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Асимптотика формфактора пиона в квантовой хромодинамике

В рамках ренормируемой квантовой теории поля развит новый подход к анализу асимптотического поведения электромагнитного формфактора связанного состояния двух частиц. Показано, что при достаточно больших передачах импульса поведение формфактора пиона в квантовой хромодинамике определяется характером взаимодействия на малых расстояниях. Получена формула, в которой асимптотика формфактора пиона выражается через фундаментальные константы теории.

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Asymptotical Behaviour of Pion Electromagnetic Form Factor in QCD

In the framework of the renormalizable quantum field theory a new approach is developed to the investigation of asymptotical behaviour of two-particle bound state electromagnetic form factor. It is shown that the behaviour of the pion EM form factor in quantum chromodynamics at sufficiently large momentum transfers is controlled by the short-distance dynamics only. The formula is obtained which expresses the asymptotical behaviour of the pion form factor in terms of the fundamental constants of the theory.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. *INTRODUCTION*

Investigation of electromagnetic form factors of hadrons at high momentum transfers raises now a considerable interest [1-8]. To a great extent, this is caused by the agreement between the quark counting formula [9,10]

$$F_H(t) \sim t^{1-n_H} \tag{1.1}$$

which relates the asymptotical behaviour of the hadron form factor F_H to the number of quarks inside hadron H, and experimentally observed power-law behaviour of the proton form factor ($G_M^p \sim t^{-2}$) and that of the pion ($F_\pi \sim t^{-1}$). The agreement indicates that for large t, the behaviour of $F_H(t)$ may be controlled by small distance dynamics. Really, eq. (1.1) can easily be obtained from the tree diagram depicted in *fig. 1a*, if one assumes that the hadron momentum is equally shared between the quarks [10], whereas the "decay" of the pion into its constituents (quarks) is described by some functions ϕ, ϕ^* which do not affect the asymptotical behaviour with respect to t.

But it is not a trivial task to prove the validity of this approximation as well as to give a recipe of calculating the corrections to it.

An analysis of bound states and of the corresponding dynamical variables is based usually on the Bethe-Salpeter formalism [11,12], that is, the hadron (e.g., the pion) is described by the BS wave function

$$\chi_P(x_1, x_2) = \langle 0|T(\psi(x_1)\bar{\psi}(x_2))|P\rangle. \tag{1.2}$$

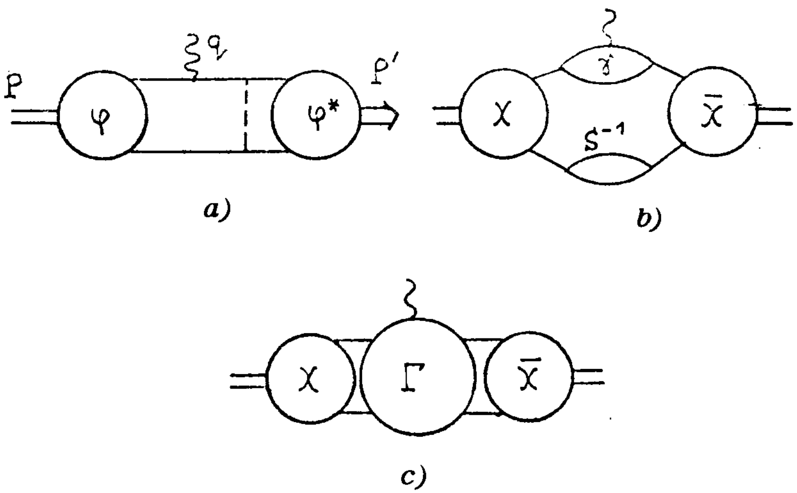

Fig. 1

To study the behaviour of the bound state form factor in such an approach one must solve the BS-equation or at least, thoroughly analyse it (or the quasipotential equation [13], see ref. [5]) to get necessary information about the corresponding wave function. As a result, the form factor is expressed in terms of $\chi, \bar{\chi}$ and of the functions γ, S^{-1}, Γ (*fig. 1b,c*) which can be obtained in perturbation theory.

It is just a simple comparison of *figs. 1a* and *1b* that shows that the function ϕ describing in *fig. 1a* the "decay" of the pion into two quarks does not coincide with the BS wave function. The authors of refs. [7, 10] have used for the function ϕ the term "soft part of the BS wave function". It was implied there that the function $\tilde{\chi}_P(p_1, p_2)$ (defined in the momentum representation) in the region $p_1^2 \to \infty$ can be represented as $\tilde{\chi}_P \sim I(p_1, p_2)\phi$. The function I corresponds to exchange of a highly virtual gluon, hence it may be related to the short-distance quark-quark interaction. Asymptotical properties of the function $\tilde{\chi}_P(p_1, p_2)$ have been investigated in refs. [1,2,4,7] with the help of the operator product expansion for $T(\psi(x_1)\bar{\psi}(x_2))$ at $(x_1 - x_2)^2 \to 0$. This

regime corresponds in fact to the limit $p_1^2 \sim p_2^2 \to \infty$ whereas in *fig. 1a* one has $p_2^2 \sim (P/2)^2 \sim m^2$. Hence one is forced to assume that taking p_1^2 small does not change the asymptotical properties radically. This assumption is not evident. Furthermore, the investigation of simple models [1] as well as a more general analysis [14] show that it is possible that not only small distances are relevant to the asymptotical behaviour of form factors. If the probability for a single quark to carry the whole momentum of the pion is large enough, then the asymptotical behaviour can be controlled by the Feynman process [15] when only a quark carrying the whole pion momentum participates in the hard scattering process, whereas the second one having a negligible fraction of the pion momentum, may be associated with the pion both in initial and final states. This corresponds in *fig. 1a* to a soft gluon exchange, hence this mechanism is explicitly dependent on large distance dynamics. The question is whether this process can dominate in a specific field theory model, e.g., in quantum chromodynamics (QCD), or not.

The main conclusion of the present paper is that for sufficiently large t the behaviour of the pion EM form factor in QCD is controlled only by short-distance properties of the theory

$$F_\pi^{(as)}(Q^2) = 8\pi\alpha_s(Q)\frac{f_\pi^2}{Q^2}, \qquad (1.3)$$

where α_s is the effective quark-gluon coupling constant, and $f_\pi = 132$ *MeV* is the pion decay constant, which describes the large-distance contribution. On the other hand, our analysis of simple scalar models $(\phi^3_{(4)}, \phi^3_{(6)})$ shows that in these theories the leading asymptotical behaviour of form factors is very sensitive to large-distance dynamics. It sounds like a paradox, but the analysis of form factors of colourless bound states in QCD is in some aspects simpler than the analysis of the analogous problem in $\phi^3_{(6)}$-theory.

The paper is organized as follows. In order to split the complicated problem of investigating the bound state form factors in QCD into separate parts, we investigate first some relatively simple models. In Sec. 2 we study the superrenormalizable $\phi^3_{(4)}$ -model using the α -representation analysis. In particular, we investigate here the specific manner in which the asymptotical behaviour of the bound state form factor can depend on large distances. In Sec. 3 we study the peculiarities of renormalizable theories taking the Yukawa type model $g\bar{\psi}\psi\phi_{(4)}$ as an example. In Sec. 3 we give a treatment of the pion EM form factor in QCD.

The approach used in the present paper was developed in refs. [16-18] (hereafter referred to as I-III), where we have studied some inclusive processes. The acquaintance with these papers will facilitate the understanding of the permanently used standard reasonings. We have supplied them here only with short comments, because they have been discussed in detail in I-III. The main results of the present paper have been published in a short form in refs. [19,20].

