ОБЪЕДИНЕННЫЙ
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ДУБНА



E2 - 12071



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*Submitted to ЯФ*

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Распад $K_{\ell 4}$ в квантовой киральной теории

В рамках квантовой киральной теории поля описаны формфакторы распада $K_{\ell 4}$. Аксиальные формфакторы вычислены в древесном приближении, определяющем основные вклады в эти величины. Векторный формфактор вычислен в однопетлевом приближении. Найденные значения формфакторов находятся в удовлетворительном согласии с известными экспериментальными данными.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

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$K_{\ell 4}$ Decay in the Chiral Quantum Field Theory

Form factors of $K_{\ell 4}$ -decay are described in the framework of chiral quantum field theory. The axial form factors are calculated in the tree approximation which defines their main contribution. The vector form factor is calculated in the one-loop approximation. The results are in satisfactory agreement with the available experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1978

## 1. Introduction

There exists already a large amount of experimental and theoretical works on the $K_{e_4}$ decay [1-8]. This process is interesting by itself and due to the fact that one may draw from it informationy on many other important physical quantities (e.g., the scattering lengths of the $\pi$-$\pi$ system). This is also the reason why it has been discussed so often in the literature. Among the four form factors appearing in the amplitude of the $K_{e_4}$ decay the three axial form factors may be treated relatively simply. Information on the behaviour of these form factors may be obtained, for instance, from the standard current algebra approach [4,7,8] or the simplest tree approximation of the chiral theory [9].

On the other hand, the description of the vector form factor is considerably more difficult. The standard soft pion techniques of the current algebra are now useless because the vector part of the amplitude vanishes for vanishing pion momenta*). Also the tree approximation of the chiral theory does not provide any information on the vector form factor. However, the one-loop approximation now begins to play a dominant role. We recall that a similar situation took place for decays of the type $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$, $\eta \to \pi^+\pi^-\gamma$, $K_L \to \gamma\gamma$, and $K_L \to \pi^+\pi^-\gamma$ [10-11]. As in the latter cases the calculation of the vector form factor of the $K_{e_4}$ decay requires the consideration of the so-called anomalous diagrams [7,12-13]. These

*) This argumentation does not refer to special models with vector meson dominance, etc.

diagrams contain the main information on the processes which they refer to. They are thus of principal importance even if there were no other arguments in favour of a reasonable perturbation theory within chiral quantum field theory.

Therefore, we hope to obtain reasonable results for the vector form factor of the $K_{e_4}$ decay by using the one-loop approximation.

Other more rough theoretical estimates of this quantity have been performed within models with vector meson dominance [5-6]. The respective results are in complete agreement with the estimates of the chiral quantum theory.

The calculations within the latter model are interesting, because they help to complete, from a unique point of view (SU(3)x xSU(3) chiral invariant quantum theory), the description of all leptonic, semileptonic, and radiative decays of the fundamental meson octet[14].

In the following section we shall quote the different parts of the chiral Lagrangians which are needed for the calculation of the $K_{e_4}$ decay. In Sec. 3 the axial form factors are calculated in the tree approximation, whereas Sec. 4 contains the one-loop calculation of the vector form factor. In Sec. 5 we calculate the probability of the $K_{e_4}$ decay and compare our results with the experimental data.

## 2. Chiral Lagrangians

A detailed discussion of SU(3)xSU(3) chiral invariant Lagrangians can be found in refs.[9,10,15,16]. In the following we shall quote only those parts of these Lagrangians which are needed for describing the $K_{e_4}$ decay in the tree approximation (axial form factor) and one-loop approximation (vector form factor).



The Lagrangians which characterize the strong interactions of the mesons and baryons have the form

$$\mathcal{L}_1 = i2g_A\frac{M}{F}\left[\alpha d_{ijk} - i(1-\alpha)f_{ijk}\right] : \overline{\Psi}_i \gamma_5 \Psi_j \Phi_k : , \qquad (1)$$

$$\mathcal{L}_2 = \frac{i}{2F^2} : \overline{\Psi}_i \gamma_\mu \Psi_e \Phi_j \partial^\mu \Phi_k \Big\{ (g_A^2-1) f_{iem} f_{kjm} + g_A^2\Big[\frac{2}{3}\alpha^2(\delta_{ij}\delta_{ke} - \delta_{ik}\delta_{je}) + 2\alpha(\alpha-1) f_{kjm}\left(f_{iem} - i d_{iem}\right)\Big]\Big\}, \qquad (2)$$

