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Плотность в релятивистской модели столкновения тяжелых ионов как системы двух невзаимодействующих газов

Предлагается релятивистская модель, описывающая столкновение тяжелых ионов как систему из двух невзаимодействующих газов, проникающих друг в друга. Оказывается, что плотность числа частиц зависит от энергии пучка и ее величина больше, чем простая сумма собственных плотностей снаряда и мишени.

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Density in a Relativistic Heavy Ion Model System Consisting of Two Freely Interpenetrating Nuclei

The relativistic determination of the density in a heavy ion model system consisting of two freely interpenetrating nuclei is presented. It is pointed out that the density becomes beam-energy-dependent and, thus, is increased beyond the sum of the rest densities of projectile and target.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

The increasing availability of high-energy heavy-ion projectiles made the production of nuclear matter under extreme conditions an experimental possibility, and the idea to observe new phenomena associated with pionic instabilities rapidly gained attention. One of the most important parameters for the possible onset of these collective phenomena is the value of attainable particle number density ρ during the heavy ion reaction process. But the problem is how to produce dense regions. At present only two mechanisms for getting an increase in density are known: i) incoherent interacting nucleons which make a basis for microscopic scattering models as the hydrodynamical model, the intranuclear cascade model on the Monte-Carlo basis or the model using classical equations of motion and ii) shock waves assumed a priori.

First attempts have been done to calculate the time-dependent density evolution in the framework of both the first models (Gudima et al.[1], Amsden et al.[2]) mentioned under i). Usually the extracted density ρ is reduced to the equilibrium nuclear density ρ_0. The ratio ρ/ρ_0 then tells us how many times the density is larger or smaller as compared to ρ_0. If during the reaction a phase transition or other collective phenomena occur, then this quantity is significant. However, if we want to get an information about the efficiency of a scattering mechanism with respect to an increase in density, this procedure is not very qualified for a density reduction. For a better understanding of the attained density we have to eliminate the initial state.

It is the aim of the present work to give a possibility of reducing the density in such a way. In doing so, at first the basic definition in the relativistic sense is stated. On that basis, the density in a model system consisting of two freely interpenetrating relativistic gases charactarizing the very early stage of the collision is treated.

2. *DENSITY IN A RELATIVISTIC HEAVY ION MODEL SYSTEM*

Taking into account the motion of particles in discontinuous matter, the proper density of the matter requires rather careful definition. In nonrelativistic systems thermodynamical quantities are usually related to a volume unit in the laboratory system in which the given fluid element is moving. But for relativistic systems we are forced to introduce another consideration. All thermodynamical quantities for each matter element have to be defined in each case in its own rest system (Landau and Lifshitz [3]). Quantities like density, internal energy, enthalpy and entropy are related to a rest volume unit. Thermodynamics is then seen to be essential a science of the rest frame, though all the variables which occur have known transformation laws.

Let ρ_o be the proper particle number density of matter moving with a definite Minkowskian velocity v_i , so that $\rho_o dV'$ is the number of particles contained in a 3-dimensional volume element of magnitude dV' orthogonal to its world line, i.e., to v_i . The rest system of each fluid element is then defined by the requirement that each element has the momentum zero and its energy is determined by all other thermodynamical quantities. The velocity v_i then coincides with the centre-of-momentum (CM) system velocity of the element which is given in connection with the corresponding gamma factor γ_{CM} in the appendix. If the particles of the element are at rest then an observer in the laboratory will measure the density n which is related to ρ_o by the well known formula

$$n = \rho_o \gamma_{CM} . \tag{1}$$

However, moving particles represent a current which has to be taken into account for the transformation. Density and current form the four-current density vector $j_i' = \rho_o u_i' = (\rho, \vec{j}')$ whose timelike component is the particle number density $\rho = \gamma_{u'} \rho_o$ and whose spacelike component is given by the space vector of the particle current $\vec{j}' = \rho \vec{u}'$. The length of the 4-vector j_i' is a Lorentz invariant quantity which turns out to be the particle number density ρ_o providing the covariance of $\rho_o = -j_i' u'^i$. The inverse Lorentz transformation of current and density from the lab frame to the rest frame moving with the velocity $\vec{v}_{CM}$ parallel to the z-axis is

$$\rho' = \gamma_{CM}(n - j v_{CM}) \quad \text{and} \quad j' = \gamma_{CM}(j - n v_{CM}) . \tag{2}$$

With these aspects, we will tackle a relativistic heavy ion reaction regarded as two perfect gases with the rest densities ρ_p^o and ρ_t^o penetrating each other without interaction. Let the γ-factor of the projectile in the lab system be $\gamma_p = 1 + t$ with $t = \frac{E_{bom}}{M}$ where E_{bom} is the laboratory bombarding energy of the projectile nucleus in *GeV* and M its total mass.