2. *ALPHA-REPRESENTATION ANALYSIS*

Let us consider the asymptotical behaviour of the form factor of a "pion" composed of two scalar quarks a, b interacting through a scalar gluon field ϕ in the 4-dimensional space-time

$$\mathcal{L}_{int}(x) = \sum_{i=a,b} g\psi_i^*(x)\psi_i(x)\phi(x). \tag{2.1}$$

We start, as usual [12], with the auxiliary 5-point Green function

$$R_5(x_1,x_2;y_1,y_2;0) = < \\ = <0|T(\psi_a(y_1)\psi_b(y_2)j(0)\psi_a^*(x_1)\psi_b^*(x_2))|0> \tag{2.2}$$

or, in momentum representation (see ref. [6])

$$(2\pi)^4\delta^4(p_1+p_2+p_1'+p_2'+q)R_5(p_1,p_2;p_1',p_2';q) = \\ = <0|T(\psi_a(p_1')\psi_b(p_2')j(q)\psi_b^*(p_2)\psi_a^*(p_1))|0>. \tag{2.3}$$

If the particles a, b can compose a bound state with mass m_π then

$$R_5(p_1,p_2;p_1',p_2';q) = i^2\frac{\chi_{P'}(p_1'-p_2')F_\pi(q)\chi^*_P(p_1-p_2)}{(P^2-m_\pi^2)(P'^2-m_\pi^2)}, \tag{2.4}$$

where χ,χ^* and F_π are, respectively, the BS wave functions and the bound state form factor, all taken in the momentum representation

$$<0|T(\psi_a(p_1)\psi_b(p_2))|P> = (2\pi)^4\delta^4(p_1+p_2-P)\chi_P(p_1-p_2),$$

$$<P'|j(q)|P> = (2\pi)^4\delta^4(P'-P+q)F_\pi(q). \tag{2.5}$$

Relations (2.4), (2.5) allow one to obtain $F_\pi(q)$ if the function R_5 is known. We will demonstrate later that the functions χ_P disappear in the final expression for F_π, so there is no need in our approach to calculate it explicitly.

To simplify the α-representation analysis we take $p_1 = p_2 = P/2$; ; $p_1' = p_2' = P'/2$ *. Then the contribution of any diagram (*fig. 2a*) can be represented as

$$R_5(P/2, P/2; P'/2, P'/2; q) = \\ = g^2\left(\frac{g}{4\pi}\right)^{z_T}\int_0^\infty \frac{\Pi d\alpha_\sigma}{D^2}\exp i[q^2(A+\frac{A_u}{2}+\frac{A_t}{2}+\frac{A_s}{4})/D + I(\alpha,m^2)], \tag{2.6}$$

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* This does not result in the loss of generality, as we will show below.

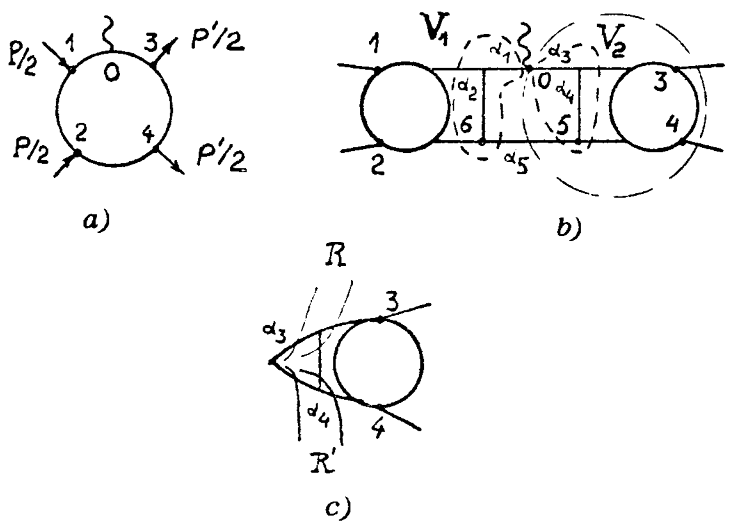

Fig. 2

where $A = B(0|1234)$, $A_t = B(01|234) + B(02|134)$, $A_u = B(03|124) + B(04|123)$; $A_s = B(24|013) + B(23|014) + B(14|023) + B(13|024)$. (for the α-representation analysis see, e.g., refs. /6,16/, the functions $B(\dots|\dots)$ are defined, in particular, in the Appendix to ref./16/). The coefficient $F(\alpha) = A + A_t/2 + A_u/2 + A_s/4$ is non-negative by construction, hence it vanishes only at the edge of the integration region, when $\alpha_\sigma = 0$ for lines σ constituting some t-subgraph V (i.e., the subgraph after the contraction of which into point the diagram loses its dependence on $t = q^2$).

Integration over $\lambda_V \to 0$ (by definition $\lambda_V = \sum_{\sigma \in V} \alpha_\sigma$) gives in the $\phi^3_{(4)}$-theory the following asymptotical contribution (see I)

$$R_{(V)}(Q) \lesssim Q^{4-\ell_{ext}-v}, \tag{2.7}$$

where ℓ_{ext} is the number of external lines of the t-subgraph V, whereas v is that of its vertices. In our case $\ell_{ext} \geq 5$, $v \geq 3$, hence $R_{(V)} \lesssim Q^{-4}$. The leading contribution is given by subgraphs V_1, V_2 (*fig. 2b*).

We introduce the Mellin transform $\Phi(J)$ of the amplitude R_5. For the diagrams of *fig. 2b* type

$$\Phi(J) = g^2\left(\frac{g}{4\pi}\right)^{z_T} \int^\infty \frac{\prod_\sigma d\alpha_\sigma}{D^2}\left(i\,\frac{(L+\frac{\ell}{2})(R+\frac{r}{2})}{D}\right)^J e^{iI(\alpha,m^2)} \tag{2.8}$$

where $R, r\ (L, \ell)$ are the functions corresponding to the right (left) hand of the diagram *2b*: $R(V_R) = B(0|345;\ V_R)$, $r = B_+(05|34;\ V_R)$ and similarly for L, ℓ. We will utilize the factorization properties:

$$R = \alpha_3 D_0(V_R') + \alpha_4 R(V_R');\ r(V_R) = \alpha_4\, r(V_R');$$
$$L = \alpha_1 D_0(V_L') + \alpha_3 L(V_L');\ \ell = \alpha_2\, \ell(V_L'), \tag{2.9}$$

where the functions $R(V_R'), r(V_R')$ are related to the subgraph V_R'. Introducing $\rho = \alpha_3 + \alpha_4$, $\lambda = \alpha_1 + \alpha_2$ and integrating in the region $\lambda, \rho \to 0$ we obtain

$$\Phi^{\rho}_{pole}{}^{\lambda}_{pole}(\mu^2) = \frac{g^4}{(J+2)^2}\left(\frac{1}{\mu^2}\right)^{2(J+2)} f_L f_R h. \tag{2.10}$$

The functions f_R have the following α-representation

$$f_R = \left(\frac{g}{4\pi}\right)^{z_{V_R'}} \int_0^\infty \prod_{\sigma \in V_R'} d\alpha_\sigma \frac{\exp iI(\alpha,m^2)}{D_0^2(V_R')} \frac{2}{1 - R_-(V_R')/D_0(V_R')}. \tag{2.11}$$

The function h corresponds to the "hanging" part:

$$h = \int_0^\infty \frac{d\alpha_5}{\alpha_5^{2+J}} e^{-\alpha_5 m^2} \Big|_{J=-2} \simeq (m^2)^{J+1}, \tag{2.12}$$

where m is quark mass.

To derive formulas (2.10)-(2.12) we have integrated over $\beta = a_4/\rho$ from 0 to 1. We have also used the relations $D = a_5 D_0(V_L^{'})D_0(V_R^{'})\{1+O(\lambda)+O(\rho)\}$ and $D_0(V_R^{'}) = R - R' + r$, where R' is the function analogous to R: $R'(V_R) = B(6|034;V_R)$. By definition $R_- = R' - R$.

Eq. (2.11) resembles formula (5) from I which describes the deep inelastic structure function in the $\phi^3_{(4)}$-model. One may introduce by analogy the parton wave functions $\tilde{\phi}(\xi)$

$$\tilde{\phi}(\xi) = \sum_{V_R'} \left(\frac{g}{4\pi}\right)^{z V_R'} \int_0^\infty \frac{\prod_\sigma d a_\sigma}{D_0^2} \delta\left(\xi - \frac{R_-(V_R')}{D_0(V_R')}\right) e^{iI(a,\, m^2)} \tag{2.13}$$

(cf. I, eq. (7)). From the equality $D_0 = R + R' + r$ it follows that $|R_-/D_0| \le 1$. Hence $\phi(\xi)$ vanishes outside the region $|\xi| \le 1$. The wave function $\phi(\xi)$ may be related to matrix elements of local operators just in the same manner it was done for the parton distribution functions. Really, the expression

$$b_n = \sum_{V_R} \left(\frac{g}{4\pi}\right)^z \int_0^\infty \frac{\prod_\sigma d a_\sigma}{D_0^2} \left(\frac{R_-}{D_0}\right)^n e^{iI(a,\, m^2)} \tag{2.14}$$

corresponds to a sum of graphs which have n derivatives in the O-vertex, consequently the quantity b_n is defined by

$$\langle 0|T\tilde{O}_{\mu_1 \dots \mu_n}(P)\psi_a^*(p_1)\psi_b^*(p_2)|0\rangle = (2\pi)^4 \delta^4(p_1 + p_2 - P)\{P_{\mu_1} \dots P_{\mu_n}\} b_n. \tag{2.15}$$