where $\Psi_i$ and $\Phi_i$ are the fields of the baryon and meson octets. $F$ is the pion decay constant ($F \approx 95$ MeV), $g_A$ is the renormalization constant of the axial current ($g_A \approx 1.25$), $M$ is an averaged mass of the baryon octet and $\alpha$ is the mixing parameter of the f-d couplings ($\alpha \approx 2/3$). The total Lagrangian contains yet a term quadratic in the meson fields without derivative coupling. However, this term does not contribute to the vector form factor and has therefore been omitted. This is analogous to the case of the $\eta \to \pi^+\pi^-\gamma$ decay.

The interactions of the pseudoscalar mesons are described by the chiral Lagrangian $\mathcal{L}_3$

$$\mathcal{L}_3 = \frac{F^2}{4} Sp\left\{\partial_\mu e^{i\xi} \partial_\mu e^{-i\xi}\right\}, \qquad (3)$$

where $\xi = \frac{1}{F}\sum_{i=1}^{8} \lambda_i \Phi_i$ ($\lambda_i$ – Gell-Mann matrices). The mesons get a finite mass by adding a symmetry breaking term to eq.(3) which, in the scheme of Gell-Mann, Oakes and Renner [27], takes the form

$$\mathcal{L}_4 = \frac{1}{2} m_{K^0}^2 F^2\left[\sin^2\theta_c \left(e^{i\xi}\right)_{22} + \cos^2\theta_c \left(e^{i\xi}\right)_{33}\right] + h.c. \qquad (4)$$

$m_{K^0}$ is the mass of the $K^0$ meson and $\theta_c$ is the Cabibbo angle.

Finally, the weak interactions of the hadrons and leptons are described by the product of two charged currents

$$\mathcal{L}_S = \frac{G}{\sqrt{2}}\left[J^{+}_{\mu,e} J^{\mu}_{h} + h.c.\right], \tag{5}$$

where $G = 10^{-5}/M^2$ is the Fermi constant. The lepton current reads

$$J_e^{\mu}(x) = :\bar{\nu}_{(\mu)}(1-\gamma_5)\gamma^{\mu}\mu: + :\bar{\nu}_{(e)}(1-\gamma_5)\gamma^{\mu}e:, \tag{6}$$

where $\mu, e, \nu_{(\mu,e)}$ are the fields of the muon, electron and neutrinos, respectively. The hadronic current is given in the usual Cabibbo form

$$J_h^{\mu} = \cos\theta_c (V-A)^{\mu}_{1+i2} + \sin\theta_c (V-A)^{\mu}_{4+i5}, \tag{7}$$

where $V^{\mu}_{k+ie} = V^{\mu}_{k} + iV^{\mu}_{e}$ , $A^{\mu}_{k+ie} = A^{\mu}_{k} + iA^{\mu}_{e}$ are the vector or axial vector currents, respectively. The hadronic current contains baryonic and mesonic parts, $J_h^{\mu} = J_B^{\mu} + J_m^{\mu}$ . The baryonic currents read

$$(V_B)^{\mu}_{i} = -i :\bar{\Psi}_k \gamma^{\mu} f_{ike} \Psi_e:, \tag{8}$$

$$(A_B)^{\mu}_{i} = -g_A :\bar{\Psi}_k \gamma^5 \gamma^{\mu}\left[\alpha d_{ike} - i(1-\alpha) f_{ike}\right]\Psi_e, \tag{9}$$

whereas the mesonic currents are defined by the formula /9,14/

$$\sum_{i=1}^{8} \lambda_i (J_m)^{\mu}_{i} = \sum_{i=1}^{8} \lambda_i (V_m - A_m)^{\mu}_{i} = -iF^2 e^{i\xi}\partial^{\mu} e^{-i\xi}. \tag{10}$$

Using the Lagrangians (1-5) we are now able to calculate the form factors of the $K_{e_4}$-decay.

