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UNIFIED GAUGE MODEL OF WEAK AND ELECTROMAGNETIC INTERACTIONS FOR A NONLINEAR CHIRAL $SU(4) \times SU(4)$ THEORY OF 0^--MESONS AND $1/2^+$-BARYONS

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UNIFIED GAUGE MODEL OF WEAK AND ELECTROMAGNETIC INTERACTIONS FOR A NONLINEAR CHIRAL $SU(4) \times SU(4)$ THEORY OF 0^--MESONS AND $1/2^+$-BARYONS

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Единая модель слабых и электромагнитных взаимодействий для нелинейной киральной $SU(4) \times SU(4)$ теории 0^--мезонов и $1/2^+$-барионов

С использованием нелинейной реализации калибровочной группы $SU(2)_L \times U(1)$ на основе кирального лагранжиана $SU(4) \times SU(4)$ описываются слабые и электромагнитные взаимодействия 0^--мезонов и $1/2^+$-барионов. Окончательный эффективный лагранжиан для слабых взаимодействий адронов имеет обычную структуру ток $\times$ ток. Очарованные заряженные токи имеют форму Кабиббо, нейтральные токи удовлетворяют условию $\Delta S = 0$ (схема ГИМ). В полученной модели описаны лептонные и полулептонные распады D- и F-мезонов.

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Unified Gauge Model of Weak and Electromagnetic Interactions for a Nonlinear Chiral $SU(4) \times SU(4)$ Theory of 0^--Mesons and $1/2^+$-Baryons

We construct a nonlinear realization of the gauge group $SU(2)_L \times U(1)$ by using a chiral $SU(4) \times SU(4)$ Lagrangian of hadrons. The obtained effective Lagrangian for the weak interaction of hadrons has the usual current $\times$ current structure. The charmed charged currents are of the Cabibbo type, the neutral current satisfies the condition $\Delta S = 0$ of the GIM-scheme. Semileptonic decays of D- and F-mesons are calculated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

It is now generally accepted that quantum chromodynamics (QCD) [1] - a Nonabelian gauge theory of coloured quarks and gluons - is a promising candidate for a theory of strong interactions. Weak and electromagnetic interactions of quarks can easily be included by extending this framework to a unified theory based upon a spontaneously broken gauge group (e.g., $SU(2)_L \times U(1)$) [2, 3].

QCD is an asymptotic free field theory. It is able to explain qualitatively many specific properties of high-energy scattering processes as, for example, the weak deviations from scaling behaviour in deep inelastic lepton-hadron scattering, etc. As to the low energy behaviour, however, some important problems still remain to be solved within QCD. This concerns first of all the explanation of the experimental non-observation of quarks (quark confinement) and the calculation of the physical hadron spectrum.

Looking forward to a solution of these complicated questions we recall that the low energy hadronic world has also successfully been described in the framework of phenomenological chiral Lagrangians [4-6]. In this approach, the hadrons are considered as approximately structureless objects which are described by their own fields. A field theory with a chiral-invariant Lagrangian has been first proposed by Gursey [7] and Gell-Mann and Levy [8]. Further the connection of current algebra with chiral Lagrangians including partial conservation of axial vector currents (PCAC) has been clarified on the basis of tree diagrams [9].

The quantum chiral field theory gives us a possibility to obtain low energy expansions for the amplitudes of different hadron processes. Thus, using the "tree" and "one-loop" approximation many important low energy characteristics of hadron physics (e.g., scattering phases and lengths, interaction radii, decay probabilities and form-factors, etc.) have been calculated /10/.

In the following we are interested in the nonlinear (nonpolynomial) version of chiral Lagrangians /5,6/ because they do not contain spurious "σ"-particles (recall that linear $SU(4)\times SU(4)$ σ-models contain, for example, 15 σ-particles /11/). As is well known, the nonlinear models are nonrenormalizable. One can, however, obtain quite reasonable results also for such theories by using special regularization methods (e.g., the superpropagator technique /12/). It is worth mentioning that for the case of a nonlinear $SU(3)\times SU(3)$ Lagrangian most of the one-loop diagrams could be handled by applying standard renormalization techniques. This concerns, e.g., the calculation of almost all decays of the $SU(3)$ meson octet /10/. As has been found there, the small number of loop diagrams requiring special regularizations yields as a rule only small contributions negligible in comparison with other diagrams. All these facts certainly illustrate the usefullness of investigating nonrenormalizable chiral Lagrangians.

The aim of this paper is to construct a unified model for the weak, electromagnetic, and strong interactions of hadrons based on a nonlinear chiral Lagrangian. Taking into account the recent discovery of charmed particles it is quite natural to extend first the $SU(3)\times SU(3)$ meson-baryon Lagrangians /5,13/ to chiral $SU(4)\times SU(4)$. The new Lagrangian contains the 15-plet and 20-plet of 0^--mesons and $1/2^+$-baryons formed by the ordinary $SU(3)$-octets of hadrons and by the charmed particles. In order to generate weak and electromagnetic interactions, we consider in the next step field transformations nonlinear with respect to the local gauge group $SU(2)_L\times U(1)$ of the Weinberg-Salam model /2/. A nonlinear unified hadron Lagrangian invariant with respect to local $SU(2)_L\times U(1)$ is then constructed by introducing gauge-covariant chiral derivatives. Finally, we derive an effective Lagrangian describing the weak and weak/radiative decays of ordinary and charmed hadrons. The weak Lagrangian obtained is of the current $\times$ current type with charged weak currents having a generalized Cabibbo structure. The neutral weak current satisfies the famous rule $\Delta S=0$ of the GIM-scheme /14/. In the end, we give some illustrative applications of this model to the description of leptonic and semileptonic decays of charmed particles.

The paper is organized as follows. In Sec. 2 we introduce the $SU(4)\times SU(4)$ invariant meson-baryon Lagrangian. In Sec. 3 we consider the gauge-covariant derivatives for the group $SU(2)_L\times U(1)$. Sec. 4 contains our chiral nonlinear Weinberg-Salam-GIM type Lagrangian together with the explicit expressions for the weak and electromagnetic hadronic currents. The resulting effective Lagrangian is contained in Sec. 5. For illustration and as a first application in Sec. 6 some two-, three- and four-body leptonic and semileptonic decays of charmed D- and F-mesons have been calculated. Finally, Sec. 7 contains a summary and a brief discussion of the results.

