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A SUPERSYMMETRIC FORM
OF THE THIRRING MODEL

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**A SUPERSYMMETRIC FORM
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Суперсимметрическая форма модели Тирринга

В настоящей работе предложено и исследовано суперсимметрическое уравнение модели Тирринга. Решение перенормированного уравнения модели Тирринга построено через экспоненты двух безмассовых скалярных суперполей. Получены представления суперконформной алгебры для скалярных суперполей. Показано, что перенормированное уравнение Тирринга, ковариантно относительно этих представлений.

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A Supersymmetric Form of the Thirring Model

In the present paper a super Thirring equation is proposed and studied. A solution of the renormalized Thirring model is constructed from two scalar massless superfields. The representations of the superconformal algebra for the scalar superfields are obtained. It is shown that the renormalized Thirring equation is covariant with respect to these representations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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INTRODUCTION

In a recent paper ^{/3/} the solutions of the quantum field problem for the free massless scalar field $\phi(x)$ have been studied and have been used to find solutions of the Thirring model. Namely the Thirring fields $\psi(x)$ are constructed as $:\exp(-i\alpha\phi(x)+i\beta\gamma^5\phi(x)):$. The representations of the conformal group for the scalar as well as for the spinor fields have been obtained. They are not the standard ones. The scalar field transforms inhomogeneously under the conformal group which is implied by the requirement for conformal covariance of the commutation function. It has also been proved that the renormalized Thirring equation is covariant with respect to the obtained representations.

In this paper we give a supersymmetric form of the considered model. The conformal algebra in two space-time dimensions is extended to a superconformal one and the transformation properties of a scalar superfield $\Phi(x, \theta)$ are examined. With the help of the free scalar massless superfield the solutions of the renormalized super Thirring equation are constructed. The latter equation is proved to be covariant with respect to the considered superconformal algebra.

TRANSFORMATION PROPERTIES OF THE SCALAR SUPERFIELDS

In the supersymmetrical approach we consider a superspace (x^μ, θ_k) , where $x^\mu, \mu=0,1$ are the usual co-ordinates and $\theta_k, k=1,2$ are anticommuting Majorana spinors - elements of Grassman algebra

$$\{\theta_k, \theta_l\} = 0. \quad (1.1)$$

The scalar superfield $\Phi(x, \theta)$ is defined on the super-space (x^μ, θ_k) . If we expand $\Phi(x, \theta)$ in a power series in θ_k

$$\Phi(x, \theta) = \phi(x) + \psi(x) \gamma^0(\theta) + S(x) \theta \gamma^0 \theta, \quad (1.2)$$

we see that it is equivalent to three ordinary * fields - the two scalar fields $\phi(x)$ and $S(x)$ and the spinor field $\psi(x)$. For the superfields one can introduce a covariant spinor derivative operator^{7,8/}:

$$D_k = i(\gamma^0 \frac{\partial}{\partial \theta})_k - (\gamma^\mu \theta)_k \partial_\mu. \quad (1.3)$$

Then the massless free field equation for the scalar superfield^{1,2/} $\Phi(x, \theta)$ is:

$$D^k D_k \Phi(x, \theta) = 0 \quad (1.4)$$

which is equivalent to a Klein-Gordon equation $\square \phi(x) = 0$, a Dirac equation $i \gamma^\mu \partial_\mu \psi(x) = 0$ together with the equation $S(x) = 0$. Before proceeding further we write some notations used throughout the paper.

The Greek letters $\mu, \nu, \dots = 0, 1$ are used to denote a Lorentz two vector, while the Latin letters $j, k, l, \dots = 1, 2$ are used for the spinor indices.