## 3. Axial form factors of the $K_{e_4}$-decay (tree approximation)

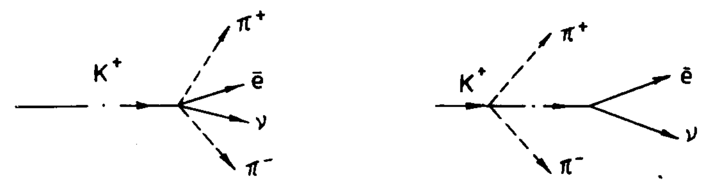
Let us consider the process $K^+ \to \pi^+ + \pi^- + \bar{e} + \nu$ . The corresponding decay amplitude is usually written in the form



$$T_{K_{e_4}} = \frac{1}{m_K}\Big\{ f(p^+ + p^-)^{\mu} + g(p^+ - p^-)^{\mu} + r(p_K - p^- - p^+)^{\mu} + i\frac{h}{m_K^2}\varepsilon^{\mu\nu\rho\sigma}(p_K)^{\nu}(p^+ + p^-)^{\rho}(p^+ - p^-)^{\sigma}\Big\}\, e^{(-)}_{\mu}, \tag{11}$$

where $p^+, p^-$ and $p_K$ are the momenta of the $\pi^+, \pi^-$ and $K^+$ mesons, respectively, and $e^{(-)}_{\mu} = \sin\theta_c \frac{G}{\sqrt{2}} \bar{u}_{\nu}(1-\gamma^5)\gamma^{\mu} u_e$. $f$ , $g$ and $r$ are the axial form factors, $h$ is the vector form factor. The calculation of the latter is the main purpose of this paper.

As the axial form factors get their main contributions from the tree approximation we shall consider here tree graphs only. The relevant diagrams are shown in fig. 1.




a b

Fig. 1

Diagram 1a can be calculated by taking into account the following part of the Lagrangian $\mathcal{L}_S$

$$\mathcal{L}_S^{(1)} = \frac{\sqrt{2}}{3F}\left[\pi^-(K^+\partial_{\mu}\pi^+ + \pi^+\partial_{\mu}K^+) - 2K^+\pi^+\partial_{\mu}\pi^-\right]L^{(-)}_{\mu}, \tag{12}$$

where

$$L^{(-)}_{\mu} = \sin\theta_c \frac{G}{\sqrt{2}} \bar{\nu}(1-\gamma^5)\gamma^{\mu} e .$$

This diagram determines nearly the whole probability of the decay and the magnitude of the form factors $f$ and $g$ . It gives rise to the amplitude

$$T^{(1)} = \frac{\sqrt{2}}{3F}\left(3p^+ + q\right)^{\mu} e^{(-)}_{\mu}, \tag{13}$$

where $q = p_K - p^+ - p^-$ . In order to get also a reasonable expression for the third form factor $\tau$ one has further to investigate the important diagram 1b. It can be calculated by using the Lagrangians

$$\mathcal{L}_5^{(2)} = -\sqrt{2}\, F\, \partial_\mu K^+ L_\mu^{(-)} , \tag{14}$$

$$\begin{aligned}\bar{\mathcal{L}}_3 + \bar{\mathcal{L}}_4 = &-\frac{1}{6F^2}\Big\{\pi^+\pi^-\partial_\mu \bar{K}^+ \partial_\mu K^+ + \partial_\mu \pi^+ \partial_\mu \pi^- \bar{K}^+ K^+ + \\ &+ (\pi^+ \overleftrightarrow{\partial}_\mu \pi^-)\times(\bar{K}^+ \overleftrightarrow{\partial}_\mu K^+) - \pi^+\partial_\mu \pi^- K^+ \partial_\mu \bar{K}^+ - \pi^- \partial_\mu \pi^+ \bar{K}^+ \partial_\mu K^+ - \\ &- (m_K^2 + m_\pi^2)\,\pi^+\pi^-\bar{K}^+K^+\Big\} ,\end{aligned} \tag{15}$$

which are parts of the Lagrangians $\mathcal{L}_5$ and $\mathcal{L}_3$ , $\mathcal{L}_4$ . This diagram contributes the following expression to the amplitude

$$T^{(2)} = \frac{\sqrt{2}}{6F}\left(1 + 6\,\frac{p_K p^-}{q^2 - m_K^2}\right) q^\mu \ell_\mu^{(-)} . \tag{16}$$

By adding the contributions $T^{(1)}$, $T^{(2)}$ we obtain the following expressions for the axial form factors

$$f = g = \frac{m_K}{\sqrt{2}F} ; \quad \tau = \sqrt{2}\,\frac{m_K}{F}\,\frac{q p^+}{m_K^2 - q^2} = \begin{cases} 0 , & p^+ = 0 \\ \frac{m_K}{\sqrt{2}F} , & p^- = 0 \end{cases} . \tag{17}$$

The results (17) are in complete agreement with earlier calculations using current algebra (see ref. /7,8/).