Let us consider in the lab system an infinitesimal volume dV which contains target particles at rest of the density $n_t = \rho_t^o$ and moving projectile particles of the density $n_p = \gamma_p \rho_p^o$. The projectile beam represents a current of $j_p = n_p v_p$. In order to determine the 4-current density vector of the element, we have to sum up all partial densities and currents, i.e.,

$$n = n_p + n_t = \rho_p^o(\gamma_p + B)$$

and $j = n_p v_p$. Here we have used the abbreviation for the ratio of the target and projectile rest densities

$$B = \rho_t^o / \rho_p^o .$$

Going to the rest system of the regarded element according to eq. (2) we find for the rest density

$$\rho = \rho_p^o \gamma_{v_p \ominus v_{CM}} + \rho_t^o \gamma_{CM}. \tag{3}$$

Here the sign $\ominus$ means the relativistic rule for the composition of velocities. An observer in the rest system therefore sees the sum of the target and projectile rest densities increased by the γ-factors corresponding to eq. (1). Introducing γ_{CM} (see the appendix) into eq. (3) we obtain

$$\rho(\gamma_p, B) = \rho_t^o \frac{(\gamma_p + \frac{B+1}{B})\gamma_p + B}{\sqrt{\gamma_p^2(1+2B) + B^2}} . \tag{4}$$

We see from this expression, the relativistic density of two gases penetrating each other without interaction is not a simple sum of the target and projectile rest densities $\rho_p^o + \rho_t^o$ as it is well known in the nonrelativistic physics.

Let us discuss the asymptotic behaviour of eq. (4). The trivial limits $\rho_t^o = 0$ and $\rho_p^o = 0$ yielding $\rho = \rho_p^o$ and $\rho = \rho_t^o$, respectively, are automatically included. If γ_p tends to $\gamma_p \approx 1$ ($v_p \ll 1$) we get $\rho = \rho_p^o + \rho_t^o$, as is expected. A power expansion of expression (4) in the vicinity of $\gamma_p = 1$ yields to a high degree of accuracy

$$\rho/\rho_t^o = \frac{B+1}{B} + \frac{t}{B+1} \quad \text{or using } B \quad \rho = \rho_p^o + \rho_t^o + \frac{\rho_p^o \rho_t^o}{\rho_p^o + \rho_t^o} t \tag{5}$$

We see, only in the limit of small energies we get the simple sum $\rho_p^o + \rho_t^o$. If γ_p is increased the rest density becomes always larger than the nonrelativistic one, i.e., it depends on the state of motion. If γ_p tends to infinity then eq. (4) displays the following behaviour:

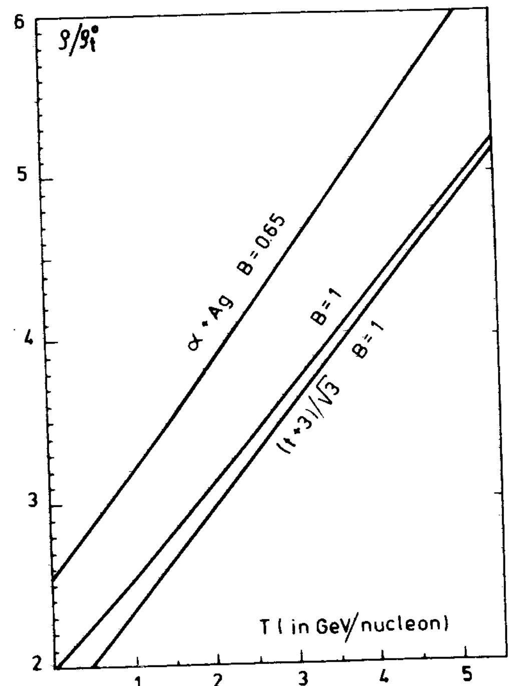
$$\rho/\rho_t^o \sim (\gamma_p + \frac{B+1}{B})/\sqrt{1+2B} \xrightarrow[\gamma_p \to \infty]{} \gamma_p . \tag{6}$$

From this expression we can deduce the following equation which provides excellent approximate values for an estimation of the density at given beam energies:

$$\rho/\rho_t^o = 2 + 0.7\,T. \tag{7}$$

The quantity T is measured in units of *GeV* per nucleon. It can be seen that the nonrelativistic density will be enlarged by a growth rate of 0.7 per *GeV/nucleon* increase in beam energy.

Let us now suppose that in the element dV there are the same densities of projectile and target particles ($n_p = n_t$ or $B = \gamma_p$). Then we get $\rho = 2\rho_p^o \gamma_{eq}^3$, where $\gamma_{eq} = \sqrt{1 + t/2}$ is the γ-factor for the equal velocity system.



