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QUANTUM FIELD THEORY WITH A MOMENTUM SPACE OF CONSTANT CURVATURE (PERTURBATION THEORY)

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ОИЯИ
БИБЛИОТЕКА

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Квантовая теория поля с пространством импульсов постоянной кривизны (теория возмущений)

В рамках теоретико-полевого подхода, в котором расширение за массовую поверхность происходит в p-пространстве постоянной кривизны, построена теория возмущений. Показано, что матричные элементы S-матрицы задаются сходящимися выражениями.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Quantum Field Theory with a Momentum Space of Constant Curvature (Perturbation Theory)

In the framework of the field-theoretical approach, in which the extension off the mass shell proceeds in the p-space of constant curvature, the perturbation theory is developed. It is shown that the S-matrix elements are given by convergent expressions.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Introduction

An essential point in the conventional procedure of the S-matrix extension off the mass shell is the assumption that the four-dimensional momentum space off the mass shell is a flat pseudoeuclidean space. In fact, this is an independent additional postulate of quantum field theory (Q.F.T.) . The analysis of the axiomatic Q.F.T. has shown /1-11/, that there exists an alternative of local Q.F.T. in which the virtual 4-momenta belong to the de Sitter space

$$p_0^2 - \vec{p}^{\,2} + M^2c^2 p_4^2 = M^2c^2 = \frac{\hbar^2}{\ell^2} . \qquad (1.1)$$

Here ℓ is the universal constant (the fundamental length). In what follows we put $\hbar = c = \ell = M = 1$.

The picture of elementary particle interaction with momenta $|p| \gtrsim 1$ differs essentially from that existing in the standard Q.F.T. . For $|p| \ll 1$ the geometry of de Sitter p-space is indistinguishable from the usual pseudoeuclidean geometry.

In this report we consider another de Sitter space with metric

$$p_0^2 - \vec{p}^{\,2} - p_4^2 = g_{KL}\, p^K p^L = -1 \qquad (1.2)$$

$$K, L = 0,1,2,3,4 \ ; \ g_{KL} = diag(1,-1,-1,-1,-1) \ .$$

Following ref./1-11/ for the space (1.1), we introduce here, through the Fourier transformation on the group of motions of the space (1.2), the configurational representation. This representation is canonically conjugated to the de Sitter p-space. The geometry of this new space at small distances differs essentially from the pseudoeuclidean one. It is remarkable that at the same time the new ξ -space possesses the "causal structure". It splits in two irreducible regions: the time-like one (continuous) with the invariant ordering in time, and the space-like one (discrete). But the light cone, i.e., the surface, which divides in the Minkovsky space these two regions, does not exist in the ξ -space. Later on we shall convince ourselves of that this fact influences essentially the problem of the multiplication of singular functions in Q.F.T.

On the basis of a natural generalization of the Bogolubov causality condition to the case of the new configurational representation, we construct a perturbation theory with the local in ξ-space Lagrangian density function. The obtained S-matrix obeys all the axioms of Q.F.T. /12-14/ , including the requirement of translation invariance.

2. The configurational ξ- representation

In the conventional Q.F.T. with the flat p-space we have two representatives of inhomogeneous pseudoeuclidean motion group, given by the transformation formulae:

$$x'_\mu = \Lambda_\mu^{\ \nu} x_\nu + a_\mu \, , \tag{2.1}$$

$$p'_\mu = \Lambda_\mu^{\ \nu} p_\nu + \lambda_\mu \, , \tag{2.2}$$

$$(\mu, \nu = 0,1,2,3) \, ,$$

where Λ denotes the homogeneous Lorentz transformations. The relativistic invariance of Q.F.T . is formulated in terms of the Poincare group (2.1), i.e., the motion group of the configuration space. The group (2.2) has no direct physical meaning; it is not the group of invariance of the physical theory. To this end, to show that vacuum is not invariant under the transformations (2.2) .

Nevertheless, certain quantities in the framework of group (2.2) have definite interpretation and, moreover, in the theory they play the key role. The Casimir operator

$$\left(-i \frac{\partial}{\partial p_\mu}\right)^2 = \hat{\sigma}^2 \tag{2.3}$$

is the interval (proper time) operator. For the unitary representations of the group (2.2) we have the following spectrum of eigenvalues of σ^2 :

1) $\sigma^2 > 0$ the time-like region,
2) $\sigma^2 = 0$ the light-cone,
3) $\sigma^2 < 0$ the space-like region.

In regions 1) and 2) we have an additional discrete invariant of the group (2.2), the sign of time.

When passing to the de Sitter p-space, the group (2.2) is replaced by the de Sitter group O(2.3), because the parallel shifts

$$p' = p + k \tag{2.4}$$

are replaced by the generalized shifts

$$p' = p (+) k \, . \tag{2.5}$$

The new operation of the parallel shift, which we denote by (+) has the following form

$$p'_\mu = p_\mu + k_\mu \left(p_4 + \frac{p_\nu k_\nu}{1 + k_4}\right), \tag{2.6}$$

$$p'_4 = p_\mu k_\mu + p_4 k_4 \, .$$

These transformations are the pseudoeuclidean rotations in the hyperplanes which contain the vacuum 5-vector $V_L = (0_\mu, 1)$ and the 5-vector k .

