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ON THE SUPERSYMMETRIC SOLITONS AND MONOPOLES

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О суперсимметричных солитонах и монополях

В этой работе представлены основные результаты нового направления в суперсимметрии и теории солитонов. Показано, что среднее по солитонным состояниям оператора энергии является массой солитона без квантовых поправок. Построена также новая модель с суперсимметричными монополями на базисе суперсимметрической модели син -Гордона.

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On the Supersymmetric Solitons and Monopoles

The basic results in a new trend in supersymmetry and soliton theory are presented. It is shown that the soliton expectation value of the energy operator is mass of the soliton without the quantum corrections. A new supersymmetric monopoles model in three dimensions is constructed by generalization of the supersymmetric sine-Gordon model in one space dimension.

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1. INTRODUCTION

Recent researches have revealed a possibility of obtaining information about the physical content of nontrivial quantum field theories by using semiclassical models where both the fermionic and bosonic sectors are present.

Great progress has been achieved in the two-dimensional field theories where only the bosonic sector is present. These field theories (such as the Schwinger model, the Thirring model, the nonlinear model, the Gross-Neveau model, or the sine-Gordon model) have proved to be a finite theoretical ground for testing many features believed to be characteristic of four-dimensional space-time. The reason for descending to two dimensions is that here the field theories very often admit of exact solutions and their properties may be immediately read off and compared with what is thought to happen in the real word.

A natural way of incorporating the fermionic sector in two-dimensional theories is via supersymmetric extension. The evolution of this is as follows:

At first such extensions have been worked for the sine-Gordon model [1-3] and for the nonlinear σ model [2,4]. For the four-dimensional case, the simplest possibility along this line was considered by N.S.Baaklini [5], who first constructed a classical solution to the field equation of the massless supersymmetric Wess-Zumino model.

The exact S-matrix for the elementary boson and fermion of the supersymmetric form of the sine-Gordon equation was constructed, as a byproduct of the exact S-matrix for the nonlinear σ model in two dimensions [6].

Some properties of the $(\bar{\psi}\psi)^2$ model in two dimensions were studied[7], and it was shown that the two-dimensional $(\bar{\psi}\psi)^2$ model is equivalent to the supersymmetric sine-Gordon model in a special case[1,6]. The supersymmetric extension has also been worked out for instantons[8], and it was shown that the eigenvalue equations for the fluctuations of scalars, fermions and gluon around any classical self-dual solution of the Yang-Mills theory have the same spectrum of non-zero eigenvalues.

The supersymmetric generalization of the t 'Hooft-Polyakov magnetic monopole and of Julia-Zee dyon was constructed[9], and there it was shown that the quantum corrections to the monopole mass were vanishing as a consequence of the supersymmetry of the theory.

All these results seem to suggest that two-dimensional field theories which correspond, at the classical level, to integrable Hamiltonian systems may retain complete integrability when fermionic fields are added in a supersymmetry way. The existence of higher order bosonic conservation laws in the supersymmetric sine-Gordon model and alike theories was solved in ref.[10,11].

Quite recently it has been shown[12,13] that in supersymmetric theories with solitons the usual superfield algebra is not valid; the superalgebra has been modified to include the topological quantum numbers as central charges. Using the corrected algebra, it has been shown that in certain four-dimensional supersymmetric gauge theories, there are no quantum corrections to the classical mass spectrum. There also was constructed certain supersymmetric form of the Georgi-Glashow model, and the exact mass spectrum was determined. The prediction of the mass formula is the following[12]: "...the mass of any particle is the vacuum expectation value of the Higgs field times $\sqrt{E^2+G^2}$, E and G being the electric and magnetic charges." In ref.[12] it is also shown that Bogomolny 's classical bound:

$$P_0 = \int dx \frac{1}{2}(\dot{\phi}^2 + \phi'^2 + V'^2_{(\phi)}) = \qquad (1.1)$$

$$= \int dx \frac{1}{2}[\dot{\phi}^2 + (\phi' \mp V'_{(\phi)})^2] \pm \int dx\, \phi' V'_{(\phi)} \geq |\int d\phi\, V'_{(\phi)}|,$$

where inequality is saturated for a soliton at rest, is valid quantum-mechanically (the prime denotes differentiation with respect to argument).

