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О неоднозначности определения тока в модели Тирринга

В работе показано, что в модели Тирринга существует более общее определение тока, которое приводит к тому, что перенормированные решения модели образуют однопараметрическое семейство. Показано, что при этом более общем определении тока конформная размерность двухточечной функции не фиксирована. Таким образом модель Тирринга принимает все основные характерные свойства градиентной модели. Вычислены также двухточечные функции между полями с одинаковой и различной перенормировкой.

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On the Nonuniqueness of the Current Definition in the Thirring Model

In the paper it is shown that for the Thirring model there exists a more general definition of the current, which implies that the renormalized solutions form a one parameter family. It is also shown that with this more general current definition the conformal dimension of the two-point functions is not fixed. Thus the Thirring model acquires all characteristic features of a gauge model. The two-point functions for fields of one and the same and of different renormalizations are calculated.

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In a series of papers [1-4] the exact solution of the renormalized massless quantum Thirring model has been constructed as an exponential from two fields, which are usually called massless scalar fields. As it has been shown in paper [4] these fields are not usual scalars, moreover the solutions of the Thirring model are not spinors (even in the sense that is accepted for a spinor in a two dimensional space-time).

As it is known the solution of the Thirring model leading off Johnson [5] is connected with the conservation of two currents (a vector one and an axial one) that can be expressed as gradients of two scalar fields. In paper [5] a correct definition of the vector current has been given too (see also [6]) and later on all authors followed this definition. However as we show in the present work this definition is not unique. The obtained here new definition of the vector current coincides in some special case with that of Johnson [5]. On the other hand it provides a possibility to establish that there exists a closer connection between the Thirring model and the gauge model. It is shown in particular that both models have analogous properties with respect to the conformal and to the gauge symmetries and the hence resulting consequences have been considered too.

1. *THE GAUGE MODEL IN TWO-DIMENSIONAL SPACE-TIME*

We call a gauge model the quantum field theory of two interacting fields - a scalar one $\phi(x)$ and a spinor one $\psi(x)$, based on the following equations*

$$i\gamma^{\mu}\partial_{\mu}\psi(x) = g{:}\,\partial_{\mu}\phi(x)\gamma^{\mu}\psi(x){:}\;;\quad \gamma^0 = \sigma_1,\ \gamma^1 = i\sigma_2, \tag{1.1}$$

$$\Box\phi(x) = 0, \tag{1.2}$$

where σ_1, σ_2 are the Pauli matrices. The field $\phi(x)$ we consider is completely identical with the massless "scalar" field of paper /4/. In particular we once more write the nonzero commutation relations which hold for the field $\phi(x)$ and its creation and annihilation parts $\phi^{\pm}(x)$:

$$[\phi(x), \phi(y)] = iD(x-y), \tag{1.3}$$

$$[\phi^{\pm}(x), \phi^{\mp}(y)] = D^{\pm}(x-y). \tag{1.4}$$

The eq. (1.2) is also satisfied by the dual field $\tilde{\phi}(x)$ (pseudoscalar). The latter is related with the field by the differential equalities:

$$\partial_{\mu}\phi(x) + \epsilon_{\mu}{}^{\nu}\partial_{\nu}\tilde{\phi}(x) = 0 \quad (\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}\ ;\ \epsilon_{01} = -\epsilon_0{}^1 = 1). \tag{1.5}$$

The dual field (sometimes it is called conjugated) and its creation and annihilation parts $\tilde{\phi}^{\pm}(x)$ satisfy the same commutation relations (1.3) and (1.4). Their commutators with the fields $\phi(x)$ and $\phi^{\pm}(x)$ are the following

$$[\tilde{\phi}(x), \phi(y)] = i\tilde{D}(x-y),$$

$$[\tilde{\phi}^{\pm}(x), \phi^{\mp}(y)] = \tilde{D}^{\pm}(x-y);\quad [\tilde{\phi}^{\pm}(x), \phi^{\pm}(y)] = 0. \tag{1.6}$$

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* The symbol : : denotes normal ordering.

The commutation functions are given in Appendix A. With the help of the fields listed here, an exact solution of eq. (1.1) can be constructed. By a direct substitution it can be verified that the expression

$$\psi(x,\alpha) = \exp\{i\beta\gamma^5\tilde{\phi}^-(x) - i\alpha\phi^-(x)\}\exp\{-i\alpha\phi^+(x) + i\beta\gamma^5\tilde{\phi}^+(x)\}u \tag{1.7}$$

$$(\gamma^5 = \gamma^0\gamma^1)$$

is such a solution. (Here $u = \binom{u_1}{u_2}$, $|u_1| = |u_2|$, $u_k = \text{const}$).