2. *THE STRONG INTERACTION LAGRANGIAN*

In this section we shall apply the techniques of nonlinear realizations of symmetry groups to a phenomenological meson-baryon Lagrangian invariant with respect to the chiral group $SU(4)\times SU(4)$. In particular, we consider the 0^--mesons and $1/2^+$-baryons belonging to the 15- or 20-dimensional representations of the algebraic subgroup $SU(4)$, respectively. Their corresponding fields are denoted by Φ_i $(i=1,2,\dots,15)$ and B_i $(i=1,2,\dots,20)$ (cf. App. A). It is further convenient to consider the dimensionless fields $\xi_i=\Phi_i/f$, where f is a parameter with the dimension of a mass the meaning of which becomes clear later on. The starting point of our analysis is the following meson-baryon Lagrangian extended from chiral $SU(3)\times SU(3)$ /5,13/ to $SU(4)\times SU(4)$

$$L_{inv}(D_\mu \xi; B, D_\mu B) = \frac{f^2}{2} D_\mu \xi_i D_\mu \xi_i + \bar{B}(i\gamma_\mu D_\mu - M)B -$$

$$- \bar{B}\gamma_\mu D_\mu \xi_i K_i B,$$

$$K_i = [\alpha D_i + (1-\alpha)F_i]\gamma_5 g_A; \quad \gamma_5 = -\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \qquad (1)$$

Here $\alpha = \frac{D}{D+F} \approx \frac{2}{3}$ is the mixing parameter of the F-D couplings, $g_A \approx 1.25$ determines the renormalization of the axial vector coupling and M is an averaged mass of the baryon multiplet. Further, F_i and D_i are 20x20 matrix representations of the two possible sets of 15-plet operators of the group $SU(4)$ (for definitions, see App. A), and $D_\mu \xi_i$, $D_\mu B = (\partial_\mu + i\Theta_\mu^k F_k)B$ are the chiral covariant derivatives. They are given in terms of Cartan forms by $(\xi \cdot A = \xi_i A_i)$

$$e^{-i\xi\cdot A}\,\partial_\mu e^{i\xi\cdot A} = i(A\cdot D_\mu \xi + V\cdot \Theta_\mu(\xi)), \qquad (2)$$

where $A_i = \frac{\lambda_i}{2}\gamma_5$, $V_i = \frac{\lambda_i}{2} I$ is the complete orthonormal set of the axial and vector generators of the chiral group $SU(4)\times SU(4)$. We use the normalization

$$\mathrm{Tr}\, A_i A_k = \mathrm{Tr}\, V_i V_k = 2\delta_{ik}\,,$$
$$\mathrm{Tr}\, A_i V_k = 0, \qquad (3)$$

where the trace is taken over internal and Lorentz indices.

Note that the chiral group is spontaneously broken down to the algebraic subgroup $SU(4)$ spanned by the vector generators V_i. The mesons ξ_i are just the massless Goldstone bosons associated with the broken axial generators A_i. To get massive mesons as well as baryon mass splittings, the chiral symmetry of the original Lagrangian (1) has further to be broken by adding a symmetry breaking term ΔL to L_{inv}. For convenience, ΔL will be included only at the end of all calculations.

Parametrizing the group elements $g \in SU(4)\times SU(4)$ by $g = e^{i\alpha\cdot A}\, e^{iu\cdot V}$, the invariance of the Lagrangian (1) under the nonlinear field transformations

$$(\xi, B) \to (\xi', B') = g(\xi, B)$$

or, explicitly,

$$g\, e^{i\xi\cdot A} = e^{i\xi'\cdot A}\, e^{iu'(\xi,g)\cdot V},$$
$$B' = D(e^{iu'(\xi,g)\cdot V})B = e^{iu'(\xi,g)\cdot F}B \qquad (4)$$

easily follows from the corresponding transformation laws of the chiral covariant derivatives /5/

$$(D_\mu \xi)' = D^{(A)}(e^{iu'(\xi,g)\cdot V})D_\mu \xi,$$
$$(D_\mu B)' = D(e^{iu'(\xi,g)\cdot V})D_\mu B. \qquad (5)$$

Here $D(\ldots)$ is a linear representation of the algebraic subgroup $SU(4)$ and $D^{(A)}(e^{iu'(\xi,g)\cdot V})$ is the linear representation defined by $A\cdot(D_\mu\xi)' = e^{iu'\cdot V}\, A\cdot D_\mu \xi\, e^{-iu'\cdot V}$. Note that the field transformations (4) become linear if g is restricted to the algebraic subgroup $SU(4)$.

3. *GAUGE-COVARIANT DERIVATIVES FOR THE GROUP $SU(2)_L\times U(1)$*

Let us now introduce the weak and electromagnetic interactions into the chiral meson-baryon Lagrangian

(1). For this aim, we require the unified Lagrangian for the strong, weak and electromagnetic interactions of hadrons be invariant with respect to the local gauge group $G_w = SU(2)_L \times U(1)$ of the Weinberg-Salam model [2]. To find the (nonlinear) transformation laws of the hadron fields (ξ, B) with respect to the gauge group G_w of the weak and electromagnetic interactions, we shall embed G_w into the global chiral group. Let us first consider the following 4x4 matrix representation of the generators of G_w:

$$SU(2)_L: \hat{C}_i = \frac{1+\gamma_5}{2}\frac{C_i}{2}, \quad C_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_1 \sigma_i \sigma_1^{-1} \end{pmatrix}$$

$$U(1): \quad \frac{\hat{Y}}{2} = \frac{y_w}{2} + \frac{1-\gamma_5}{2}\frac{C_3}{2}, \quad [\hat{C}_i, \frac{\hat{Y}}{2}] = 0, \tag{6}$$

where σ_i are usual Pauli matrices and y_w denotes the weak hypercharge. The operator of the electromagnetic charge may be expressed by the operators of the weak isospin and the weak hypercharge $\hat{C}_3$, $\hat{Y}/2$ or by the operators of the "strong" isospin, hypercharge and charm I_3, Y_s, C respectively. We have

$$Q = \hat{C}_3 + \frac{\hat{Y}}{2} = \frac{C_3}{2} + \frac{y_w}{2} \tag{7}$$

or

$$Q = I_3 + \frac{Y_s}{2} + \frac{2}{3}C, \tag{8}$$

where

$$Y_s = \frac{1}{\sqrt{3}}\lambda_8, \quad C = \frac{1}{4}(1-\sqrt{6}\lambda_{15}).$$

In order to get the (generalized) Cabibbo structure of the weak interactions, we next rewrite the Lagrangian (1) in terms of the Cabibbo rotated fields

$$e^{i\xi^c \cdot A} = U e^{i\xi\cdot A} U^{-1},$$

$$B^c = D(U)B, \tag{9}$$

where $U = e^{i2\theta V_7}$ and θ is the Cabibbo angle.

Let us now consider the nonlinear realization of the (global) group G_w defined by the following field transformation laws (cf. eq. (4))

$$(\xi^c, B^c) \to (\xi'^c, B'^c) = h(\xi^c, B^c),$$

$$h = e^{i\eta\frac{y_w}{2}} e^{i\eta\frac{1-\gamma_5}{2}\frac{C_3}{2}} e^{i\epsilon\cdot\hat{C}} \in G_w, \tag{10}$$

where*

$$h e^{i\xi^c \cdot A} = e^{i\eta\frac{y_w}{2}}[e^{i\xi'^c\cdot A} e^{iu'(\xi^c,h)\cdot V}],$$

$$B'^c = D(e^{iu'(\xi^c,h)\cdot V})B^c \tag{11}$$

We next use coordinate-dependent gauge transformations (10). As usual, the construction of a Lagrangian invariant under local group transformations requires a set of gauge fields W_μ^i, B_μ associated to the generators $\hat{C}_i$, $\hat{Y}/2$ of the local group G_w. Let their transformation laws be given by

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*Strictly speaking, the group G_w must be embedded into the enlarged group $U(4)\times U(4)$ since the generator $\hat{Y}/2$ contains the unit matrix. The unit matrix gives here, however, an irrelevant phase factor only which drops out in the transformation law (11). The embedding of more general "weak" groups G_w into a global "strong" group $U(N)\times U(N)$ has been discussed by Weinberg [15].