Finally, we want to draw attention to the reasons for which we recommend here the theories with supersymmetric solitons and monopoles. The reasons are as follows:

i) They provide a definite prescription for introducing the fermions;

ii) Infinite number of conservation laws would not exist if the fermions were introduced in an arbitrary fashion[10,11];

iii) In the supersymmetric model the axial vector current has an anomaly proportional to instanton density[4], thus making the analogy with quark-gluon theory in four dimensions stronger. Also, quark confinement is obtained in the supersymmetric sine-Gordon model[3];

iv) For the model which also exhibits dimensional transmutation[14], we were able to obtain an S-matrix with no free parameters[6,15];

v) The generalization of the two-dimensional soliton to four space-time dimensions is a magnetic monopole[12,13,23], and a special supersymmetric soliton theory unifies gauge particle with monopole solitons via a dual symmetric mass formula[12], encompassing the Higgs mechanism, and apparently exact in quantum mechanics;

vi) Possible applications of the special supersymmetry non-linear models for supergravity[16,17].

In this work we shall limit the discussion to the question why in theories with supersymmetric solitons or monopoles there are no quantum corrections. The answer is given in Sec. 2 and, in general, in Sec. 3.

A new supersymmetric monopole model is constructed by generalization of the supersymmetric sine-Gordon model and an extension of t 'Hooft-Polyakov 's model is obtained in Sec. 4, and also a zero-energy fermion solution is found.

2. *PROPERTIES OF THE SUPERSYMMETRIC SINE-GORDON MODEL*

We shall use the supersymmetric sine-Gordon model as a working example for the supersymmetric two-dimensional models with solitons. By this example we want to show only basic properties of these theories.

The dynamics of the supersymmetric sine-Gordon model will be determined by the Lagrangian density which is the coefficient of $\bar{\theta}\theta$ in the following expression[1,3]:

$$\frac{1}{2} i S(x,\theta)\bar{D}DS(x,\theta) + V(S(x,\theta)), \tag{2.1}$$

where the potential, the scalar superfield in four-dimensional superspace, and the covariant derivative are given by the usual expressions:

$$V(S(x,\theta)) = \frac{a}{b^2}(\cos bS(x,\theta) - 1), \tag{2.2a}$$

$$S(x_\mu, \theta_\alpha) = \phi(x) + i\bar{\theta}\psi(x) + \frac{1}{2} i\bar{\theta}\theta F(x), \tag{2.2b}$$

$$\bar{D}_\alpha = \frac{\partial}{\partial\theta} + i\bar{\theta}\gamma.\partial, \tag{2.2c}$$

where a, b are parameters, $\mu=0,1$; $\alpha=0,1$ and the supermultiplet $\{\phi(x), \psi(x), F(x)\}$ contains the Majorana fermion $\psi(x)$ and the Bose fields $\phi(x)$ and $F(x)$. The expression (2.1) results in the supersymmetric form of the Lagrangian density in two-dimensional space:

$$L = \frac{1}{2}\phi\Box\phi + \frac{1}{2}F^2 - \frac{1}{2}i\bar{\psi}\partial\!\!\!/\psi - \frac{a}{2b}F\sin b\phi - \frac{1}{2}ia\bar{\psi}\psi\cos b\phi. \tag{2.3}$$

There is no kinetic term for F; hence this field can be eliminated by using the equation of motion which includes

$$F - \frac{a}{2b}\sin b\phi = 0 \tag{2.4}$$

and we thus get the Lagrangian density in the form:

$$L = \frac{1}{2}\phi\Box\phi - \frac{1}{2}i\bar{\psi}\partial\!\!\!/\psi - \frac{1}{8}\frac{a^2}{b^2}\sin^2 b\phi - \frac{1}{2}ia\bar{\psi}\psi\cos b\phi. \tag{2.5}$$

So, we obtain two basic equations of motion coupling the Fermi field ψ and the Bose field ϕ:

$$(i\partial\!\!\!/ + ia\cos b\phi)\psi = 0, \tag{2.6a}$$

$$\Box\phi - \frac{1}{4}\frac{a^2}{b^2}\sin b\phi\cos b\phi + \frac{1}{2}iab\bar{\psi}\psi\sin b\phi = 0. \tag{2.6b}$$