The metric tensor $\eta_{\mu\nu}$ is chosen as: $\eta_{00} = -1 = -\eta_{11}$. The γ -matrices are taken in Majorana representation

$$\gamma^0 = i\sigma_2, \gamma^1 = \sigma_1, \gamma^5 = \gamma^0 \gamma^1 = \sigma_3,$$

where $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices

$$\sigma_{\mu\nu} = \frac{1}{4} [\gamma_\mu, \gamma_\nu] = \frac{1}{2} \epsilon_{\mu\nu} \gamma_5 \quad (1.5)$$

with $\epsilon_{\mu\nu}$ totally antisymmetric and $\epsilon_{01} = 1$

$$\theta \gamma^0 \theta = \bar{\theta} \theta = \theta^k \theta_k = -\theta_k \theta^k, \quad \theta_k = (\gamma^0)_{kl} \theta^l.$$

Just as in the ordinary case there exists another scalar superfield $\tilde{\Phi}(x, \theta)$ (we call it a dual field) which also satisfies eq. (1.4):

$$\tilde{\Phi}(x, \theta) = \tilde{\phi}(x) + \psi(x) \gamma^0 \gamma_5 \theta + P(x) \theta \gamma^0 \theta. \quad (1.6)$$

The two superfields are related by the differential equalities

$$D_k \Phi(x, \theta) + (\gamma^5 D)_k \tilde{\Phi}(x, \theta) = 0. \quad (1.7)$$

The fields $\Phi(x, \theta)$ and $\tilde{\Phi}(x, \theta)$ satisfy the following commutation relations:

$$\begin{aligned} [\Phi(x, \theta), \Phi(y, \xi)] &= i \Lambda(x-y, \theta, \xi) \\ [\tilde{\Phi}(x, \theta), \tilde{\Phi}(y, \xi)] &= i \Lambda(x-y, \theta, \xi) \\ [\tilde{\Phi}(x, \theta), \Phi(y, \xi)] &= i \tilde{\Lambda}(x-y, \theta, \xi). \end{aligned} \quad (1.8)$$

The superfunctions $\Lambda(x-y, \theta, \xi)$ and $\tilde{\Lambda}(x-y, \theta, \xi)$ have the form:

$$\Lambda(z, \theta, \xi) = e^{i \theta \gamma^0 \gamma^\mu \xi \partial_\mu} \tilde{D}(z), \quad (1.9)$$

where $D(z)$ and $\tilde{D}(z)$ are the corresponding commutation functions of the ordinary scalar fields considered in^{3/}

$$iD(z) = [\phi(x), \phi(y)] \quad i\tilde{D}(x-y) = [\tilde{\phi}(x), \phi(y)].$$

For the fields $\Phi(x, \theta)$ and $\tilde{\Phi}(x, \theta)$ one can define the creation and annihilation parts $\Phi^\pm(x, \theta)$ and $\tilde{\Phi}^\pm(x, \theta)$ respectively

$$\Phi^\pm(x, \theta) = \phi^\pm(x) + \psi^\pm(x) \gamma^0 \theta \quad (1.10)$$

$$\tilde{\Phi}^\pm(x, \theta) = \tilde{\phi}^\pm(x) + \psi^\pm(x) \gamma^0 \gamma_5 \theta.$$

* By ordinary we mean not super.

We do not discuss here the decomposition of the ordinary scalar fields $\phi(x)$ and $\bar{\phi}(x)$ into creation and annihilation parts since it is carried out precisely in ^{/3/}. The commutation relations between $\Phi^\pm(x, \theta)$ and $\tilde{\Phi}^\pm(x, \theta)$ are the following:

$$[\Phi^\pm(x, \theta), \Phi^\mp(y, \xi)] = \Lambda^\pm(x-y, \theta, \xi), \quad (1.11)$$

$$[\tilde{\Phi}^\pm(x, \theta), \Phi^\mp(y, \xi)] = \tilde{\Lambda}^\pm(x-y, \theta, \xi)$$

where

$$\begin{aligned} i\tilde{\Lambda}^\pm(z, \theta, \xi) &= \tilde{\Lambda}^{\pm(+)}(z, \theta, \xi) + \tilde{\Lambda}^{\pm(-)}(z, \theta, \xi) \\ \tilde{\Lambda}^{\pm(+)}(z, \theta, \xi) &= e^{i\theta\gamma^\sigma\gamma^\mu\xi\partial_\mu} \tilde{\Lambda}^{\pm(+)}(z) \end{aligned} \quad (1.12)$$

Again $D^\pm(z)$ and $\tilde{D}^\pm(z)$ are the corresponding functions from paper ^{/3/}. All other commutators are equal to zero.