$$ig W'_\mu \cdot \hat{C} = e^{i\epsilon(x)\cdot \hat{C}}[\partial_\mu + ig W_\mu \cdot \hat{C}]e^{-i\epsilon(x)\cdot\hat{C}} ,$$

$$B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu \eta(x). \tag{12}$$

The new unified Lagrangian $L_{unif}(\bar{D}_\mu \xi^c; B^c, \bar{D}_\mu B^c)$ invariant with respect to the gauge transformations (11) and (12) follows now from eq. (1) by replacing the chiral covariant derivatives by gauge-covariant ones, i.e., $D_\mu \xi_i \to \bar{D}_\mu \xi^c_i$, $D_\mu B \to \bar{D}_\mu B^c = (\partial_\mu + i\bar{\Theta}^k_\mu F_k)B^c$. The gauge-covariant Cartan forms $\bar{D}_\mu \xi^c_i$, $\bar{\Theta}^k_\mu(\xi^c)$ are defined by

$$e^{-i\xi^c\cdot A}[\partial_\mu + igW_\mu\cdot\hat{C} + ig'B_\mu \frac{\hat{Y}}{2}]e^{i\xi^c\cdot A} =$$

$$= i(A\cdot\bar{D}_\mu\xi^c + V\cdot\bar{\Theta}_\mu(\xi^c)) + ig'B_\mu\frac{y_w}{2}. \tag{13}$$

The invariance of $L_{unif}(\bar{D}_\mu\xi^c; B^c, \bar{D}_\mu B^c)$ with respect to the local group G_w immediately follows from the fact that the gauge-covariant derivatives $\bar{D}_\mu\xi^c$, $\bar{D}_\mu B^c$ obey the same transformation laws as the old ones (cf. eq. (5)). (On the other hand, the original $SU(4)\times SU(4)$ symmetry of the theory will now intrinsically be broken by order g^2, g'^2 perturbations arising from the emission and absorption of virtual gauge bosons).

For subsequent considerations it is convenient to introduce the fields $W^\pm_\mu$, Z_μ and A_μ of the charged and neutral vector bosons and of the photon, respectively,

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu),$$

$$Z_\mu = \cos\theta_w W^3_\mu - \sin\theta_w B_\mu, \tag{14}$$

$$A_\mu = \sin\theta_w W^3_\mu + \cos\theta_w B_\mu,$$

where θ_w is the Weinberg mixing angle defined by $\tan\theta_w = g'/g$. Introducing the physical fields (14), into eq. (13) and reexpressing the Cabibbo rotated hadron fields in terms of the unrotated fields (ξ, B) we finally get the explicit formulae

$$\begin{pmatrix} \bar{D}_\mu \xi^c_i \\ \bar{\Theta}^i_\mu(\xi^c) \end{pmatrix} = D(e^{-i2\theta V_7})_{ij} \begin{pmatrix} \bar{D}_\mu \xi_j \\ \bar{\Theta}^j_\mu(\xi) \end{pmatrix},$$

$$\begin{pmatrix} \bar{D}_\mu \xi_i \\ \bar{\Theta}^i_\mu(\xi) \end{pmatrix} = \left\{ \begin{pmatrix} D_\mu \xi_i \\ \Theta^i_\mu(\xi) \end{pmatrix} + \frac{g}{2\sqrt{2}} \langle (2\hat{C}_+(\theta)W^+_\mu + h.c.) \begin{pmatrix} A_i(\xi) \\ V_i(\xi) \end{pmatrix} \rangle + \right.$$

$$+ \frac{g}{2\cos\theta_w} Z_\mu \langle (2\hat{C}_3 - 2\sin^2\theta_w Q) \cdot \begin{pmatrix} A_i(\xi) \\ V_i(\xi) \end{pmatrix} \rangle +$$

$$\left. + eA_\mu \langle Q \begin{pmatrix} A_i(\xi) \\ V_i(\xi) \end{pmatrix} \rangle \right\}. \tag{15}$$

Here $\hat{C}_\pm(\theta)$ $(\hat{C}_\pm = \hat{C}_1 \pm i\hat{C}_2)$ are the Cabibbo rotated charged generators of the weak isospin group $SU(2)_L$

$$\hat{C}_\pm(\theta) = U^{-1}\hat{C}_\pm U, \quad \hat{C}_3(\theta) = \hat{C}_3 \tag{16}$$

and $e = g\sin\theta_w$ is the electromagnetic charge. For convenience, we use henceforth the notations

$$\langle \ldots \rangle = \frac{1}{2}\mathrm{Tr}(\ldots)$$

$$X_i(\xi) = e^{i\xi\cdot A} X_i e^{-i\xi\cdot A}, \quad X_i = (A_i, V_i).$$

It is worth remarking that eq. (15) provides us with a generalized minimal substitution rule $(D_\mu\xi, D_\mu B) \to (\bar{D}_\mu\xi^c, \bar{D}_\mu B^c)$ for introducing the unified weak and electromagnetic interactions into the strong interaction chiral Lagrangian (1). Our result (15) containes, as a special case, the minimal substitution rule of ref. /16/ for introducing electromagnetism into the group $SU(3)\times SU(3)$.

4. *THE NONLINEAR UNIFIED LAGRANGIAN*

4.1. *Currents*

Taking into account the explicit expressions for the gauge-covariant derivatives (15) the unified Lagrangian may be written in the form

$$L_{unif}(\bar{D}_\mu \xi^c; B^c, \bar{D}_\mu B^c) = L_{inv}(D_\mu \xi, B, D_\mu B) - \frac{g}{2\sqrt{2}}(W^+_\mu j^W_\mu + h.c.) - \frac{g}{2\cos\theta_w} Z_\mu j^Z_\mu - eA_\mu j^A_\mu + L_{bil}(A_\mu, W^\pm_\mu, Z_\mu). \qquad (17)$$

Here j^W_μ, j^Z_μ and j^A_μ are the weak hadronic charged and neutral currents and the electromagnetic current, respectively. The term L_{bil} contains expressions bilinear in the vector fields. With the definition *

$$j_\mu = -\frac{\delta L_{unif}}{\delta \Phi_\mu}, \quad \Phi_\mu = \left(\frac{g}{2\sqrt{2}} W^\pm_\mu ; \frac{g}{2\cos\theta_w} Z_\mu ; eA_\mu\right)$$

we get

$$j^W_\mu = -f^2 \bar{D}_\mu \xi_i \langle 2\hat{C}_+(\theta) A_i(\xi)\rangle + \bar{B}\gamma_\mu(\langle 2\hat{C}_+(\theta) V_i(\xi)\rangle F_i + \langle 2\hat{C}_+(\theta) A_i(\xi)\rangle K_i)B, \qquad (18)$$

$$j^Z_\mu = -f^2 \bar{D}_\mu \xi_i \langle (2\hat{C}_3 - 2\sin^2\theta_w Q) A_i(\xi)\rangle + \bar{B}\gamma_\mu(\langle (2\hat{C}_3 - 2\sin^2\theta_w Q) V_i(\xi)\rangle F_i + \langle (2\hat{C}_3 - 2\sin^2\theta_w Q) A_i(\xi)\rangle K_i)B, \qquad (19)$$

$$j^A_\mu = -f^2 \bar{D}_\mu \xi_i \langle Q A_i(\xi)\rangle + \bar{B}\gamma_\mu(\langle Q V_i(\xi)\rangle F_i + \langle Q A_i(\xi)\rangle K_i)B. \qquad (20)$$

