \cdot J[\frac{u^2}{z^2}] = \frac{X^2 \cdot V}{P_X^2},$$

$$J[\frac{u}{z}] = -\frac{eV}{P_X^2 r} + \frac{XV}{P_X^2}\cdot\tilde{\ell}_A,\ P_X = X+Y,\ \tilde{\ell}_A = \ln\frac{P_X^2}{m^2\cdot r},$$

$$J[\frac{u^2}{z}] = \frac{V^2}{r}\cdot[\frac{X(YS-e)}{P_X^3} - \frac{e\cdot S_X}{2rP_X^2}] + \frac{V^2\cdot X^2}{P_X^3}\tilde{\ell}_A. \qquad (A.3)$$

?. Formulae for Cross Section

Using the Table of Integrals one can calculate $d\sigma_R^F$ for processes

$$\nu q_1 \to \ell q_2\gamma,\quad \bar{\nu}\bar{q}_1 \to \bar{\ell}\bar{q}_2\gamma \qquad (A.4)$$

and

$$\bar{\nu}q_1 \to \bar{\ell}q_2\gamma,\quad \nu\bar{q}_1 \to \ell\bar{q}_2\gamma. \qquad (A.5)$$

We begin by splitting up the $d\sigma_R^F$ into parts according to the powers of charges of particles q_1 and q_2

$$d\sigma_R^F = d\sigma_{12} + d\sigma_1 + d\sigma_2 + d\sigma_0. \qquad (A.6)$$

With the use of the system SCHOONSCHIP [22] we obtain for processes (A.4)

$$\frac{d^2\sigma_{12}}{dXdY} = K\cdot\frac{1}{S^2}\cdot\frac{\alpha}{\pi}\{f_2^2\cdot\frac{SP_X}{4r} + f_1^2\cdot[\frac{Y-S}{4} + \frac{5S^2+2SY-Y^2}{4S_X} - \frac{YS(S+Y)}{2S_X^2} +$$

$$+\frac{3}{2}\cdot\frac{S^2Y^2}{S_X^3} + \frac{S^2Yv}{2S_X^3}\tilde{\ell}_u] + |f_1|\cdot|f_2|\frac{S}{S_X^2}\cdot(SY\tilde{\ell}_u + \frac{e-XY}{2} + \frac{SY^2}{r})\},$$

(A.7)

$$\frac{d^2\sigma_2}{dX\,dY} = K \cdot \frac{1}{S^2} \cdot \frac{\alpha}{\pi} \{ |f_2| \cdot (-\frac{SP_X}{\tau} - \frac{S^2}{P_X} \cdot \tilde{\rho}_A)\}, \qquad (A.8)$$

$$\frac{d^2\sigma_1}{dX\,dY} = K \cdot \frac{1}{S^2} \cdot \frac{\alpha}{\pi} \{ |f_1| \cdot [-\frac{SX(v-Y)}{S_X^2} + \frac{S+P_X}{2} \ln \frac{P_X^2 \cdot m_1^2}{X^2 \tau} + (\frac{S+P_X}{2} - \frac{S^2}{S_X} - \frac{YS^2}{S_X^2})\tilde{\rho}_u]\}, \qquad (A.9)$$

$$\frac{d^2\sigma_0}{dX\,dY} = K \cdot \frac{1}{S^2} \cdot \frac{\alpha}{\pi} \{ \frac{1}{2} \cdot \frac{Sv}{P_X} \cdot \tilde{\rho}_A \}. \qquad (A.10)$$

For processes (A.5) the contribution $d^2\sigma_{12}$ can be obtained from (A.7) by changing $S \leftrightarrow -X$, and the remaining contributions are

$$\frac{d^2\sigma_1}{dXdY} = K \cdot \frac{1}{S^2} \cdot \frac{\alpha}{\pi} \{ |f_1| \cdot [X^2 (1+\frac{S}{P_X}) \frac{1}{P_X} \tilde{\rho}_A - X^2 (1+\frac{Y}{S_X}) \frac{1}{S_X} \tilde{\rho}_u + \frac{Y-S}{2} - 2X - Xe(\frac{1}{S_X^2} + \frac{1}{P_X^2})]\}, \qquad (A.11)$$

$$\frac{d^2\sigma_2}{dXdY} = K \cdot \frac{1}{S^2} \cdot \frac{\alpha}{\pi} \cdot \{ |f_2| \cdot (\frac{X^2 S}{P_X^2} \tilde{\rho}_A - \frac{X^2}{\tau} - \frac{5X^2 S}{2P_X^2} + \frac{2X \cdot S - X^2}{2P_X} - \frac{X}{2})\}, \qquad (A.12)$$

$$\frac{d^2\sigma_0}{dXdY} = K \cdot \frac{1}{S^2} \cdot \frac{\alpha}{\pi} \{ \frac{SvX^2}{2P_X^3} \tilde{\rho}_A - \frac{3}{2} \cdot \frac{X^2 S^2}{P_X^3} + \frac{SX(S+X)}{2P_X^2} + \frac{S^2 x - 6X^2}{4P_X} - \frac{S_X}{4} \}. \qquad (A.13)$$