The functions $\Lambda(x, \theta, \xi)$ and $\tilde{\Lambda}(x, \theta, \xi)$ as well as $\Lambda^\pm(x, \theta, \xi)$ and $\tilde{\Lambda}^\pm(x, \theta, \xi)$ are related by the differential equation

$$D_k \Lambda(x, \theta, \xi) + (\gamma^5 D)_k \tilde{\Lambda}(x, \theta, \xi) = 0. \quad (1.13)$$

In paper ^{/3/} the transformation properties of the free massless scalar field $\phi(x)$ as well as of the dual field $\bar{\phi}(x)$ with respect to the ordinary conformal group have been obtained. The inhomogeneity of the transformation laws has been implied by the requirement for covariance of the commutator functions. For the purpose of this paper we need a supersymmetric generalization of such representations.

The superconformal algebra in two dimensions is the algebra of $SO(2,2)$ extended with two spinor operators T_k and S_k , $k=1,2$. A realization of the algebra can be constructed by a straight-forward adaptation of the exploited technique in four dimensions ^{/4,5,6/}. However the standard linear representations do not preserve the covariance of the commutation functions. Having in mind their explicit form (see paper ^{/3/}) it is readily verified

that for instance under dilations the functions $\Delta^\pm(x, \theta, \xi)$ acquire nonhomogeneous terms. We follow closely the procedure discussed in paper ^{/3/} to obtain the following transformation laws for the superfields $\Phi^\pm(x, \theta)$ and $\tilde{\Phi}^\pm(x, \theta)$:

$$[P_\mu, \Phi^\pm(x, \theta)] = i\partial_\mu \Phi^\pm(x, \theta) \quad [P_\mu, \tilde{\Phi}^\pm(x, \theta)] = i\partial_\mu \tilde{\Phi}^\pm(x, \theta)$$

$$[M_{\mu\nu}, \Phi^\pm(x, \theta)] = -i(x_\mu \partial_\nu - x_\nu \partial_\mu) \Phi^\pm(x, \theta) +$$

$$+i\theta\gamma^\sigma\sigma_{\mu\nu}\gamma^\sigma\frac{\partial}{\partial\theta}\Phi^\pm(x, \theta) + \frac{i}{2\sqrt{2\pi}}\epsilon_{\mu\nu}b^\pm(0)$$

$$[M_{\mu\nu}, \tilde{\Phi}^\pm(x, \theta)] = -i(x_\mu \partial_\nu - x_\nu \partial_\mu) \tilde{\Phi}^\pm(x, \theta) +$$

$$+i\theta\gamma^\sigma\sigma_{\mu\nu}\gamma^\sigma\frac{\partial}{\partial\theta}\tilde{\Phi}^\pm(x, \theta) - \frac{i}{2\sqrt{2\pi}}\epsilon_{\mu\nu}a^\pm(0)$$

$$[D, \Phi^\pm(x, \theta)] = i(x_\mu \partial^\mu + \frac{1}{2}\theta\frac{\partial}{\partial\theta})\Phi^\pm(x, \theta) + \frac{i}{2\sqrt{2\pi}}a^\pm(0)$$

$$[D, \tilde{\Phi}^\pm(x, \theta)] = i(x_\mu \partial^\mu + \frac{1}{2}\theta\frac{\partial}{\partial\theta})\tilde{\Phi}^\pm(x, \theta) - \frac{i}{2\sqrt{2\pi}}b^\pm(0)$$

$$[K_\mu, \Phi^\pm(x, \theta)] = i(x^2\partial_\mu - 2x_\mu x_\nu \partial^\nu - x_\mu \theta\frac{\partial}{\partial\theta} +$$

$$+ 2x^\nu \theta\gamma^\sigma\sigma_{\mu\nu}\gamma^\sigma\frac{\partial}{\partial\theta})\Phi^\pm(x, \theta) - \frac{i}{\sqrt{2\pi}}x_\mu a^\pm(0) + \frac{i\epsilon_{\mu\nu}x^\nu}{\sqrt{2\pi}}b^\pm(0)$$