In the coordinate representation $O_{\mu_1 \dots \mu_n}(x) = (2i)^n \psi^*(x)\overleftrightarrow{\partial}_{\mu_1} \dots \overleftrightarrow{\partial}_{\mu_n} \psi(x)$. If there exists a bound state of particles a, b with mass m_π, then

$$b_n = \frac{i}{P^2 - m_\pi^2} a_n \chi_P(p_1 - p_2). \tag{2.16}$$

The function a_n is defined now by the matrix element

$$\langle 0|O_{\mu_1 \dots \mu_n}(0)|P\rangle = \{P_{\mu_1} \dots P_{\mu_n}\} a_n. \tag{2.17}$$

Expanding the expression $(1 - R_-/D_0)^{-1}$ (entering into eq. (2.11)) into power series in (R_-/D_0) and using eqs. (2.14), (2.15) we obtain

$$R^{\lambda\ \ \rho}_{pole\ pole}(\mu^2) = \frac{g^4}{m^2 Q^4} \ln\frac{Q^2 m^2}{\mu^4} \left(2\sum_{n=0}^\infty b_n\right)\left(2\sum_{\ell=0}^\infty b_\ell\right). \tag{2.18}$$

Comparing eqs. (2.4), (2.16) and (2.18) we find that the contribution into the form factor is given by eq. (2.18), where one must change $R \to F$, $b_{n,\ell} \to a_{n,\ell}$.

Thus we have seen that, as promised, the BS function $\chi_P(p_1, p_2) \sim \langle 0|T\psi(p_1)\psi(p_2)|P\rangle$ disappears in the final expression for the form factor. It means that one may use in place of $\psi(p_1)\psi(p_2)$ any combination $C(\psi, \psi^*, \phi)$ of quark and gluon fields satisfying $\langle 0|C|P\rangle \neq 0$. In particular, the only restriction for p_1, p_2 in eq. (2.4) is $p_1 + p_2 = P$. We are free to utilize this arbitrariness for simplifying the investigation of R_5 in the a-representation.

The wave function $\tilde{\phi}$ (see eq. (2.13)) also possesses the pole at $P^2 = m_\pi^2$:

$$\tilde{\phi} = \phi \frac{\chi_P(p_1 - p_2)}{P^2 - m_\pi^2}. \tag{2.19}$$

According to eqs. (2.13-2.17), (2.19) the coefficients a_n are the moments of the function ϕ:

$$a_n = \int_{-1}^{1} \phi(\xi)\xi^n d\xi. \tag{2.20}$$

Eq. (2.18) may be rewritten in terms of the ϕ-functions

$$F^{\lambda\ \ \rho}_{pole\ pole}(Q) = \frac{g^4}{m^2 Q^4} \ln\frac{Q^2 m^2}{\mu^4} \left(\int_{-1}^{1} \frac{\phi(\xi)}{(1-\xi)/2}\right)^2. \tag{2.21}$$

The pole contribution (2.21) is due to integration over the region $\lambda \sim 0$, $\rho \sim 0$. The remaining domain of integration over the α-parameters also yields the leading poles in J as a result of integration over $\rho \sim 0$ (then $\lambda > 1/\mu^2$) or over $\lambda \sim 0$ (then $\rho > 1/\mu^2$). To calculate the corresponding contribution $F^{\lambda}_{pole}\,{}^{\rho}_{reg}$ one has to use the formulas $D = D_0(V_R)D_0(V_L')[1 + O(\lambda)]$ and $R = r/2 = \frac{1}{2}(D_0(V_R) \cdot R_-)$. This gives the following representation for the right half of the diagram

$$\left(\frac{g}{4\pi}\right)^z \int^{\infty} \frac{\prod_\sigma d\alpha_\sigma}{D^2}\left(\frac{1}{2}\left(1 - \frac{R_-}{D_0}\right)\right)^J e^{I(\alpha, m^2)} \tag{2.22}$$

It possesses a pole $(J+2)^{-1}$ due to integration over $\rho \sim 0$. One must subtract this pole and then put $J = -2$. This can be written formally as

$$F^{\lambda}_{pole}\,{}^{\rho}_{reg} = \frac{g^2}{Q^4}\left(\int_{-1}^{1} \frac{\phi(\xi)d\xi}{(1-\xi)/2}\right) \mathrm{Reg}_{\mu^2}\left(\int_{-1}^{1} \frac{\phi(\eta)}{\left(\frac{1-\eta}{2}\right)^2} d\eta\right) =$$

$$= \frac{g^2}{Q^4}\left(2\sum_{n=0}^{\infty} a_n\right) \mathrm{Reg}_{\mu^2}\left(4\sum_{m=0}^{\infty}(m+1)a_m\right). \tag{2.23}$$

Eq. (2.23) has a simple parton interpretation dictated by *fig. 1a*: the function $\phi(\xi)$ describes the "decay" of the pion into two quarks with momenta $\frac{1+\xi}{2}P$ and $\frac{1-\xi}{2}P$, whereas the function $\phi^*(\eta)$ describes the "fusion" of quarks with momenta $\frac{1+\eta}{2}P'$, $\frac{1-\eta}{2}P'$ into pion. Amplitude for a parton subprocess is constructed according to ordinary rules of perturbation theory: it is just the propagators which give the factor $\frac{2}{1-\xi}\left(\frac{2}{1-\eta}\right)^2\frac{1}{Q^4}$. The

sum of eqs. (2.21), (2.23) must not depend on μ. One may always substitute the μ-independent sum $\ln(Q^2m^2/\mu^4) + \mathrm{const}(\mu^2)$ by $\ln(Q^2/M^2)$. This gives

$$F_\pi(Q) = \frac{g^4}{m^2Q^4}\ln\frac{Q^2}{M^2}\left(\int_{-1}^{1}\frac{\phi(\xi)\,d\xi}{(1-\xi)/2}\right)^2. \tag{2.24}$$

It is worth noting that in each order of perturbation theory the contribution into $\tilde{\phi}(\xi)$ behaves like $(1-\xi)$ as $\xi \to 1$. Really, taking $\xi = 1 - \zeta$, we obtain

$$\delta\left(\xi - \frac{R_-}{D_0}\right) = \delta\left(\frac{r - 2R}{D} - \zeta\right).$$

The equality $r - 2R = 0$ may be fulfilled only due to vanishing of at least two α-parameters. For instance, one must take $\alpha_3 = \alpha_4 = 0$ in the diagram shown in *fig. 2c*, because of $r = \alpha_1\tilde{r}$, $R = \alpha_3\tilde{D} + \alpha_4\tilde{R}$. Integrating over α_3, α_4 we obtain that $\tilde{\phi}(\xi) \sim \zeta = 1-\xi$ in the region $\xi \to 1$. Such a behaviour produces a logarithmic divergence of the η-integral in eq. (2.23). This reflects the fact that the right-half contribution has a pole at $J = -2$. Formula (2.24) obtained as a result of singling out this pole is a meaningful expression if the function $\tilde{\phi}(\xi)$ behaves like $1 - \xi$ as $\xi \to 1$, i.e., in each finite order of perturbation theory. However, there exists a possibility that the "full" function $\tilde{\phi}(\xi)$ resulting from the summation over all relevant diagrams does not approach zero as $\xi \to 1$. In this case the ξ-integral (2.24) diverges. The parton interpretation allows one to expect that an account of quark transverse momentum leads to the change $xyQ^2 \to xyQ^2 + (k_\perp - k_\perp')^2$ which provides the convergence of the aforementioned integral. But as a result, there appears an explicit dependence on $\langle k_\perp^2 \rangle$ which characterizes the pion size.

In the $\phi^3_{(4)}$-model we have no a priori suggestions about the behaviour of the wave function $\phi(\xi)$ for ξ close to 1. But in renormalizable theories, e.g., in $\phi^3_{(6)}$, matrix elements of operators (and, hence, the wave function $\phi(\xi)$)) depend on an additional renormalization parameter μ: $\phi(\xi) \to \phi(\xi, \mu^2)$ and it is possible to calculate the limiting form of $\phi(\xi, \mu^2)$ as $\mu^2 \to \infty$.