The constants α and β are related only by the equation

$$\alpha - \beta = g. \tag{1.8}$$

The solutions (1.7) form a one-parameter family. We choose α as a parameter and consider β expressed by it due to (1.8). The transformation laws of the fields $\phi^{\pm}(x)$ and $\tilde{\phi}^{\pm}(x)$ under the two-dimensional conformal group have been found in /4/. With the help of these transformations the corresponding transformations of the "spinor" field (1.7) can be obtained. The latter coincide in form with the transformations of the Thirring field, given in paper /4/. The only difference is that α and β satisfy a single equality (1.8). The generators of these transformations for any α and β are given in Appendix B. As in /4/ it can be shown that eq. (1.1) is covariant with respect to the infinitesimal transformations of the two-dimensional conformal group and with respect to the global transformations of its universal covering group. As far as the proof of this statement does not differ at all from the proof of the analogous statement for the Thirring case given in /4/ we do not consider it here. However we remark that the parameter α is connected with the conformal dimension of the two-point function which is hence not fixed. Indeed starting from eq. (1.7) and using eqs. (1.4) and (1.6) we can readily calculate the two point function of the fields $\psi(x)$:

$$\Lambda_{ij}(x-y) = \langle 0|\psi_i(x)\bar{\psi}_j(y)|0\rangle \quad i,j = 1,2. \tag{1.9}$$

Here $\bar{\psi}(y)$ is the Dirac conjugated field. If we denote $z = x - y$ then the functions (1.9) have the following form

$$\Delta_{ij}(z) = u_i \bar{u}_j (-\mu^2 z^2 + i0z^0)^{\frac{1}{4\pi}[\beta^2(-1)^{i+j} - \alpha^2]} \times$$

$$\times \left(\frac{z^0 - z^1 - i0}{z^0 + z^1 + i0}\right)^{\frac{\alpha\beta}{4\pi}[(-1)^j - (-1)^i]} \qquad (1.10)$$

In this formula $\bar{u}_i = (u^* \gamma^0)$ where u^* is the hermitian conjugated quantity. The functions $\Lambda_{ij}(z)$ are obviously homogeneous in z, but the degree of homogeneity depends on the indices i and j. Indeed the diagonal terms of the matrix $\Lambda_{ij}(z)$ have one homogeneity degree

$$\frac{\beta^2 - \alpha^2}{2\pi} = \frac{g^2 - 2\alpha g}{2\pi}$$

and the nondiagonal - another one

$$-\frac{\alpha^2 + \beta^2}{2\pi} \equiv -\frac{2\alpha^2 + g^2 - 2\alpha g}{2\pi}.$$

Nevertheless, as we see further, the two-point function (1.9) of the spinor field $\psi(x)$ is conformal invariant. Apparently this invariance is due to the corresponding invariance of eq. (1.1) although such statement is not always true. The presence of diagonal terms in the two-point function namely points out that in the quantum theory of the gauge model a spontaneous breakdown of some symmetry takes place. To prove the latter we consider the transformation

$$\psi(x) \to e^{i\chi\gamma^5}\psi(x); \quad \phi(x) \to \phi(x). \qquad (1.11)$$

(χ is a numerical parameter and $\gamma^5 = \gamma^0\gamma^1$). Obviously the eq. (1.1) remains invariant under this transformation. Now if we would suppose invariance of the vacuum state $|0\rangle$ too, then the two-point function $\Delta_{ij}(z)$ from (1.9) should satisfy the following equation

$$\Lambda_{ij}(z) = [e^{i\chi\gamma^5}\Lambda(z)e^{i\chi\gamma^5}]_{ij}. \qquad (1.12)$$

We should have in particular for the diagonal terms (taking into account that the matrix γ^5 is diagonal)

$$\Lambda_{kk}(z) = \exp[2i\chi(-1)^k]\Lambda\ (z). \qquad (1.13)$$

Hence however it follows

$$\Lambda_{kk}(z) = 0,$$

which is in contradiction with eq. (1.10). Thus the vacuum is not invariant with respect to the symmetry (1.11), so this symmetry is spontaneously broken.