We can see that if we have no spinor field (i.e., $\psi = 0$) or if $\bar{\psi}\psi=0$, then (2.6b) is equivalent to the sine-Gordon equation:

$$\Box\phi - \frac{a^2}{8b}\sin 2b\phi = 0 = \Box\phi + \frac{\alpha_0}{\beta}\sin\beta\phi \tag{2.7}$$

and we obtain the following expression for the parameters a, b in the usual notation[18]:

$$b = \frac{1}{2}\beta, \quad a = 2i\sqrt{\alpha_0}.$$

We know the static classical solution of eq. (2.7), namely, the soliton solution[18]:

$$\phi_c = \frac{4}{\beta}\,\mathrm{tg}^{-1}\exp\sqrt{\alpha_0}\,x. \tag{2.8}$$

Now we want to obtain the static solution ψ. For this purpose we shall consider the soliton ϕ_c as an input potential in eq. (2.6a), and for the static solution ψ we get

$$i\gamma^1\partial_1\psi = 2\sqrt{\alpha_0}\cos[2\,\mathrm{tg}^{-1}\exp\sqrt{\alpha_0}\,x]\,\psi. \tag{2.9}$$

The static solution of eq. (2.9) has the form[3]:

$$\psi_c(x) = C(\cosh\sqrt{\alpha_0}\,x)^{-2}\begin{pmatrix}1\\ -i\end{pmatrix}, \tag{2.10}$$

where C is a constant.

The static solutions ϕ_c and ψ_c will be called the soliton solutions, because they are both permanently confined in space[19]. These solutions ϕ_c and ψ_c are also solutions of the supersymmetric sine-Gordon theory, so we shall call these solutions "supersymmetric solitons" and the superfield $S(x,\theta)$ -"supersoliton"[1]. Moreover, because the field equations (2.6a,b) are Lorentz invariant, once we have the static solutions $\phi_c(x)$ and $\psi_c(x)$, we also have the boosted solutions $\phi_c(\frac{x-vt}{\sqrt{1-v^2}})$ and $S(\Lambda)\psi_c(\frac{x-vt}{\sqrt{1-v^2}})$ for arbitrary v, $|v|<1$. The supersymmetric solitons can move in space.

It can be easily shown that for $\psi_c(x)$ we get

$$\bar{\psi}\psi = \psi^+\gamma^o\psi = C^2(\cosh\sqrt{a_o}\,x)^{-4}(1,\,i)\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\-i\end{pmatrix} = 0$$

and also

$$\psi\gamma^1\frac{d}{dx}\psi = C^2(\cosh\sqrt{a_o}\,x)^{-4}(1,\,i)\begin{pmatrix}-1&0\\0&1\end{pmatrix}\begin{pmatrix}1\\-i\end{pmatrix} = 0.$$

Hence the field ψ_c is of no importance in energy in the supersymmetric sine-Gordon theory, it is

$$E_c(\phi_c,\psi_c) = E_c(\phi_c). \tag{2.11}$$

This property is typical of a "bag" model[3] where the Higgs field ϕ and the confinement of the field ψ represents "kink with a trapped quark". This is the case of nontopological binding which has been shown to arise from the interaction of the fermion with the Higgs field.

Our result (2.11) means that the field ψ_c is unessential in the classical mass of the Bose soliton ϕ_c, because the integral of the mass term vanishes.

This result

$$E_c(\phi_c,\psi_c) = E_c(\psi_c) = M$$

can be obtained also for the lowest quantum correction for an arbitrary model with supersymmetric solitons[8].

For this purpose we shall assume the most general supersymmetric Lagrangian for an arbitrary function $V(\phi)$ in the form:

$$L(\phi,\psi) = \frac{1}{2}\phi\Box\phi - \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi - \frac{1}{2}[V'(\phi)]^2 - \frac{1}{2}i\bar{\psi}\psi V''(\phi). \tag{2.12}$$

For the time-independent solutions the following first order equations are valid:

$$\frac{d}{dx}\phi_c(x) = V'(\phi_c), \tag{2.13a}$$

$$\frac{d}{dx}\psi_{\mp}(x) = \pm V''(\phi_c)\,\psi_{\mp}(x), \tag{2.13b}$$

where for the static solution $\psi(x)$ we have defined [2, 8]:
$\psi_{\pm}(x) = \frac{1}{2}(1\pm\gamma_1)\psi(x)$, $\psi(x) = \psi^+(x) + \psi^-(x)$.