In the "flat limit" $p, k \ll 1$ shifts (2.5) transform to the usual parallel shifts (2.4).

The generalization of the operator $\hat{\sigma}^2$ (2.3) to the case of the de Sitter p-space is the Casimir operator of the group SO(4.1):

$$\frac{1}{2} M_{KL} M^{KL} = -\frac{1}{\sqrt{|g|}} \frac{\partial}{\partial p_\mu} \left(\sqrt{|g|}\, g^{-1}_{\mu\nu} \frac{\partial}{\partial p_\nu} \right) = \hat{S}$$

$$K, L = 0, 1, 2, 3, 4\ , \tag{2.7}$$

where

$$g_{\mu\nu} = \delta_{\mu\nu} + \frac{p_\mu p_\nu}{1 - p^2}\ , \quad g = \det \| g_{\mu\nu} \| \ . \tag{2.8}$$

Two series of unitary irreducible representations of the group SO(4,1) exist which correspond to the following eigenvalues of the operator $\hat{S}$ (2.7):

1) The continuous Λ -series

$$S = \sigma(\sigma + 3) = \frac{9}{4} + \Lambda^2\ , \quad \sigma = i\Lambda - \tfrac{3}{2}\ , \quad 0 < \Lambda < \infty \tag{2.9a}$$

2) The discrete L -series

$$S = \sigma(\sigma + 3) \qquad \sigma = L = -1, 0, 1, 2, \ldots \tag{2.9b}$$

In the flat limit the Λ -series turns into the time-like region of the usual Minkowskian x-space ($\sigma^2 > 0$), the L -series into the space-like region. Later on we shall see, that in the Λ -series the additional discrete invariant (the "sign of time") also does exist. Let us emphasize that there is no analog of the light cone for the curved p-space. The new time-like and space-like regions are divided by the finite interval.

Let us consider the eigenfunctions $\langle \xi | p \rangle$ of the Casimir operator(2.7) in the time-like Λ -region

$$\frac{1}{2} M_{KL} M^{KL} \langle \xi | p \rangle = \left[\left(\tfrac{3}{2}\right)^2 + \Lambda^2 \right] \langle \xi | p \rangle . \tag{2.10}$$

The flat limit of this equation is

$$\left(-i \frac{\partial}{\partial p_\mu} \right)^2 e^{ipx} = x^2 \cdot e^{ipx} . \tag{2.11}$$

Let us solve now the important problem of parametrization of the quantity ξ , i.e.,of the point of the new configurational space. Consider the set of 5-vectors N_L belonging to the cone:

$$g_{KL} N^K N^L = 0 .$$

It is easy to see that the quantity

$$\langle \xi | p \rangle = \left(N_L p^L \right)_+^{i\Lambda - 3/2} \tag{2.12}$$

satisfies equation (2.9). The subscript $+$ in expression (2.12) means that we should treat $\langle \xi | p \rangle$ as a generalized function x) (comp. /15/). Each of the poles of the cone (2.11), corresponding to the positive and negative signs of the time component $N_0 \gtrless 0$ is transitive under de Sitter group transformations. We conclude from here that in Λ -series a discrete invariant (the sign of the time) exists.

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x) Later on when analysing the matrix elements of S-matrix we shall see that in the Q.F.T. with the curved p-space the ultraviolet divergences are absent and the problem of regularization of the field-theoretical objects, propagators, does not arise.

It looks like that this problem in the developed scheme is transferred to the plane waves $\langle \xi | p \rangle$. It is important to stress, that here this problem is solved uniquely with the help of group-theoretical considerations and the requirement of the proper flat limit (see (2.16)).

The quantities $\langle \xi | p \rangle$ serve as kernels for the Fourier transformation on the de Sitter space. By the point of the configurational ξ-space we call the set of 4 variables

$$\xi = (\Lambda, N_\mu), \tag{2.13}$$

where N_μ is the four-dimensional part of the five-dimensional isotropic vector belonging to the "contour Γ which crosses all generatrices of the cone" (2.11) (cf. /15/). We choose the equation of the contour Γ in the form

$$N_4^2 = 1 \tag{2.14}$$

hence it follows that N_μ is the unit time-like 4-vector

$$N_0^2 - \vec{N}^2 = 1 . \tag{2.15}$$

In the flat limit we have

$$\langle \xi | p \rangle \Big|_{N_4=-1} = e^{\left(\frac{i\Lambda}{\ell} - \frac{3}{2}\right)\ln\left(\frac{\frac{\hbar}{\ell}p_4 + p_\mu N^\mu}{\hbar/\ell}\right)} \to \tag{2.16}$$

$$\to e^{i\frac{(p_\mu N^\mu)\Lambda}{\hbar}} = e^{i\frac{p_\mu x^\mu}{\hbar}}$$

$$(x_\mu = \Lambda \cdot N_\mu) .$$

The region $N_4 = +1$ has no analog in the usual theory. When $\ell \to 0$ its contribution vanishes.