$$[K_\mu, \tilde{\Phi}^\pm(x, \theta)] = i(x^2\partial_\mu - 2x_\mu x_\nu \partial^\nu - x_\mu \theta\frac{\partial}{\partial\theta} +$$

$$+ 2x^\nu \theta\gamma^\sigma\sigma_{\mu\nu}\gamma^\sigma\frac{\partial}{\partial\theta})\tilde{\Phi}^\pm(x, \theta) + \frac{i}{\sqrt{2\pi}}x_\mu b^\pm(0) - \frac{i\epsilon_{\mu\nu}x^\nu}{\sqrt{2\pi}}a^\pm(0)$$

$$[T_k, \Phi^\pm(x, \theta)] = -i(\gamma^0 \frac{\partial}{\partial \theta})_k \Phi^\pm(x, \theta) - (\gamma^\mu \theta)_k \partial_\mu \Phi^\pm(x, \theta)$$

$$[T_k, \tilde{\Phi}^\pm(x, \theta)] = -i(\gamma^0 \frac{\partial}{\partial \theta})_k \tilde{\Phi}^\pm(x, \theta) - (\gamma^\mu \theta)_k \partial_\mu \tilde{\Phi}^\pm(x, \theta)$$

$$[S_k, \Phi^\pm(x, \theta)] = i(\hat{x} \gamma^0 \frac{\partial}{\partial \theta})_k \Phi^\pm(x, \theta) + (\hat{x} \gamma^\mu \theta)_k \partial_\mu \Phi^\pm(x, \theta)$$

$$- (\sigma_{\mu\nu} \theta)_k \epsilon^{\mu\nu} \frac{b^\pm(0)}{2\sqrt{2\pi}} + \theta_k \frac{a^\pm(0)}{2\sqrt{2\pi}}$$

$$[S_k, \tilde{\Phi}^\pm(x, \theta)] = i(\hat{x} \gamma^0 \frac{\partial}{\partial \theta})_k \tilde{\Phi}^\pm(x, \theta) + (\hat{x} \gamma^\mu \theta)_k \partial_\mu \tilde{\Phi}^\pm(x, \theta)$$

$$+ (\sigma_{\mu\nu} \theta)_k \epsilon^{\mu\nu} \frac{a^\pm(0)}{2\sqrt{2\pi}} - \theta_k \frac{b^\pm(0)}{2\sqrt{2\pi}}, \quad (1.14)$$

where $a^\pm(0)$ and $b^\pm(0)$ are constant operators determined by the commutators:

$$[a^\pm(0), \Phi^\mp(x, \theta)] = \pm \frac{1}{\sqrt{2\pi}}, \quad [b^\pm(0), \tilde{\Phi}^\mp(x, \theta)] = \mp \frac{1}{\sqrt{2\pi}}$$

$$[a^\pm(0), \tilde{\Phi}^\mp(x, \theta)] = [a^\pm(0), \tilde{\Phi}^\pm(x, \theta)] = [a^\pm(0), \Phi^\pm(x, \theta)] = 0$$

$$[b^\pm(0), \Phi^\mp(x, \theta)] = [b^\pm(0), \Phi^\pm(x, \theta)] = [b^\pm(0), \tilde{\Phi}^\pm(x, \theta)] = 0$$

$$[a^+(0), b^-(0)] = [b^+(0), a^-(0)] = 0. \quad (1.15)$$

It is easy to verify that the commutation relations between the superfields $\Phi(x, \theta)$ and $\tilde{\Phi}(x, \theta)$ (1.8) as well as between their creation and annihilation parts (1.11)

and also the generalized Klein-Gordon equation (1.4) are covariant with respect to the written above infinitesimal transformations of the superconformal algebra.