In ref.[20] it is shown that the Bose soliton mass in the lowest quantum correction is the classical energy plus half sum of the small fluctuation frequencies, a completely reasonable quantum-mechanical formula. For our Fermi-Bose system such a formula reads:

$$M = E_C(\phi_C) + \frac{1}{2}\Sigma\,\omega_B - \frac{1}{2}\Sigma\, i\omega_F. \tag{2.14}$$

In order to evaluate the lowest quantum corrections, we have to solve the following eigenvalue equations [8, 19, 20]:

$$\{-\frac{d^2}{dx^2} + [V''(\phi_C)]^2 + V'(\phi_C)V'''(\phi_C)\}\phi(x) = \omega_B^2\,\phi(x), \tag{2.15a}$$

$$\{\gamma_1\frac{d}{dx} + V''(\phi_C)\}\psi(x) = i\omega_F\gamma_0\psi(x). \tag{2.15b}$$

By using the components $\psi_{\pm}$ eq. (2.15b) becomes:

$$\{\frac{d}{dx} + V''(\phi_C)\}\psi_+ = i\omega_F\gamma_0\psi_-, \tag{2.16a}$$

$$\{-\frac{d}{dx} + V''(\phi_C)\}\psi_- = i\omega_F\gamma_0\psi_+. \tag{2.16b}$$

It is easy to prove that

$$\{\frac{d}{dx} + V''(\phi_C)\}\{-\frac{d}{dx} + V''(\phi_C)\}\psi_- = \tag{2.17}$$

$$= \{-\frac{d^2}{dx^2} + [V''(\phi_C)]^2 + V'(\phi_C)V'''(\phi_C)\}\psi_- = -\omega_F^2\psi_- ,$$

where eqs. (2.13a,b) have been used.

So, we can see that eq. (2.17) for ψ_- is identical to eq. (2.15a) for ϕ, when $\omega_B^2 = -\omega_F^2$. If $\phi(x)$ is an eigenfunction of eq. (2.15a) with eigenvalue $\omega_B^2 \neq 0$, then one can construct the following eigenfunction of the fermion equation:

$$\psi_-(x) = \phi(x)\, u_- , \quad \psi_+(x) = -(\omega_B)^{-1}\gamma^0\{-\frac{d}{dx} + V''(\phi_C)\}\psi_-(x),$$

where eq. (2.16b) and $\omega_B = i\omega_F$ have been used and u_- is an arbitrary x-independent spinor.

As a consequence of the equality $\omega_B = i\omega_F$ one gets that the one loop correction to the soliton mass is identically vanishing and formula (2.14) has the form

$$M = E_C(\phi_C)$$

equivalent to (2.11) but now quantum-mechanically at lowest quantum correction. Why in theories with supersymmetric solitons there are no quantum corrections to the mass of the Bose soliton we shall see in Sec. 3.

3. *THE SUPERSYMMETRY ALGEBRA AND A NEW FORMULA*

In this section we shall start from a very interesting result which is obtained by E.Witten and D.Olive[12].

They have shown how in theories with supersymmetric solitons the superalgebra is modified to include central charges because certain surface terms, customarily discarded in deriving the algebra, are actually non-vanishing.

In theories with supersymmetric solitons a conserved supercurrent

$$\partial_\mu\{[\not\partial\phi - V'(\phi)]\gamma^\mu\psi\} = 0 \tag{3.1}$$

exists. From (3.1) it is possible to obtain the supercharges Q_α and show that the usual anticommutation relation between supercharges Q_α is invalid. The true relation has the form

$$\{Q_\alpha, Q_\beta\} = 2(\gamma.P\gamma^0)_{\alpha\beta} + 2iT(\gamma_5\gamma_0)_{\alpha\beta} , \tag{3.2}$$

where

$$T = \int d\phi\, V'(\phi) .$$

The extra term T appears in (3.2) due to the nontrivial boundary condition. T, being a surface term, must commute with ϕ, ψ and all other generators of the algebra, and so is what is called a "central charge"[21].

The relation (3.2) has the form[13]:

$$Q_1^2 = P^0 + P^1, \tag{3.3a}$$

$$Q_2^2 = P^0 - P^1 \tag{3.3b}$$

$$\{Q_1, Q_2\} = 2T . \tag{3.3c}$$

If the Hermitean Lorentz scalar operator

$$\bar{Q}Q = i(Q_2Q_1 - Q_1Q_2)$$

is considered and squared it, using relations (3.3) E.Witten and D.Olive[12] obtained exact Lorentz invariant "mass formula"

$$M^2 = P_0^2 - P_1^2 = T^2 + \frac{1}{4}(\bar{Q}Q)^2. \tag{3.4}$$

The mass formula implies Bogomolny's bound:

$$M \geq T . \tag{3.5}$$

In this way Bogomolny's classical bound is shown to be true in quantum field theory.