Let us introduce in the de Sitter p-space (1.2) the coordinate system

$$\begin{aligned} p_0 &= sh\,\chi \\ \vec{p} &= ch\,\chi \cdot \sin\omega \cdot \vec{n} \\ p_4 &= ch\,\chi \cdot \cos\omega \\ \vec{n} &= (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta) \\ -\infty < \chi &< \infty , \quad 0 < \omega, \vartheta < \pi , \quad 0 < \varphi < 2\pi \end{aligned} \qquad d\Omega_p = ch^3\chi\, d\chi \cdot \sin^2\omega\, d\omega \cdot \sin\vartheta\, d\vartheta\, d\varphi \tag{2.17}$$

Separating the variables in this coordinate system, we obtain the solutions of (2.7), the matrix elements of the continuous series of unitary irreducible representations of the de Sitter group:

$$\langle \Lambda, n, \ell, m | \chi, \omega, \vartheta, \varphi \rangle_\varepsilon = \langle \Lambda, n | \chi \rangle_\varepsilon \langle n, \ell, m | \omega, \vartheta, \varphi \rangle \tag{2.18}$$

$$\langle \Lambda, n | \chi \rangle_\varepsilon = (ch\,\chi)^{-3/2} P^{-i\Lambda}_{n+1/2}(\mp th\,\chi) \tag{2.19}$$

$$\langle n, \ell, m | \omega, \vartheta, \varphi \rangle = \sqrt{\frac{2^{2\ell+1} \cdot (n-\ell)!\, (\ell!)^2 \cdot (n+1)}{\pi \cdot \Gamma(\ell+n+2)}} \cdot \tag{2.20}$$

$$\cdot (\sin\omega)^\ell \cdot C^{\ell+1}_{n-\ell}(\cos\omega) \cdot Y_{\ell m}(\vartheta, \varphi) = Y_{n\ell m}(\omega, \vartheta, \varphi) ,$$

where P^μ_ν is the Legendre function of the first kind, C^p_q the Gegenbauer polynomial, $Y_{\ell m}(\vartheta, \varphi)$ the spherical function, $\varepsilon = \pm = sgn\, N_0$.

The basic functions (2.18) obey the orthogonality condition:

$$\int \langle \Lambda, n, \ell, m | \chi, \omega, \vartheta, \varphi \rangle_\varepsilon \, d\Omega_p \langle \chi, \omega, \vartheta, \varphi | \Lambda', n', \ell', m' \rangle_{\varepsilon'} =$$

$$= 2\, ch\,\pi\Lambda \cdot \frac{cth\,\pi\Lambda}{\Lambda}\, \delta(\Lambda - \Lambda') \cdot \delta_{\varepsilon\varepsilon'} \cdot \delta_{nn'} \delta_{\ell\ell'} \delta_{mm'} \cdot \tag{2.21}$$

The expansion of the "plane wave" (2.12) over the basic functions (2.18) has the form:

$$\langle \xi | p \rangle =$$

$$= \sqrt{\frac{2}{\pi}} \cdot \Gamma(i\Lambda - \tfrac{1}{2}) (|N_0|)^{i\Lambda - \frac{1}{2}} \cdot \tag{2.22}$$

$$\cdot \sum_{n,l,m} (-1)^n (n+1) \langle \Lambda, n, l, m | \chi, \omega, \vartheta, \varphi \rangle_{\operatorname{sg} N_0} \cdot \langle \delta, \vartheta_N, \varphi_N | n, l, m \rangle$$

The parameters $\delta, \vartheta_N, \varphi_N$ are connected with the four-vector N_μ

$$N_0 = \frac{1}{\cos\delta}$$

$$\vec{N} = tg\delta \cdot \vec{m} \tag{2.23}$$

$$\vec{m} = (\sin\vartheta_N \cos\varphi_N, \sin\vartheta_N \sin\varphi_N, \cos\vartheta_N)$$

The plane waves (2.22) submit to the following orthogonality condition

$$\frac{1}{(2\pi)^4} \int \langle \xi | p \rangle d\Omega_p \langle p | \xi' \rangle = \delta(\xi, \xi') =$$

$$= \frac{cth\, \pi\Lambda}{\Lambda(\Lambda^2 + 1/4)} \delta(\Lambda - \Lambda') \cdot \delta(N, N') . \tag{2.24}$$

The plane waves in the discrete space-time region are constructed in a similar way. The orthogonality condition for them has the form

$$\frac{1}{(2\pi)^4} \int \langle \xi | p \rangle d\Omega_p \langle p | \xi' \rangle = \frac{1}{(L+\frac{3}{2})(L+1)(L+2)} \delta_{LL'} \cdot \delta(N, N') , \tag{2.25}$$

where N, N' are unit space-like vectors.