To calculate the contribution to process (2) from bremsstrahlung (5) on i-th parton scattering $(d^2 \Sigma_R^F)_i^{\nu,\bar{\nu}}$, we follow the usual scheme (see (31)) and obtain

$$(\frac{d^2\Sigma_R^F}{dx\,dy})_i^{\nu,\bar{\nu}} = \sigma_0 \cdot \frac{\alpha}{\pi} \cdot S^{\nu,\bar{\nu}} (f_i) \qquad (A.14)$$

with

$$S^{\nu,\bar{\nu}}(f_i) = \frac{y}{S_N} \int_x^1 \frac{d\xi}{\xi} \cdot f_i(\xi) \cdot E^{\nu,\bar{\nu}}(\xi),$$

where $E^{\nu,\bar{\nu}}(\xi)$ is a sum of terms which result from the terms in braces of (A.7) to (A.10) or (A.7) and (A.11) with the change (31).

REFERENCES

1. Mo L.W., Tsai Y.S. Rev.Mod.Phys., 1969,41,205. Tsai Y.S. SLAC-PUB-848, 1971.
2. Ахундов А.А., Бардин Д.Ю., Шумейко Н.М., ЯФ, 1977,26,с.1251.
3. Stein S. et al. Phys.Rev., 1975, D12, 1884. Atwood W.B. SLAC-PUB-185, 1975. Riordan E.M. et al. SLAC-PUB-1634, 1975.
4. Anderson H.L et al.Phys.Rev.Lett., 1977,38,1450; Phys.Rev.Lett., 1978,41,615; Gabathuler E. Rapporteur talk at XIX Intern.Conf.on High Energy Physics, Tokyo, August, 1978.
5. Barich B.C. Phys.Reports, 1973,30,278.
6. Kleinknecht.K.Proc.1977 Intern.Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 1977, p.271. Tittel K. Rapporteur talk at XIX Intern.Conf. on High Energy Physics, Tokyo, August, 1978.
7. Бардин Д.Ю., Шумейко Н.М., ОИЯИ, Р2-10872, Р2-10873, Дубна, 1977; ЯФ, 1978, вып.4.
8. Kiskis J. Phys.Rev., 1973, D8, 2129.
9. Barlow R., Wolfram. Preprint of University of Oxford 24 78, 1978.
10. Weinberg S. Phys.Rev.Lett., 1967, 19,1264; Rev.Mod.Phys., 1974, 46, 255. Salam A. In Elementary particle theory. Ed. Svartholm W. p.367, Stockholm, 1969.
11. Prescott C.Y. et al. Phys.Lett., 1978,77,347. Барков Л.М., Золоторев М.С. Письма в ЖЭТФ,1978,28,544. Weinberg S. Rapporteur talk at XIX Intern.Conf. on High Energy Physics, Tokyo, August, 1978.
12. Bartels J. Nucl.Phys., 1974, B82, 1972. Bodwin G.T., Stockham C.D. Phys.Rev.,1975,D11, 3324.

13. Бардин Д.Ю., Федоренко О.М. ОИЯИ, Р2-11461, Дубна,1978.
14. Бардин Д.Ю., Федоренко О.М. ОИЯИ, Р2-11414, Дубна, 1978.
15. Бардин Д.Ю., Федоренко О.М. ОИЯИ, Р2-11413, Дубна, 1978.
16. Particle Data Group. Phys.Lett., 1978, 75B.
17. Kinoshita T., Sirlin A. Phys.Rev., 1959, 113, 1652.
18. Bollini C.G., Giambiagi J.J., Silin A. Nuovo Cimento, 1973, 16A, 423.
Marciano W.J., Sirlin A. Phys.Rev., 1973,D8, 3612.
19. Salomonson P., Ueda Y. Phys.Rev., 1975,D11,2606.
20. Appelquist T.W., Pritmack J.R., Quinn H.R. Phys.Rev., 1973, D7, 2998.
21. Bardin D.Yu., Shumeiko N.M. Nucl.Phys., 1977, B127, 242.
22. Strubbe H.S. Comp.Phys.Com., 1974, 8.,1.
23. Barger V., Phillips R.J.N. Nucl.Phys., 1974, B73, 268.
24. Bosetti P.S. et al. Phys.Lett., 1978, 76B,505.
Faissner H. Neutrino 77, M., 1978, p.142,p.164.
Baltay C. Rapporteur talk at XIX Intern.Conf. on High Energy Physics, Tokyo, August, 1978.
25. Okada J., Pakvasa S., Tuan S.F. Lett.Nuovo Cim., 1976, 16, 555.
26. Drees J. Preprint of University of Wuppertal, WU-B78-16, 1978.

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