Dependence of the coefficients a_n on μ, i.e., the anomalous dimensions, is calculated in the standard way /21/, with some obvious modifications In the $\phi^3_{(6)}$-theory we have

$$(\mu \frac{\partial}{\partial \mu} + \beta(g)\frac{\partial}{\partial g}) a_n(\mu^2, g) =$$

$$= \kappa g^2 \{ -a_n + \frac{6}{(n+1)(n+2)} \sum_{k=0}^{n} \frac{1+(-1)^k}{2} a_k \}. \tag{2.25}$$

If one chooses another basis, namely, the conformal one (see the Appendix)

$$K_{\mu_1 \dots \mu_n} = i^n [\psi^* \partial_+^n C_n^{3/2} (2\partial / \partial_+) \psi]_{\{\mu_1 \dots \mu_n\}} \tag{2.26}$$

then the anomalous dimension matrix is diagonalized

$$(\mu \frac{\partial}{\partial \mu} + \beta(g)\frac{\partial}{\partial g}) k_n(\mu^2, g) = \kappa g^2 (-1 + \frac{6}{(n+1)(n+2)}) k_n(\mu^2, g) \equiv$$

$$\equiv \gamma_n(g) k_n(\mu^2, g), \tag{2.27}$$

where

$$<0|K_{\mu_1 \dots \mu_n}|P> = \{P_{\mu_1} \dots P_{\mu_n}\} k_n(\mu^2, g). \tag{2.28}$$

As a result, we have (see eq. (A.9))

$$\phi(\xi, \mu^2) = (1-\xi^2) \sum_{n=0}^{\infty} k_n(\mu^2, g) \frac{n+3/2}{(n+1)(n+2)} C_n^{3/2}(\xi), \tag{2.29}$$

where $k_n(\mu^2, g) \sim (\ln \mu^2)^{-\gamma_n/B}$ (by definition $\beta(g) = -Bg^3 \dots$).

From the representation (2.29) it follows that the term with minimal n (for which $k_n \neq 0$) dominates for $\mu^2 \to \infty$, whereas the contribution from higher harmonics responsible for a "bad" behaviour at $\xi \sim 1$ diminishes with growing μ^2:

$$\phi(\xi, \mu^2) \to (1-\xi^2) C_{n_0}^{3/2}(\xi) (\ln \mu^2)^{-\gamma_{n_0}/B}. \tag{2.30}$$

The coupling constant g in $g\phi^3_{(4)}$-theory has dimension of mass. This results in a "good" behaviour in the ultraviolet region and in a rather "bad" behaviour in the infrared one. In particular, eqs. (2.18), (2.24), are meaningless in the limit $m \to 0$. The limits $m \to 0$ and $Q^2 \to \infty$ do not commute in this case. Large-Q^2 asymptotical behaviour of the form factor in the massless $\phi^3_{(4)}$-theory, in distinction with the theory with $m \neq 0$, is governed by $a_\sigma \to \infty$, $F(a)/D(a) \to 0$ regime (where F, D is the q^2-coefficient in the a-representation eq. (2.6)). Note, that by construction F as well as D are linear functions of any chosen a_σ-parameter

$$F(a) = a_\sigma f_\sigma(a) + \phi_\sigma(a),$$

$$D(a) = a_\sigma d_\sigma(a) + \delta_\sigma(a). \tag{2.31}$$

Furthermore, the D-function contains the parameters of all the lines of the diagram (we consider now only 1-particle irreducible diagrams) i.e., $d_\sigma(a) \neq 0$ for any line σ, whereas it is possible that $f_\sigma(a) \equiv 0$ for some line σ. Really, the function $F(a)$ is a sum of products of $a_{\sigma_1} \dots a_{\sigma_k}$ type, where the lines $\sigma_1 \dots \sigma_k$ are subjected to the requirement that after removing these lines we obtain a 2-tree, and the square of the total momentum k entering into each component of the 2-tree is large: $k^2 \sim O(Q^2)$. As a result, the function $F(a)$ does not include the a_{σ_i}-parameter, if for any 2-tree obtained after removing (among others) the σ_i-line, we have k^2 small (i.e., k^2 does not contain the $O(Q^2)$ term). In other words the diagram looses the Q-dependence after removing the σ_i-line. In the configuration *fig. 2b* such a line is σ_5. Hence in the massless $\phi^3_{(4)}$-theory the integration of the expression

$$\int_0^\infty \frac{d a_5}{(a_5 d_5(a) + \delta_5(a))^{2+J}} (\phi_5(a))^J \exp i \frac{Q(a, p)}{a_5 d_5 + \delta_5} \tag{2.32}$$

over the $a_5 \to \infty$ region gives a pole at $J = -1$, and the diagram *2b* gives $O(1/Q^2)$ contribution. The limit $a_5 \to \infty$ corresponds to a zero-momentum flow through the σ_5 -line, i.e., to the Feynman mechanism. It follows from the representation (2.32) that the only possibility to damp such an "infrared" contribution is to introduce the non-zero mass for a particle corresponding to the σ_5 -line.

The infrared regime $\alpha_i \to \infty$ can contribute also when $f_i(\alpha) \neq 0$, if one performs a simultaneous integration over $\alpha_i \to \infty$ and over the region of the α-parameters space where $f_i/d_i \sim 0$. The meaning of such a combined integration is easily demonstrated without using the Mellin transformation

$$\int_0^\infty \frac{d\alpha_i \, \Pi \, d\alpha_\sigma}{(\alpha_i d_i + \delta_i)^2} \exp[-iQ^2 \frac{f_i \alpha_i + \phi_i}{d_i \alpha_i + \delta_i} + I] \sim$$

$$\sim \frac{1}{Q^2} \int_0^\infty \frac{\Pi d\alpha_\sigma}{d_i \phi_i - \delta_i f_i} \exp[iQ^2 \frac{f_i}{d_i} + \tilde{I}]. \qquad (2.33)$$

The Q^2-dependence is trivial only if $f_i(\alpha) = 0$. For $f_i(\alpha) \neq 0$ the r.h.s. of eq. (2.33) has a form similar to the ordinary α-representation, and the large-Q^2 behaviour is governed by the region, where $f_i/d_i \sim 0$. This can be realized either by $\lambda_V \to 0$ regime, when the α-parameters which correspond to the lines of some subgraph V tend to zero (the subgraph V must be t-subgraph for the diagram with the line σ_i removed), or by $\alpha_k \to \infty$, $f_i(\alpha)/d_i(\alpha) \to 0$ regime. In the second case $f_i/d_i = 0$ may be provided either by the fact that after removing the lines σ_i, σ_k the diagram loses the Q^2-dependence, or by integration over the region where $f_{ik}/d_{ik} \sim 0$, and so on, until all the possibilities will be exhausted.

For example, *fig. 3a* gives a contribution into the form factor of a quark participating in the Feynman process. *Fig. 3b* describes a modification of the Feynman process when one has several wee partons. (The dashed lines correspond to the $\alpha \to \infty$ integration). *Fig. 3c* describes a situation when the hard scattering process is accompanied by the wee parton exchange between initial

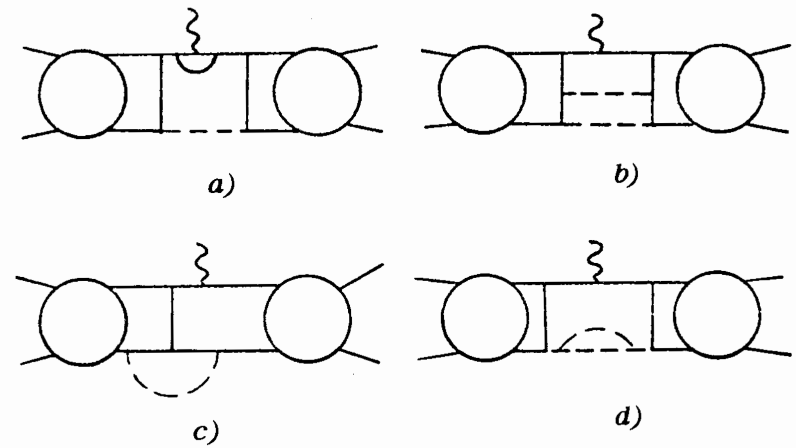

Fig. 3

and final states. In the $\phi^3_{(4)}$-model, however, all these contributions give the corrections which vanish more rapidly than $1/Q^2$. Hence, one may neglect them. On the other hand, higher order corrections result in a more singular behaviour of the propagator $D^c(k)$ in the small-k^2 limit: $D^c(k) = 1/k^2 - (g^2/k^4)\ln k^2 + \ldots$. As a consequence, the integration over $\rho \to \infty$ for the diagram *3d* (where $\rho = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$) gives $F \sim$ const contribution, whereas the diagrams containing the $(g^2)^n$-order corrections to the propagator, may give as large contribution as $F_\pi \sim g^{2n}(Q^2)^{n-1}$. It is very probable that the expansion for $D^c(k)$ is a series expansion in g^2 for an expression like $(k^2 + g^2 L(k^2))^{-1}$. If this is true, then the rising powers of Q^2 correspond to the expansion like