We can construct the Fourier analysis on the de Sitter group, expanding the functions $f(p)$ given on the space (1.2) over the orthogonal system of plane waves $\langle \xi | p \rangle$. A detailed treatment of this scheme requiring the analytic continuation in components of the N-vector, is outside the scope of this report. We only mention here that the representation of the function $f(p)$ in terms of its images $\tilde{f}(\xi)$ on the ξ-space contains both the integral over the continuous spectrum and the sum over the discrete one:

$$f(p) = \frac{1}{(2\pi)^{5/2}} \int d\mu(\Lambda)\, d\Omega_N \langle p | \xi \rangle_\Lambda \tilde{f}_\Lambda(\xi) +$$

$$+ \frac{1}{2} \frac{1}{(2\pi)^{5/2}} \int \sum_L \mu(L)\, d\Omega_N \langle p | \xi \rangle_L \tilde{f}_L(\xi) \equiv$$

$$\equiv \frac{1}{(2\pi)^{5/2}} \int d\Omega_\xi \langle \xi | p \rangle \tilde{f}(\xi) \tag{2.26}$$

$$\mu(\Lambda) = \frac{cth\, \pi\Lambda}{\Lambda(\Lambda^2 + 1/4)} d\Lambda , \quad \mu(L) = (L + \tfrac{3}{2})(L+1)(L+2) .$$

We can pass from the summation over L in (2.26) to the integration over the complex variable σ along the contour which goes near the real axis and encloses points of the discrete spectrum in the positive direction with the measure.

$$\mu(\sigma) = \frac{1}{2i} ctg\, \pi\sigma \cdot (\sigma + \tfrac{3}{2})(\sigma+1)(\sigma+2)\, d\sigma \tag{2.27}$$

possessing the simple poles at points of discrete spectrum (2.9b).

Concluding this section, we write down the differential-difference equation in the ξ-space whose solution is the plane wave

$$\hat{K} \langle \xi | p \rangle = \left[2 ch \frac{\partial}{\partial\sigma} + \frac{3}{\sigma + 3/2} sh \frac{\partial}{\partial\sigma} - \frac{e^{\frac{\partial}{\partial\sigma}}}{(\sigma+\frac{3}{2})(\sigma+2)} \Delta_N^{(3)} \right] \langle \xi | p \rangle = \tag{2.28}$$

$$= 2 p_4 \langle \xi | p \rangle ,$$

where $\Delta_N^{(3)}$ is the angular part of the d'Alambert operator corresponding to the space like or time like case.

In the flat limit

$$2p_4 \to 2+p^2$$
$$\hat{K} \to 2+\left(i\frac{\partial}{\partial x_\mu}\right)^2, \tag{2.29}$$

i.e., the operator $\hat{K}$ is an analog of the Klein-Gordon operator in the ξ-space.

3. The theory of free fields

Let us consider the free scalar fields with mass m /9/. In the p-spaoe they are on the three-dimensional mass shell

$$p^2-m^2=0. \tag{3.1}$$

This equation does not contradict the equation (1.2). Consequently, the mass shell could be embedded into the 4-dimensional de Sitter space.

Let us denote by μ the non-Euclidean distance between origin and point $(m, \vec{o}, \sqrt{1+m^2})$ of the de Sitter space, i.e.,

$$ch\mu=\sqrt{1+m^2}=m_4, \quad sh\mu=m. \tag{3.2}$$

Basing upon equation (1.2) and the fact that to each value of p there correspond two values of p_4, which differ only in sign, we come to the two conditions

$$ch\mu-p_4=0, \tag{3.3a}$$

$$ch\mu+p_4=0. \tag{3.3.b}$$

In what follows we suppose that the free fields obey the equation which is a consequence of (3.3a):

$$2(ch\mu-p_4)\varphi(p,p_4)=0. \tag{3.4}$$

In the flat limit equation (3.4) turns into the Klein-Gordon equation

$$(m^2-p^2)\varphi(p)=0. \tag{3.5}$$

On the mass shell (3.1) the equality /3/ :

$$\tilde{\varphi}(p,m_4)=\tilde{\varphi}(p) \tag{3.6}$$

holds, where

$$\varphi(p,p_4)=\delta(2p_4-2m_4)\tilde{\varphi}(p,p_4), \tag{3.7a}$$

$$\varphi(p)=\delta(p^2-m^2)\tilde{\varphi}(p). \tag{3.7b}$$

There is no connection between $\varphi(p,p_4)$ and $\varphi(p)$ off the mass shell, and the role of the geometry of the 4-dimensional p-space becomes significant.