4. Vector form factor of the $K_{e4}$ decay (one-loop approximation).

The calculation of the vector form factor $h$ may be done in complete analogy with the calculation of the decay amplitudes for the processes $\eta \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-\gamma$ /10,11/. These amplitudes are contributed by the box diagrams with strong



vertices given by $\mathcal{L}_1$ and triangle diagrams with one vertex of the type $\mathcal{L}_1$ and one vertex of the type $\mathcal{L}_2$ (comp. fig. 2a,b,c,d).

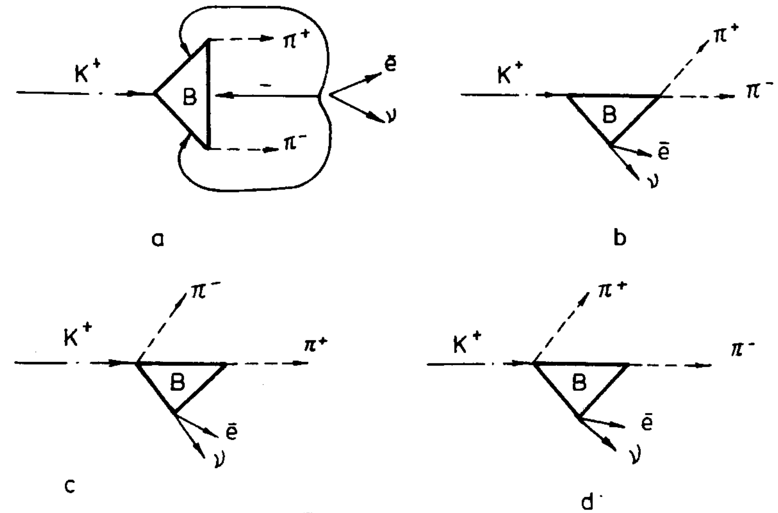



Fig. 2

Fig. 2a represents schematically three types of box diagrams where the lepton pairs are emitted from a baryon propagator between the $K^+$ and $\pi^+$ mesons, the $K^+$ and $\pi^-$ mesons, and two neighbouring pions. The Lagrangians associated to the vertices of this diagram are given by

$$\begin{aligned}\mathcal{L}_1^{(1)} = i\sqrt{2}\, g_A \frac{M}{F}\pi^+\Big[\bar{p}\cdot n + (1-2\alpha)\,\bar{\Xi}^\circ\cdot\Xi^- + \sqrt{\tfrac{2}{3}}\,\alpha\,(\bar{\Lambda}\cdot\Sigma^- + \bar{\Sigma}^+\cdot\Lambda) + \\ + \sqrt{2}\,(1-\alpha)\,(\bar{\Sigma}^\circ\cdot\Sigma^- - \bar{\Sigma}^+\cdot\Sigma^\circ)\Big] + h.c. ,\end{aligned} \tag{18}$$

$$\begin{aligned}\mathcal{L}_1^{(2)} = i\sqrt{2}\, g_A \frac{M}{F} K^+\Big[(2\alpha-1)\,\bar{n}\cdot\Sigma^- + \frac{2\alpha-1}{\sqrt{2}}\,\bar{p}\cdot\Sigma^\circ - \frac{3-2\alpha}{\sqrt{6}}\,\bar{p}\cdot\Lambda + \bar{\Sigma}^+\cdot\Xi^\circ - \\ - \frac{\bar{\Sigma}^\circ\cdot\Xi^-}{\sqrt{2}} + \frac{4\alpha-3}{\sqrt{6}}\,\bar{\Lambda}\cdot\Xi^-\Big] ,\end{aligned} \tag{19}$$

$$\mathcal{L}_5^{(3)} = \left[\bar{\Xi}^0 \times \Sigma^+ - \bar{\Sigma}^- \times n - \frac{1}{\sqrt{2}}\left(\bar{\Sigma}^0 \times p + \bar{\Xi}^- \times \Sigma^0\right) - \sqrt{\tfrac{3}{2}}\left(\bar{\Lambda} \times p + \bar{\Xi}^- \times \Lambda\right)\right] \cdot L_\mu^{(-)} \qquad (20)$$

We have used the notation: $\cdot \equiv \gamma_5 = -\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$ and $\times \equiv \gamma^\mu$ .