$$\frac{1}{Q^2} \frac{1}{1 + g^2 Q^2/M^4} = \frac{1}{Q^2} \sum_{n=0}^{\infty} (-\frac{g^2 Q^2}{M^4})^n, \qquad (2.34)$$

where $M^2(g^2, Q^2)$ is a slowly varying function. As a result, the sum of infrared contributions behaves asymptotically like $1/Q^4$, i.e., just like the $m \neq 0$ formula (2.24). This is natural because adding $g^2 L$ is equivalent to introducing the effective k^2-dependent mass for a quark. Hence, the result (2.24) can be justified in this sense, but one must assume now that the mass m entering into eq. (2.24) is the effective quark mass averaged over the pion volume, i.e., a phenomenological constant accumulating the (nonperturbative) large-distance contribution. This parameter reflects the dynamical cut-off for integrals over large α (or small k) at $1/m^2$ as if the system were enclosed into the bag having a radius $1/m$. These reasonings, of course, pretend only to a rough qualitative description of such theories where the behaviour of the exact propagator is less singular at $k\to0$ than dictated by perturbation theory. It is quite possible that QCD is a theory of this type. But, as we will see later, the situation in QCD is simpler: infrared contributions cancel out in the leading term for colourless pion form factor.

3. SCALAR GLUON MODEL

Renormalizable theories are more singular than the $\phi^3_{(4)}$-model in the ultraviolet region, but less singular in the infrared one. For example, in $\phi^3_{(6)}$-theory, the terms rising with Q^2 are absent: all infrared contributions behave like Q^{-4} modulo logarithms. Small-λ_V integration gives also Q^{-4} contribution for any t-subgraph V having 5 external lines. There appear also logarithms due to ultraviolet divergences of diagrams inherent to renormalizable theories. The Born term $(1-\xi)^{-1}(1-\eta)^{-2}$ behaves just like in the $\phi^3_{(4)}$-model. As a result, both "left" and "right" subgraphs may contribute simultaneously in the $\phi^3_{(6)}$-model. This leads, in particular, to $\ln^2 Q^2$-behaviour of the one-loop diagram.

Fortunately, the amplitude of the leading parton subprocess (i.e., of those giving $O(1/Q^2)$ contribution) in theories with the spin-$1/2$ quarks in 4-dimensional space-time is as singular for $y=(1-\xi)/2\to0$ as $1/y$ modulo logarithms y. This follows from simple dimensional considerations. Really, taking into account that both the amplitude T and the propagator S (*fig. 4a*)

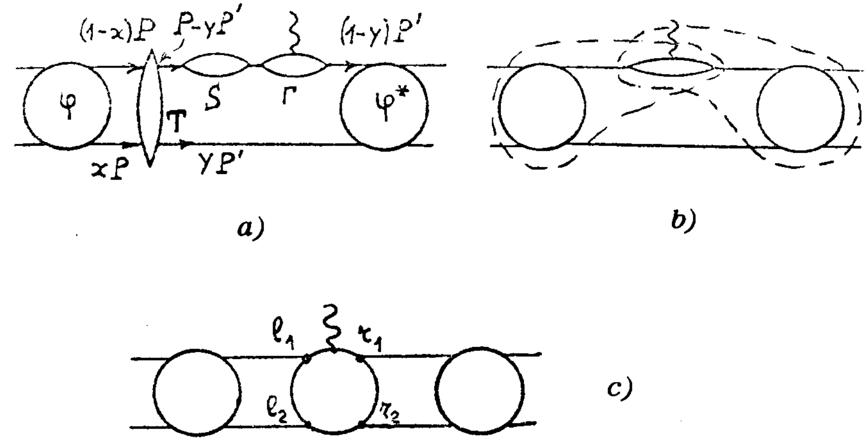

Fig. 4

depend on the momentum P' only through the product yP' we obtain

$$T\sim\gamma_5\,\frac{\alpha\hat{P}+\beta(y\hat{P}')}{y(P'P)};\quad S\sim\frac{\tilde{\alpha}\hat{P}+\beta(y\hat{P}')}{y(P'P)}. \qquad (3.1)$$

This gives $TS\Gamma\sim1/y$ for $y\to0$, since the vertex function $\Gamma(q,p_1,p_2)=\Gamma(P'-P,P'-yP,(1-y)P')$ may possess only logarithmic singularities as $p_1^2/Q^2\to0$. Thus, for spin-$1/2$ quarks only one subgraph in the configuration *4b* may produce a pole at $J=-1$, whereas the second one gives the contribution which is regular at $J=-1$. Henceforth it is always assumed that the form factor $F_\pi(Q)$ is given by

$$F_\pi(Q)=\frac{\langle P'|P'_\mu J^\mu(0)|P\rangle}{m_\pi^2+(PP')}. \qquad (3.2)$$

Hence, the contributions which must be taken into account in the Yukawa type theory (i.e., for scalar gluons) are those shown in *fig. 4c*. We have also taken into account that the estimate (2.7) for theories with dimensionless coupling constants is

$$F_\pi^{(V)}(Q)\lesssim Q^{2-\Sigma t_i} \qquad (3.3)$$

(see the Appendix to paper I), where the sum is taken over the external lines of the t-subgraph V, excluding the photon one; t_i is the twist of the field describing the i-th external line (remind, that $t = 1$ for particles with spin equal to 0 or 1/2, and $t = 0$ for a vector field in the Feynman gauge).

The $\alpha_\sigma \to \infty$ regime gives $O(1/Q^4)$ contribution for lines σ corresponding to spin-1/2 particles. This allows one to neglect the configurations shown in *fig. 3a,b,d*. The configuration of *fig. 3c* gives $O(1/Q^2)$ only if the wee parton is a vector particle. Our proof of these statements is based on the α-representation analysis of all possible combinations of the preexponential factors. We will not present it here because it is rather lengthy whereas the final result - that the contribution of the infrared domain ($\alpha \to \infty$, or $k \to 0$) is not damped only for vector particles - is well-known (see, e.g., ref. [22]).

We consider first the scalar gluon model. To factorize the contribution of spinor numerators one must use the Fierz identity

$$\delta_{\alpha'}^{\alpha} \delta_{\beta}^{\beta'} = \sum_i (\Gamma_i)_{\beta}^{\alpha} (\Gamma_i)_{\alpha'}^{\beta'}; \quad i = S, V, T, A, P. \tag{3.3}$$

The S-, V-, T-projections give bilocal operators which have zero matrix elements: $\langle 0|O_i|P\rangle$. The A-projection gives factors $\hat{P}, \hat{P}'$ which combine into an additional factor $(PP') \approx Q^2/2$ absent for the P-projection. That is why the axial projection is responsible for the leading contribution. We write it in the coordinate representation (see *fig. 4c*):

$$\int d\ell_1 d\ell_2 dr_1 dr_2 E_{\rho\lambda}(\ell_1, \ell_2, r_1, r_2; 0, \mu^2) \times$$
$$\times \langle 0|\mathcal{O}^{\lambda}(\ell_1, \ell_2; \mu^2)|P\rangle\langle P'|\mathcal{O}^{\rho}(r_1, r_2; \mu^2)|0\rangle. \tag{3.4}$$

The parameter μ^2 characterizes the subtraction procedure which is constructed just in the same way as it was done for massive lepton-pair production process in II. It provides the necessary infrared regularization of the E-functions and the recipe of the renormalization for

local operators $O_{\rho\rho_1 \ldots \rho_n}$ which result after the Taylor expansion

$$\langle P'|\mathcal{O}_{\rho}(r_1, r_2; \mu^2)|0\rangle = e^{-iP'\frac{r_1+r_2}{2}} \cdot \frac{1}{n!} \times$$

$$\times \langle P'|O_{\rho\rho_1\ldots\rho_n}(0, \mu^2)|0\rangle (r_1 - r_2)^{\rho_1} \ldots (r_1 - r_2)^{\rho_n}. \tag{3.5}$$

Integrating over ℓ_i, r_i gives

$$F_\pi(Q) = \frac{1}{Q^2}\{\sum_{m,n=0}^{\infty} f_m^*(\mu^2) E_{mn}(\frac{Q^2}{\mu^2}, g(\mu)) f_n(\mu^2) + O(1/Q^2)\}, \tag{3.6}$$

where f_n are defined by

$$(2i)^n \langle 0|\bar{\psi}\gamma_5\{\gamma_\mu \partial_{\mu_1} \ldots \partial_{\mu_n}\}\psi|P\rangle = f_n(\mu^2)\{P_\mu P_{\mu_1} \ldots P_{\mu_n}\}. \tag{3.7}$$