From (3.6) we obtain the commutator of free fields

$$[\varphi(p_1,p_{14}),\varphi(p_2,p_{24})]=-\delta(p_1,-p_2)\varepsilon(p_1^0)\delta(2p_{14}-2m_4) \tag{3.8}$$

and the normal pairing

$$\underbrace{\varphi(p_1,p_{14})\varphi(p_2,p_{24})}=\delta(p_1,-p_2)\vartheta(-p_1^0)\delta(2p_{14}-2m_4)\equiv \tag{3.9}$$

$$\equiv\delta(p_1,-p_2)\mathcal{D}^{(-)}(p_1)$$

$$\delta(p_1,p_2)\,d\Omega_{p_1}=\delta(p_1-p_2)\,d^4p.$$

It follows from (3.6) that the operator of 4-momentum $\hat{P}_\mu$ could be defined also in a usual way

$$e^{i\hat{P}_\mu x^\mu}\varphi(p,p_4)e^{-i\hat{P}_\mu x^\mu}=e^{ip_\mu x^\mu}\cdot\varphi(p,p_4). \tag{3.10}$$

Let us consider now the scalar field $\varphi(\xi)$ in the configurational ξ-representation:

$$\varphi(\xi) = \frac{1}{(2\pi)^{3/2}} \int \langle \xi | p \rangle \, \varphi(p, p_4) \, d\Omega_p \quad (3.11)$$

Applying the transformation (3.10) to (3.11) we arrive at the field with bilocal dependence on X and ξ

$$\varphi_x(\xi) = e^{i\hat{P}x} \varphi(\xi) e^{-i\hat{P}x} = \frac{1}{(2\pi)^{3/2}} \int \langle \xi | p \rangle e^{ipx} \cdot \varphi(p, p_4) \, d\Omega_p . \quad (3.12)$$

The nature, of variables X and ξ is different. Only in the flat limit $\varphi_x(\xi) \to \varphi(x+\xi)$. It is important that ξ is translation - invariant:

$$e^{i\hat{P}y} \varphi_x(\xi) e^{-i\hat{P}y} = \varphi_{x+y}(\xi) \quad (3.13)$$

which allows us to interprete it as some relative, inner, variable.

In fact, the physical meaning of this variable folllows from the role of ξ in the dynamical equations x).

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x) The notion "bilocal dependence" is barrowed from the book /16/. Despite the physical and geometrical considerations which have introduced the quantity $\varphi_x(\xi)$ differ from /16/, nevertheless the ideological resemblance of two approaches is obvious. This is clear from the following quotation from /16/: " ... This way the oncept of two spaces, space x and space ξ, arises. It is natural, that the ξ -space could possess also other peculiar geometrical properties, which differ from properties of the x-space and with which it would be possible to connect the specific features of the weak interactions".

Let us calculate now the frequency parts of the commutator funotion. We note that due to (3.8) and (3.9) the commutation function

$$\left[\varphi_x^{out}(\xi), \varphi_x^{out}(0) \right] = -\frac{i}{(2\pi)^3} \int \langle \xi | p \rangle \varepsilon(p_0) \delta(2p_4 - 2m_4) \, d\Omega_p \equiv \mathcal{D}(\xi) \quad (3.14)$$

and its frequency parts

$$\frac{1}{(2\pi)^3 i} \int \langle \xi | p \rangle \vartheta(p_0) \delta(2p_4 - 2m_4) \, d\Omega_p \equiv \mathcal{D}^{(+)}(\xi), \quad (3.15)$$

$$\frac{i}{(2\pi)^3} \int \langle \xi | p \rangle \vartheta(-p_0) \delta(2p_4 - 2m_4) \, d\Omega_p^{\circ} \equiv \mathcal{D}^{(-)}(\xi) \quad (3.16)$$

do not depend on x.

Each step of calculating the integrals (3.15) and (3.16) has an analog in the usual Q.F.T. (see§16 in /12/). So, on integrating the "angular" part in (3.15) in the continuous time--like part of the spectrum, we get $\mathcal{D}^{(+)}(\xi)$ in the form

$$\mathcal{D}^{(+)}(\xi) = -\frac{1}{(i\Lambda - 1/2)} \frac{1}{sh\Delta} \cdot \hat{p} \cdot \varphi(N_0, \Lambda) , \quad (3.17)$$

where

$$\varphi(N_0, \Lambda) = \frac{1}{(2\pi i)} \int_{-\infty}^{\infty} \left(-N_4 m_4 + \varepsilon \cdot m \cdot ch(\chi - \varepsilon \cdot \Delta) \right)_+^{i\Lambda - 1/2} d\chi , \quad (3.18)$$

$$N_4 = \pm 1, \quad N_0 = \pm ch\Delta ; \; |\vec{N}| = sh\Delta , \; \varepsilon = sgn N_0 .$$

The operator $\hat{p}$ has the form

$$\hat{p} = sh\Delta \left(e^{-i\frac{\partial}{\partial\Lambda}} - \frac{\hat{K}}{2} \right) - \frac{ch\Delta}{i\Lambda + 1/2} e^{-i\frac{\partial}{\partial\Lambda}} \cdot \frac{\partial}{\partial\Delta} . \quad (3.19)$$

In the flat limit x)

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x) For all the quantities of the conventional Q.F.T. we use notations of the book /12/ .

$$\hat{p} \to 2i\sqrt{\lambda}\, sh\Lambda \frac{\partial}{\partial \lambda}, \qquad (3.20)$$

where $\lambda = x_0^2 - \vec{x}^2$ and $\varphi(N_0,\Lambda)$ goes over to the integral representation for functions $H_0^{(1)}(m\sqrt{\lambda})$ or $H_0^{(2)}(m\sqrt{\lambda})$ for $x_0>0$ and $x_0<0$, resp., (cf. /12/).

