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Спектроскопия мезонов, смешивание кварков и квантовая хромодинамика

Изложена полуфеноменологическая теория спектра масс мезонов, состоящих из кварка и антикварка. Учитываются релятивистские кинематические эффекты неравных масс кварков, нарушение SU_3 -симметрии в наклонах траекторий Редже и в радиально возбужденных состояниях. Нарушение правила ОЦИ учтено с помощью матрицы смешивания кварковых волновых функций, вид которой подсказывается квантовой хромодинамикой. Для описания зависимости параметров смешивания от масс мезонов предложена простая экстраполяция выражений, даваемых квантовой хромодинамикой, из области "асимптотической свободы" в область "инфракрасного рабства". Для вычисления масс и углов смешивания псевдоскалярных мезонов предложено условие минимальности массы π -мезона. При этом масса η -мезона оказывается близкой к максимальной. Предсказания теории для масс и углов смешивания мезонов хорошо согласуются с экспериментом.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Meson Spectroscopy, Mixing of Quark Configurations and QCD

A semi-phenomenological theory of the quark-antiquark meson mass spectrum is presented in which relativistic kinematic effects of unequal quark masses as well as SU_3-breaking in Regge trajectories and in radial excitations are properly taken into account. OZI breaking effects, suggested by s-channel gluon exchange or by t-channel meson exchange, are introduced by means of an SU_3 -symmetric mixing matrix for the quark wave functions. A simple generalization and extrapolation of the QCD expressions for mixing parameters from the domain of "asymptotic freedom" into the domain of "infrared slavery" is proposed to describe a dependence of the mixing parameters on meson masses. A condition of a minimum of the pion mass is used for calculating the pseudoscalar masses and mixing angles, which prove to be somewhat different for η and η' : $\theta_p(\eta) \cong -17.5$, $\theta_p(\eta') \cong -20.5$. The η meson mass is observed to be maximum possible. The prediction for meson masses and mixing angles are in good agreement with experiment.

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1. Introduction

The discovery of the charmed particles requires to make some more precise concepts of the "old" particle physics, such as SU_3 breaking, chiral symmetry breaking, the quark line rule (or the OZI rule), and mixing of quark configurations in isoscalar states $\eta-\eta'$, $\omega-\varphi$, $f-f'$,etc. (which is the phenomenological manifestation of the OZI rule breaking). The necessity of a revision of these and some other phenomena is dictated by the fact that in "new" particle physics they define principal effects rather than give small corrections. For example, SU_4 symmetry breaking is much larger than SU_3 breaking, and the slope of the J/ψ Regge trajectory is two or three times smaller than the average slope of the "old" particle trajectories. The OZI rule results in extremely small widths of the J/ψ and ψ' and a very good theory of OZI breaking is necessary to understand the decays of these particles. Our ideas of "radial" excitations (such as $\rho'(1.6)$, $\psi'(3.7)$), and of "exotic" many-quark systems (which probably spoil the generally simple picture of the charmonium levels) should also be clarified and made more quantitative.

A consistent approach to spectroscopy of new and old particles should be based on QCD (see,e.g.,/1/-/6/). However, to construct a complete theory, we should first understand the structure of QCD at large distances, where the coupling is large, and to solve the notorious quark confinement problem. In first attempts to employ QCD in constructing hadron spectroscopy the quark confinement was used as a fundamental hypothesis. The simplest idea is to write down some equation for quark bound-states with a binding potential, which is $\sim 1/r$ for $r \to 0$ and is indefinitely rising for $\tau \to \infty$ (see, e.g. /3/, /5/). A more consistent approach was realized in the MIT and SLAC "bag" models and in some other similar models /4/. The spectrum of low-lying hadrons in these models is approximately the same as in earlier approaches based on some sort of confinement (e.g., "Dubna bag", oscillator potential, etc., for a comparative review see /7/)[1].

A serious attempt to develop the hadron spectroscopy starting from the QCD Lagrangian with built-in confinement hypothesis has recently been enterprised in refs. /10/, /11/. While the results are very promising, much more remains to be done. In fact, the problems mentioned above were not considered up to now (see, however, a very interesting descussion of the η-η' problem in ref. /11/).

The aim of the present paper is to give a general and simple enough treatment of the meson spectrum, based on a relativistic dynamic quark model, but using minimum detailed information on interactions of the quarks. The following discussion will in general be restricted to considering the old mesons, consisting of the quark u, d, s and of the antiquarks $\bar{u}, \bar{d}, \bar{s}$. Charmed, exotic and radially excited states will be treated in subsequent publications. In the next paper a phenomenological consideration of the radiative decays of the vector and pseudoscalar mesons, using the results of the present paper, will be given.

Our approach to meson spectroscopy is frankly phenomenological and as close as possible to standard treatments (see, e.g. /12/, /13/)[2] though, when necessary, we shall use some basic concepts of QCD, dual models, etc. Our principal aim is to obtain a phenomenology of all quark-antiquark meson states which is free of shortcomings of the standard phenomenological models. This phenomenology should not be a substitute for potential models, bag theories, or QCD, it is rather aimed at elucidating which of the results are independent of detailed dynamical considerations, and at finding simple empirical relations to be explained by a future consistent theory.

Some of the ideas employed in what follows were first formulated in refs. /17-21/, /7/, they are briefly summarized in

[1] Some new ideas in bag theories can be found in refs. /4/, /8/, /9/.

[2] An extensive review may be found in refs. /7/, /14/, /16/.

the next section. The main new result of this paper is the good description of the quark configuration mixing in the pseudoscalar nonet. In the standard treatments the η-η' mixing angle is calculated by using all the pseudoscalar masses as an imput. Here we not only calculate the angle but also successfully predict one or two of the pseudoscalar masses, thus verifying our ideas of mixing mechanisms.

Let us introduce the notation used below

$$|Q_0\rangle = \frac{1}{\sqrt{3}}\{|u\bar{u}\rangle_Q + |d\bar{d}\rangle_Q + |s\bar{s}\rangle_Q\}, \qquad (1.1)$$
$$|Q_8\rangle = \frac{1}{\sqrt{6}}\{|u\bar{u}\rangle_Q + |d\bar{d}\rangle_Q - 2|s\bar{s}\rangle_Q\}.$$

Here Q is the set of the quantum numbers (J, L, S, N) defining the state of the quarks. For the pseudoscalar (P), vector (V), tensor (T), axial (A), and scalar (S) multiplets, resp. $Q = P, V, T, A, S$. The states $|M'_Q\rangle$ and $|M_Q\rangle$ describing the heavy (M'_Q) and the light (M_Q) isoscalar mesons of the multiplet Q are some superpositions of $|Q_0\rangle$ and $|Q_8\rangle$. One usually defines the pseudoscalar states as

$$|\eta'\rangle = |P_0\rangle \cos\Theta_P(\eta') + |P_8\rangle \sin\Theta_P(\eta'), \qquad (1.2)$$
$$|\eta\rangle = -|P_0\rangle \sin\Theta_P(\eta) + |P_8\rangle \cos\Theta_P(\eta).$$

Our result for the angles $\Theta_P(\eta')$ and $\Theta_P(\eta)$ is the following

$$\Theta_P(\eta) \cong -17.2^\circ, \quad \Theta_P(\eta') \cong -20.6^\circ,$$

i.e., the states $|\eta'\rangle$ and $|\eta\rangle$ are slightly non-orthogonal. To obtain this result, we have used the value of

$$\Delta^2 = \Delta^2_{su} = s^2 - u^2 \qquad (1.3)$$

derived from the vector meson masses, and the values of masses m_π, η (the masses of the mesons and quarks are denoted by their respective symbols, with the exception of the pion mass). The masses of K and η' are predicted to be $K \cong .49$, $\eta' \cong .96$ (all masses in 1 GeV units)

In the standard mixing models[3] $\Theta_P^{(Q)} = \Theta_P(\eta) = \Theta_P(\eta') \approx -10^\circ$ for quadratic (Q) mass formulae, and $\Theta_P^{(L)} \cong -24^\circ$ for linear (L) mass formulae. Remark that our angle $\Theta_P(\eta)$ is intermediate between these extremes:

$$\Theta_P(\eta) = \frac{1}{2}(\Theta_P^{(L)} + \Theta_P^{(Q)}).$$

[3] For a fresh review of different mixing models and of their comparison with experiment see /22/, for further references see also /7/.