The contributions from higher twist operators have additional factors $(M/Q)^{t_i-2}$, where M is a characteristic scale inherent to the matrix element of a higher twist operator

$$\langle 0|O^{(i)}_{\mu_1\ldots\mu_n}|P\rangle = \{P_{\mu_1} \ldots P_{\mu_n}\} M^{t_i-2} b_n^{(i)}(\mu). \tag{3.8}$$

Eq. (3.6) resembles the expansion for the virtual Compton amplitude

$$T(\omega, Q^2) = \sum_{n=0}^{\infty} \omega^n \frac{1+(-1)^n}{2}\{E_n(\frac{Q^2}{\mu^2}, g) A_n(\mu^2) + O(M^2/Q^2)\} \tag{3.9}$$

for $\omega = 1$. It is well-known [23] that the terms denoted as $O(M^2/Q^2)$ give for $\omega \to 1$ the contribution which exceeds the scaling term. The corrections $O(M^2/Q^2)$ are in this case responsible for the resonance structure in the region where $\sqrt{s} = \sqrt{(\omega-1)Q^2 - m_N^2}$ is close to the masses of the low-lying resonances. In the kinematical situation characteristic for the form factor problem, we have no large time-like invariant variable, and the presence of resonances having photon quantum numbers and the ρ-dominance indicate the change $1/Q^2 \to 1/(Q^2 + m_\rho^2)$. The contributions $O(M^2/Q^2)$ correspond to the nonzero value of the primordial transverse momentum of partons, or (as argued earlier) to the change $xy \to xy + O(k_\perp^2/Q^2)$ for integrals over x, y. In both cases the corrections $O(M^2/Q^2)$ may be neglected for Q^2 sufficiently large. The detailed treatment of the higher twist contributions, of course, would be very useful for understanding the behaviour of the form factor at moderately large Q^2. In the present paper, however, we will concentrate only on the contribution of operators having minimal twist.

The wave functions $\phi(\xi, \mu^2)$ may be introduced in the same way as in $\phi^3_{(6)}$-model (with the change $\psi^* \partial^n \psi \to \bar{\psi} \gamma_5 \gamma_\mu \partial^n \psi$).). The function E_{nm} for $\mu = Q$ is given by a series expansion in $\bar{g}^2(Q)$ whereas all the logarithmic corrections are absorbed by the wave functions $\phi(\xi, Q^2)$. *Fig. 5a* represents the Born approximation $E_{mn} = -g^2$. This results in

$$F_\pi(Q) = -\frac{\bar{g}^2(Q)}{Q^2}\left(\int_{-1}^{1} \frac{\phi(\xi, Q^2)}{1-\xi^2} d\xi\right)^2 \{1 + O(\bar{g}^2)\}. \tag{3.10}$$

We have used here the fact that only operators with even number of derivatives have nonzero matrix element $\langle 0 | \bar{\psi} \gamma_5 \gamma_\mu \overleftrightarrow{\partial}^n \psi | P$ due to parity conservation. Hence, the function $\phi(\xi)$ is symmetric $\phi(\xi) = \phi(-\xi)$. The change $(1-\xi)^{-1} \to [(1-\xi)^{-1} + (1+\xi)^{-1}]/2$ gives $(1-\xi^2)^{-1}$ in (3.10).

The dependence of $\phi(\xi, Q^2)$ on Q^2 is given by the formula

$$\phi(\xi, Q^2) = (1-\xi^2) \sum_{n=0}^{\infty} k_n(\mu_0^2) \frac{1+(-1)^n}{2} \frac{n+3/2}{(n+1)(n+2)} C_n^{3/2}(\xi) \times$$

$$\times \exp\left(\int_{\mu_0}^{Q} \gamma_n(\bar{g}^2(\lambda)) \frac{d\lambda}{\lambda}\right), \tag{3.11}$$

where $\gamma_n = -\frac{g^2}{16\pi^2}\left(1 + \frac{2}{(n+1)(n+2)}\right)$ (see the Appendix).

The $\bar{\psi}\psi\phi_{(4)}$-theory has a range of unpleasant properties. First, the coupling constant $\bar{g}(Q)$ grows with growing Q (null-charge situation). Second, eq. (3.10) predicts that the form factor is negative for large Q. This probably indicates that the repulsion dominates in the $q\bar{q}$-system, because in the nonrelativistic approximation the EM form factor is positive for $q^2 < 0$ /24/ if the potential is attractive. Hence, there arises a question about the very existence of the bound state. Third, the anomalous dimensions γ_n approach their limiting value from below, i.e., the contribution of higher harmonics responsible for a "bad" behaviour at $\xi \to 1$ is enlarged with growing Q. This also indicates that there are no bound states in the $q\bar{q}$-system.

4. *QUANTUM CHROMODYNAMICS*

The leading t-subgraph may possess in vector gluon theories an arbitrary number of external gluon lines because the vector gluon field has zero twist (in the Feynman gauge). Hence one must sum over the gluons participating in the parton subprocess. In III we have developed a rather simple technique of such a summation. The diagrams describing the cross section for the $AB \to \mu^+\mu^- X$ process have the structure analogous to the diagrams describing the pion EM form factor, and we will not repeat the reasonings presented in III. Rather, we formulate only the final result that the summation over the gluons in configurations shown in *fig. 5b* gives (for the colourless pion) the gauge-invariant bilocal operators

$$\mathcal{O}_\nu(\ell_1, \ell_2; \mu^2) = N_{\mu^2}[\bar{\psi}(\ell_1)\gamma_5\gamma_\nu T_c \exp(ig \int_{\ell_2}^{\ell_1} \hat{A}_\mu(z)dz^\mu)\psi(\ell_2)] \tag{4.1}$$

in place of the operator $\bar{\psi}(\ell_1)\gamma_5\gamma_\mu\psi(\ell_2)$. In the configurations *5b* we also obtain the contributions which correspond to operators containing the gluon field tensor $G_{\mu\nu}$. These operators have a higher twist and, as a consequence, they give power corrections $\sim \frac{1}{Q^2}(\frac{M^2}{Q^2})^N$.

We have stressed earlier that in theories describing massless vector particles one can obtain the pole at $J = -1$ by a simultaneous integration over $\lambda_V \to 0$, $\alpha_{\sigma_1}, \dots, \alpha_{\sigma_n} \to \infty$; where V is some subgraph which becomes a leading t-subgraph after removing the lines $\sigma_1, \dots, \sigma_n$, corresponding to massless vector particles (cf. ref. /18/). Such a configuration (*fig. 5c*) describes the wee gluon exchange between the initial and final state. These exchanges spoil the factorization we have observed studying the scalar gluon model (see eq. (3.6)). But, as it was argued in III (cf. also with the results of ref. /25/) the wee exchanges give power corrections ($O(M^2/Q^2)$ rather than logarithmic ones ($\sim \ln Q^2/p^2$, where p^2 is the parameter which is responsible for an infrared regularization), provided that only colourless particles are present in the initial and final states. Note that the elementary colourless particles are implied rather than colourless bound states. But the choice of fundamental fields describing the $\bar{q}q$ system in the auxiliary Green function (2.2) is rather arbitrary. If we substitute $\bar{\psi}(a_1)\gamma_5\psi(a_2)$ by the product of colourless currents $j(a_1)j_5(a_2)$, where $j = \bar{\psi}\psi$, $j_5 = \bar{\psi}\gamma_5\psi$ then the requirement formulated above is fulfilled. This corresponds to the transition from $\bar{\psi}(a_1)\gamma_5\psi(a_2)$ to the following gauge-invariant superposition of quark and gluon fields (*fig.* 6)

$$C(\bar{\psi}, \psi, A) = \bar{\psi}(a_1)\gamma_5 \mathcal{S}^c(a_1, a_2)\psi(a_2), \tag{4.2}$$

where $\mathcal{S}^c(a_1, a_2)$ coincides formally with the quark propagator in an external gluon field (see III)

$$\mathcal{S}^c(a_1, a_2) = S^c(a_1 - a_2) + g\int d^4\xi\, S^c(a_1 - \xi)\gamma^\mu \hat{A}_\mu(\xi) S^c(\xi - a_2) + \dots =$$

$$= T_c \exp(ig\int_{a_2}^{a_1} \hat{A}_\mu dz^\mu)[S^c(a_1 - a_2) + O(G_{\mu\nu})], \tag{4.3}$$