In our case in the time-like region we have also different expressions for $N_0>0$ and $N_0<0$. Besides, we must consider separately the expressions for $\mathcal{D}^{(+)}(\xi)$ in "classical" $N_4=-1$ and "non-classical" $N_4=+1$ regions.

We have the following expressions for $\mathcal{D}^{(+)}(\xi) \equiv \mathcal{D}^{(+)}(\Lambda,\xi_4,sgn\,\xi_0)$

$$\mathcal{D}^{(+)}(\Lambda,-1,+) = \frac{\sqrt{m}\,\Gamma(-i\Lambda-\frac{1}{2})}{2i\,(2\pi)^{3/2}} P_{1/2}^{i\Lambda}\left(\frac{m_4}{m}\right) \qquad (3.21a)$$

$$\mathcal{D}^{(+)}(\Lambda,-1,-) = \frac{\sqrt{m}\,\Gamma(i\Lambda-\frac{1}{2})}{2i\,(2\pi)^{3/2}} P_{1/2}^{-i\Lambda}\left(\frac{m_4}{m}\right) \qquad (3.21b)$$

$$\mathcal{D}^{(+)}(\Lambda,+1,+) = \frac{2i\sqrt{m}\,\Gamma(-i\Lambda-\frac{1}{2})}{(2\pi)^{5/2}} e^{\pi\Lambda} Q_{1/2}^{i\Lambda}\left(\frac{m_4}{m}\right) \qquad (3.21c)$$

$$\mathcal{D}^{(+)}(\Lambda,+1,-1) \equiv 0. \qquad (3.21d)$$

The function $\mathcal{D}^{(+)}(\xi)$ has no singularities in the continuous spectrum.

The expression for $\mathcal{D}^{(+)}(\xi)$ in the discrete space-like region could be obtained in straightforward way by analytic continuation with respect to interval. An analog of this continuation exists in the usual Q.F.T. for $\lambda \neq 0$. Due to the relations

$$K_1(z) = \frac{i}{2}\pi e^{i\frac{\pi}{2}} H_1^{(1)}(ze^{i\frac{\pi}{2}}) = -\frac{i}{2}\pi e^{-i\frac{\pi}{2}} H_1^{(2)}(ze^{-i\frac{\pi}{2}}) \qquad (3.22)$$

between McDonald K_1 and Hankel H_1 /1,2/ functions, the expressions for $\mathcal{D}^{(+)}(x)$ in the space-like region are obtained from $\mathcal{D}^{(+)}(x)$ in the time-like region by the substitution

$$\sqrt{\lambda} \to e^{\pm i\frac{\pi}{2}}\sqrt{-\lambda} \qquad (3.23)$$

($\pm = sgn\, x^0$).

The detailed discussion of this calculation in the case of the curved p-space goes out of this report. We only write down here the prescription of the analytic continuation

$$i\Lambda - 3/2 \to -\sigma - 3 \qquad \text{for } N_0 > 0$$
$$i\Lambda - 3/2 \to \sigma \qquad \text{for } N_0 < 0 \qquad (3.24)$$

where σ are in the neighbourhood of the discrete spectrum $\sigma = L$.

We get the following expression for $\mathcal{D}^{(+)}(\xi)\ (N_4 = -1)$

$$\mathcal{D}^{(+)}(\sigma) = \frac{\sqrt{m}\,\Gamma(\sigma+1)}{2i\,(2\pi)^{3/2}} P_{1/2}^{-\sigma-3/2}\left(\frac{m_4}{m}\right) \qquad (3.25)$$

The quantity $\mathcal{D}^{(+)}(\sigma)$ has a simple pole at the point $\sigma=-1$. This is the only singularity which contributes to the integrals of perturbation theory (see §4). For large Λ and σ $\mathcal{D}^{(+)}(\xi) \to 0$

In the flat limit $\mathcal{D}^{(+)}(\xi)$ passes into the $\mathcal{D}^{(+)}(x)$ of local Q.F.T.:

$$\mathcal{D}^{(+)}(x) = \frac{1}{4\pi}\varepsilon(x^0)\delta(\lambda) - \frac{im}{8\pi\sqrt{\lambda}}\vartheta(\lambda)\, H_1^{(2,1)}(m\sqrt{\lambda}) - \vartheta(-\lambda)\frac{im}{4\pi^2\sqrt{-\lambda}} K_1(m\sqrt{-\lambda}). \qquad (3.26)$$

The values of $\mathcal{D}^{(+)}(\xi)$ in region $N_4 = +1$ vanish in the flat limit.

We emphasize that in the ξ-space no term like $\frac{1}{4\pi}\varepsilon(x^0)\delta(\lambda)$ does exist. This result is a clear manifestation of the absence of an analog of the light cone in the ξ-space.