In the quark model another definition of mixing angles is obviously more natural

$$|M'_Q\rangle = |Q_s\rangle\cos\theta_{M'_Q} + |Q_u\rangle\sin\theta_{M'_Q}, \qquad (1.4)$$
$$|M_Q\rangle = -|Q_s\rangle\sin\theta_{M_Q} + |Q_u\rangle\cos\theta_{M_Q},$$

where

$$|Q_s\rangle = |s\bar{s}\rangle_Q \;, \quad |Q_u\rangle = \frac{1}{\sqrt{2}}\{|u\bar{u}\rangle_Q + |d\bar{d}\rangle_Q\} \qquad (1.5)$$

are the wave functions of quarks with zero isospin $(I=0)$. Note that the standard definition of the mixing angles for $Q\neq P$ is different from Eq.(1.2):

$$|M'_Q\rangle = |Q_8\rangle\cos\theta_Q(M') - |Q_0\rangle\sin\theta_Q(M'), \qquad (1.6)$$
$$|M_Q\rangle = |Q_8\rangle\sin\theta_Q(M) + |Q_0\rangle\cos\theta_Q(M),$$

which is rather inconvenient. The relation between our angles and the standard ones is given by

$$\theta_P(\eta) = \theta_\eta + \theta_0 - \frac{\pi}{2}\;; \quad \theta_Q(M) = \theta_M + \theta_0, \; Q = V,T,A,S \qquad (1.7)$$
$$tg\,\theta_0 = 1/\sqrt{2}\;, \quad \theta_0 \cong 35.26^\circ$$

(the same for η' and M'_Q). The angle θ_0 is usually called the "ideal" mixing angle.

Our excuse for a somewhat lengthy discussion of the trivial matter of defining the mixing angles is in that the inconvenient standard definitions formerly led some authors to wrong conclusions. Besides that, the equation (1.6) for $\theta_Q = \theta_0$ gives $|M'_Q\rangle = -|s\bar{s}\rangle_Q$, and the minus sign is easy to lose. The more important advantage of Eq.(1.4) over Eqs. (1.2), (1.6) is a somewhat more clear relation of the angles $\theta_{M_Q}, \theta_{M'_Q}$ to OZI breaking. For the exact OZI rule $\theta_M = \theta_{M'} = 0$, $|M'_Q\rangle = |Q_s\rangle$, $|M_Q\rangle = |Q_u\rangle$, and $|\theta_M|, |\theta_{M'}|$ are growing with growing OZI violating amplitudes.

2. Mass Formulae Without Mixing

As in refs./18-20/, let us suppose that the wave function Ψ_{ijQ} of the quarks q_i and $\bar{q}_j$ in the state Q satisfy the following equations

$$\{\hat{\mathcal{K}}^2_{ijQ} - \kappa^2(M^2_{ijQ}, m_i^2, m_j^2)\}\Psi_{ijQ} = \sum_{k,\ell} M^Q_{ij,k\ell}\,\Psi_{k\ell Q}\;, \qquad (2.1)$$

where the right-hand side describes the quark mixing. The wave function Ψ_{ijQ} depends on the relative coordinates of q_i and $\bar{q}_j$ and on $Q=(J,L,S,N)$, where J is the total angular momentum, L is the relative orbital momentum, S is the total spin angular momentum, and $N=0,1,2,\ldots$ is the radial quantum number. We do not use any explicit form of the operator $\hat{\mathcal{K}}^q_{ijQ}$ making, instead, some simple assumptions on the dependence of its eigenvalues $\mathcal{K}^2_{ijQ}$ on i,j and Q. Finally, κ^2 is the centrum-of-mass momentum squared of quarks $q_i, \bar{q}_j$:

$$\kappa^2(M^2, m_i^2, m_j^2) = \frac{1}{4}M^2 - \frac{1}{2}(m_i^2+m_j^2) + \frac{(m_i^2-m_j^2)^2}{4M^2}, \qquad (2.2)$$

where m_i is the mass of the i-th quark, $i=u,d,s$, M is the mass of the bound state to be determined by solving the equations. Even if $\hat{\mathcal{K}}^2$ is independent of i and j, some symmetry breaking is implicit in eq.(2.1) due to the dependence of κ^2 on quark masses. As will be shown below, some symmetry breaking, effective for the states with $L\neq 0$ or $N\neq 0$, must be present in $\hat{\mathcal{K}}^2$.

For the sake of completeness let us mention two simple dynamical realizations of our phenomenological scheme. For example, we may assume that

$$\hat{\mathcal{K}}^2_{ijQ} = -\frac{d^2}{dr^2} + L(L+1)r^{-2} + V_{ijQ}(r)\,, \qquad (2.3)$$

where $r = |\vec{r}_i - \vec{r}_j|$ is the distance between the quarks. Then eq. (2.1) essentially coincides with the Schroedinger equation, up to a different energy momentum relation. Equations of the form (2.1) with $\hat{\mathcal{K}}^2$ given by eq. (2.3) can be derived in some relativistic quasipotential models. In some cases the Bethe-Salpeter equation can be approximately reduced to similar equations. For example, the tightly bound pseudoscalar $(q\bar{q})$ wave function approximately satisfies the equation (2.1) with

$$\hat{\mathcal{K}}^2_{ijP} \cong -\frac{d^2}{dr^2} + [L(L+1) + \frac{3}{4}]r^{-2} + V_{ijP}(r), \qquad (2.4)$$

where $r^2 = (x_{i\mu} - x_{j\mu})^2$ is the squared distance between the quarks in the four-dimensional Euclidean coordinate space. A more detailed discussion of these and other realizations of the general equation (2.1) as well as numerous references may be found in refs. /7/,/20/.

Let us temporarily forget about mixing, i.e., assume that $M^Q_{ij,k\ell} = 0$. Then, supposing that the dependence of $\mathcal{K}^2_{ijQ}$ is given by

$$\mathcal{K}^2_{ijQ} = k^2_Q + \frac{1}{2}(\mu_i^2+\mu_j^2)\,\ell_Q \,, \tag{2.5}$$

it is easy to obtain simple mass formulae. Postponing the discussion of the Q -dependence of k^2_Q and ℓ_Q , and using the isospin invariance relations $u=d$, $\mu_u=\mu_d$ we derive the mass relations from the eigenvalue condition

$$\mathcal{K}^2_{ijQ} = K^2(M^2_{ijQ}, m_i^2, m_j^2)\,. \tag{2.6}$$

To make the derivation more transparent introduce effective quark masses m_{iQ}:

$$m^2_{iQ} = m_i^2 + k^2_Q + \mu_i^2\ell_Q\,,\quad m^2_{iQ}-m^2_{jQ} = m_i^2-m_j^2+(\mu_i^2-\mu_j^2)\ell_Q \equiv \Delta^2_{ij,Q}\,, \tag{2.7}$$

i.e.,

$$\Delta^2_{ijQ} = \Delta^2_{ij} + (\mu_i^2-\mu_j^2)\ell_Q\,,\quad \Delta^2_{ij} \equiv m_i^2 - m_j^2 \,. \tag{2.8}$$

Now, a simple calculation gives

$$M_{ijQ} = m_{iQ} + m_{jQ} \tag{2.9}$$

and consequently

$$\rho=\omega\,,\quad K^* \equiv K_V = \frac{1}{2}(\varphi+\rho)\,, \tag{2.10}$$

$$A_2 = f\,,\quad K^{**} \equiv K_T = \frac{1}{2}(f'+A_2)\,. \tag{2.11}$$

The observed vector masses (see /23/) are in good agreement with eq. (2.10) for the tensor mesons the agreement is reasonable. The corresponding relations for the pseudoscalar mesons are badly violated. The conclusion is that mixing (or OZI violation) is very small for $Q=V$, somewhat larger for $Q=T$ and very large for $Q=P$.