We know that the inequality (3.5) is saturated for a soliton at rest. But in this case

$$\bar{Q}Q = i(Q_1 + Q_2)(Q_1 - Q_2) = -i(Q_1 - Q_2)(Q_1 + Q_2),$$

where

$$Q_2 \pm Q_1 = \int dx\, [(\pm\partial_1 \phi + V'(\phi))(\psi_1 \mp \psi_2)] = 0 \qquad (3.6)$$

for any soliton (antisoliton) case, and so $\bar{Q}Q$ vanish.

Now we show why in theories with supersymmetric solitons are no quantum corrections to the mass of the soliton.

It is shown[22] that, in a supersymmetric theory, the sum of all vacuum diagrams vanishes identically as a consequence of compensation among contributions involving Bose and Fermi fields of the supermultiplet. This means that the zero energy-point energy momentum density is zero to all orders in the supersymmetric interaction.

If the supercharge annihilates vacuum, then the vacuum expectation value of the energy momentum tensor is zero $\langle 0|T_{\lambda\mu}(x)|0\rangle = 0$ and especially

$$\langle 0|\int d^3x\, T_{00}(x)|0\rangle = 0.$$

In theories with supersymmetric solitons a beautiful analogy exists. We shall assume that the role of the vacuum state, the soliton state $|s\rangle$ will play. Then we obtain:

$$\begin{aligned} \langle s|P_0|s\rangle &= \frac{1}{2}\langle s|(Q_1^2 + Q_2^2)|s\rangle \\ &= \frac{1}{2}\langle s|[\mp\{Q_1, Q_2\} + (Q_1 \pm Q_2)^2|s\rangle \\ &= \langle s|T|s\rangle + \frac{1}{2}\langle s|(Q_2 \pm Q_1)^2|s\rangle \qquad (3.7) \\ &= \langle s|T|s\rangle, \end{aligned}$$

where we used the relations (3.3a,b,c), and (3.6).

Now, because for solitons from Bogomolny's bound (3.5) $T = M$ follows, we obtain from (3.7) the formula for the mass of the Bose soliton:

$$\langle s|P_0|s\rangle = M, \qquad (3.8)$$

where we suppose normalization $\langle s|s\rangle = 1$.

The formula (3.8) is the reason why the mass of the Bose soliton in theories like the supersymmetric sine-Gordon model one is unchanged not only classically but also quantum mechanically and there are no quantum corrections to the mass of the soliton.

4. A MODEL WITH SUPERSYMMETRIC MONOPOLES

One of the ideas based on a permanent theoretical progress in classical field theory concerns the existence of the monopole type solutions in the classical gauge theory[23]. It is well known that monopoles exist in field theories which contain the massless photon field and charged matter fields carrying a positive anomalous magnetic moment.

These matter fields can be both Fermi and Bose fields, and, correspondingly, there will exist monopoles with magnetic charges $G = \pm 1/2e, \pm 1/e \ldots$.

The generalization of the two-dimensional soliton to four space-time dimensions is a magnetic monopole. If we want to obtain supersymmetric monopoles, we would start with a two-dimensional supersymmetric model with solitons, and generalizing such a model we can obtain the Fermi and Bose monopole solutions.

Now we shall show that the supersymmetric sine-Gordon model is a good candidate for obtaining supersymmetric monopoles.

We shall proceed as follows:

at first we construct the Lagrangian density from relation (2.1) for the scalar superfield $\bar{D}V$, where the covariant derivative is given by (2.2c) and the spinor superfield is defined as

$$V_\alpha(x,\Theta) = \xi_\alpha + \frac{1}{2}\gamma_5\gamma_\mu\Theta_\alpha A^\mu + \frac{1}{2}\phi\Theta_\alpha + \frac{1}{2}\sigma(\gamma_5\Theta)_\alpha + \frac{i}{2}\bar{\theta}\theta\zeta_\alpha . \quad (4.1)$$

The ordinary fields ξ_α, ζ_α are Majorana spinors, A_μ is a vector, σ and ϕ, respectively, a pseudoscalar and a scalar.

