

ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ  
ДУБНА



19/11-79

E2 - 11935

Z-36

L.G.Zastavenko

869 / 2-79

ELEMENTARY PROOF  
OF THE EXTREMUM PROPERTY  
OF THE REAL NODELESS SOLUTION  
OF THE SCHRÖDINGER EQUATION

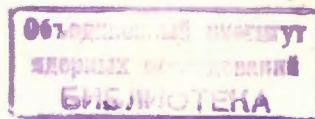
**1978**

E2 - 11935

L.G.Zastavenko

**ELEMENTARY PROOF  
OF THE EXTREMUM PROPERTY  
OF THE REAL NODELESS SOLUTION  
OF THE SCHRÖDINGER EQUATION**

*Submitted to "Communications  
in Mathematical Physics"*



Заставенко Л.Г.

E2 - 11935

Элементарное доказательство экстремального свойства  
безузлового решения уравнения Шредингера

Дано элементарное доказательство экстремального свойства  
безузлового решения уравнения Шредингера.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

Zastavenko L.G.

E2 - 11935

Elementary Proof of the Extremum Property  
of the Real Nodeless Solution of the  
Schrödinger Equation

A simple proof of the extremal property (3) of  
the real nodeless solution  $\Omega_0(x)$  of Schrödinger equation  
(1) is given.

The investigation has been performed at the  
Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research.

Dubna 1978

The real nodeless solution  $\Omega_0(x)$  of the  
Schrödinger equation in the nonrelativistic  
quantum mechanics

$$H\Omega_0 = E_0\Omega_0, \quad (1)$$

$$H = -\frac{1}{2} \frac{\partial^2}{\partial x^2} + V(x), \quad (2)$$

is well known (see, e.g., ref.<sup>/1/</sup>, volume I,  
chapter VI) to give to the functional of  
the energy

$$\epsilon(\Omega) = \int \Omega^* H \Omega dx / \int \Omega^* \Omega dx \quad (3)$$

its minimal value  $E_0$ :

$$\epsilon(\Omega) > E_0 = \epsilon(\Omega_0) \quad (4)$$

if the function  $U(x)$ ,

$$U(x) = \Omega(x)/\Omega_0(x), \quad (5)$$

is not constant. The proof<sup>/1/</sup> of this proper-  
ty, though being not very complicated, is  
not simple enough to be included in the text-  
book of quantum mechanics.

We give here the elementary proof of  
inequality (4).

1. From (5), (2) and (1) it follows

$$(H - E_0)\Omega = -\frac{1}{2}\Omega_0^2 \frac{\partial^2 U}{\partial x^2} - \frac{\partial U}{\partial x} \frac{\partial \Omega_0}{\partial x} \quad (6)$$

and

$$\begin{aligned} & \{\epsilon(U\Omega_0) - E_0\} \int \Omega_0^2 U^* U dx = \\ & = -\int dx \left[ \frac{1}{2}\Omega_0^2 U^* \frac{\partial^2 U}{\partial x^2} + \Omega_0^2 U^* \frac{\partial U}{\partial x} \frac{\partial \Omega_0}{\partial x} \right]. \end{aligned} \quad (7)$$

Integration by parts here gives

$$\epsilon(\Omega) = E_0 + \frac{1}{2} \int dx \Omega_0^2 \frac{\partial U^*}{\partial x} \frac{\partial U}{\partial x} / \int dx \Omega_0^2 U^* U, \quad (8)$$

this equality results in (4).

2. Let us now take as  $\Omega_0$  the real solution of eq. (1), which has zeroes. Then (8) gives that  $\epsilon(\Omega) > \epsilon(\Omega_0)$  if functions  $\Omega$  and  $\Omega_0$  have the same zeroes. If, on the contrary,  $\Omega_0(x_0) = 0$  and  $\Omega(x_0) \neq 0$  for some  $x_0$ , then the function  $U(x)$  has a singularity at the point  $x = x_0$ ; now the transition from (7) to (8) is incorrect and so may be eq. (4).

#### REFERENCES

1. Courant R., Gilbert D. *Metody Matematicheskoi Fiziki*, vol. I, Gostekhizdat, Moscow, 1951.

Received by Publishing Department  
on October 5 1978.