As we observe from eq. (18) the weak charged current exhibits the generalized Cabibbo structure of the GIM-scheme /14/. Indeed, taking into account the representation

$$\hat{C}_+(\theta) = \frac{1}{2}\cos\theta[(V^{1+i2} + A^{1+i2}) + (V^{13-i14} + A^{13-i14})] + \frac{1}{2}\sin\theta[(V^{4+i5} + A^{4+i5}) - (V^{11-i12} + A^{11-i12})],$$

$$V^{k\pm i\ell} = V^k \pm iV^\ell \quad \text{etc.} \qquad (21)$$

we get

$$j^W_\mu = \cos\theta[j^{1+i2}_\mu + j^{13-i14}_\mu] + \sin\theta[j^{4+i5}_\mu - j^{11-i12}_\mu]. \qquad (22)$$

The currents $j^{k+i\ell}_\mu$ have the usual $V+A$-form. They are defined by a formula analogous to eq. (18) where $2\hat{C}_+(\theta)$ is replaced by $(V^{k+i\ell} + A^{k+i\ell})$. By analogy with the quark model, the currents j^{1+i2}_μ, j^{13-i14}_μ, j^{4+i5}_μ and j^{11-i12}_μ describe weak transitions with the following

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*These expressions for j_μ agree with the expressions obtained from the standard definition /8/ $j_\mu = -\delta L/\delta\partial_\mu \epsilon$. There, the hadron fields (ξ^c, B^c) have to be varied according to eq. (11); W^i_μ, B_μ have to be varied as in eq. (12) with derivative terms excluded.

changes of the strangeness and charm, respectively: $\Delta S = \Delta C = 0$, $\Delta S = \Delta C = 1$, $\Delta S = 1 \neq \Delta C = 0$ and $\Delta S = 0 \neq \Delta C = 1$. Finally, the neutral current obeys the well-known relation

$$j_\mu^Z = j_\mu^{(2\hat{C}_3)} - 2\sin^2\theta_w \, j_\mu^A \qquad (23)$$

of the (linear) Weinberg-Salam-model with $j_\mu^{(2\hat{C}_3)}$ the current belonging to the third component of the weak isospin. For illustration and further applications, we quote in Appendix B the first terms of a power series expansion in ξ of the mesonic part of the weak currents. It should be mentioned that (after having added a symmetry breaking term ΔL cf. eq. (35)) the total axial vector currents can be shown to obey the PCAC-relations

$$\partial_\mu j_\mu^W = \sqrt{2} f \cos\theta (m_\pi^2 \pi^- + m_F^2 F^-) + \sqrt{2} f \sin\theta (m_K^2 K^- - m_D^2 D^-) + O(\xi^3), \qquad (24)$$

$$\partial_\mu j_\mu^Z = f(m_\pi^2 \pi^\circ + \frac{1}{\sqrt{3}} m_\eta^2 \eta - \sqrt{\frac{2}{3}} m_{\eta_c}^2 \eta) + O(\xi^3). \qquad (25)$$

As we now see from eqs. (24), (25) the parameter f introduced in eq. (1) for dimensional reasons is recognized as an averaged meson decay constant (from pion decay we have $f = f_\pi = 95$ *MeV*).

4.2. Discussion of Bilinear Terms

In this section we quote an explicit formula for the expressions bilinear in the vector fields that appear in L_{unif} (note that we include here also contributions arising from the currents). After some algebra we get ($C_\pm = \frac{C_1 \pm iC_2}{2}$.)

$$L_{unif}^{bil.} = (\frac{fg}{\sqrt{2}})^2 |W_\mu^+|^2 + (\frac{gf}{2\cos\theta_w})^2 Z_\mu^2 +$$

$$+ (\frac{g}{2\cos\theta_w})^2 Z_\mu^2 (2\sin^2\theta_w) \frac{f^2}{8} \{H(\frac{C_3}{2}) - 2\sin^2\theta_w \langle [Q, e^{i2\xi\cdot A}][Q, e^{-i2\xi\cdot A}] \rangle\} -$$

$$- e^2 A_\mu^2 \frac{f^2}{8} \langle [Q, e^{i2\xi\cdot A}][Q, e^{-i2\xi\cdot A}] \rangle -$$

$$- e(\frac{g}{2\sqrt{2}}) \frac{f^2}{8} (A_\mu W_\mu^+ H(C_+) + h.c.) +$$

$$+ (\frac{g}{2\sqrt{2}})(\frac{g}{2\cos\theta_w}) 2\sin^2\theta_w \frac{f^2}{8} (Z_\mu W_\mu^+ H(C_+) + h.c.) -$$

$$- e(\frac{g}{2\cos\theta_w}) \frac{f^2}{8} Z_\mu A_\mu \{H(\frac{C_3}{2}) - 4\sin^2\theta_w \langle [Q, e^{i2\xi\cdot A}][Q, e^{-i2\xi\cdot A}] \rangle\}. \qquad (26)$$

Here the function $H(X)$ describes the coupling of the vector bosons and pseudoscalar mesons. It reads

$$H(X) = \langle [Q, e^{i2\xi\cdot A}]([X, e^{i2\xi\cdot A}]_- - \gamma_5 \{X, e^{i2\xi\cdot A}\}_+) + (\gamma_5, A) \to (-\gamma_5, -A) \rangle.$$

As we observe from eq. (26) there appear mass terms for the W and Z bosons due to their interactions with the hadron sector *

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* Note that this generation of vector meson masses arises from the inherent mechanism of the spontaneous breakdown of the gauge group G_W embedded into the spontaneously broken chiral group $SU(4) \times SU(4)$.

The remaining terms in L^{bil}_{unif} describe the interaction of the pseudoscalar meson 15-plet (π, K, η, F, D, η_c) with the gauge bosons. Our model contains the following 3-particle vertices

$$\underline{AW^{\pm}\xi\text{-vertex}}: \; ie\left(-\frac{g}{2\sqrt{2}}\right)\frac{f^2}{2}\langle C_{\pm}(\theta)[Q,V\cdot\xi]\rangle A_\mu W^{\pm}_\mu$$

$$\underline{ZW^{\pm}\xi\text{-vertex}}: \; -i\left(\frac{g}{2\sqrt{2}}\right)\left(\frac{g}{2\cos\theta_W}\right)2\sin^2\theta_W\frac{f^2}{2}\langle C_{\pm}(\theta)[Q,V\cdot\xi]\rangle Z_\mu W^{\pm}_\mu . \qquad (27)$$

It should be remarked that the bilinear expressions (26) contain no "seagull" terms of the form $|W^{\pm}_\mu|^2F(\xi)$ for the charged vector boson.