Retaining in the integrals only those terms which do not depend on momenta we get from fig. 2a the following contribution to the amplitude

$$T^{\sqcap} = i \frac{g_A C^{\sqcap}}{\sqrt{2}\, 8\pi^2 F^3} \varepsilon^{\mu\nu\rho\sigma} (p_K)^\nu (p^-)^\rho (p^+)^\sigma \ell_\mu^{(-)} , \qquad (21)$$

where $C^{\sqcap} = -24 g_A^2 \alpha \left(1 - 2\alpha + \frac{4}{3}\alpha^2\right) \approx -6.5$ .

The diagrams 2b-d contain vertices of the type $\mathcal{L}_2$ . The corresponding Lagrangians read

$$\mathcal{L}_2^{(1)} = i \frac{\pi^- \overleftrightarrow{\partial}_\mu \pi^+}{(2F)^2} \Big\{ (g_A^2 - 1)(\bar{p} \times p - \bar{n} \times n) + \left[1 - g_A^2 \left(1 - 4\alpha(1-\alpha)\right)\right]\left(\bar{\Xi}^- \times \Xi^- - \right.$$
$$\left. - \bar{\Xi}^0 \times \Xi^0\right) + 2\left[g_A^2\left(1 - 2\alpha + \tfrac{4}{3}\alpha^2\right) - 1\right]\left(\bar{\Sigma}^+ \times \Sigma^+ - \bar{\Sigma}^- \times \Sigma^-\right) +$$
$$+ \frac{4}{\sqrt{3}} \alpha (1-\alpha) g_A^2 \left(\bar{\Lambda} \times \Sigma^0 + \bar{\Sigma}^0 \times \Lambda\right) \Big\} , \qquad (22)$$

$$\mathcal{L}_2^{(2)} = i \frac{\pi^- \overleftrightarrow{\partial}_\mu K^+}{\sqrt{2}\,(2F)^2} \Big\{ \sqrt{2}\left[g_A^2 \left(1 + 4\alpha\left(\tfrac{2}{3}\alpha - 1\right)\right) - 1\right] \bar{p} \times \Sigma^+ +$$
$$+ \sqrt{2}\left[g_A^2\left(1 - \tfrac{4}{3}\alpha^2\right) - 1\right] \bar{\Sigma}^- \times \Xi^- - \left[g_A^2\left(1 + 4\alpha(\alpha - 1)\right) - 1\right] \bar{n} \times \Sigma^0 + (g_A^2 - 1)\bar{\Sigma}^0 \times \Xi^0 -$$
$$- \sqrt{3}\left[g_A^2\left(1 + \tfrac{8}{3}\alpha(\alpha - 1)\right) - 1\right] \bar{\Lambda} \times \Xi^0 + \sqrt{3}\left[g_A^2\left(1 + \tfrac{4}{3}\alpha(\alpha - 1)\right) - 1\right] \bar{n} \times \Lambda \Big\} +$$
$$+ i \frac{g_A^2 \alpha^2}{3F^2} K^+ \overleftrightarrow{\partial}_\mu \pi^+ \left(\bar{\Sigma}^+ \times \Xi^- + \bar{p} \times \Sigma^-\right) . \qquad (23)$$

We again retain only constant terms in the integrals for the baryon triangle diagrams. As a result, we obtain the following expression for the coefficients $C$



$$C_b^{\Delta} = C_c^{\Delta} = 6\alpha\left[-1 + g_A^2\left(3 - 6\alpha + \tfrac{32}{9}\alpha^2\right)\right] ;$$
$$C_d^{\Delta} = \frac{16}{3} g_A^2 \alpha^3 ; \qquad C_{b+c+d}^{\Delta} = 1.7 . \qquad (24)$$