CLASSICAL SUPERSYMMETRIC THIRRING EQUATION

We propose the following supersymmetric generalization of the classical Thirring^{/9/} equation

$$i\gamma^k D_k \psi(x, \theta) = -g J_k \gamma^k \psi(x, \theta), \quad (2.1)$$

where $J_k(x, \theta) = \bar{\psi}(x, \theta) \gamma_k \psi(x, \theta)$, $k=1, 2$, $\bar{\psi} = \psi^* \gamma^0$

is the spinor current. The matrices γ_k ($k=1, 2$) are linear combinations of the matrices γ^μ ($\mu=0, 1$)

$$\gamma_k = \frac{1}{2}(\gamma^0 + (-1)^{k+1} \gamma^1). \quad (2.2)$$

Both γ_k and γ^μ form Clifford algebra but γ^μ is a Lorentz two vector and γ_k is a $SO(1,1)$ "bispinor". Here we have used the fact that in two dimensions the vector and the spinor Lorentz representations are equivalent to two scalar representations. Therefore it is possible to write the eq. (2.1) also in a "vector" form

$$i\gamma^\mu D_\mu \psi(x, \theta) = -g J_\mu(x, \theta) \gamma^\mu \psi(x, \theta) \quad (2.3)$$

with a "vector" derivative D^μ , $\mu=0, 1$ constructed out of the spinor derivative D_k , $k=1, 2$

$$D^\mu = \frac{1}{2} \sum_{k=1}^2 (-1)^{\mu(k+1)} D_k.$$

The choice of the spinor form of eq. (2.1) is for convenience only. If we define the operation hermitian conjugation for a superfunction as

$$[D_k f(x, \theta)]^* = \epsilon D_k f^*(x, \theta), \quad (2.4)$$

where $\epsilon = \pm 1$ for an even or odd in θ_k function $f(x, \theta)$ respectively, then it follows from eq. (2.1)

$$D^k J_k(x, \theta) = 0 \quad (2.5)$$

and $D^k \tilde{J}_k(x, \theta) = 0$,

where $J_k(x, \theta) = \tilde{\psi}(x, \theta) \gamma^5 \gamma_k \psi(x, \theta)$.

The latter equations imply that J_k has the form

$$J_k(x, \theta) = D_k \Phi(x, \theta), \quad (2.6)$$

where $\Phi(x, \theta)$ satisfies eq. (1.4) due to eq. (2.5). Therefore as in the ordinary case one can solve eq. (2.1) making the ansatz

$$\psi(x, \theta) = \psi_0(x, \theta) e^{-i\alpha \Phi(x, \theta) + i\beta \gamma^5 \tilde{\Phi}(x, \theta)} \quad (2.7)$$

where $\psi_0(x, \theta)$ is a free massless spinor superfield with equation of motion

$$i\gamma^k D_k \psi_0(x, \theta) = 0 \quad (2.8)$$

and $\Phi(x, \theta)$ and $\tilde{\Phi}(x, \theta)$ are scalar superfields satisfying the generalized Klein-Gordon equation (1.4).

If we expand $\psi_0(x, \theta)$ in a power series in θ_k

$$\psi_0(x, \theta) = u_0(x) + (\gamma_\mu \theta) F_0^\mu + \theta S_0(x) + (\gamma_5 \theta) P_0(x) + \chi_0(x) \bar{\theta} \theta \quad (2.9)$$

and substitute it in eq. (2.8) we find that it is equivalent to the following equations for the ordinary fields in the expansion

$$i\gamma^\mu \partial_\mu u_0(x) = 0 \quad F_0^\mu(x) = 0 \quad (2.10)$$

$$\partial_\mu S_0(x) + \epsilon_\mu^\nu P_{\nu 0}(x) = 0 \quad \chi_0(x) = 0.$$

QUANTIZATION

Let us now consider the quantum case. For this purpose we first have to define in a proper way the current $J_k(x, \theta)$ since the product

$$\tilde{\psi}(x, \theta) \gamma_k \psi(y, \xi)$$

is not well defined for $x=y$ and $\theta=\xi$. In full analogy with the ordinary case (see paper^[3]) we discuss the following statement. Let $\Phi^\pm(x, \theta)$ and $\tilde{\Phi}^\pm(x, \theta)$ be quantum superfields satisfying eq. (1.4). Then the two component quantity