5. *THE EFFECTIVE LAGRANGIAN*

Our final aim is to obtain from eq. (17) an effective Lagrangian describing weak and weak/radiative decays of ordinary and charmed hadrons to first order in the weak interaction constant $G \sim 10^{-5}/m_p^2$. In order to get leptonic and semileptonic decays of hadrons we must also include the leptonic charged and neutral currents of the standard Weinberg-Salam-model. They are given by [2,14] (cf. Appendix C)

$$(j^W_\mu)_{lept} = \bar{\ell}\gamma_\mu(2\hat{C}_+)\ell \quad , \qquad \ell = \begin{pmatrix} \nu_e \\ \mu \\ \nu' \end{pmatrix} ,$$

$$(j^Z_\mu)_{lept} = \bar{\ell}\gamma_\mu(2\hat{C}_3 - 2\sin^2\theta_W Q)\ell . \qquad (28)$$

The effective Lagrangian describes processes with W- and Z-boson exchange in second order of weak perturbation theory. In a general (R_β) gauge the vector propagators read (cf. Appendix C)

$$D^W_{\mu\nu}(x) = \int\frac{d^4k}{(2\pi)^4}e^{-ikx}\left(-g_{\mu\nu} + \left(1-\frac{1}{\beta}\right)\frac{k_\mu k_\nu}{k^2 - \frac{\bar{M}^2_W}{\beta} + i\epsilon}\right)\frac{1}{k^2 - \bar{M}^2_W + i\epsilon},$$

$$(29)$$

$$D^Z_{\mu\nu}(x) = \int\frac{d^4k}{(2\pi)^4}e^{-ikx}\left(-g_{\mu\nu} + \left(1-\frac{1}{\gamma}\right)\frac{k_\mu k_\nu}{k^2 - \frac{\bar{M}^2_Z}{\gamma} + i\epsilon}\right)\frac{1}{k^2 - \bar{M}^2_Z + i\epsilon},$$

where the total masses of the vector bosons get contributions both from the hadron sector and the Higgs mechanism of the lepton sector. We have

$$\bar{M}^2_W = (M_W)^2_{Higgs} + \left(\frac{fg}{\sqrt{2}}\right)^2,$$

$$(30)$$

$$\bar{M}^2_Z = (M^2_Z)_{Higgs} + 2\left(\frac{fg}{2\cos\theta_W}\right)^2.$$

Using the approximations $D_{\mu\nu}(x) \approx \delta(x)\frac{g_{\mu\nu}}{\bar{M}^2}$ valid for large masses of the intermediate vector bosons we, finally, obtain

$$L_{eff} = L_{weak} + L_{weak/elm}, \qquad (31)$$

where

$$L_{weak} = -\frac{\bar{G}}{\sqrt{2}}\{J^Z_\mu(x)J^Z_\mu(x) + J^{W^+}_\mu(x)J^W_\mu(x)\}, \qquad (32)$$

$$L_{weak/elm} = -\frac{\bar{G}}{\sqrt{2}}e\frac{f^2}{8}A_\mu(x)\{(H(C_+)(j^{0W^+}_\mu(x))_{mes} + h.c.) +$$

$$+ (H(\frac{C_3}{2}) - 4\sin^2\theta_W\langle[Q,e^{i2\xi\cdot A}][Q,e^{-i2\xi\cdot A}]\rangle)(j^{0Z}_\mu(x))_{mes}\}. \qquad (33)$$

Here

$$J_\mu(x) = (j_\mu(x))_{lept} + j^0_\mu(x) \qquad (34)$$

is the sum of the leptonic currents (28) and of the "free" hadronic currents $j^0_\mu = j_\mu(e = g = 0)$. $\bar{G} = \sqrt{2} g^2/8\bar{M}^2_W$ is a corrected Fermi constant which coincides up to a negligible term $O(f^2(\frac{G}{\sqrt{2}})^2)$ with the usual expression $G = \sqrt{2} g^2/8M^2_W$. In writing down eq. (33) we have omitted terms describing purely leptonic weak/radiative processes.

Note that the effective weak Lagrangian (32) has the current × current form of the conventional weak interaction theory. Some of its hadronic $SU(3) \times SU(3)$ or $SU(4) \times SU(4)$ substructures have previously been derived only empirically [10,17,18]. Finally, the expression $L_{weak/elm}$ describes "inner" weak/radiative processes with participating mesons (in addition, perturbation theory yields also "bremsstrahlung" contributions arising from the interaction of photons with "external" hadron lines).

We remark that the Lagrangian $L_{weak/elm.}$ could also be obtained empirically from the Lagrangian L_{weak} by applying the minimal substitution rule $\partial_\mu \to \partial_\mu + ieA_\mu Q$ to the currents*.

Up till now the pseudoscalar mesons of our model are to lowest order in the coupling constant e, G massless. (Mass corrections of order e^2, G arising from the emission or absorption of virtual vector bosons lead to an intrinsic breakdown of the global $SU(4) \times SU(4)$ symmetry [15]). Disregarding the small electromagnetic-

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* Considering the charged current, such an independent treatment of the weak and electromagnetic interaction must, however, fail in higher order processes with an internal $W-W-\gamma$ vertex. Recall that the W-boson of the unified theory possesses an anomalous magnetic moment arising from a nonminimal term $F_{\mu\nu} W^+_\mu W^-_\nu$.

weak mass corrections, finite meson masses may easily be included into the model by adding a $SU(4) \times SU(4)$ -breaking term ΔL. In the scheme of Gell-Mann, Oakes and Renner [19,20] ΔL transforms according to the representation $(4, 4^*) + (4^*, 4)$ of $SU(4) \times SU(4)$. We use the explicit expression

$$\Delta L = \frac{f^2}{4} \langle (aV_0 + bV_8 + cV_{15}) e^{i2\xi \cdot A} \rangle, \qquad (35)$$

where the parameters a, b, c have to be chosen in such a way that the physical meson masses are reproduced *. Similarly, baryon mass splittings can be taken into account by adding matrix elements of the following baryon mass operator [21] to the Lagrangian (17)

$$\Delta M = (b' V_8 + c' V_{15}). \qquad (36)$$

6. *SOME APPLICATIONS: LEPTONIC AND SEMILEPTONIC DECAYS OF CHARMED MESONS*

It has been shown in ref. [10] that the $SU(3) \times SU(3)$ part of the effective Lagrangian (31) provides us already in the tree and one-loop approximation with a satisfactory description of the leptonic, semileptonic and radiative decays of the $SU(3)$ meson octet (π, K, η). Analogous results may now be obtained for the decay of charmed particles, too. For illustration and first applications,

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* One gets: $m^2_\pi = (\frac{a}{\sqrt{2}} + \frac{b}{\sqrt{3}} + \frac{c}{\sqrt{6}})$; $m^2_k = (\frac{a}{\sqrt{2}} - \frac{b}{2\sqrt{3}} + \frac{c}{\sqrt{6}})$;

$m^2_\eta = (\frac{a}{\sqrt{2}} - \frac{b}{\sqrt{3}} + \frac{c}{\sqrt{6}})$; $m^2_{\eta c} = (\frac{a}{\sqrt{2}} - \sqrt{\frac{2}{3}} c)$;

$m^2_F = (\frac{a}{\sqrt{2}} - \frac{b}{\sqrt{3}} - \frac{c}{\sqrt{6}})$; $m^2_D = (\frac{a}{\sqrt{2}} + \frac{b}{2\sqrt{3}} - \frac{c}{\sqrt{6}})$.

we shall give in this section a few typical calculation examples for the leptonic and semileptonic decay rates of F-and D-mesons. A detailed investigation of charmed meson and baryon decays including tree and one-loop contributions will be given elsewhere

i) $D_{\ell 2}$, $F_{\ell 2}$ decays

Let us first consider the leptonic decays $F^- \to \mu\bar{\nu}$, $D^- \to \mu\bar{\nu}$. The relevant part of the effective Lagrangian (32) is given by

$$L_1 = Gf(\cos\theta\,\partial_\mu F^- - \sin\theta\,\partial_\mu D^-)(j_\mu^{W+})_{lept.} \qquad (37)$$