Let us consider also the following operator gauge transformation:

$$\psi(x,\alpha) \to :\exp\{i\kappa[\gamma^5\tilde{\phi}(x) - \phi(x)]\}\psi(x,\alpha): \qquad (1.14)$$

Taking into account the explicit form of the $\psi(x,\alpha)$ it is easily verified that the transformation (1.14) is actually reduced to the transformation

$$\psi(x,\alpha) \to \psi(x, \alpha + \kappa), \qquad (1.15)$$

i.e., to a translation of a parameter α. Hence β also translates in such a way that eq. (1.8) holds. Therefore eq. (1.1) is covariant with respect to the gauge transformation (1.14).

Here it can be shown that the vacuum is not invariant under the given symmetry (1.14). If we suppose the contrary, then the two-point function should not change under the substitutions $\alpha \to \alpha + \kappa$ and $\beta \to \beta + \kappa$. As it is seen from (1.10) this is not the case, which proves our statement.

At the end we note that the considered above one-parameter solution of eq. (1.1) is a special case of a more general two-parameter solution

$$\psi(x, \alpha, \alpha') = \exp\{i\beta'\gamma^5 \tilde{\phi}^-(x) - i\alpha'\phi^-(x)\} \times$$
$$\times \exp\{-i\alpha\phi^+(x) + i\beta\gamma^5 \tilde{\phi}^+(x)\} u, \tag{1.16}$$

where

$$\alpha' - \beta' = \alpha - \beta = g. \tag{1.17}$$

The discussion of this solution leads only to more cumbersome calculations without any principle difference from the given above results, so we omit, it here.

2. *THE CURRENT IN THE THIRRING MODEL*

The current in the Thirring model has been defined before according to Johnson /5,6/. We remind this definition. One constructs the following two expressions

$$j_\mu(x) = \lim_{\substack{\epsilon^0 = 0 \\ \epsilon^1 \to 0}} j_\mu(x, \epsilon); \quad \tilde{j}_\mu = \lim_{\substack{\tilde{\epsilon}^0 = 0 \\ \tilde{\epsilon}^1 \to 1}} j_\mu(x, \tilde{\epsilon}), \tag{2.1}$$

where

$$j_\mu(x,\epsilon) = (-\epsilon^2)^{\frac{1}{4\pi}(\alpha^2+\beta^2) - \frac{1}{2}} [\bar{\psi}(x+\epsilon)\gamma_\mu\psi(x) - \psi(x)\bar{\psi}(x-\epsilon)\gamma_\mu], \tag{2.2}$$

and

$$\tilde{\epsilon}^2 = -\epsilon^2, \quad \tilde{\epsilon}\epsilon = 0.$$

(We consider here, as in /4/, the renormalized Thirring model). Then one takes their linear combination and finds the current in the form

$$J_\mu(x) = \frac{1}{2}[j_\mu(x) + \tilde{j}_\mu(x)]. \tag{2.3}$$

The renormalized Thirring model with such a current leads to solutions of the type (1.7) but with different relations for α and β:

$$\alpha\beta = \pi, \quad \beta - \alpha = \frac{g}{2\pi}(\alpha + \beta). \tag{2.4}$$

The current is expressed further with the help of the field

$$J_\mu(x) = \frac{1}{2}(\alpha + \beta)\partial_\mu \phi(x). \tag{2.5}$$

This current namely reduces the Thirring model to the gauge model, considered in Sec. 2.

On the other hand it is not difficult to calculate the expression (2.1) with the help of (1.7) without fixing α and β

$$j_0(x) = -\frac{i}{2\pi}(-1)^{\frac{\alpha\beta}{2\pi}}\{(-1)^{-\frac{\alpha\beta}{\pi}}[-\alpha\partial_1\phi - \beta\partial_1\tilde{\phi}] - \alpha\partial_1\phi + \beta\partial_1\tilde{\phi}\},$$

$$j_1(x) = -\frac{i}{2\pi}(-1)^{\frac{\alpha\beta}{2\pi}}\{(-1)^{-\frac{\alpha\beta}{\pi}}[-\alpha\partial_1\phi - \beta\partial_1\tilde{\phi}] + \alpha\partial_1\phi - \beta\partial_1\tilde{\phi}\}, \tag{2.6}$$

$$\tilde{j}_0(x) = \frac{1}{\pi}\alpha\partial_1\tilde{\phi},$$

$$\tilde{j}_1(x) = \frac{1}{\pi}\beta\partial_1\phi, \tag{2.7}$$

where for simplicity we have set $(-1)^A = e^{i\pi A}$ and

$$(\mu^2)^{-\frac{\alpha^2+\beta^2}{4\pi}} |u|^2 = \frac{1}{2\pi}.$$

It is easy to note that eqs. (2.6) and (2.7) give the mutual inverse relations between the quantities $j_\mu(x)$ and $\tilde{j}_\mu(x)$ on one hand and between $\alpha\partial_1\phi$, $\beta\partial_1\phi$ and $\alpha\partial_1\tilde{\phi}$, $\beta\partial_1\tilde{\phi}$ on the other hand. Therefore we can solve eqs. (2.6) and