Note that the corresponding equations for the charmed particles

$$\mathcal{D}^* \equiv \mathcal{D}_V = \frac{1}{2}(J/\psi+\rho)\,,\quad F^* \equiv F_V = \frac{1}{2}(J/\psi+\varphi) \tag{2.12}$$

predict the masses $\mathcal{D}_V, F_V$ to be $\sim$ 70 MeV lower than observed ones, in spite of the expected smallness of mixing. This effect is probably related to a larger SU_4 breaking as compared to the SU_3 breaking - the parameter k^2_Q may be different for the charmed particles and the "old" ones. (see /19/,/20/ where the intriguing problem of the η_c meson is also discussed). Here we will not apply our mass formulae to the charmed particles, but

emphasize that the quadratic mass formulae are satisfied significantly worse, for example, the $\mathcal{D}^*$ mass is predicted to be $\sim$ 250 MeV larger than the observed one. The obvious conclusion is that the linear mass formulae obtained in our model without mixing, are better than the quadratic formulae. For the old particles the linear formulae are approximately as good as quadratic, though the former are slightly better.

Note that our mass formulae are linear in spite of the fact that all the equations depend on squared masses. This is due to the relativistic kinematic relation (2.2) and to the neglect of the mixing effects. In the presence of such effects the mass formulae are neither linear nor quadratic.

Now we introduce a dependence of k_Q and ℓ_Q (see eq.(2.5)) on the quantum numbers Q which in the non-relativistic limit corresponds to a simple picture of orbitally and radially excited quarks with spin-spin (S-S) and spin-orbit (L-S) level splitting. We supplement this simple picture by certain relation between radial and orbital excitations.

Let us call the radial splitting the quantity

$$\delta_R(M^2_{ijQ}) = M^2_{ijQ}(N=1) - M^2_{ijQ}(N=0) \equiv M^2_{ijQ'} - M^2_{ijQ}\,, \tag{2.13}$$

and the "orbital splitting" the quantity

$$\delta_L(M^2_{ijQ}) = M^2_{ijQ}(L=1) - M^2_{ijQ}(L=0)\,. \tag{2.14}$$

It is implied in eq.(2.13) that all the quantum numbers Q , except N , are unchanged. In eq. (2.14) $Q=(J=L, L, S=0, N)$, so as the $L-S$ term vanishes for all L . For all models (e.g., eq.(2.3), (2.4)) and for all potentials there exists a certain relation between δ_L and δ_R . The reason is very simple: both quantities are defined by an average "radius" R_{ij} of the bound state $(q_i\bar{q}_j)$:

$$\delta_R(M^2_{ijQ}) \sim \left\langle \frac{d^2}{dr^2}\right\rangle_{ijQ} \sim \left\langle \frac{1}{r^2}\right\rangle_{ijQ} \sim \frac{1}{R^2_{ij}}\,,\quad \delta_L(M^2_{ijQ}) \sim \left\langle \frac{1}{r^2}\right\rangle_{ijQ} \sim \frac{1}{R^2_{ij}}\,,$$

where the brackets denote averaging over the state ψ_{ijQ} . For simple potentials (linear, oscillator, Coulomb, square well, etc.) δ_L and δ_R are proportional to R^{-2}_{ij} and are approximately independent of Q .

For the oscillator potential $\delta_R/\delta_L = 2$, for the linear potential δ_R/δ_L is slightly less than 2, and for Coulomb potential $\delta_R/\delta_L = 1$. If $V=R^{-2}(r/R)^a$, then the ratio δ_R/δ_L

defines the value of a, and the magnitude of δ (or δ_L) defines the dimensional parameter R. Any realistic potential must be of a more complicated form, and some additional information (e.g., on decays of the vector and pseudoscalar particles) is required for estimating the form of the potential.

Taking into account all these considerations we finally suppose that the eigenvalues of $\hat{\mathcal{K}}^2_{ijQ}$ are of the form

$$4\hat{\mathcal{K}}^2_{ijQ} = m_0^2 + 2(\mu_i^2+\mu_j^2)(L+\beta N) + \mu_L^2(\vec{L}\vec{S}) + 4[\mu_F^2\delta_{L0} + \mu_F'^2(1-\delta_{L0})](\vec{s}_i\vec{s}_j), \quad (2.15)$$

where $\vec{S} = \vec{s}_i + \vec{s}_j$. We have tacitly supplimented the above-mentioned assumptions by the hypothesis of linearly rising Regge trajectories and used the simplest possible expressions for the $L-S$ and $S-S$ splittings. For simplicity we neglect the tensor forces which mix states with different values of Q. At first sight, the assumption $R_{ij}^{-2} \sim (\mu_i^2+\mu_j^2)$, used in eq. (2.15), does not seem to be natural. In fact, the multiplicative relation between the Regge slopes $\alpha_{ii}\alpha_{jj} = \alpha_{ij}^2$ seems to be more justified/24/. However, for the "old" mesons the distinction between multiplicative relations for α_{ij} and additive relations for α_{ij}^{-1} is practically negligible, while the latter is more convenient in the context of mass formulae. In the potential models the additive relation is easier to obtain (see, e.g.,/25/).

Combining the equations (2.1), (2.2) and (2.15), one easily obtains the universal mass formula for the mesons

$$M^2_{ijQ} = m_0^2 + 2(\mu_i^2+\mu_j^2)(L+\beta N) + 2(m_i^2+m_j^2) - (m_i^2-m_j^2)^2/M^2_{ijQ} + \\ +4[\mu_F^2\delta_{L0}+\mu_F'^2(1-\delta_{L0})](\vec{s}_i\vec{s}_j) + \mu_L^2(\vec{L}\vec{S}) - \tfrac{4}{3}\varepsilon_Q^2(M^2_{ijQ}). \quad (2.16)$$

A new term $-\frac{4}{3}\varepsilon_Q^2$ has been added here, the explanation of its origin to be given later. For the moment, the reader is advised to forget about this term[4)]. This formula is applicable to $I=1/2$ and $I=1$ states. For $I=0$ states it can be used, provided mixing is very small, as, e.g., in the vector nonet. In that case, as explained above, it gives the simple linear mass relations (2.9)-(2.11).

By fixing the parameters in eq. (2.16) one can predict the masses of all $I=\frac{1}{2}, 1$ mesons and, for negligible mixing, the masses of the isoscalar particles. Taking into account the isospin symmetry, there are 8 independent free parameters in eq. (2.16).

4) The magnitude of ε_Q^2 is very small $|\varepsilon_Q^2| \lesssim .01$, for all Q except $Q=P$, $|\varepsilon_P^2| \lesssim .1$.