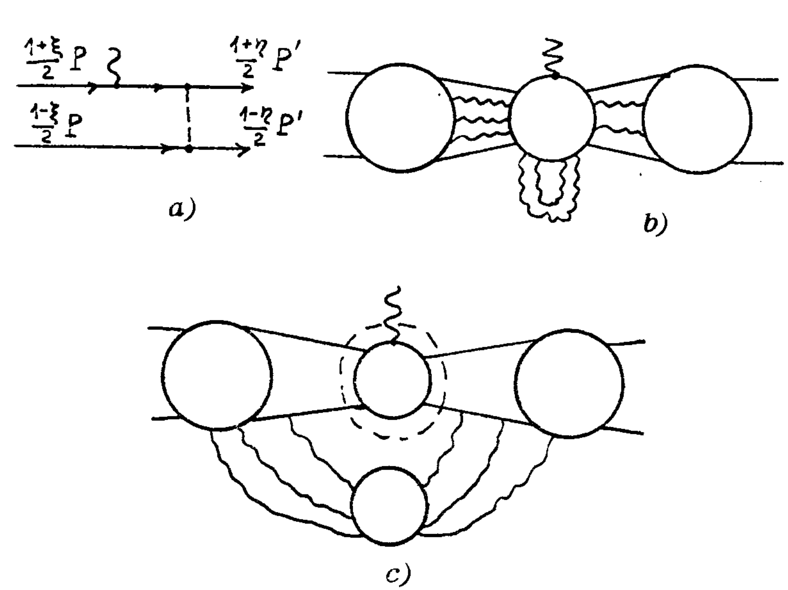

Fig. 5

where $\hat{A}_\mu = \tau_a A^a_\mu$, and τ_a is the gauge group matrix in the fundamental representation. Note, that up to $O(G_{\mu\nu})$ term and a numerical factor $S^c(a_1 - a_2)$, this change corresponds to the description of the pion by the gauge-invariant combination

$$\tilde{C}(\bar{\psi}, \psi, A) = \bar{\psi}(a_1)\gamma_5 T_c \exp(ig\int_{a_2}^{a_1} \hat{A}_\mu dz^\mu)\psi(a_2). \tag{4.4}$$

It was argued in ref. /26/ that if one assumes the quark confinement, then $\langle 0 | \bar{\psi}(a_1)\gamma_5\psi(a_2) | P \rangle = 0$, but $\langle 0 | \tilde{C}(\bar{\psi}, \psi, A) | P \rangle \neq 0$ nevertheless, and only $\tilde{C}$ is the right combination to be used as a pion interpolating field.

The gauge-invariant bilocal operators $\mathcal{O}_\nu(\xi, \eta; \mu^2)$ may be expanded in the ordinary way into Taylor series over the local gauge-invariant operators (see III)

$$O_{\nu\nu_1 \dots \nu_m}(0, \mu^2) = N_{\mu^2} \bar{\psi}(0)\gamma_5 \{\gamma_\nu \overleftrightarrow{D}_{\nu_1} \dots \overleftrightarrow{D}_{\nu_m}\} \psi(0) \tag{4.5}$$

Thus, the representation (3.6) is valid in QCD also. As usual, $D_\mu = \partial_\mu - ig\hat{A}_\mu$ denotes in eq. (4.5) the covariant derivative acting on the quark field.

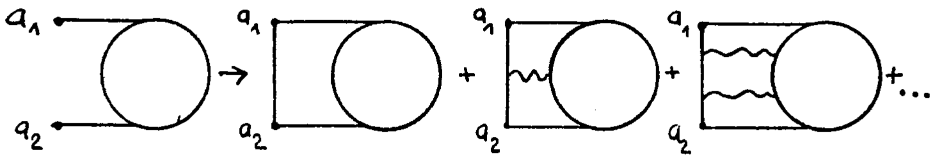

Fig. 6.

The matrix element of operators (4.5) depends on the renormalization parameter μ, as $(g^2 \ln\mu)^N$. hence, the validity of the representation (3.6) means that the double-logarithmic contributions $(g^2 \ln^2 Q^2/\mu^2)^N$ which appear in some diagrams, are cancelled after summation over all diagrams of the given order (cf. ref. /27/).

We emphasize that it is just the colourlessness of the pion that is responsible for cancellation of the wee gluon exchanges which spoil the factorization, as well as for cancellation of the double logs. If one considers the form factor of the coloured particle, no cancellation will be observed (cf. III, part 3).

Further analysis proceeds in the same way as in sections 2,3. The change $\partial_\mu \to D_\mu$ does not affect the conformal property of a tensor in the free-field approximation*. It is natural, hence, to expect that in the conformal basis

$$K_{\mu\mu_1\ldots\mu_n} = \bar\psi\gamma_5\{\gamma_\mu(\partial_+^n C_n^{3/2}(2D/\partial_+))_{\mu_1\ldots\mu_n}\}\psi \qquad (4.6)$$

the matrix of anomalous dimensions is diagonal in the one-loop approximation. Straightforward (but cumbersome) calculations support this view.

The parton wave functions $\phi(\xi,\mu^2)$ satisfy the very specific normalization condition

$$iP_\nu \int_{-1}^{1} \phi(\xi,\mu^2)d\xi = \langle 0|\bar\psi\gamma_5\gamma_\nu\psi|P\rangle = iP_\nu f_\pi \qquad (4.7)$$

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* We are grateful to A.A.Vladimirov for clarifying this point.

because matrix element of the axial current is known from the decay $\pi\to\mu\nu$: $f_\pi = 132$ *MeV*. This property (in a rather different way) was utilized also in refs. /7,8,28/. Eq. (4.7) holds for all μ^2, because the axial current has zero anomalous dimension. It is worth-while to introduce dimensionless functions $a(\xi,\mu^2) = \phi(\xi,\mu^2)/f_\pi$. Then

$$a(\xi,Q^2) = (1-\xi^2)\sum_{n=0}^{\infty}\frac{1+(-1)^n}{2}\,\frac{n+3/2}{(n+1)(n+2)}k_n(\mu_0^2)C_n^{3/2}(\xi)\times$$
$$\times(\ln(Q_0^2/\Lambda^2)/\ln(Q^2/\Lambda^2))^{\gamma_n/B}, \qquad (4.8)$$

where

$$\gamma_n = c_F\Big(1-\frac{2}{(n+1)(n+2)}+4\sum_{j=2}^{n+1}\frac{1}{j}\Big);\ B = 11-\frac{2}{3}N_f, \qquad (4.9)$$

$c_F = 4/3$ and N_f is the number of quark flavours. The coefficients $k_n(Q_0^2)$ are defined by

$$k_n(Q_0^2) = \int_{-1}^{1} a(\xi,Q_0^2)C_n^{3/2}(\xi)d\xi. \qquad (4.10)$$

Now we can express the pion form factor in terms of the wave functions

$$F_\pi(Q) = \frac{1}{Q^2}\int_{-1}^{1}d\xi\int_{-1}^{1}d\eta\,\phi^*(\eta,\mu^2)E(\xi,\eta;\frac{Q^2}{\mu^2},g)\phi(\xi,\mu^2). \qquad (4.11)$$

Taking $\mu = Q$ and using the Born approximation for E

$$E(\xi,\eta;1,\bar g) = \frac{2\bar g^2}{(1-\xi)(1-\eta)}\cdot\frac{c_F}{N_c}\{1+O(\bar g^2)\}, \qquad (4.12)$$

where $N_c = 3$, we obtain the final expression for the asymptotical behaviour of the pion form factor in QCD /19/

$$F_\pi(Q) = 8\pi\alpha_s(Q)\frac{f_\pi^2}{Q^2}\frac{c_F}{N_c}(\gamma(Q))^2\{1+O(\alpha_s(Q))\}. \qquad (4.13)$$

The function $\gamma(Q)$ is given by

$$\gamma(Q) = \frac{3}{2}+\sum_{n=2}^{\infty}\frac{1+(-1)^n}{2}k_n(Q_0^2)\frac{2n+3}{(n+1)(n+2)}\Big(\frac{\ln Q_0^2/\Lambda^2}{\ln Q^2/\Lambda^2}\Big)^{\gamma_n/B}. \qquad (4.14)$$