An Abelian gauge transformation on the spinor superfield is defined/24/ as:

$$V_\alpha \longrightarrow V_\alpha + i(\gamma_5 D)_\alpha \Lambda ,$$

where Λ is a scalar superfield: $\Lambda(x,\Theta) = \lambda + \dots$.

Therefore ϕ and $\psi = \zeta - \gamma\cdot\partial\xi$ are gauge invariant fields. The vector field A_μ transforms as usual $A_\mu \to A_\mu + \partial_\mu\lambda$ and ξ, σ are submitted to an arbitrary translation under gauge transformation. Two-dimensional Wess-Zumino gauges are therefore defined as follows/24/:

$$\xi = \sigma = 0, \ \delta\phi = \delta\psi = 0, \quad \delta A_\mu = \partial_\mu\lambda , \quad (4.2)$$

λ being a gauge function.

So the quantity $\bar{D}V$ can be written as

$$\bar{D}_\alpha V_\alpha = \phi - i\Theta\psi + \frac{i}{2}F\bar{\Theta}\Theta , \quad (4.3)$$

where

$$F = \epsilon_{\mu\nu}\partial^\mu A^\nu . \quad (4.4)$$

The quantity $\bar{D}V$ is manifestly gauge invariant because $\bar{D}\gamma_5 D = 0$.

Using the quantity $\bar{D}V$ in relation (2.1) we obtain the following expression

$$\frac{i}{2}\bar{D}V\,\bar{D}D\,\bar{D}V + \frac{a}{b}(\cos b\bar{D}V - 1)$$

and hence the Lagrangian density for ordinary fields in expansion (4.3) has the form (2.3) from Sec. 2. Using the equation of motion for the field F and the expression (4.4) we get the supersymmetric Lagrangian density:

$$L = \frac{1}{2}\phi\Box\phi - \frac{1}{4}F^2_{\mu\nu} - \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi - \frac{a^2}{4b^2}\sin^2 b\phi - \frac{i}{2}a\bar{\psi}\psi\cos b\phi . \quad (4.5)$$

Now we shall study small oscillations about the ground state $\phi = 0$. We can expand the cosine and the sine in power series and obtain:

$$L = \frac{1}{2}\phi\Box\phi - \frac{1}{4}F^2_{\mu\nu} - \frac{i}{2}\psi\partial\!\!\!/\psi - \frac{a^2}{4}\phi^2 + \frac{a^2b^2}{12}\phi^4 - \frac{i}{2}a\bar{\psi}\psi + \frac{i}{4}a\bar{\psi}\psi b^2\phi^2 . \quad (4.6)$$

We shall specify the parameters a, b by the usual notation/3/:

$$g = i\frac{ab^2}{2}, \quad \mu^2 = -\frac{a^2}{4}, \quad \lambda = -\frac{a^2b^2}{6} ,$$

where λ and μ^2 are the positive parameters in the familiar ϕ^4 theory and g is the coupling constant. So, we obtain the supersymmetric Lagrangian density in two dimensions:

$$L = -\frac{1}{4}F^2_{\mu\nu} + \frac{1}{2}\phi\Box\phi - \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi - \frac{\lambda}{2}(\phi^2 - \frac{\mu^2}{\lambda}) + \mu\bar{\psi}\psi + \frac{g}{2}\phi^2\bar{\psi}\psi . \quad (4.7)$$

The Fermi-Bose monopole model in four space-time dimensions is obtained from the Lagrangian density (4.7) with supersymmetric solitons by extending (4.7) to the Yang-Mills-Higgs model, which is constructed on SU(2) group.

We get:

$$L = -\frac{1}{4}F^2_{a\mu\nu} + \frac{1}{2}(D^\mu_\phi)_a(D^\mu_\phi)_a - \frac{\lambda}{2}(\phi^2_a - \frac{\mu^2}{\lambda}) - \frac{i}{2}\bar{\psi}D\!\!\!/\psi + \mu\bar{\psi}\psi + \frac{g}{2}\phi^2_a\bar{\psi}\psi . \quad (4.8)$$

