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D.T.Stoyanov

REMARKS ON THE PAPER "TWO-DIMENSIONAL QUANTUM FIELD THEORIES INVOLVING MASSLESS PARTICLES" BY N.NAKANISHI

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Стоянов Д.Ц., E2 - 11884

Замечания по поводу статьи Н.Наканиши "Двумерные теории квантованных полей, включающие безмассовые частицы"

Работа излагает некоторые критические замечания по поводу статьи Н.Наканиши "Двумерные теории квантованных полей, включающие безмассовые частицы". Утверждается, что вследствие установленных коммутационных соотношений присутствующие в теории безмассовые скалярные поля не могут иметь того асимптотического поведения, которое требует для них Н.Наканиши. Выявлено противоречие, существующее в доказательстве неприводимости скалярного поля. Таким образом, построенная Наканиши теория, в которой сделана попытка обойтись единственным скалярным полем и соответственно одним топологическим зарядом, противоречива. Показано, что статистика построенных решений не является фиксированной и что решения, удовлетворяющие бозе- или ферми-статистикам, отличаются друг от друга постоянными операторными множителями.

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Remarks on the Paper "Two-Dimensional Quantum Field Theories Involving Massless Particles" by N.Nakanishi

The present paper contains some critical remarks on the paper by N.Nakanishi "Two-Dimensional Quantum Field Theories Involving Massless Particles". It is stated that because of the obtained commutation relations the massless scalar fields of the theory cannot have the asymptotic behaviour assumed by N.Nakanishi. The contradiction, appearing in the proof of the irreducibility of the scalar field, is demonstrated. Therefore, the theory constructed by Nakanishi, in which an attempt is made to formulate it with the help of one scalar field and correspondingly with one topological charge, is contradictory. It is shown that the statistics of the solutions is not fixed and the solutions satisfying Bose or Fermi statistics differ by constant operator factors.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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N.Nakanishi has published a number of papers [1,5] devoted to the construction of a singlet scalar massless field in the two-dimensional space-time. His approach cannot be regarded as a mathematically flawless one due to some contradictions presented in refs. [1,5]. On the other hand, a consistent theory for the same object (the two-dimensional scalar massless field) has been constructed in refs. [2-4]. There some comments on the papers [1] have also been made.

The review paper [5] submitted to the XIX International Conference on High Energy Physics shows that a few changes made by the author do not eliminate the main contradictions in refs. [1]. Moreover, this paper [5] contains two critical remarks on refs. [2,3] so we think we should give an answer. At the same time we wish to point out some of the contradictions in refs. [1,5] we have found.

First of all, the definition of the field *

$$\tilde{\Phi}(x) = \int_{-\infty}^{x'} dz' \partial_0 \Phi(x^0, z') \qquad (3.16)$$

and the assumed asymptotic condition for $\partial_0 \Phi(x)$

$$\partial \Phi(x)$$

$$\partial_0 \Phi(x) \sim (x')^{-2} \qquad (3.9)$$
$$|x'| \to \infty$$

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* The numeration of the formulae here follows the one of ref. [5].

lead to the following conclusion:

$$\tilde{\Phi}(x) \underset{x' \to -\infty}{\to} 0 \qquad \text{for any } x^0 \tag{A}$$

The asymptotic condition (A) contradicts also the commutation relations (3.22-3.25) themselves:

$$[\Phi^{(\pm)}(x), \tilde{\Phi}^{(\pm)}(y)] = \frac{i}{8}, \tag{3.22}$$

$$[\Phi^{(\pm)}(x), \tilde{\Phi}^{(\mp)}(y)] = \tilde{D}(x-y) + \frac{i}{8}, \tag{3.23}$$

$$[\tilde{\Phi}^{(\pm)}(x), \tilde{\Phi}^{(\pm)}(y)] = 0, \tag{3.24}$$

$$[\tilde{\Phi}^{(\pm)}(x), \tilde{\Phi}^{(\mp)}(y)] = D^{\pm}(x-y). \tag{3.25}$$

Indeed, from Eqs. (3.24) and (3.25) it follows that

$$[\tilde{\Phi}^{(\pm)}(x), \tilde{\Phi}(y)] = D^{(\pm)}(x-y). \tag{D}$$

This equation contradicts Eq. (A) because $D^{(\pm)}(x-y)$ do not vanish at infinity.