Consider this in more detail. In the conventional Q.F.T. we have the following expression for $\mathcal{D}^{(+)}(x)$

$$\mathcal{D}^{(+)}(x) = -\frac{1}{2\pi}\frac{\partial}{\partial\lambda}\varphi(x^0,\lambda). \qquad (3.27)$$

For $\varphi(x^0,\lambda)$ we have different expressions in the time-like and space-like regions:

$$\varphi(x^0,\lambda) = \vartheta(\lambda)\cdot\frac{1}{2} H_0^{(2,1)}(m\sqrt{\lambda}) + \qquad (3.28)$$
$$+\, \vartheta(-\lambda)\cdot\frac{i}{\pi}\cdot K_0(m\sqrt{-\lambda}),$$

where the superscript of H-function depends on $\mathrm{sgn}\, x^0$.

The differentiation with respect to λ produces the term $\frac{1}{4\pi}\varepsilon(x^0)\,\delta(\lambda)$.

In the ξ-space, the differential operator $\frac{d}{d\lambda}$ is replaced by the finite-difference operator $\hat{p}$. The application of this operator to $\varphi(N^0,\Lambda)$ does not introduce new singularity. The recurrence operator $\hat{p}$ expresses $\mathcal{D}^{(+)}(\xi)$ in terms of φ at shifted points.

The function $\mathcal{D}^{(-)}(\xi)$ is connected with $\mathcal{D}^{(+)}(\xi)$ by the relation

$$\mathcal{D}^{(-)}(\sigma,N_\mu) = -\mathcal{D}^{(+)}(\sigma,-N_\mu)\,. \qquad (3.29)$$

For $\mathcal{D}^{c}(\xi)$ we get

$$\mathcal{D}^{c}(\xi) = \vartheta(N^0)\,\mathcal{D}^{(-)}(\xi) - \vartheta(-N^0)\,\mathcal{D}^{(+)}(\xi) =$$

$$= \begin{cases} -\dfrac{\sqrt{m}\,\Gamma(i\Lambda-\frac{1}{2})}{2i\,(2\pi)^{3/2}}\,P_{1/2}^{-i\Lambda}\left(\dfrac{m_4}{m}\right) & (3.30a) \\ -\dfrac{\sqrt{m}\,\Gamma(\sigma+1)}{2i\,(2\pi)^{3/2}}\,P_{1/2}^{-\sigma-3/2}\left(\dfrac{m_4}{m}\right). & (3.30b) \end{cases}$$

For massless particles in the time-like region we have (cf. /17/)

$$\mathcal{D}^{c}(\xi)\Big|_{m=0} = \frac{1}{4\pi^2 i\,(\Lambda^2+\frac{1}{4})}\,. \qquad (3.31)$$

The expression for $\mathcal{D}(\xi) = \mathcal{D}^{(+)}(\xi) + \mathcal{D}^{(-)}(\xi)$ is as follows:

$$\mathcal{D}(\xi) = \varepsilon(N_0)\,\frac{2\sqrt{m}}{(2\pi)^{5/2}}\,\Gamma(i\Lambda-\tfrac{1}{2})\,e^{-\pi\Lambda}\cdot \mathrm{sh}\,\pi\Lambda\cdot Q_{1/2}^{-i\Lambda}\left(\frac{m_4}{m}\right) \qquad (3.32a)$$

for the time-like Λ- region

$\mathcal{D}(\xi) = 0$ for the space-like L region. (3.32b)

Equation (3.32) provides the following locality condition for the free fields

$$\left[\varphi_x^{out}(\xi),\ \varphi_x^{out}(0)\right] = 0\,, \qquad (3.33)$$

if $\xi \in$ space-like L- region.

4. <u>Formulation of the perturbation theory</u>

In papers /3-5/ the Bogolubov causality condition was generalized to the ξ-space, corresponding to the de Sitter p-space (1.1) in the form

$$\frac{\delta j_x(\xi)}{\delta\varphi_x(0)} = i\,\vartheta(-N_0)\left[j_x(\xi), j_x(0)\right] - \qquad (4.1)$$
$$-\,\Lambda_x(\xi,0),$$

where $j_x(\xi)$ is the current operator, $\Lambda_x(\xi,0)$, the quasi-local terms.

Though in our case (1.2) the structure of the ξ-space differs from that considered in papers /3-5/, the causality condition is written in the same form, but ξ means now the configurational representation, conjugated to the p-space (1.2).

In the conventional Q.F.T. the locality condition for free fields and for current operators, describing the interacting systems have the same form. In the scheme developed, it is naturally therefore to formulate the locality condition for the currents as the requirement that their commutators vanish in the space-like region:

$$[j_x(\xi), j_x(0)] = 0 \tag{4.2}$$

for $\xi \in L$ - region.