Summing up all baryon loop diagrams the total contribution to the vector part of the $K_{e_4}$ decay amplitude reads

$$T^V = \cancel{T^V \ell_\mu^{(-)}} - i \frac{g_A \bar{C}}{\sqrt{2}\, 8\pi^2 F^3} \varepsilon^{\mu\nu\rho\sigma} (p_K)^\nu (p^+ + p^-)^\rho (p^+ - p^-)^\sigma \ell_\mu^{(-)} , \qquad (25)$$

where

$$\bar{C} = 6\alpha - g_A^2\left[1 + (2\alpha - 1)^3\right] \approx 2.4 \text{ *)} . \qquad (26)$$

Eq. (25) provides the following expression for the vector form factor $h$ (comp. eq.(11))

$$h = - \frac{g_A \bar{C}}{\sqrt{2}\,\pi^2}\left(\frac{m_K}{2F}\right)^3 . \qquad (27)$$

5. <u>Discussion of the results</u>

In order to compare the above results with the experimental data (see ref.[/1-3/]) we shall give first of all a rough estimate of the probability of the $K_{e_4}$ decay. To this end we consider only those parts of the amplitude (11) which contain the form factors $f$ and $g$ .

This yields the formula

$$W_{K_{e_4}} \simeq 10^{-10} \frac{1}{6\pi m_K}\left[\frac{\sin\Theta_c \; m_\pi^5}{(8\pi)^2 m_K M_\rho^2 F}\right]^2 J , \qquad (28)$$

---

*) Note that this value of $\bar{C}$ is equal to the value of an analogous coefficient arising in the description of the decay $\eta \to \pi^+\pi^-\gamma$ [/10/]. The amplitudes $T^V_{K_{e_4}}$ and $T_{\eta\to\pi^+\pi^-\gamma}$ are connected by $T^\mu_{K_{e_4}} = \sqrt{6}\, \ell_\mu\, T^\mu_{\eta\to\pi^+\pi^-\gamma}$ , following from the Lagrangian $\mathcal{L}_1$ , when $K$ field is replaced by $\eta$ .

where $\mathcal{J}$ is a phase space integral given by

$$\mathcal{J} = \int_1^{(a-1)^2} dx \sqrt{(a^2-1)^2 + x[x-2(a^2+1)]}\,\left[x^3 - 8(x^2-1) - \frac{1}{x} + 12x \ln x\right] \approx 355,$$

$$(a = m_K/m_\pi).$$

To get eq. (28) we have used the values for $f$ and $g$ obtained in the tree approximation (eq.(17)). With the value $\theta_c \simeq 0.26$, eq.(28) yields *)

$$W^{th}_{Ke_4} \simeq 1.7 \cdot 10^3\ s^{-1} . \tag{29}$$

The experimental value is (see ref./1/)

$$W^{exp}_{Ke_4} = (3.26 \pm 0.15) \cdot 10^3\ s^{-1} . \tag{30}$$

Keeping in mind that the tree approximation is rather crude the theoretical value (29) may be considered to be quite satisfactory. The numerical estimates for $f, g$ and $h$ give

$$f = g \simeq 4 \quad , \quad h = -3.7 . \tag{31}$$

We recall that the main task of this paper was to estimate the vector form factor $h$ of the $K_{e_4}$ decay. In this case, as already mentioned, the one-loop approximation is of principal importance. On the other hand, the form factors $f$ and $g$ get already a large contribution from the tree approximation. The one-loop approximation should then only slightly modify this estimate (formulae (17), (31)). A similar situation has been met for the case of the $\pi$-$\pi$ $S$-wave scattering length /9,18/. There the esti-

*) We use always the approximate relation $F_\pi \approx F_K = F_\eta \approx F = 95$ MeV. In this case, with $\theta_c \approx 0.26$, the theoretical estimates for the decay rates of the decays $\pi^\pm \to \mu^\pm \nu$ and $K^\pm \to \mu^\pm \nu$ are in good agreement with experiment (see ref. /14/).



mates for $a_0^0$ in the tree and one-loop approximation are as follows *)

$$a_0^0\Big|_{tree} \approx 0.15\, m_\pi^{-1}$$
$$a_0^0\Big|_{tree+1\text{-}loop} \approx 0.18\, m_\pi^{-1} . \tag{32}$$

In the case of the axial form factor the number of the baryon loops considerably exceeds the number of the baryon loops for the vector form factor, and many of the loop diagrams even diverge. Using special regularization methods which are characteristic for nonpolynomial theories (e.g.,superpropagator methods/19/) we may calculate also these diagrams. They will give a more complete information on the form factors $f$, $g$ and $\tau$ and determine, in particular, the slope parameter $\lambda$ of the form factor $f$. (The experimental value of $\lambda$ can be found in ref./1/).