From eq. (37) we obtain the decay amplitudes $(S = 1 - iT)$

$$T_{F\to\mu\bar{\nu}} = iGf\cos\theta\, p_{F\mu}\,\ell_\mu^{(+)}, \quad T_{D\to\mu\bar{\nu}} = -iGf\sin\theta\, p_{D\mu}\,\ell_\mu^{(+)}, \qquad (38)$$

where $\ell_\mu^{(+)} = \bar{u}(\mu)\,\gamma_\mu(1+\gamma_5)u_\nu$ and $p_{F,D}$ are the momenta of the F- and D-mesons, respectively. The charged meson rate for the decay $X \to \mu\bar{\nu}$ is

$$W_{X\to\mu\bar{\nu}} = \frac{(Gfm_\mu)^2}{4\pi}\, m_X \begin{pmatrix} \sin^2\theta \\ \cos^2\theta \end{pmatrix}. \qquad (39)$$

With $f = f_\pi = 95$ *MeV*, $\theta = 0.22$, $m_D \simeq 1.87$ *GeV* and $m_F \simeq 2.03$ *GeV* we have

$$W_{D^-\to\mu\bar{\nu}} = 2\cdot 10^8 s^{-1}, \quad W_{F\to\mu\bar{\nu}} = 3\cdot 10^9 s^{-1}. \qquad (40)$$

ii) $D_{\ell 3}$, $F_{\ell 3}$ decays

We now estimate $D_{\ell 3}$ and $F_{\ell 3}$ decays described by the following part of the effective Lagrangian

$$L_2 = -i\frac{G}{\sqrt{2}}\{\cos\theta(\bar{D}^\circ \overleftrightarrow{\partial}_\mu K^- + \sqrt{\frac{2}{3}}\,\eta\,\overleftrightarrow{\partial}_\mu F^-) +$$

$$+ \sin\theta(-\bar{D}^\circ\overleftrightarrow{\partial}_\mu \pi^- + \frac{1}{\sqrt{6}}\,\eta\,\overleftrightarrow{\partial}_\mu D^- + K^\circ\overleftrightarrow{\partial}_\mu F^-)\}(j_\mu^{W+})_{lept.} \qquad (41)$$

$(\bar{D}^\circ\overleftrightarrow{\partial}_\mu K^- = \bar{D}^\circ\partial_\mu K^- - \partial_\mu\bar{D}^\circ \cdot K^-$, etc..).

The amplitudes of the Cabibbo favoured reactions $\bar{D}^\circ \to K^+ e\bar{\nu}$ and $F^- \to \eta e\bar{\nu}$ follow from eq. (41) to be

$$T_{\bar{D}^\circ\to K^+ e\bar{\nu}} = -\frac{G}{\sqrt{2}}\cos\theta(p_D + p_K)_\mu\,\ell_\mu^{(+)}, \qquad (42)$$

$$T_{F^-\to\eta e\bar{\nu}} = -\frac{G}{\sqrt{3}}\cos\theta\,(p_F + p_\eta)_\mu\,\ell_\mu^{(+)}. \qquad (43)$$

We have

$$W_{\bar{D}^\circ\to K^+ e\bar{\nu}} = \frac{G^2 m_D m_K^4 \cos^2\theta}{12\pi^3}\, I(\kappa_{D,K}), \qquad (44)$$

where

$$I(\kappa) = \int_1^\kappa dx(x^2-1)^{3/2} = \frac{1}{4}[\kappa\sqrt{\kappa^2-1}(\kappa^2 - \frac{5}{2}) + \frac{3}{2}\ln(\kappa + \sqrt{\kappa^2-1})]$$

and $\kappa_{D,K} = \frac{m_D^2 + m_K^2}{2m_K m_D}$. Thus, we predict

$$W_{\bar{D}^\circ\to K^+ e\bar{\nu}} = 1.0\cdot 10^{11} s^{-1}. \qquad (45)$$

The same estimate can be obtained for the decay $D^- \to K^\circ e\bar{\nu}$. Similarly, we have*

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* Since the function $I(\kappa)$ is approximately constant for the decays $\bar{D}^\circ \to K^+ e\bar{\nu}$, $F^- \to \eta e\bar{\nu}$ and $\bar{K}^\circ \to \pi^+ e\bar{\nu}$ the following approximate relations

$$W_{\bar{D}^\circ\to K^+ e\bar{\nu}} \approx \frac{m_D}{m_K}\left(\frac{m_K}{m_\pi}\right)^4 ctg^2\theta\, W_{\bar{K}^\circ\to\pi^+ e\bar{\nu}},$$

$$W_{F^-\to\eta e\bar{\nu}} \approx \frac{2}{3}\frac{m_F}{m_K}\left(\frac{m_\eta}{m_\pi}\right)^4 ctg^2\theta\, W_{\bar{K}^\circ\to\pi^+ e\bar{\nu}}$$

are valid.

$$W_{F^-\to\eta e\bar\nu} = 1\cdot 10^{11} s^{-1} . \qquad (46)$$

Finally, we estimate the Cabibbo forbidden decays $\bar D^\circ \to \pi^+ e\bar\nu$, , $D^- \to \eta e\bar\nu$ and $F^- \to \bar K^\circ e\bar\nu$. These decays are suppressed by a factor $\tan^2\theta \approx 0.05$ relative to the favoured reactions. We have

$$W_{\bar D^\circ \to \pi^+ e\bar\nu} \simeq 1.2 \cdot 10^{10} s^{-1} ,$$

$$W_{D^- \to \eta e\bar\nu} \simeq 1.1 \cdot 10^{9} s^{-1} , \qquad (47)$$

$$W_{F^- \to \bar K^\circ e\bar\nu} \simeq 1.2 \cdot 10^{10} s^{-1} ,$$

The above predictions are in agreement with earlier results obtained by using the quark currents of the GIM-scheme /22/. Analogous results have also been obtained within a chiral $SU(4)\times SU(4)$ meson theory /18/ using, an empirical current $\times$ current Lagrangian.

iii) $D_{\ell 4}$, $F_{\ell 4}$ decays

The $D_{\ell 4}$ and $F_{\ell 4}$ decays $\bar D^\circ \to K^\circ \pi^+ e\bar\nu$, $F^- \to K^+K^- e\bar\nu$ can be calculated in the tree approximation from the effective Lagrangian

$$L_3 = \frac{G\cos\theta}{3f}[(2\bar D^\circ \bar K^\circ \partial_\mu \pi^- - \pi^- \bar D^\circ \partial_\mu \bar K^\circ - \pi^- \bar K^\circ \partial_\mu \bar D^\circ) + \qquad (48)$$
$$+ (2F^- K^- \partial_\mu K^+ - F^- K^+ \partial_\mu K^- - K^- K^+ \partial_\mu F^-)](j_\mu^{W+})_{lept.}$$

The amplitude of the $D_{\ell 4}$ decay is

$$T_{\bar D^\circ \to K^\circ \pi^+ e\bar\nu} = - i \frac{G\cos\theta}{3f} p_\mu \ell_\mu^{(+)} , \qquad (49)$$

where

$$p = p_{\bar D^\circ} + 2p_{\pi^+} - p_{K^\circ} = 3p_{\pi^+} + p_\ell .$$

Here p_ℓ denotes the momentum of the lepton pair. From eq. (49) we obtain

$$W_{\bar D^\circ \to K^\circ \pi^+ e\bar\nu} = \frac{G^2\cos^2\theta\, m_\pi^4 m_K^6}{24(4\pi)^5 f^2 m_D^3} I(\frac{m_D}{m_K}), \qquad (50)$$

where

$$I(\Delta) = \int_1^{(\Delta-1)^2} dx\sqrt{(\Delta^2-1)^2 - 2(\Delta^2+1)x + x^2}(x^3 - 8(x^2-1) - \frac{1}{x} + 12x\ln x). \qquad (51)$$