To find them, let us first use the masses ρ, $K^* = K_V$, $A_1(1.1)$, A_2, $K^{**}=K_T$, B (1.231)[5)]. Then

$$m_0^2 + 2(u^2+s^2) + \mu_F^2 \cong .814, \quad \Delta^2_{su} = \Delta^2 \cong .108, \\ \mu_F'^2 \cong -.012, \ \mu_L^2 \cong .255, \ 4\mu_u^2 - \mu_F^2 \cong .880, \ 2\mu_u^2 + 2\mu_s^2 - \mu_F^2 \cong .968. \quad (2.17)$$

With these values of the parameters the masses of other states can be predicted. For example, $g(J=3, L=2, S=1) \cong 1.690$, $K^*(J=3, L=2, S=0) \cong 1.80$, $K^*(J=1, L=1, S=1) \cong 1.23$, $K^*(J=1, L=1, S=0) \cong 1.35$, $\rho_1(J=1, L=2, S=1) \cong 1.26$.
The experimental masses are respectively $g = 1.688$, $K^* = 1.784$, $Q_1 = 1.28$, $Q_2 \sim 1.4$. Some arguments in favour of the ρ_1-particle with mass $\rho_1 \sim 1.25$ are discussed in ref./26/.

The parameter μ_F will be fixed after considering the mixing effects; later on we shall show that $\mu_F^2 \cong .111$. With this value of μ_F we can fix the parameters μ_u, μ_s and β. Identifying $\rho'(1.6)$ with the first radial excitation of the ρ-meson ($N=1$), we now find $\beta = 1.98$, which is rather close to the oscillator potential prediction. Knowing this parameter one can predict the masses of other radially excited mesons, but any comparison with experiment would be premature[6)].

As the data on the scalar mesons are somewhat controversial we do not mention our predictions for the scalar multiplet. In addition, there are good reasons to believe that the simple quark picture is, in this case, spoiled by mixing of the $q\bar{q}$ states with exotic $q\bar{q}q\bar{q}$ states (see/9/), similar effects are expected for the charmed particles/3/,/7/,/19/,/20/. Neglecting such a mixing one can obtain $4\mu_c^2 \cong 2.0$ and predict the radially excited states of the J/ψ particle: $\psi' = 3.7$, $\psi'' = 4.19$, etc.

A more detailed comparison of the predictions of eq. (2.16) with experiment will be given in another communication. Here we make only some general remarks. It has been shown that these formulae give a good description of the mass spectrum of the "usual" mesons and a reasonable first approximation for treating the charmed particles (see/19/,/20/ for more detail). The main symmetry violation is due to mass differences of quarks,

5) As justified in the next section, we use $\rho = .773$, $K_T = 1.421$. For K_V we take $\frac{1}{2}(K_V^+ + K_V^0)$.

6) For a discussion of the present status of the data on the mass spectrum in the interval 1÷2 GeV see, e.g., the reviews/26,27/.

and some symmetry breaking is observed in the Regge slopes (in our treatment these are $(4\mu_u^2)^{-1}, (2\mu_u^2+2\mu_s^2)^{-1}, (4\mu_s^2)^{-1}, (4\mu_c^2)^{-1}$). For the old particles this breaking is $\lesssim$ 10%, and for charmed particles it is $\lesssim$ 40%. The same pattern is to be observed in the spectrum of the radial excitations. For the S-S and L-S splittings the symmetry violation is expected to be significantly smaller as far as the corresponding parameters depend on the behaviour of the wave function at small distances. In fact, the spectrum of the old mesons exhibits no SU_3 symmetry breaking in the S-S and L-S splittings, and the SU_4 violation is $\lesssim 20\%$ for the charmed particles. (see/20/). Above we have tacitly assumed no dependence on i,j and Q of the parameter m_o^2. For the old particles this is justified by good agreement of predicted masses with experiment. However, some effects not included in our scheme (e.g., tensor forces) may give an effective dependence of m_o^2 on Q. With present data such a possibility is difficult to exclude for the charmed particles.

We conclude by summarizing the main features of our approach as distinct from the standard treatments:/12/-/16/: 1) the relativistic energy-momentum relations, esp. for unequal masses; 2) a relation between orbital (L) and radial (N) excitations; 3) inclusion of some symmetry breaking in the dependence of the meson masses on L and N. Similar formulae can be employed for describing the diquark (d_{ij}) mass spectrum, the baryon mass spectrum, and the exotic meson $(d\bar{d}_{ij})$ mass spectrum/18/. The mass formulae are always linear.

3. Mixing Effects

As has been pointed out in refs./28/-/38/, simple QCD arguments give the mixing matrix of the form

$$M^{Q}_{ij,k\ell} = -\varepsilon_Q^2\, \delta_{ij}\, \delta_{k\ell}. \tag{3.1}$$

This corresponds to annihilation of two or three gluons in the s-channel of the system $q_i\bar{q}_j$ and to further transition of the gluons into $q_k\bar{q}_\ell$. The effective Lagrangian for such a transition may be written as $\mathcal{L}_{eff} \sim (\bar{q}\lambda_n q')(\bar{q}'\lambda_n q)$, where $\lambda_n\ (n=1,\ldots,8)$ are the Gell-Mann matrices, $\lambda_0=\sqrt{\frac{2}{3}}\,I$, and $q', \bar{q}'$ are referred to the quarks in the final state. We suppress here the γ-matrices describing a spin dependence of the mixing matrix and neglect the colour structure. All these effects are included in the parameter ε_Q^2.

meter ε_Q^2. Introducing, for example, a nonet of vector mesons V_n^μ one may interpret $\mathcal{L}_{eff}$ in terms of the exchange of these mesons in the t-channel, with the effective quark-meson Lagrangian $\mathcal{L}_{Vqq} \sim (\bar{q}\lambda_n\gamma_\mu q)\, V_n^\mu$. The Lagrangians $\mathcal{L}_{eff}$ and $\mathcal{L}_{Vqq}$ are not only SU_3-invariant but also U_3-invariant. Remark that the assumption of the U_3-invariance was first used by Schwinger/31/, who invented the mixing matrix (3.1) and derived on its basis his well-known mass formula 7). Schwinger's mass formula is quadratic in meson masses and is in reasonable agreement with present data, its prediction for the ρ-mass being $\rho_{(q)} \cong .760$. The linear version of the formula is better: $\rho_{(l)} \cong .774$.

Both linear and quadratic Schwinger's mass formulae, however, fail to describe the pseudoscalar nonet. The predictions of η' with the input masses m_π, K and η are resp. $\eta'_{(q)} = 1.61$, $\eta'_{(l)} = 2.34$. To avoid this difficulty it was proposed/29/ that ε_Q^2 is strongly dependent on the bound state mass M. This is very natural in QCD as $\varepsilon_P^2(M^2) \sim \alpha_s^2(M^2)$ and $\varepsilon_V^2(M^2) \sim \alpha_s^3(M^2)$ (see, e.g./2/,/3/). However, this dependence for the linear mass formulae/29/ proves to be unreasonably strong - $\varepsilon_P^2(\eta^2)/\varepsilon_P^2(\eta'^2) \simeq 7.6$, which is difficult to reconcile with $\varepsilon_V^2(\varphi^2)/\varepsilon_V^2(\omega^2) \sim 1$. This difficulty has not been discussed in refs./28/,/30/ treating the same mixing mechanism in the context of the quadratic mass formulae. Assuming a dependence of ε_Q^2 on M^2 to exist also in this case, one can find that $\varepsilon_P^2(\eta^2)/\varepsilon_P^2(\eta'^2) \simeq 3.6$, and $\varepsilon_V^2(\varphi^2)/\varepsilon_V^2(\omega^2) \sim 1$, leading to the same difficulty. In addition, both "linear" and "quadratic" predictions for the pseudoscalar mixing angle are in poor agreement with present data on radiative decays of the vector mesons and with data on η/η' production at high energies (see/22/). For the quadratic formulae $\theta_P(\eta) \cong -5.4°$, $\theta_P(\eta') \cong -19.8°$, for the linear - $\theta_P(\eta) \cong -11.1°$, $\theta_P(\eta') \cong -44.7°$. In the next paper it will be shown that these angles are inconsistent with the decay data unless OZI violating terms not included in η-η', ω-φ mixing are unreasonably large. The linear angles badly disagree with the high-energy production data, and the quadratic angles give the value of $\sigma(\eta')/\sigma(\eta)$ which is $\sim\frac{3}{2}$ larger than the experimental result.