$$\psi_k(x, \theta) = (e^{i\beta \gamma^5 \tilde{\Phi}^-(x, \theta) - i\alpha \Phi^-(x, \theta)} e^{-i\alpha \Phi^+(x, \theta) + i\beta \gamma^5 \tilde{\Phi}^+(x, \theta)})_{u_k} \quad (3.1)$$

where u_k , $k=1,2$ are complex numbers and $|u_1|=|u_2|$, satisfies the renormalized quantum equation of the super Thirring model

$$i\gamma^k D_k \psi(x, \theta) = -g: J_k(x, \theta) \gamma^k \psi(x, \theta):, \quad (3.2)$$

where γ^k are the matrices from eq. (2.2).

The renormalized current is defined as a bilinear form of the fields $\psi(x, \theta)$ in the following way

$$J_k(x, \theta) = \int d\eta \gamma^\alpha \gamma_k J = \int (d\eta \gamma^\alpha \gamma_k)^\ell J_\ell \quad (3.3)$$

where $J_\ell(x, \theta, \eta)$, $\ell=1,2$ is a spinor formed from a vector current $j^\mu(x, \theta, \epsilon, \eta)$, $\mu=0,1$

$$J_\ell(x, \theta, \eta) = \lim_{\epsilon \rightarrow 0} e^{(-1)^\ell \alpha \beta \tilde{\Phi}^+(\epsilon^\mu + 2i\eta \gamma^\alpha \gamma^\mu \theta)} \times \quad (3.4)$$

$$\times \frac{1}{2} [j^0(x, \theta, \epsilon, \eta) + (-1)^{k+1} j^1(x, \theta, \epsilon, \eta)].$$

The vector current is given by:

$$j_{\mu}(x, \theta, \epsilon, \eta) = \frac{1}{2} e^{-(a^2 + \beta^2) D^+ (\epsilon^{\mu} + 2i\eta \gamma^{\circ} \gamma^{\mu} \theta)} \times$$

$$\times (\bar{\psi}(x^{\mu} + \epsilon^{\mu}, \theta) \gamma^{\mu} \psi(x^{\mu} - i\eta \gamma^{\circ} \gamma^{\mu} \theta, \theta - \eta) -$$

$$- \psi(x^{\mu} + i\eta \gamma^{\circ} \gamma^{\mu} \theta) \gamma^{\mu} \psi^*(x^{\mu} - \epsilon^{\mu}, \theta)). \quad (3.5)$$

The proof of the statement follows closely the analogous procedure of the ordinary Thirring model³⁾. First one must calculate the current, then substitute eq. (3.1) into eq. (3.2) and verify that it is a solution of the equation. To calculate the current we substitute the quantity (3.1) into eq. (3.5) and reorder the operator exponentials. We have:

$$j_{\mu}(x, \theta, \epsilon, \eta) = \sum_{\ell=1}^2 (-1)^{\mu(\ell+1)} |u_{\ell}|^2 \exp \{ (-1)^k 2a\beta \tilde{D}^+ (\epsilon^{\mu} + 2i\eta \gamma^{\circ} \gamma^{\mu} \theta) \} \times$$

$$\times (\epsilon^{\mu} \partial_{\mu} - i\eta^k D_k) F_{\ell}(x, \theta) \quad (3.6)$$

with

$$F_{\ell}(x, \theta) = i\alpha \Phi(x, \theta) + i\beta (-1)^k \tilde{\Phi}(x, \theta).$$

Then using eq. (3.4) we form the spinor current $J_k(x, \theta, \epsilon, \eta)$ which in the limit $\epsilon \rightarrow 0$ gives

$$J_k(x, \theta, \eta) = |u|^2 \eta^{\ell} D_{\ell} (-\alpha \Phi(x, \theta) + (-1)^k \beta \tilde{\Phi}(x, \theta)). \quad (3.7)$$

After integrating* over η according to (3.3) we obtain:

$$J_k(x, \theta) = (a + \beta) D_k \Phi(x, \theta). \quad (3.8)$$

* The integration is according to the rule $\int d\eta_k = 0$,

$$\int \eta_k d\eta_{\ell} = \delta_{k\ell}.$$

On the other hand one can easily verify the identity

$$i\gamma^k D_k \psi(x, \theta) = (a - \beta) : D_k \Phi(x, \theta) \gamma^k \psi(x, \theta) : \quad (3.9)$$

Inserting eqs. (3.8) and (3.9) into (3.2) we find an algebraic equation for the coefficients a and β .

$$\beta - a = g(a + \beta) \quad (3.10)$$

which is the only relation between them.