Rest density (in units of the target density ρ_t^o) in a heavy ion reaction during the diving phase as a function of the beam energy in units of GeV per nucleon. The curves reflect the cases: i) α-particle and silver (upper), ii) two nuclei with the same densities (B=1) and iii) the same as in ii) but the asymptotic behaviour for large energies (lower).

In the *figure* the function eq. (4) is represented for some coefficients B. Finally, it should be referred to an analogue in the electrodynamics (Born /4/). Consider a long straight wire at rest. If electrons are moving in it, they give rise to an electric current. Because of charge conservation the wire is electrically neutral, for there are as many positive ions at rest as moving negative electrons. However, an observer moving in the direction of the wire now finds it positively charged, i.e., the total charge density has been increased in the observer's system in consequence of motion.

3. SUMMARY

In this paper the determination of the density in a relativistic heavy ion model system consisting of two freely interpenetrating nuclear characterizing the very early stage of the collision has been presented. It has been pointed out that just at that moment, if the projectile enters the target without any interaction, the density is changed due to a relativistic effect. All nucleons in the overlap region between projectile and target feel in their rest system an enlarged neighbourhood of other nucleons which results during the diving phase in an increased density beyond the sum of the rest densities of projectile and target. The density becomes beam-energy-dependent. It is shown that the nonrelativistic density $2\rho_o$ will be enlarged by a growth rate of $0.7\,\rho_o$ per *GeV/nucleon* increase in beam energy. Further, collisions with α-particle projectiles which are the densest known complex particles provide the highest densities.

APPENDIX. DETERMINATION OF THE γ-FACTOR

A. Generalities

Consider first Lorentz transformations between the CMS and the target system (TS) where $\vec{v}_{CM}$ is the velocity

of the CMS in the TS. To determine $\vec{v}_{CM}$, we proceed as follows. We form the 4-momentum vector P_i of the group of particles in question and use then its invariant. The timelike component is given by the total energy $E = \Sigma E_k$ and the spacelike one by the total momentum $\vec{P} = \Sigma \vec{p}_k$. We have, for the group of particles regarding, that the velocity of their CMS in the TS is given by

$$\vec{v}_{CM} = \vec{P}/E \; . \tag{A1}$$

Further,

$$\gamma_{CM} = E/m \; ,$$

where m is the invariant mass of the group of particles which is defined by

$$m^2 = (P_i)^2 = E^2 - \vec{P}^2 \; .$$

It has the same value in all frames. Inserting this equation into the foregoing one we find

$$\gamma = \frac{E}{\sqrt{E^2 - \vec{P}^2}} \tag{A2}$$

B. The γ-factor of two gases penetrating each other without interaction

Consider the case when all particles of a gas with the same kinetic energy T and momentum $\vec{p}$ are moving parallelly against a second one at rest. The gases penetrate each other. Let the volume element dV in the lab system have $N = N_p + N_t$ particles whereby N_p are of the projectile type and N_t of the target type leading to the lab densities n_p and n_t, respectively. For the total energy we find

$$E = N_p m \gamma_p + N_t m \; .$$

The total momentum can be written as $\vec{P} = N_p \vec{p}$ or

$$|\vec{P}| = N_p m \sqrt{\gamma_p^2 - 1} \; .$$

According to eq. (A1) we obtain

$$v_{CM} = \frac{\gamma_p \sqrt{\gamma_p^2 - 1}}{\gamma_p^2 + B} \tag{B1}$$

and using eq. (A2)

$$\gamma_{CM} = \frac{\gamma_p^2 + B}{\sqrt{B^2 + \gamma_p^2 (1 + 2B)}} \; . \tag{B2}$$

We see that γ_{CM} as a function of γ_p depends only on the ratio of the projectile and target rest density B. Let us now consider the asymptotic behaviour of formula (B2). If only target particles are in the volume ($\rho_p^0 = 0$, $\gamma_p = 1$), we get $\gamma_{CM} = 1$. If, on the other hand, there are only projectile particles, we obtain $\gamma_{CM} = \gamma_p$. Now let in the volume element there be the same partial densities ($n_p = n_t$ or $B = \gamma_p$). Then we have $\gamma_{CM} = \gamma_{eq} = \sqrt{\frac{1+\gamma_p}{2}}$, i.e., the γ-factor equals that one for arbitrary colliding partners in the "equal velocity coordinate system". The particles own rest frame agrees with the particles equal velocity system. The asymptotic behaviour with respect to γ_p provides for the non-relativistic case ($\gamma_p \longrightarrow 1$) that $\gamma_{CM} \longrightarrow 1$ and for $\gamma_p \longrightarrow \infty$ that $\gamma_{CM} \sim \gamma_p$ as is expected.

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