The mixing mechanisms in question share with the standard mechanisms (see/12/-/16/) the following deficiency. Either none of the pseudoscalar masses is predicted, or the prediction is very bad. An exterme of this feature is presented in the inte-

7) This fact was overlooked in refs. /28/-/30/ as well as by the present author who independently introduced a similar mechanism in refs. /7/,/18/.

resting paper[32], where a large SU_3 breaking has been introduced into the mixing matrix. As a result, all the pseudoscalar masses and the mixing angle $\theta_P(\eta)=\theta_P(\eta')\cong-10^\circ$ should be used for a description of the pseudoscalar nonet. In addition to poor agreement of this angle with experiment, there is no way to check up the consistency of the assumptions within the old particle family.

We will not discuss other, more exotic mixing schemes as our approach is a direct generalization of those based on eq. (2.1). In ref.[7] we have developed a mixing scheme starting from eqs. (2.1), (2.2), (2.15) with the mixing matrix (3.1). Even assuming no dependence of ε_Q^2 on M^2 we have succeeded in describing V and T multiplets and the masses of the pseudoscalar mesons η, η' and K. For the optimum value of ε_P^2 ($\varepsilon_P^2\cong .524$) the mixing angle is predicted to be $\theta_P(\eta)=\theta_P(\eta')\cong$ -20.1° and the fitted masses of η and η' are $\eta=.542$, $\eta'=.963$. In spite of the bad prediction for the pion mass ($m_\pi\cong .28$) the description seems to be more successful than those discussed above, as the dependence of ε^2 on M^2 is rather weak and the mixing angle is in nice agreement with experiment. With regard for such a dependence, we have obtained $\varepsilon_P^2(\eta')\cong .0605$, $\varepsilon_P^2(\eta')\cong .0503$, $\theta_P(\eta)\cong$ -17.6°, $\theta_P(\eta')\cong$ -20.9°. This improvement is solely due to the relativistic kinematic relation (2.2). If we omit the last term in this equation, we reproduce the quadratic formule of refs.[28],[30],[31]. Substituting, in addition, masses for squared masses, we arrive at the linear relations of ref.[29].

To successfully describe also the pion it has been suggested[19],[20] to generalize the mixing mechanism (3.1) as follows. As is well known, the U_3 symmetry, implied in eq.(3.1), results in unpleasant consequences for the pseudoscalar mass spectrum (the so-called U_1 -problem, see,e.g.[33]). Recently, a possible solution of the problem has been outlined in the course of developing new ideas on the infrared properties of QCD[34]. It seems quite probable that the two-gluon anomaly, along with a rearrangement of the vacuum due to instanton contributions, reduce the undesirable U_3 -symmetry to U_1 -symmetry. One may therefore tentatively assume that the mixing matrix is SU_3 symmetric rather than U_3 -symmetric.

Apparently different arguments in favour of using SU_3-symmetr mixing were earlier given in refs.[19],[20]. It is con-

ceivable that the effect of soft gluon exchanges can be represented by an exchange of the Regge trajectories (R) of the observed mesons in the t -channel (see fig. 1). Then the SU_3 -flavour-exchange amplitude is decreasing with s while the amplitude with the exchange of the SU_3 -singlet "trajectories" R is approximately "constant". The latter is the same for all meson states ($\mathcal{L}_{eff}\sim(\bar q\lambda_0 q')(\bar q'\lambda_0 q)$) in the s- channel and can be included in the $\hat{\mathcal{K}}^2$. The octet-R -exchange can be described by an effective Lagrangian $\mathcal{L}_{eff}\sim g(s)(\bar q\lambda_n q')(\bar q'\lambda_n q)_{,n=1,..8}$, that strongly depends on $s=M_{ij}^2$ and gives the mixing matrix

$$M^Q_{ij,k\ell}=-\varepsilon_Q^2(M^2_{ijQ})\left[\delta_{ij}\delta_{k\ell}-\frac{1}{3}\delta_{ik}\delta_{j\ell}\right]. \tag{3.2}$$

In the following descussion we consider the mixing mechanism based on eqs.(2.1), (2.2), (2.15) with the mixing matrix (3.2).

The mixing parameter ε_Q^2 is large only for $Q=P$, an elegant explanation of this fact in terms of QCD has recently been advanced by Friedberg and Lee[11]. Owing to the smallness of ε_Q^2 for $Q=V,T,..$ and to the approximate degeneracy of masses in these multiplets, the M^2-dependence of ε_Q^2 can be neglected in the first approximation. To save the space we shall write the relevant formulae for the general case not making such an approximation.

With the mixing matrix (3.2) the equations for calculating the masses of the $I=1/2,1$ mesons belonging to the multiplet Q, are

$$M^2_{Q,1}=m_Q^2-2\Delta^2-\frac{4}{3}\varepsilon_Q^2(M^2_{Q,1}), \tag{3.3}$$

$$M^2_{Q,1/2}=m_Q^2-\frac{\Delta^4}{M^2_{Q,1/2}}-\frac{4}{3}\varepsilon_Q^2(M^2_{Q,1/2}), \tag{3.4}$$

where m_Q^2 is to be determined from eq.(2.15), $\Delta^2=\Delta^2_{su}$. As $\varepsilon_Q^2(M^2)$ is a decreasing function of M^2 the equations have two solutions at most. Only one of them is stable in each case[8]. The easiest way to see this is to draw the picture version of the equations as presented in fig. 2. The curves correspond to the right member of the equations, the stable solution is denoted by S, the unstable one- by A, the dotted lines correspond to iterations

[8] A solution is stable if and only if the derivatives of the right-hand-sides of eqs.(3.3), (3.4) are <1 and >-1 in the respective points. We shall show that this condition is fulfilled for the observed masses.