With eq. (50) we predict

$$W_{\bar D^\circ \to K^\circ \pi^+ e\bar\nu} = 2.1 \cdot 10^6 s^{-1} . \qquad (52)$$

Analogously we have

$$W_{F^- \to K^+K^- e\bar\nu} = \frac{G^2\cos^2\theta\, m_K^{10}}{24(4\pi)^5 f^2 m_F^3} I(\frac{m_F}{m_K}) . \qquad (53)$$

This yields

$$W_{F^- \to K^+ K^- e\bar\nu} = 1.1 \cdot 10^9 s^{-1}. \qquad (54)$$

7. *SUMMARY AND DISCUSSIONS*

The main purpose of this paper was the construction of a non-linear realization of the Weinberg-Salam-model starting from a phenomenological chiral $SU(4)\times SU(4)$ meson-baryon Lagrangian. The Lagrangian we have used consists of a chiral symmetric main part and a small but important mass term which breaks chiral $SU(4)\times SU(4)$. The weak and electromagnetic interactions were then introduced into this "strong" Lagrangian by a principle of minimal coupling using gauge-covariant derivatives of the gauge group $SU(2)_L\times U(1)$. The chiral unified model thus obtained provides us with an effective current $\times$ $\times$ current Lagrangian for weak processes as well as with a Lagrangian describing weak/radiative processes.

In particular, our effective weak Lagrangian involves $SU(3)\times SU(3)$ [10,17] and $SU(4)\times SU(4)$ substructures [18] that have previously been derived only in a heuristic way.

The weak charged currents of the above nonlinear model exhibit the generalized Cabibbo structure of the GIM-scheme first derived in the framework of the quark model. Furthermore, the weak neutral current does not contain strangeness changing terms like $\bar{f}\,\partial_\mu K^\circ$, $K^- \overleftrightarrow{\partial}_\mu \pi^+$, etc. Thus, the decays $K^\circ \to \mu^+\mu^-$, $K^- \to \pi^- e e^+$ etc, are forbidden in agreement with the experimental data.

As has been shown in ref. [10], the $SU(3)\times SU(3)$ substructure of the effective weak Lagrangian (31) ensures, even in the tree and one-loop approximation, a satisfactory description of the leptonic, semileptonic and radiative decays of the $SU(3)$ meson octet (π, K, η). In order to show how these ideas work for the larger group $SU(4)\times SU(4)$, we have calculated some typical leptonic and semileptonic decays of charmed D- and F-mesons in the tree approximation. The predictions agree as a rule with similar calculations based on the quark model, the PCAC hypothesis and certain assumptions on the behaviour of form-factors [22].

Finally, let us comment on the non-leptonic part of the weak interaction Lagrangian (32). It can easily be seen from the explicit expression of the neutral current that in the GIM-scheme the decay $K_S^\circ \to \pi^\circ\pi^\circ$ cannot be described by a current × current Lagrangian. This decay can, however, in principle proceed in our model via a baryon loop with exchange of a W-boson. Similarly, two-point weak vertices, e.g., $D^\circ \bar{K}^\circ$, required for explaining the decay $D^\circ \to \bar{K}^\circ \pi^+\pi^-$ [18] may in principle be generated via a baryon loop.

There is also the old problem of the $|\Delta I| = 1/2$ rule in non-leptonic decays according to which $\Delta S \neq 0$ transitions with $|\Delta I| = 1/2$ are strongly enhanced in comparison with $|\Delta I| = 3/2$ transitions. Usually such an enhancement is taken into account by multiplying the $|\Delta I| = 1/2$ amplitudes by a factor $x \sim \frac{1}{\sin\theta\cos\theta}$. An explanation of this enhancement factor requires, however, additional dynamical assumptions (e.g., octet or 20-plet dominance [22,23], inclusion of renormalization effects from the strong interaction [24], etc.), a discussion of which is outside the scope of this paper. In a forthcoming publication we shall present a comprehensive investigation of weak and weak/radiative decays of charmed mesons/baryons on the basis of tree and one-loop calculations.

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APPENDIX A

The 15-plets of the 20x20 matrix operators F_i, $D_i = \frac{1}{2} d_{ijk} F_j F_k$ satisfy the commutation relations

$$[F_i\,, F_j] = i f_{ijk} F_k\,, \tag{A1}$$

$$[D_i\,, F_j] = i f_{ijk} D_k\,, \tag{A2}$$

where f_{ijk} and d_{ijk} are the antisymmetric or symmetric structure constants of the group $SU(4)$, respectively. The F and D matrices are defined by the relation

$$\frac{1}{2}(\bar{B}^a_{[m,n]}(\lambda_i)^b_a B_b^{[m,n]} \pm \bar{B}^m_{[b,n]} B_m^{[a,n]} (\lambda_i)^b_a) =$$

$$= \bar{B} D_i B \ (\text{or } \bar{B} F_i B). \tag{A3}$$

Here $B_c^{[a,b]}$ is a tensor representation of the $1/2^+$-baryon quark wave functions in the representation of mixed symmetry 20_m of the group SU(4), and B is the wave function in the vector representation. Using the notations of ref. /21/ we assign the vector B to the representation 20_m as follows

a) octet $(C=0)$: $\{B_i\} \to (p, n, \Lambda, \Sigma^{(+,0,-)}, \Xi^{(0,-)})$ $(i = 1,2,\ldots,8)$

b) triplet $(C=1)$: $\{B_i\} \to (A_2^{(0,+)}, A_1^{+})$ $(i = 9,10,11)$

c) sextet $(C=1)$: $\{B_i\} \to (B_3^{(0,+,++)}, B_2^{(0,+)}, B_1^{(0)})$ $(i = 12,\ldots,17)$

d) triplet $(C=2)$: $\{B_i\} \to (C_2^{(+,++)}, C_1^{+})$ $(i = 18, 19, 20)$.

(A4)

For completeness, we quote also the explicit expression for the 4x4 meson matrix $P^a_b = \sum_{i=1}^{15} (\lambda_i)^a_b \Phi_i \frac{1}{\sqrt{2}}$,

$$P^a_b = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta + \frac{\eta_c}{\sqrt{12}} & F^- \\ D^0 & D^+ & F^+ & -\frac{\sqrt{3}}{2}\eta_c \end{pmatrix}$$

(A5)