Another contradiction is found in the proof of the theorem about the irreducibility of the field $\Phi(x)$. From Eqs. (3.22), (3.23) we have

$$[\Phi^{(\pm)}(x), \tilde{\Phi}(y)] = \tilde{D}^{(\pm)}(x-y) + \frac{i}{4}. \tag{E}$$

Let us define the quantity B

$$B = \lim_{\substack{y^2 < 0 \\ y' \to \infty}} \frac{\tilde{\Phi}(y)}{\pm D^{\pm}(y)}.$$

Using Eq. (E) one can find that B commutes with $\Phi^{(+)}(x)$ and $\Phi^{(-)}(x)$

$$[\Phi^{(\pm)}(x), B] = 0. \tag{F}$$

This means that

$$[\Phi(x), B] = 0 \tag{G}$$

for any x.

On the other hand, for any finite x Eq. (D) has the following corollary

$$[\tilde{\Phi}^{(\pm)}(x), B] = \pm 1. \tag{H}$$

This implies that B cannot be a c-number. Indeed, according to Eq. (F) B commutes with the "charges"

$$\Phi^{(\pm)} = \int_{-\infty}^{\infty} dx' \partial_0 \Phi^{(\pm)}(x). \tag{3.27}$$

Then Eq. (H) can be rewritten in the form

$$[\tilde{\Phi}^{(+)}(x) - \frac{1}{4}\Phi^{(-)}, B] = 1. \tag{I}$$

Further, from Eq. (3.22) one sees that the vacuum state obeys the condition

$$[\tilde{\Phi}^{(+)}(x) - \frac{1}{4}\Phi^{(-)}]|0\rangle = 0. \tag{3.32}$$

This fact together with the commutator (I) means that

$$(\tilde{\Phi}^{(+)}(x) - \frac{1}{4}\Phi^{(-)})B|0\rangle = |0\rangle \tag{J}$$

and finally

$$B|0\rangle \neq \lambda|0\rangle. \tag{K}$$

So one can conclude that either

1. The field $\Phi(x)$ is not irreducible, or
2. B does not exist. In the second case, however, Eq. (A) is not possible and hence Eqs. (3.9), (3.10) and (3.16) cannot be true. We think that the latter conclusion clarifies the problem about the singlet character of the field $\Phi(x)$.

The second part of our note concerns the statistics of the Thirring model solutions. As it is well known, the two-dimensional spinor representation is reducible, and it decomposes into two scalar ones. For this reason the equation of motion does not fix the statistics.

Let the fields $\Phi_{HS}(x)$ be the massless scalar fields considered in refs./2,3/. We introduce new fields

$$\tilde{\Phi}_N^{(\pm)}(x) = \tilde{\Phi}_{HS}^{(\pm)}(x) \pm \frac{i}{8}\sqrt{2\pi}\,[a^+(0) + a^-(0)]$$

$$\Phi_N^{(\pm)}(x) = \Phi_{HS}^{(\pm)}(x) \qquad \text{(L)}$$

(here and below we use the definitions and the notation of refs./2-4/). With the help of the commutators

$$[\Phi_{HS}^{(\pm)}(x), a^{\mp}(0)] = \pm\frac{1}{\sqrt{2\pi}} \qquad \text{(M)}$$

one can easily show that $\Phi_N^{(\pm)}(x)$ satisfy the commutation relations (3.22)-(3.25).

Thus, we see that the solutions of the Thirring model with one or another statistics differ by constant operator factor. This follows from the form of $\tilde{\Phi}_N^{\pm}(x)$ (Eq. (L)) and from the expression for $\Psi_K(x)$ in refs./2,3/. The same statement follows also from refs./6,7/ where the operator factors mentioned above are introduced by hand.

Just for the reasons explained here the question about the statistics of the Thirring model solutions has not been discussed in refs./2-4/ (moreover, these solutions are not in fact spinorial in the proper sense of the word).

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