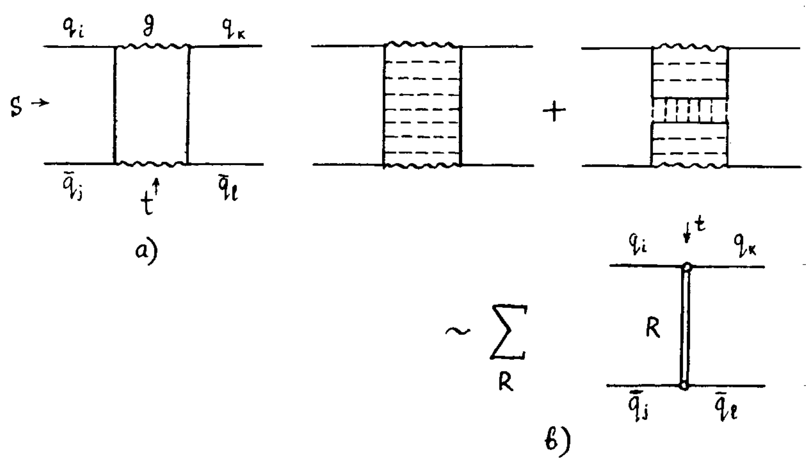

Fig. 1

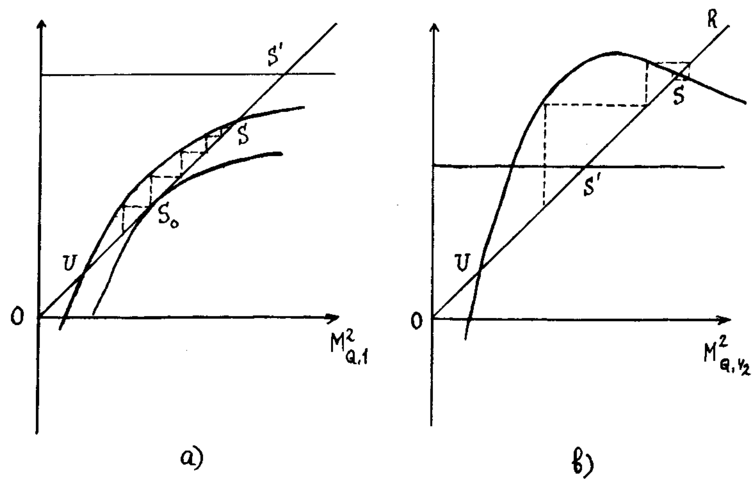

Fig. 2

always giving the stable solutions. If ε_Q^2 is independent of M^2 , the solution is unique (the point S'). In the case $Q=P$ the pion mass, $M_{P,1}=m_\pi$, is very small, and this requires $\varepsilon_P^2(M^2)$ to be rather large and rather strongly dependent on M^2 . By inspecting fig. 2a one can infer that m_π^2 has a minimum if the curve representing the right-member of eq.(3.3) is tangent to the straight line OR at the point S_0 . Then

$$d\varepsilon_P^2/dm_\pi^2 = -\frac{3}{4}\,. \tag{3.5}$$

This hypothesis of the <u>minimum mass of the pion,</u> or of the maximum mixing will be used later to calculate the magnitude of the mixing parameter $\varepsilon_\pi^2 \equiv \varepsilon_P^2(m_\pi^2)$.

The isoscalar wave functions satisfy the equation

$$(\hat{\mathcal{K}}^2_{uuQ}-k_u^2)\Psi_{u,Q} = \varepsilon_Q^2\left[-\frac{5}{3}\Psi_{u,Q}-\sqrt{2}\,\Psi_{s,Q}\right], \tag{3.6}$$

$$(\hat{\mathcal{K}}^2_{ssQ}-k_s^2)\Psi_{s,Q} = \varepsilon_Q^2\left[-\frac{2}{3}\Psi_{s,Q}-\sqrt{2}\,\Psi_{u,Q}\right], \tag{3.7}$$

where (see eq.(2.2))

$$k_u^2=\frac{1}{4}M^2-u^2,\quad k_s^2=\frac{1}{4}M^2-s^2,\quad M^2=M_Q'^2 \text{ or } M_Q^2\,,$$

$\Psi_{u,Q}$, $\Psi_{s,Q}$ are the wave functions in the quark basis (1.5), and the isospin invariance has been used. Solving this system we find the equations for M_Q' and M_Q

$$M_Q'^2 = m_Q^2+\frac{14}{3}\varepsilon_Q'^2+2\sqrt{(\Delta^2-\varepsilon_Q'^2)^2+8\varepsilon_Q'^4}\,, \tag{3.8}$$

$$M_Q^2 = m_Q^2+\frac{14}{3}\varepsilon_Q^2-2\sqrt{(\Delta^2-\varepsilon_Q^2)^2+8\varepsilon_Q^4}\,, \tag{3.9}$$

where $\varepsilon_Q=\varepsilon_Q(M_Q^2)$, $\varepsilon_Q'=\varepsilon_Q(M_Q'^2)$. The eigenvectors corresponding to these eigenvalues are to be determined by eqs. (1.4), and

$$\operatorname{tg}\theta_{M_Q} = 2\sqrt{2}\,\varepsilon_Q^2\left[\Delta^2-\varepsilon_Q^2+\sqrt{(\Delta^2-\varepsilon_Q^2)^2+8\varepsilon_Q^4}\,\right]^{-1}. \tag{3.10}$$

To obtain $\operatorname{tg}\theta_{M_Q'}$ one has simply to substitute here ε_Q by ε_Q'.

The solutions of eqs.(3.8), (3.9) are stable for all observed particles.

Eqs. (3.3), (3.4), (3,8), (3.9) constitute the solution of the mixing problem for all multiplets. Now assume ε_Q to be independent of M_Q and introduce the notation

$$M_{Q,0}\equiv 2M_{Q,1/2}-M_{Q,1}\,,\quad \delta_Q^2\equiv M_Q^2-M_{Q,1}^2\,,\quad \delta_Q'^2\equiv M_Q'^2-M_{Q,0}^2\,. \tag{3.11}$$

After simple calculations the following mass formula can be derived[9)]

$$\delta_Q^2\left(1+\frac{\delta_Q^2}{M_Q'^2-M_Q^2}\right) = 2\delta_Q'^2\left(1-\frac{\delta_Q'^2}{M_Q'^2-M_Q^2}\right) \tag{3.12}$$

By substituting masses in the first of the definitions (3.11) by squared masses the Schwinger's formula can be reproduced.

If the mixing parameter is small, the δ_Q^2 and $\delta_Q'^2$ are very small due to the approximate equalities $M_Q \cong M_{Q,1}$ (e.g., $\omega \cong \rho^{\cdot}$, $A_2 \cong f$) and $M_Q' \cong M_{Q,0}$ which follow from eqs. (2.9)-(2.11). Then, neglecting in eq.(3.12) the terms of second order in δ_Q^2 and $\delta_Q'^2$, we obtain an approximate mass formula which for the vector mesons is

$$2\varphi^2 + \rho^2 \cong 2(2K_V - \rho)^2 + \omega^2. \tag{3.13}$$

The prediction of this formula for the ρ mass is $\rho = .7726$, while the exact formula predicts $\rho = .7728$. With regard for the uncertainty of the ρ mass (electromagnetic splitting) the best prediction for the ρ -mass is $\rho = .773(\pm).004$, (K_V). Hereafter we employ the notation $(\pm)$, $(...)$ for errors correlated to the errors in the variable (e.g.,(K_V)) written in the parentheses.

Now, the masses of other vector mesons allow us to fix other free parameter

$$\Delta^2 = \varsigma^2 - u^2 = .1086(\mp).0007, \quad \varepsilon_V^2 = .0020(\mp).0008,$$
$$m_V^2 = .817(\pm).004, \quad \theta_\omega = \theta_\varphi = (1.5(\mp).6)^\circ, \quad \theta_V = (36.8(\mp).6)^\circ. \tag{3.14}$$

Here all the errors are correlated to the errors in K_V.

Applying eq. (3.12) to the tensor meson one can predict the mass of the K_T by using the input masses A_2, f, f': $K_T = 1.421 \pm .006$. This is slightly different from the mass quoted in ref./23/ ($K_T = 1.434 \pm .005$)., but the statistical average (with Student's distribution) of the world data gives another result, $K_T = 1.4237 \pm .0015$/23/, which is in good agreement with our value. For this reason we take $K_T = 1.421$ as an "experimental" K_T -mass. For similar reasons we consider $\rho = .773$ as an "experimental" ρ -mass.