APPENDIX B

We collect here some useful formulae for the meson currents obtained by expanding them in powers of the ξ field. We restrict ourselves to zero order contributions in e and g. Using some simple algebra the meson currents may be put in the form

$$(j_\mu^{0,W})_{mes} = -i\frac{f^2}{2} < 2\hat{C}_+(\theta)\, e^{i2\xi\cdot A}\partial_\mu e^{-i2\xi\cdot A} >, \qquad \text{(B1)}$$

$$(j_\mu^{0,Z})_{mes} = -i\frac{f^2}{2} < (2\hat{C}_3 - 2\sin^2\theta_W Q)\, e^{i2\xi\cdot A}\partial_\mu e^{-i2\xi\cdot A} >, \qquad \text{(B2)}$$

$$(j_\mu^{0,A})_{mes} = -i\frac{f^2}{2} < Q\, e^{i2\xi\cdot A}\partial_\mu e^{-i2\xi\cdot A} >, \qquad \text{(B3)}$$

or using

$$e^{i2\xi\cdot A}\partial_\mu e^{-i2\xi\cdot A} = \frac{1}{f^2}[-if\partial_\mu \Phi_k \gamma_5 + if_{ijk}\Phi_i \partial_\mu \Phi_j]\lambda_k + O(\Phi^3),$$

$$(j_\mu^{0,W})_{mes} = \cos\theta[-f\sqrt{2}\partial_\mu \frac{\Phi_1 + i\Phi_2}{\sqrt{2}} - f\sqrt{2}\partial_\mu \frac{\Phi_{13} - i\Phi_{14}}{\sqrt{2}} +$$
$$+ (f_{ij1} + if_{ij2})\Phi_i \partial_\mu \Phi_j + (f_{ij13} - if_{ij14})\Phi_i \partial_\mu \Phi_j + O(\Phi^3)] +$$
$$+ \sin\theta[-f\sqrt{2}\partial_\mu \frac{\Phi_4 + i\Phi_5}{\sqrt{2}} + f\sqrt{2}\partial_\mu \frac{\Phi_{11} - i\Phi_{12}}{\sqrt{2}} +$$
$$+ (f_{ij4} + if_{ij5})\Phi_i \partial_\mu \Phi_j - (f_{ij11} - if_{ij12})\Phi_i \partial_\mu \Phi_j + O(\Phi^3)],$$

(B1')

$$(j_\mu^{0,Z})_{mes} = -f\partial_\mu(\Phi_3 + \frac{1}{\sqrt{3}}\Phi_8 - \sqrt{\frac{2}{3}}\Phi_{15}) +$$
$$+ (1 - 2\sin^2\theta_W)(f_{ij3} + \frac{1}{\sqrt{3}}f_{ij8} - \sqrt{\frac{2}{3}}f_{ij15})\Phi_i \partial_\mu \Phi_j + O(\Phi^3),$$

(B2')

$$(j_\mu^{0,A})_{mes} = (f_{ij3} + \frac{1}{\sqrt{3}}f_{ij8} - \sqrt{\frac{2}{3}}f_{ij15})\Phi_i \partial_\mu \Phi_j + O(\Phi^4).$$

(B3')

APPENDIX C

For completeness, we include in this appendix the lepton sector of the Weinberg-Salam-model written in a compact four-dimensional notation. Using the generators (6) the covariant derivatives of the lepton and Higgs multiplets ℓ, Φ_L

$$\ell = \begin{pmatrix} \nu \\ e \\ \mu \\ \nu' \end{pmatrix}, \quad \Phi_L = \frac{1+\gamma_5}{2} \begin{pmatrix} \phi_+ \\ \phi_0 \\ \phi_0 \\ \phi_+ \end{pmatrix} \tag{C1}$$

may be written as

$$\nabla_\mu(\ell;\Phi_L) = [\partial_\mu + igW_\mu \cdot \hat{C} + ig' B_\mu \frac{\hat{Y}}{2}](\ell\ ;\Phi_L), \tag{C2}$$

where y_w takes the values -1 or $+1$ for ℓ and Φ_L, respectively. The Lagrangian of the Weinberg-Salam-model then reads $(\ell_L = \frac{1+\gamma_5}{2}\ell)$

$$L_{WS} = -\frac{1}{4}(W_{k\mu\nu})^2 - \frac{1}{4}(B_{\mu\nu})^2 + \frac{1}{4}\mathrm{Tr}(\nabla_\mu \Phi_L)^+(\nabla_\mu \Phi_L) -$$

$$-V(\Phi_L^+\Phi_L) + i\bar{\ell}\gamma_\mu\nabla_\mu\ell - \frac{1}{2}\mathrm{Tr}(\bar{\ell}_L A \Phi_L + \Phi_L^+ \bar{A}\ell_L)., \tag{C3}$$

where $W_{k\mu\nu}$, $B_{\mu\nu}$ are the usual covariant curls of the gauge bosons $W_{k\mu}$, B_μ respectively. The Higgs potential may be cast into the form

$$V(\Phi_L^+\Phi_L) = \frac{\lambda}{8}\mathrm{Tr}(\Phi_L^+\Phi_L - \frac{1+\gamma_5}{2}v^2)^2. \tag{C4}$$

The last term in eq. (C3) generates the lepton masses. Using the expression

$$A = \begin{pmatrix} f_1 e_R I & 0 \\ 0 & f_2 \mu_R I \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{C5}$$

and

$$(\Phi_L)_0 = \frac{1+\gamma_5}{2} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \\ \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} \tag{C6}$$

we get $m_e = f_1 \frac{v}{\sqrt{2}}$, $m_\mu = f_2 \frac{v}{\sqrt{2}}$. In addition, there arise the following vector boson masses

$$(M_W^2)_{Higgs} = (\frac{gv}{2})^2, \quad (M_Z^2)_{Higgs} = (g^2 + g'^2)(\frac{v}{2})^2. \tag{C7}$$

Let us consider the quadratic part of the Lagrangian (C3) $(\phi_0 = \frac{1}{\sqrt{2}}v + (\sigma + i\chi))$.

$$L_{quadr} = |\partial_\mu\phi_+|^2 + \frac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\chi)^2] + \bar{M}_W^2|W_\mu^+|^2 +$$

$$+ \frac{1}{2}\bar{M}_Z^2 Z_\mu^2 - iM_W[\partial_\mu\phi^+ W_\mu^- - \partial_\mu\phi^- W_\mu^+] - M_Z\partial_\mu\chi Z_\mu. \tag{C8}$$

In eq. (C8) we have included the mass terms of the vector bosons arising from their interaction with the 0^--mesons. To get the vector propagators in the R-gauge we choose the gauge conditions

$$L_g = -\frac{1}{2\alpha}(\partial_\mu A_\mu)^2 - \beta|\partial_\mu W_\mu^+ + i\frac{M_W}{\beta}\phi^+|^2 - \frac{1}{\beta}(\frac{fg}{\sqrt{2}})^2|\phi^+|^2 -$$

$$- \frac{\gamma}{2}(\partial_\mu Z_\mu + \frac{M_Z}{\gamma}\chi)^2 - \frac{1}{4\gamma}(\frac{fg}{\cos\theta_W})^2\chi^2. \tag{C9}$$

This yields the standard expressions

$$iD^{W}_{\mu\nu} = (-g_{\mu\nu} + (1-\frac{1}{\beta})\frac{k_\mu k_\nu}{k^2 - \frac{\bar{M}^2_W}{\beta} + i\epsilon})\frac{i}{k^2 - \bar{M}^2_W + i\epsilon},$$

$$iD^{\phi^+} = \frac{i}{k^2 - \frac{\bar{M}^2_W}{\beta}}, \text{ etc.} \qquad (C10)$$

As usual, the ghost pole at $k^2 = \frac{\bar{M}^2_w}{\beta}$ in the vector propagator cancels the pole in the Goldstone (ϕ^+) propagator.

Note also that the gauge condition (C9) leaves the ("pseudo") Goldstone mesons of the strong interaction unaffected (they become massive at the end of our calculations).

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