Expanding the field $\psi(x, \theta)$ in a power series in

$$\psi_k(x, \theta) = u_k(x) + (\gamma_{\mu} \theta)_k F^{\mu}(x) + \theta_k S(x) + (\gamma_5 \theta)_k P(x) +$$

$$+ \chi_k(x) \theta \gamma^{\circ} \theta \quad (3.11)$$

and inserting it into eq. (3.2) we find that the renormalized Thirring equation is equivalent to the following equations between the ordinary fields in the expansion

$$F_{\mu}(x) = \frac{1}{2} g \sum_{k=1}^2 (-1)^{\mu k} \bar{u}_k(x) u_k(x) (\gamma^{\circ} u)_k$$

$$\chi_k(x) = g (\gamma^{\circ} \gamma^{\mu})_{kk} [\bar{u}_k u_k F_{\mu} - u_k (\gamma^{\circ} u)_k F_{\mu}^*] +$$

$$+ g \bar{u}_k (\gamma^{\circ} u)_k [S - (\gamma^5)_{kk} P] \quad (\text{no summation over } k)$$

$$(\gamma^{\circ} \gamma^{\mu})_{kk} \partial_{\mu} u_k = g \bar{u}_k u_k (S - \gamma_{kk}^5 P) + g u_k (\gamma^{\circ} u)_k (-S^* + \gamma_{kk}^5 P^*) -$$

$$- g (\gamma^{\circ} \gamma^{\mu})_{kk} F_{\mu} \bar{u}_k (\gamma^{\circ} u)_k \quad (\text{no summation over } k)$$

$$\partial_{\mu} S + \epsilon_{\mu}^{\nu} \partial_{\nu} P = g \sum_{k=1}^2 (-1)^{\mu(k+1)+1} (\bar{u}_k u_k + \bar{u}_k \chi_k + \bar{\chi}_k u_k) (\gamma^{\circ} u)_k$$

$$+ g \sum_{k=1}^2 (-1)^{\mu(k+1)} (F_{\mu}^* F^{\mu} - S^* S + P^* P) (\gamma^{\circ} u)_k$$

$$+ g \sum_{k=1}^2 (-1)^{(\mu+1)(k+1)} (-\epsilon^{\mu\nu} F_{\mu} F_{\nu}^* - S^* P + P^* S) (\gamma^{\circ} u)_k. \quad (3.12)$$

Due to the covariance of the commutators (1.8) and (1.11) as well of the eq. (1.4) with respect to the infinitesimal transformations of the superconformal algebra it can be proved analogously to the ordinary Thirring model studied in ^{/3/} that the renormalized super Thirring equation is also covariant with respect to the considered representations.

REFERENCES

1. Ferrara S. CERN Preprint TH 2035, Geneve, 1975.
2. Zumino B. CERN Preprint TH 1779, Geneve, 1973.
3. Hadjiivanov L., Mikhov S., Stoyanov D. Trieste Preprint IC/77/149, 1977.
4. Wess J., Zumino B. Nucl.Phys., 1974, B70, p.39.
5. Molotkov V., Petrova S., Stoyanov D. Theor. and Math. Phys., 1976, v. 26, No. 2.
6. Aneva B., Mikhov S., Stoyanov D. Theor. and Math. Phys., 1976, v.27, No. 3.
7. Salam A., Strathdee J. Phys.Rev., 1975,, D11, p.521.
8. Ferrara S., Di Vecchia P. Trieste preprint IC/77/63, 1977.
9. Wightman A. Problems in Relativistic Dynamics of Quantum Fields (in Russian), "Nauka", Moscow, 1968.

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