(2.7) with respect to $\partial_\mu \phi(x)$ and $\partial_\mu \tilde{\phi}(x)$, without fixing α and β, then construct the current from eq. (2.5), considering this equality as its definition.

$$\frac{1}{2\pi}(\alpha+\beta)\partial_0\phi \equiv J_0(x) =$$

$$= \frac{1}{2}\{j_0(x) + \frac{i}{2}(-1)^{-\frac{\alpha\beta}{2\pi}}[j_0(x) - j_1(x)] - $$

$$- \frac{i}{2}(-1)^{\frac{\alpha\beta}{2\pi}}[j_0(x) + j_1(x)]\}, \tag{2.8}$$

$$\frac{1}{2\pi}(\alpha+\beta)\partial_1\phi = J_1(x) =$$

$$= \frac{1}{2}\{\tilde{j}_1(x) - \frac{i}{2}(-1)^{-\frac{\alpha\beta}{2\pi}}[j_0(x) - j_1(x)] - $$

$$- \frac{i}{2}(-1)^{\frac{\alpha\beta}{2\pi}}[j_0(x) + j_1(x)]\}. \tag{2.9}$$

Thus, if we define the current in the Thirring model by eqs. (2.8) and (2.9), one of the relations between the coefficients α and β no longer arises, and instead of (2.4) we have only

$$\beta - \alpha = \frac{g}{2\pi}(\alpha+\beta). \tag{2.10}$$

The definition (2.8)-(2.9) differs from that of Johnson, but it is reduced to it by setting $\alpha\beta = \pi$.

The eq. (2.10) does not determine the constant α and β uniquely and therefore eq. (1.7) represents a one-parameter family of renormalized solutions of the Thirring model. As is known the factor $(-\epsilon^2)^{-\frac{\alpha^2+\beta^2}{8\pi}+\frac{1}{4}}$ $(\epsilon \to 0)$ has the meaning of a renormalization constant for the field operators. More exactly the relation between the renormalized fields $\psi(x)$ and the unrenormalized ones $\psi_0(x)$ is of the type:

$$\psi(x) \sim (-\epsilon^2)^{-\frac{1}{8\pi}(\alpha^2+\beta^2)+\frac{1}{4}}_{\epsilon\to 0}\ \psi_0(x). \tag{2.11}$$

As far as α and β are related only by eq. (2.10), then eq. (2.11) means that there exists a one-parameter family of renormalizations in the Thirring model. We choose α as a parameter of this family.

Any solution belonging to this family with given α is expressed by the scalar fields $\phi(x)$ and $\tilde{\phi}(x)$ according to eq. (1.7). Therefore the Thirring equation with any α is covariant with respect to the representation of the conformal algebra with generators given in Appendix B. Considering the whole family of renormalized solutions we find a closer connection between the Thirring model and the gauge model.

In particular here as it is in the case of the gauge model there is no sense to assign any conformal dimension to the two-point functions of the fields $\psi(x)$, because it depends on α and hence on the chosen renormalization. Besides, it is possible, with the help of a transformation analogous to (1.14) to change the value of the parameter α, thus passing over from one renormalized solution to another, i.e., roughly speaking from one dimension to another. In particular, the transformation

$$\psi(x,\alpha) \to :\exp\{i(\kappa-1)[\beta\gamma^5\tilde{\phi}(x) - \alpha\phi(x)]\}\psi(x,\alpha)\!: \tag{2.12}$$

which is simply a multiplication of the constant α and β with κ

$$\psi(x,\alpha) \to \psi(x,\kappa\alpha) \tag{2.13}$$

leaves eq. (2.10) unchanged. Therefore the whole family of renormalized Thirring equations is invariant with respect to the transformations (2.12).