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9) Strictly speaking this formula is valid only for $L=0$. If $L \neq 0$, eqs. (3.3), (3.8), (3.9), (3.10) depend on $\Delta^2_{ef} = \Delta^2_{su,Q}$ instead of Δ^2_{su} (see eq.(2.8)). As far as the difference $\mu_u^2 - \mu_1^2$ is very small ($\sim .022$) with respect to the squared masses of the $L=1$ mesons we can approximate the Δ^2 in eq.(3.4) by Δ^2_{ef} without loosing the accuracy. Then we arrive at the mass formula (3.12) for the $L=1$ multiplets.

As the mixing parameters $\varepsilon_f^2, \varepsilon_{f'}^2$ are slightly different and are larger than ε_V^2 we calculate an average mixing parameter ε_T^2 by taking half-sum of eqs.(3.8) and (3.9), the parameter m_T^2 being fixed by the A_2 mass. The result is

$$\varepsilon_T^2 \cong -.0114, \quad \theta_f \cong \theta_{f'} \cong -5.6^\circ \tag{3.15}$$

Due to the approximate relation $\theta_f \simeq \frac{\pi}{6} - \theta_0$, where $tg\,\theta_0 = \frac{1}{\sqrt{2}}$ (see (1.7)), we can write the simple approximate expression

$$tg\,\theta_f \cong (\sqrt{2} - \sqrt{3})(1+\sqrt{6})^{-1}, \tag{3.16}$$

which is easy to remember.

A more careful analysis of the vector and tensor multiplets indicates that $\varepsilon_V^2(M^2)$ and $\varepsilon_T^2(M^2)$ are smooth-decreasing functions of M^2. However, the present data on masses do not allow us to extract this dependence with good precision. On the contrary, this dependence is very pronounced in the pseudoscalar nonet.[10)].

In this case there are 4 masses and 5 unknown parameters $m_P^2, \varepsilon_\pi^2, \varepsilon_K^2, \varepsilon_\eta^2$ and $\varepsilon_{\eta'}^2$. As pointed out above, the independence of ε^2 on M^2 results in a too large prediction for the pion mass. This is easy to understand if $\varepsilon_\pi^2 \neq \varepsilon_\eta^2$. In fact, assuming $\varepsilon_K \simeq \varepsilon_\eta \simeq \varepsilon_{\eta'}$ one can obtain $\varepsilon_\eta^2 \simeq .5$, $\varepsilon_\pi^2 \simeq .1$. It is interesting to observe that $\varepsilon_\pi^2 \sim \Delta^2$, $\varepsilon_\eta^2 \sim \Delta^2/2$. Very probably, the last relation is not accidental. Indeed, regarding η^2 as a function of ε_η^2, one can obtain from eq.(3.9) the remarkable result: the mass of the η meson has a maximum for $\varepsilon_\eta^2 = \Delta^2/2$ ($d\eta^2/d\varepsilon_\eta^2 = 0$, $d^2\eta^2/d(\varepsilon_\eta^2)^2 > 0$ for $\varepsilon_\eta^2 = \Delta^2/2$), and

$$tg\,\theta_\eta = 1/\sqrt{2}, \quad \theta_\eta = \theta_0, \quad \theta_P(\eta) \cong -19.47^\circ.$$

Thus, a simple pattern of mixing in the pseudoscalar nonet emerges - <u>the pion mass is minimum</u> and <u>the η mass is maximum possible</u>.

With the observed dependence of ε_P^2 on M^2, the last statement is only approximately realized. If we take $\varepsilon_\eta^2 = \frac{\Delta^2}{2}$ and calculate the unknown parameters by using the pseudoscalar masses we find a variation of $\varepsilon^2(M^2)$ in the interval $K^2 \leq M^2 \leq \eta^2$ to be too large:

$$\varepsilon_\pi^2 \cong .1032, \quad \varepsilon_K^2 \cong .0509, \quad \varepsilon_\eta^2 \cong .0543, \quad \varepsilon_{\eta'}^2 \cong .0504.$$

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10) We are not discussing other multiplets as their experimental status is unclear.

Assuming a more realistic approximation $\varepsilon_\eta \cong \varepsilon_K$ we obtain

$$\varepsilon_\pi^2 = .1038\,, \quad \varepsilon_K^2 \cong \varepsilon_\eta^2 = .0605\,, \quad \varepsilon_{\eta'}^2 = .0503 \tag{3.17}$$

$$\theta_\eta = 37.15^\circ, \quad \theta_{\eta'} = 33.86^\circ, \quad \theta_P(\eta) = -17.59^\circ, \quad \theta_P(\eta') = -20.88^\circ$$

These values of the parameters satisfy the simple relation $\varepsilon_\eta^2 + \varepsilon_{\eta'}^2 \cong \Delta^2$. With this relation as an input the parameters are

$$\varepsilon_\pi^2 = .1035\,, \quad \varepsilon_K^2 = .0602\,, \quad \varepsilon_\eta^2 = .0582\,, \quad \varepsilon_{\eta'}^2 = .0504 \tag{3.18}$$

$$\theta_\eta = 36.49^\circ, \quad \theta_{\eta'} = 33.90^\circ, \quad \theta_P(\eta) = -18.25^\circ, \quad \theta_P(\eta') = -20.84^\circ.$$

We regard this result as a most reliable description of mixing in the pseudoscalar multiplet.

The next section is devoted to an attempt to explain the M^2-dependence of ε_P^2 in QCD. All the above procedures of calculating the mixing angles are in fair mutual agreement and give $\theta_P(\eta) = (17.5 \div 20)^\circ$, and $\theta_P(\eta') = -(20 \div 21)^\circ$. The data discussed by Okubo /22/, are in very good agreement with these mixing angles. With due regard for the large experimental errors, the tensor and vector angles are not at variance with the data.

4. Mixing and QCD

The dependence of $\varepsilon_P^2(M^2)$ can be qualitatively explained by the relation $\varepsilon^2(M^2) \sim \alpha_S^2(M^2)$, where $\alpha_S(M^2)$ is the "Sommerfeld constant" for the quark-gluon interaction (related to the invariant "charge") /1/-/6/. As far as we are trying to apply QCD in the resonance region, where this constant is not small, we have to somewhow take into account the higher order contributions. With this in mind, we assume that

$$\varepsilon(M^2) \sim \alpha_S(M^2)\,[1 + \lambda\alpha_S(M^2)]^{-1}, \tag{4.1}$$

where λ is some unknown constant, responsible for these contributions to $\varepsilon(M^2)$. To make this relation useful, some explicit expression for $\alpha_S(M^2)$ is needed. The renorminvariant perturbation theory result for the $\alpha_S(M^2)$ is (see e.g. /2/,/6/)

$$\alpha_S(M^2) = \alpha_0\,[1 + \alpha_0 \sigma_0(M^2)]^{-1}, \tag{4.2}$$

where $\sigma_0(M_0^2) = 0$, $\alpha_0 = \alpha_S(M_0^2) > 0$, and $\sigma_0(M^2)$ represents the second-order-quark-loop contribution. For $M^2 \to 0$ the $\sigma_0(M^2)$ is logarithmically divergent so as for some finite M^2 the denominator in eq.(4.2) has a zero. If we believe in deriving a confinement mechanism from infrared singularities of QCD,

a more natural assumption would be diverging $\alpha_S(M^2)$ for $M^2 = 0$. Starting from this observation we suggest to regularize $\sigma_0(M^2)$ in such a way that the denominator in eq.(4.2) would vanish only at $M^2 = 0$.