In the present case our calculations of $f, g$ and $\tau$ **) have, however, been performed within the tree approximation. In order to compare with the experimental data we consider the ratio $h/f$, quoted in many experimental works. Using for $f$ the recent results of ref./1/ and for $h$ our theoretical value, we get

$$h^T = -3.7\ ;\ f_{exp}^{[1]} \approx 6.15 \pm 0.15\ ;\ \frac{h^T}{f_{exp}} \approx -0.6 \tag{33}$$

*) In ref./18/ the pion mass terms have been introduced by a scheme proposed by Gürsey. In this case one obtained $a_0^0\big|_{tree} \approx 0.12\, m_\pi^{-1}$. The above value (32) refers, on the other hand, to the mass term (4).

**) Preliminary calculations taking into account convergent box diagrams only (in general the main contributions) yield an increase of $f$ and $g$ of about 30%.

The experimental values are

1) ref. /1/: $h = -2.95 \pm 0.75$; $h/f = -0.48 \pm 0.12$; $a_0^0 = (0.28 \pm 0.05)\, m_\pi^{-1}$

2) ref. /2/: $h/f = -0.71 \pm 0.23$; $a_0^0 = (0.17 \pm 0.13)\, m_\pi^{-1}$

3) ref. /3/: $h/f = -0.97 \pm 0.46$ .

For completeness, let us also quote the theoretical estimates for $h$ obtained in a model with $\rho$ -dominance /5,6/

$$1.5 \lesssim |h| \lesssim 5 .$$

Thus, the above calculated value of $h$ is in good agreement with both the available experimental data and other theoretical estimates.

## References


1. L.Rosselet et al. Phys.Rev. D15, (1977) 574.
2. E.W.Beier et al. Phys.Rev.Lett. 30 (1973) 399.
3. P.Basile et al. Phys.Rev. 36B (1971) 619.
4. L.M.Chounet, J.M.Gaillard and M.K.Gaillard. Phys.Rev. 4C (1972) 199.
5. F.A.Berends, A.Donnachie and G.C.Oades. Phys.Rev. 171 (1968) 1457.
6. A.K.Mohant, R.E.Marshak, Nuovo Cim. 52A (1967) 915.
7. S.B.Treiman, R.Jackiw, D.J.Gross "Lectures on current algebra and its applications", Princeton University Press, Princeton, New Jersey, 1972.





8. V. De Alfaro, S.Fubini, G.Furlan, C.Rossetti. "Currents in hadron physics"North-Holland Publishing Company, Amsterdam - London, 1973.
9. M.K.Volkov, V.N.Pervushin. "Essentially nonlinear quantum theories, dynamic symmetries and physics of mesons". Moscow, Atomizdat, 1978.
10. M.K.Volkov. Yader. Fiz., 27 (1978) 758.
11. M.K.Volkov. Yader. Fiz., 28 (1978) 162.
12. S.Adler, W.Bardeen. Phys.Rev. 182 (1969) 1517.
13. M.V.Terentjev. UFN, 112 (1974) 37.
14. M.K.Volkov. Particles and Nuclei (to be published).
15. S.Cheman, C.Wess, B.Zumino, Phys.Rev. 177 (1969) 2239; G.C.Callan et al. ibid. 177 (1969) 2447.
16. M.K.Volkov, V.N.Pervushin. UFN, 120 (1976) 363.
17. R.J.Oakes. Phys.Lett. B29 (1969) 683; M.Gell-Mann, R.I.Oakes and B.Renner. Phys.Rev. 175 (1968) 2195.
18. M.K.Volkov, V.N.Pervushin. Yader.Fiz. 20 (1974) 762.
19. M.K.Volkov. Theor.Math.Phys. 6 (1971) 21.


Received by Publishing Department
on December 7 1978.