At the end we show that the expressions for the current components (2.8) and (2.9) can be written in manifestly covariant form. For this purpose we introduce the quantities

$$T_\mu(x,\epsilon) = \frac{1}{2}[V_1(\epsilon)(\delta_\mu^{\ \nu} - \epsilon_\mu^{\ \nu}) + V_2(\epsilon)(\delta_\mu^{\ \nu} + \epsilon_\mu^{\ \nu})] j_\nu(x,\epsilon), \quad (2.14)$$

where $j_\nu(x,\epsilon)$ is given by eq. (2.2) (α and β arbitrary, and $V_1(\epsilon)$ and $V_2(\epsilon)$ are given by

$$V_1(\epsilon) = \left(\frac{\epsilon^0 - \epsilon^1 - i0}{\epsilon^0 + \epsilon^1 - i0}\right)^{-\frac{\alpha\beta}{2\pi}} = [V_2(\epsilon)]^{-1}. \quad (2.15)$$

It is easily found that

$$\lim_{\substack{\epsilon^0 \to 0 \\ \epsilon^1 \to 0}} T_\mu(x,\epsilon) = \frac{1}{2}\{(-1)^{-\frac{\alpha\beta}{2\pi}}[j_0(x) + j_1(x)] + (-1)^{\frac{\alpha\beta}{2\pi}+\mu}[j_0(x) - j_1(x)]\} \equiv T_\mu(x),$$

$$\lim_{\substack{\epsilon^0 \to 0 \\ \epsilon^1 \to 0}} T_\mu(x,\tilde{\epsilon}) = \tilde{j}_\mu(x) \equiv \tilde{T}_\mu(x).$$

Therefore we can write the current (2.8) and (2.9) in the following covariant form

$$J_\mu(x) = \frac{1}{2}[\tilde{T}_\mu(x) + i\epsilon_\mu^{\ \nu} T_\nu(x)]. \quad (2.16)$$

3. *CONFORMAL INVARIANT TWO-POINT FUNCTIONS*

We show in this section that the two-point functions of the type (1.10) are invariant with respect to the representation of the conformal group with generators given in Appendix B.

Let the fields $\psi^1(x)$ and $\psi^2(x)$ transform under the above-mentioned representations with parameters α_1, β_1 and α_2, β_2 respectively. We calculate the two-point function of these fields defined by the formula

$$\Delta_{ij}(x) = \langle 0|\psi^1_i(x)\bar{\psi}^2_j(0)|0\rangle, \quad (3.1)$$

($\bar{\psi}$ denotes the Dirac conjugated spinor). The condition for Lorentz invariance leads to the equation

$$(M_{\mu\nu}\Delta)_{ij}(x) \equiv -i(x_\mu\partial_\nu - x_\nu\partial_\mu)\Delta_{ij}(x) + \frac{i\epsilon_{\mu\nu}\alpha_2\beta_1}{2\pi}(\gamma^5\Delta(x))_{ij} - \frac{i\epsilon_{\mu\nu}\alpha_1\beta_2}{2\pi}(\Delta(x)\gamma^5)_{ij} = 0. \quad (3.2)$$

The latter is obtained in the standard way using eq. (B_2) (see Appendix B). Analogously using eq. (B_3) one can find the condition for scale invariant of the two-point function (3.1)

$$(D\Delta)_{ij}(x) \equiv ix^\mu\partial_\mu\Delta_{ij}(x) + i\frac{\alpha_1\alpha_2}{2\pi}\Delta_{ij}(x) - i\frac{\beta_1\beta_2}{2\pi}(\gamma^5\Delta(x)\gamma^5)_{ij} = 0. \quad (3.3)$$

The condition for invariance under the special conformal transformations $(K_\mu\Delta)_{ij}(x) = 0$ can be obtained using (B_4). This condition however does not lead to new equations for the functions $\Delta_{ij}(x)$, because it is reduced to the eqs. (3.2) and (3.3). Actually, if we write the expression $(K_\mu\Delta)_{ij}(x)$ explicitly, it can be readily verified that the identity holds

$$(K_\mu\Delta)_{ij}(x) = x^\nu(M_{\mu\nu}\Delta)_{ij}(x) - x_\mu(D\Delta)_{ij}(x), \quad (3.4)$$

which proves our statement. The system of eqs. (3.2) and (3.3) is simple enough, so it is not difficult to solve. To make sure of this one has to introduce new variables

$$u = x^{+}x^{-}, \quad v = \frac{x^{-}}{x^{+}}, \quad \Lambda_{ij}(x) = \Lambda_{ij}(u, v), \tag{3.5}$$

where $x^{\pm} = x^{0} \pm x^{1} - i0$.