The symbols in the Lagrangian density (4.8) mean:

$$F^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + e\epsilon_{abc}A^\mu_b A^\nu_c ,$$

$$(D^\mu_\phi)_a = \partial^\mu\phi_a + e\epsilon_{abc}A^\mu_b\phi_c ,$$

$$D\!\!\!/\psi = \gamma^\mu(\partial_\mu\psi - i\frac{e}{2}\sigma^a A^\mu_a\psi), \quad \mu,\nu = 0, 1, 2, 3,$$

where σ^a are the Pauli matrices; a is the isospin index

We can see that our Lagrangian density (4.8) consists of fermionic and bosonic part. The bosonic part is exactly t'Hooft-Polyakov's model[25] which has the monopole solutions in the form:

$$A_a^0 = 0,$$

$$A_a^i = \epsilon^{aij} \hat{r}_j \, a(r), \qquad (4.9)$$

$$\phi_a = \hat{r}^a f(r).$$

Symbols in the monopole solution (4.9) mean: $\hat{r}$ is the radial unit vector, $a(r)$ and $f(r)$ depend only on the radial magnitude; the vector field A_a^μ and the scalar field ϕ_a have the isospin index a, whereas for $\hat{r}_a$ and ϵ^{aij} the index a is a spatial index.

We shall now consider the solution (4.9) as a potential in the fermionic part of our model. So, we obtain for the static solution the following equation (for the notation see ref.[26]):

$$[-i\vec{\alpha}\cdot\vec{\nabla}\delta_{nm} + \frac{e}{2}\sigma^a_{nm} a(r)(\vec{\alpha}\times\hat{r})_a + (2\beta\mu + g\beta f^2(r))\delta_{nm}]\psi_m = E\psi_n, \qquad (4.10)$$

where isospin indeces n, m take values $1, 2$; $\gamma_i = \beta\alpha_i$.

We want to get the zero-energy mode of the Dirac equation (4.10) using the methods of ref.[26]. We know that the Hamiltonian of (4.10) commutes with the generator of rotations in the sum of orbital and the spin angular momenta plus isospin. This gives a possibility of expressing the problem in terms of scalar and vector functions, because the true spin of our Dirac particle is not 1/2 but rather 0 or 1.

Separate ψ into upper and lower components as usual, we can define the two scalar and two vectors fields uniquely related to $\psi^\pm$ by the linear relation:

$$\psi^\pm_{im} = (g^\pm \delta_{im} + \vec{g}^\pm \cdot \vec{\sigma}_{in})\sigma^2_{nm}, \qquad (4.11)$$

where i, m are spin and isospin indices respectively.

For the zero-mode energy it is shown in ref.[26] that and $\vec{g}^\pm(r) = 0$, and our problem thus is reduced to solve the equation:

$$(\frac{d}{dr} - a(r) + 2\mu + g f^2(r)\, g^+(r) = 0. \qquad (4.12)$$

For the static spinor solution, using (4.11) and the solution of the eq. (4.12), we get:

$$\psi^+_{im} = N \exp\{\int_0^r dr'[a(r') - 2\mu - g f^2(r')]\} \times (S_i^+ S_m^- - S_i^- S_m^+), \qquad (4.13)$$

$$\psi^-_{im} = 0,$$

where $S^+(S^-)$ is the positive-(negative)-eigenvalue eigenvector of σ^3 and N is a constant.

So, we obtained the static fermion solution (4.13) and the static Bose monopole solution (4.9) - the supersymmetric monopoles.

In our model we get the massive spinor field as is assumed for the fermion-monopole systems.

5. COMMENTS

We have discussed the realization of theories with supersymmetric solitons and monopoles and also presented some new trends in these theories. We have studied the interesting property of these theories:

The mass of the Bose soliton is unchanged when the Fermi field is incorporated via the supersymmetric extension. If we use the Bose soliton as a potential in these theories, then, as was explicitly shown, $\bar{\psi}\psi$ and $\bar{Q}Q$ vanish. The main goal of this work is the formula

$$\langle s|P_0|s\rangle = M,$$

which means that the soliton expectation value of the energy is mass of the Bose soliton and there are no quantum corrections to the mass of the soliton.

This is the basic results because it can be a starting point for the theories in four space-time dimensions whose mass spectra could be exactly known[12].

We have also constructed a new supersymmetric model and obtained exact static solutions. It is easy to show that in this case $\bar{\psi}\psi$ vanish also.

At the end we want to add that theories in which there are no quantum corrections to the classical mass spectrum are often completely integrated at the classical level. It was shown[11] that theories with supersymmetric solitons and monopoles are in that category.

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