As in the conventional Q.F.T. /12-14/, we shall consider equation (4.1) as an equation of motion for the current. In the case, when it is possible to look for the solution of equation (4.1) as an expansion in small coupling constant g:

$$j_x(\xi) = \sum_{n=1}^{\infty} g^n j_x^{(n)}(\xi), \tag{4.3}$$

and the quasi-local term in the lowest perturbation order has the form

$$\Lambda_x^{(0)}(\xi,\zeta) = :\varphi_x^{(n-2)}(\xi):\delta(\xi,\zeta). \tag{4.4}$$

We deduce the solution

$$j_x(\xi) = i\frac{\delta\sigma_x}{\delta\varphi_x(\xi)}\sigma_x^+, \tag{4.5}$$

where

$$\sigma_x = T_\xi \exp\left(ig\int \mathcal{L}_x(\xi)d\Omega_\xi\right) \tag{4.6}$$

and the local lagrangian density function $\mathcal{L}_x(\xi)$ has the form:

$$\mathcal{L}_x(\xi) = \frac{1}{n(n-1)}:\varphi_x^n(\xi):. \tag{4.7}$$

The operator σ_x resembles the scattering operator in the local Q.F.T. The symbol T_ξ represents the invariant ordering based on the existing step function $\vartheta(N^0)$.

But σ_x does not obey the condition of the translation invariance.

To construct the translation-invariant S-matrix, we introduce the switching on function $g_x(\xi)$ (cf. /12/) with bilocal dependence on X and ξ. Because x and ξ are independent variables, there exist the functional derivatives of two types: $\delta/\delta g_x(\xi)$ corresponding to the variation of g as a function of ξ $(g_x(\xi) \to g_x(\xi) + \delta(\xi))$ and $\hat{\delta}/\hat{\delta} g_x(\xi)$ corresponding to the variation of g as a function of x $(g_x(\xi) \to g_x(\xi) + \hat{\delta}(x))$.

Introduce now the operator

$$H_x(\xi) = i\frac{\delta\sigma_x}{\delta g_x(\xi)}\sigma_x^+ \tag{4.8}$$

and define the S-matrix via the relation

$$i\frac{\hat{\delta} S}{\hat{\delta} g_x(0)} S^+ = H_x(0). \tag{4.9}$$

The S-matrix extracted from this relation satisfies all the requirements of Q.F.T.

In the lowest perturbation orders it has the form

$$S_1 = g\int \mathcal{L}_x(0)d^4x \tag{4.10a}$$

$$S_2 = \frac{1}{2}g^2\left(\int\mathcal{L}_x(0)d^4x\right)^2 - \tag{4.10b}$$
$$- \frac{1}{2}g^2\int\vartheta(-N^0)[\mathcal{L}_x(0),\mathcal{L}_x(\xi)]d^4x\,d\Omega_\xi.$$

We do not obtain here the closed expression in the form of the ordered T_ξ exponent. Each term contains the integration over d^4x which garantees the translation invariance (the averaging over the translation group).

As an example, we consider the matrix element $\langle p_1|S_2|p_2\rangle$ in the second perturbation order which would correspond, in the usual approach, to the self energy diagram in the φ^3 theory.

Using the relations

$$[\varphi_x(\xi), a^+(\vec{p})] = \frac{1}{(2\pi)^{3/2}}\frac{e^{-ipx}}{\sqrt{2p^0}}\langle\xi|-p_\mu, p_4\rangle, \tag{4.11a}$$

$$[a^{(-)}(p), \varphi_x(\xi)] = \frac{1}{(2\pi)^{3/2}}\frac{e^{ipx}}{\sqrt{2p^0}}\langle\xi|p_\mu, p_4\rangle, \tag{4.11b}$$

we oome to the following expressions:

$$
\begin{aligned}
\langle p_1 | S_2 | p_2 \rangle = &- \frac{g^2 (2\pi)^2}{2(2p_1^0)} \delta(p_1 - p_2) \cdot \\
&\cdot \Big\{ 2 \int d^4x \, e^{ip_1x} \left[\left(\mathcal{D}^{(+)}(x)\right)^2 - \left(\mathcal{D}^{(-)}(x)\right)^2 \right] - \\
&- \int d\Omega_\xi \, \vartheta(-N^0) \left[\left(\mathcal{D}^{(+)}(\xi)\right)^2 - \left(\mathcal{D}^{(-)}(\xi)\right)^2 \right] \cdot \\
&\cdot \left[\langle \xi | -p_{1\mu}, p_{14} \rangle + \langle \xi | p_{1\mu}, p_{14} \rangle \right].
\end{aligned}
\tag{4.12}
$$

The first term represents the imaginary part of the matrix element, and $\mathcal{D}^{(+)}(x)$ and $\mathcal{D}^{(-)}(x)$ are usual frequency parts of the commutation function. This term coincides with the imaginary part of the usual matrix element and reduces in the momentum space to the convergent integral over the mass shell. The second term, oontaining the integration over the ξ -space in the flat limit coinoides with the real part of the polarization operator which diverges because the integrand contains the non-integrable produot of singular functions.

In the curved case, as we know, the only singularities of $\mathcal{D}^{(\pm)}(\xi)$ are the simple poles of these as functions of complex interval σ, at the point $\sigma = -1$. The integration over the "radial" part σ runs along the contour around this pole and reduces to taking the residue. (In the case (4.12) remembering that the volume element (2.27) also contains the poles, we have the residue of third order).

So, we have shown that in this approach no non-integrable expressions enter into integrals. The rule of integration of singularities follows straghtforward from geometrical, group theoretical apparatus which makes a basis of this scheme.

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