Then eqs. (3.2) and (3.3) take the following simple form

$$v\frac{\partial}{\partial v}\Lambda_{ij}(u,v) = \frac{1}{4\pi}[\alpha_1\beta_2(-1)^{j} - \alpha_2\beta_1(-1)^{i}]\Lambda_{ij}(u, v), \tag{3.6}$$

$$u\frac{\partial}{\partial u}\Lambda_{ij}(u, v) = -\frac{1}{4\pi}[\beta_1\beta_2(-1)^{i+j} - \alpha_1\alpha_2]\Lambda_{ij}(u, v). \tag{3.7}$$

In finding the latter equations we made use of the concrete form of the matrix $\gamma^{5}_{kl} = (-1)^{k}\delta_{kl}$. Solving the system and going back to the variables x_μ, we obtain the following end result

$$\Lambda_{ij}(x) = H_{ij}(-\mu^{2}x^{2} + i0x^{0})^{-\frac{1}{4\pi}[\beta_1\beta_2(-1)^{i+j} - \alpha_1\alpha_2]} \times$$

$$\times\left(\frac{x^{0} - x^{1} - i0}{x^{0} + x^{1} + i0}\right)^{\frac{1}{4\pi}[\alpha_1\beta_2(-1)^{j} - \beta_1\alpha_2(-1)^{i}]} \tag{3.8}$$

Here H_{ij} are arbitrary integration constants. It is easy to observe now that if $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ the expression (3.8) coincides with the expression (1.10) which proves the conformal invariance of the latter.

The functions (3.8) differ from zero for any α_1, β_1 and α_2, β_2. Therefore the two-point functions of the Thirring fields with different remormalization constants are not zero too.

At the end we note that two-point functions of the type

$$\tilde{\Delta}_{ij}(x) = \langle 0|\overset{1}{\psi}_{i}(x)\overset{2}{\psi}_{j}(0)|0\rangle \tag{3.9}$$

are also different from zero. The explicit form of the latter is easily obtained if in eq. (3.8) we set $\alpha_2 = -\alpha_2$.

APPENDIX A

In the paper the following commutation functions have been used

$$D(x) = -\frac{1}{2}\epsilon(x^{0})\Theta(x^{2}), \quad D^{\pm}(x) = \mp\frac{1}{4\pi}\ln(-\mu^{2}x^{2} \pm i0x^{0}),$$

$$D^{+}(x) + D^{-}(x) = iD(x),$$

$$\tilde{D}(x) = -\frac{1}{2}\epsilon(x^{1})\Theta(-x^{2}), \tilde{D}^{\pm}(x) = \pm\frac{1}{4\pi}\ln\frac{x^{0} - x^{1} \mp i0}{x^{0} + x^{1} \mp i0},$$

$$\tilde{D}^{+}(x) + \tilde{D}^{-}(x) = i\tilde{D}(x).$$

APPENDIX B

We write the commutation relations of the conformal group generators with the "spinor" field

$$[P_\mu, \psi(x)] = i\partial_\mu\psi(x), \tag{B.1}$$

$$[M_{\mu\nu}, \psi(x)] = -i(x_\mu\partial_\nu - x_\nu\partial_\mu)\psi(x) +$$

$$+ \epsilon_{\mu\nu} :(\alpha S + \beta L\gamma^{5})\psi(x):, \tag{B.2}$$

$$[D, \psi(x)] = ix^{\mu}\partial_\mu\psi(x) + :(\alpha L + \beta S\gamma^{5})\psi(x): \tag{B.3}$$

$$[K_\mu, \psi(x)] = -i(2x_\mu x_\nu - g_{\mu\nu}x^2)\partial^\nu \psi(x) + 2x_\mu :(aL + \beta S\gamma^5)\psi(x): +$$

$$+ 2\epsilon_{\mu\nu}x^\nu : (aS + \beta L\gamma^5)\psi(x): , \qquad \text{(B.4)}$$

where a and β are arbitrary constants and

$$S = \frac{1}{2\sqrt{2\pi}}[b^+(0) + b^-(0)], \quad L = \frac{1}{2\sqrt{2\pi}}[a^+(0) + a^-(0)].$$

The constant operators $a^\pm(0)$ and $b^\pm(0)$ are defined in paper /4/. They have the following commutation relations with the fields

$$[a^\pm(0), \psi(x)] = \mp i\frac{a}{\sqrt{2\pi}}\psi(x),$$

$$[b^\pm(0), \psi(x)] = \mp i\frac{\beta}{\sqrt{2\pi}}\gamma^5 \psi(x).$$

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