Only the first term in the r.h.s. of eq. (4.14) remains in the limit $Q^2 \to \infty$, and we obtain eq. (1.3). Eq. (1.3) is nothing but the result of substituting the Born approximation (4.12) for E_3 and the limiting form of the wave function $a(\xi,\infty) = \frac{3}{4}(1-\xi^2)$ into eq. (4.11). One can find the limiting form in a rather simple way, namely, solving the equation

$$\sum_{n'=0}^{n} z_{nn'} a_{n'} = 0, \tag{4.15}$$

where $z_{nn'}$ is the anomalous dimension matrix, and using the normalization condition (4.7). The limiting curve $a(\xi,\infty) = \frac{3}{4}(1-\xi^2)$ is analogous to the limiting form $f(x,\infty) \sim \delta(x)$ of the parton distribution functions. It is well-known that at accessible energies the functions $f(x,Q^2)$ differ strongly from its limiting form. The wave function $a(\xi, Q^2)$ at moderately large Q^2 may also differ from $\frac{3}{4}(1-\xi^2)$. Let us examine, however, what predicts eq. (1.3) if one interpolates it into the region $Q^2 = 2 \div 4$ GeV^2. If one takes the ordinary QCD formula $\alpha_s = 4\pi/9\ln Q^2/\Lambda^2)$ with $\Lambda \simeq 0.5$ GeV, then the curve (1.3) crosses the curve $F_\pi^{(\rho)} = (1 + Q^2/m_\rho^2)^{-1}$ (which is in agreement with experimental data) approximately at $Q^2 = 2$ GeV^2. For $Q^2 > 2$ GeV^2 the curve (1.3) goes lower, mainly due to decrease of the coupling constant $\alpha_s(Q)$. Anyway, the asymptotical formula (1.3) predicts a magnitude of the right order for $F_\pi(Q)$ in the region $Q^2 \gtrsim 1$ GeV^2, and this indicates that a better agreement between the QCD predictions and experimental data can be achieved by using the wave function that differs from $a(\xi,\infty)$, and also by taking into account some higher twist operators and next order corrections in $\alpha_s(Q)$ for the E-function.

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APPENDIX

We investigate here the diagonalization of the anomalous dimension matrix. First we consider $\phi^3_{(6)}$-theory. The anomalous dimensions in this theory are given by eq. (2.26). To simplify the calculations, it makes sense to consider the operators $\psi^* \vec{\partial}^n \psi$, with the derivative $\vec{\partial}$ rather than $\overleftrightarrow{\partial}$. The sum in eq. (2.25) runs then over all k, not only over even k. In terms of these operators

$$(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}) \langle 0|\psi^* \vec{\partial}^n \psi|P\rangle = \sum_{k=0}^{n} z_{nk} \langle 0|\partial^{n-k}(\psi^* \vec{\partial}^k \psi)|P\rangle, \tag{A.1}$$

where

$$z_{nk} = \kappa g^2 \{-\delta_{nk} + \frac{6}{(n+1)(n+2)}\theta_{nk}\}; \quad \theta_{nk} = \begin{cases} 1 & n \geq k \\ 0 & n < k \end{cases}. \tag{A.2}$$

The eigenvalues of the matrix z_{nk} are given by its diagonal elements $\lambda^{(i)} = \kappa g^2(-1 + 6/(i+1)(i+2))$, because z_{nk} is a triangular matrix. Hence, we must find the vectors k_n

$$k_n = \sum_{m=0}^{\infty} d_{nm} a_m \tag{A.3}$$

which satisfy the equation

$$(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}) k_n = \lambda^{(n)} k_n. \tag{A.4}$$

Using the explicit form of $\lambda^{(n)}$, we obtain the equation for d_{nm}.

$$\sum_{\ell=m}^{\infty} \frac{d_{n\ell}}{(\ell+1)(\ell+2)} = \frac{d_{nm}}{(n+1)(n+2)}. \tag{A.5}$$

The form of this equation is determined by the structure of the coefficient in front of θ_{nk}. Subtracting from (A.5) the equation for $d_{n,m+1}$ gives the recurrent relation, from which it follows that

$$d_{nm} = (-1)^m \frac{(m+n+2)!}{m!(m+1)!(n-m)!} d^{(n)}, \qquad (A.6)$$

where $d^{(n)}$ is an arbitrary normalization constant. We choose it in such a way that the multiplicatively renormalizable operators have the following form

$$K_{\mu_1 \ldots \mu_n} = \sum_{m=0}^{n} \frac{(m+n+2)!(-1)^m}{2m!(m+1)!(n-m)!} \partial^{n-m} (\psi^* \vec{\partial}^m \psi)_{\{\mu_1 \ldots \mu_n\}} =$$

$$= (\psi^* \partial_+^n C_n^{3/2} (2\overleftrightarrow{\partial}/\partial_+)\psi)_{\{\mu_1 \ldots \mu_n\}}, \qquad (A.7)$$

where $C_n^{3/2}(x)$ are Gegenbauer polynomials (see ref. /29/, formula (10.9.20)); $2\overleftrightarrow{\partial} = \vec{\partial} - \overleftarrow{\partial}$, $\partial_+ = \vec{\partial} + \overleftarrow{\partial}$; $\partial_+^n(\partial/\partial_+)^k = \partial_+^{n-k} \partial^k$. The tensors $K_{\mu_1 \ldots \mu_n}$ are conformal in the free-field approximation /30/. In a somewhat different context the conformal invariance was widely used in the earlier studies of asymptotic properties of the form factors /2-4,31/.

Matrix elements of the operators $K_{\mu_1 \ldots \mu_n}$, according to eq. (A.7), are related to $\phi(\xi)$ in the following way

$$\int_{-1}^{1} \phi(\xi, \mu^2) C_n^{3/2}(\xi) d\xi \{P_{\mu_1} \ldots P_{\mu_n}\} =$$

$$= \langle 0 | K_{\mu_1 \ldots \mu_n} | P \rangle \equiv k_n(\mu^2) \{P_{\mu_1} \ldots P_{\mu_n}\}. \qquad (A.8)$$

The polynomials $C_n^{3/2}(\xi)$ are orthogonal on the segment (-1.1) with the weight $(1-\xi^2)$. Taking into account their normalization /29/ gives

$$\phi(\xi, \mu^2) = (1-\xi^2) \sum_{n=0}^{\infty} k_n(\mu^2) \frac{n+3/2}{(n+1)(n+2)} C_n^{3/2}(\xi). \qquad (A.9)$$

In the $\bar{\psi}\psi\phi_{(4)}$-theory the anomalous dimension matrix for operators $\bar{\psi}\gamma_5\gamma_\mu \vec{\partial}^n \psi$ has the following form

$$z_{nk} = -\frac{g^2}{16\pi^2}(\delta_{nk} + \frac{2}{(n+1)(n+2)} \theta_{nk}). \qquad (A.10)$$

The coefficient in front of θ_{nk} has the same structure as in the $\phi^3_{(6)}$-model. Hence, the conformal tensors

$$\bar{\psi}\gamma_5 \{\gamma_\mu \partial_+^n C_n^{3/2} (2\partial/\partial_+)_{\mu_1 \ldots \mu_n}\} \psi \qquad (A.11)$$

are multiplicatively renormalizable.

In QCD we have obtained for the operators $\bar{\psi}\gamma_5\gamma_\mu \vec{D}^n \psi$ that $z_{nk} = z_{nk}^{(1)} + z_{nk}^{(2)}$, where

$$z_{nk}^{(1)} = c_F \frac{g^2}{8\pi^2}\{-\delta_{nk} + \frac{1}{(n+1)(n+2)} \theta_{nk}\}, \qquad (A.12)$$

$$z_{nk}^{(2)} = c_F \frac{g^2}{8\pi^2}[-4\delta_{nk} \sum_{j=2}^{n+1} (\frac{1}{j}) + 2(\frac{1}{n-k} - \frac{1}{n+1})(\theta_{nk} - \delta_{nk})]. \qquad (A.13)$$

The term $z_{nk}^{(2)}$ corresponds (in the Feynman gauge) to the prolongation of the derivative.

To find the limiting form of the wave function from the equation $\sum z_{nk} a_k = 0$ one may use the following trick: one solves first the equation $\sum z_{nk}^{(1)} a_k = 0$ and then sees that the solution obtained satisfies also the equation $\sum z_{nk}^{(2)} a_k = 0$. This reflects the fact that in the free field approximation the prolongation of the derivative does not affect the conformal properties of a tensor, i.e., it does not change the structure of eq. (4.8). The account of D changes only the magnitude of anomalous dimensions for $n \geq 1$. This explains the coincidence between eq.(1.3) and the result of ref. /7/ obtained in the ladder approximation where only the term $z_{nk}^{(1)}$ works.

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