Assume that

$$\alpha_S(M^2) = \alpha_0\,[1 + \alpha_0 \sigma_\mu(M^2)]^{-1}, \tag{4.3}$$

where $\sigma_\mu(M^2) \equiv \sigma_0(M^2 + \mu^2)$, i.e.,

$$\sigma_\mu(M^2) = \frac{1}{12\pi}\Big\{ 33 \ln \frac{M^2+\mu^2}{\psi^2+\mu^2} - 4\ln\Big(\frac{5u^2+M^2+\mu^2}{5u^2+\psi^2+\mu^2}\Big) - 2\ln\Big(\frac{5s^2+M^2+\mu^2}{5s^2+\psi^2+\mu^2}\Big) - 2\ln\Big(\frac{5c^2+M^2+\mu^2}{5c^2+\psi^2+\mu^2}\Big)\Big\}. \tag{4.4}$$

Here ψ is the mass of the J/ψ particle, c is the c-quark mass, μ is a regularizing mass, and $\sigma_\mu(\psi^2) = 0$ (the normalization condition).

As in refs./20/,/21/ we assume $u \sim m_\pi/2$, then the masses of other quarks are defined in terms of observed meson masses, our results for Δ^2_{su} and Δ^2_{cu} leading to $s \sim .33$, $c \sim 1.6$. The final results are weakly dependent on quark masses. The variation of the most sensitive to the quark masses quantity $\sigma'_\mu(m_\pi^2)$ is $\lesssim 5\%$ on the interval $0 \leq u \leq m_\pi/2$, $\sigma_\mu(0)$ varies within 2%, and other quantities are stable up to 1%.

The commonly used value of $\alpha_0 \equiv \alpha_S(\psi^2)$ is $\alpha_0 \sim .2$ (see, e.g., /2/,/3/). Solving the equation $1 + \alpha_0\sigma_\mu(0) = 0$ with this value of α_0 we find $\mu = .115$, very close to $m_\pi \simeq .137$, as should be expected. In the following we simply take $\mu = m_\pi$, so as $\alpha_0 = -[\sigma_{m_\pi}(0)]^{-1} \cong .2126$. We shall use the following values of $\sigma_{m_\pi}(M^2)$, where the index m_π will be suppressed

$$\sigma(m_\pi^2) = -4.137, \quad \sigma(K^2) = -2.611, \quad \sigma(\eta^2) = -2.466, \tag{4.6}$$
$$\sigma(\eta'^2) = -1.666, \quad \sigma'(m_\pi^2) = 21.5(\mp).8, \quad (m_\pi).$$

Now we can attempt to fix the only unknown parameter λ to reproduce the obtained above values of $\varepsilon_P(M^2)$ (e.g., eq.(3.18)). To achieve this we observe that eqs. (4.1), (4.2) imply the relation $(\beta_0 \equiv \lambda + \alpha_0^{-1})$

$$\varepsilon(m_1^2)/\varepsilon(m_2^2) = [\beta_0 + \sigma(m_2^2)][\beta_0 + \sigma(m_1^2)]^{-1}. \tag{4.7}$$

For $\beta_0 \sim 9.5$ the result (3.18) is fairly reproduced by this relation with the error $\lesssim 4\%$.

We can, however, obtain more interesting predictions, by deriving the three unknown parameters m_ρ^2, β_0 and ε_π^2

from eqs. (3.3), (3.9) (for $M_Q = \eta$) and (3.5). By using eq.(4.7) (or (4.1) and (4.3)) the last equation can be rewritten in the form

$$\varepsilon_\pi^2 \left[\beta_0 + \sigma(m_\pi^2)\right]^{-1} = \frac{3}{8}\left[\sigma'(m_\pi^2)\right]^{-1} = .0175\,(\pm).0006\,,\ (m_\pi). \tag{4.8}$$

The right member of this is approximately equal to m_π (remark that $\sigma'(m_\pi^2) \simeq .407/m_\pi^2$). Numerically solving all the equations we have

$$m_\rho^2 = .3736 \pm .0005, \quad \varepsilon_\pi^2 = .103 \pm .001,$$
$$\beta_0 + \sigma(m_\pi^2) = 5{,}9 \pm .2\,, \quad \beta_0 = 10{,}0 \pm .2\,. \tag{4.9}$$

and the predictions for $\varepsilon_P(M^2)$ are

$$\varepsilon_K^2 = .065 \pm .001\,, \quad \varepsilon_\eta^2 = .062 \pm .001, \quad \varepsilon_{\eta'}^2 = .051 \pm .001; \tag{4.10}$$
$$m_K = .488 \pm .002, \quad m_{\eta'} = .960 \pm .004\,,$$

$$\theta_\eta = 37{,}6^\circ, \quad \theta_{\eta'} = 34{,}12^\circ, \quad \theta_P(\eta) = -17.18^\circ\,, \quad \theta_P(\eta') = -20{,}62^\circ \tag{4.11}$$

These values are in good agreement with eqs.(3.17) and (3.18), and the predicted masses are close to the observed ones. The average of eqs.(3.17), (3.18) and (4.11)

$$\theta_\eta = (37{,}07 \pm .54)^\circ\,, \quad \theta_{\eta'} = (33.96 \pm .14)^\circ,$$
$$\theta_P(\eta) = -(17{,}67 \pm .54)^\circ\,, \quad \theta_P(\eta') = -(20.78 \pm .14)^\circ \tag{4.12}$$

is the final result of our analysis to be later confronted with experiment.

The success of the naive extrapolation of the simplest QCD relations into the domain where they are certainly not applicable cries for a discussion. Note that the results are rather sensitive to the choice of α_0 because of the strong dependence of $\sigma'(m_\pi^2)$ on this choice. As the above two estimations of β_0 are close to each other ($\beta_0 \sim 9.5$, $\beta_0 \simeq 10$) the choice $\alpha_0 \sim .2$ seems to be quite reasonable. Only ε_π^2 and m_π are sensitive to α_0 , while other parameters remain fairly stable.

There exist two important dimensional parameters in our theory - $\Delta^2 = \rho^2 - u^2 \sim .11$, corresponding to the mass parameter $\Delta \sim \sim .33$, and m_π , or the u -quark mass. For $M > \Delta$ the value $\alpha_s(M^2)$ is small ($\lesssim .5$) so as applying the equation (4.3) is justifiable. For $M \lesssim m_\pi$ we have $\alpha_s(M^2) \gtrsim 2$, and the confinement mechanisms are most important. These mechanisms are implicit in our phenomenological parameters μ and λ , introduced for "correcting" the "asymptotically free" expressions.

The problem of a more profound derivation of such an extrapolation lies far beyond the scope of the present paper. We only mention that the modern investigations of the confinement in QCD /34/ indicate that main large distance ($\gtrsim m_\pi^{-1}$) effects are related to a rearrangement of the vacuum state and to a quark bag formation. Then the qualitative feature of perturbative results, may be used, with due modifications, for describing the residual interactions of quarks inside hadrons (a picture of "free" quarks in a bag).

In conclusion we would like to mention that the results of this section allow us to improve the treatment of the vector and tensor nonets. In the first approximation $\varepsilon_V^2(M^2)$ satisfies the relation (4.1) with the right member risen to the 3/2 power (three gluon exchange). Then

$$\Delta^2 = .1091\,, \quad \varepsilon_\omega^2 = .0021, \quad \varepsilon_{K_V}^2 = .0019, \quad \varepsilon_\varphi^2 = .0017, \tag{4.13}$$

m_V^2 is practically unchanged, and the prediction for the ρ mass is $\rho = .772$. These numbers lie within the errors of the approximate values (3.14), and, most important, the Δ^2 is practically the same. The improved values of $\varepsilon_T^2(M^2)$ can be calculated quite similarly. We leave this to the reader.

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