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FIELD-THEORETIC TREATMENT OF HIGH MOMENTUM TRANSFER PROCESSES.

III. Gauge Theories

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Теоретико-полевой подход к процессам с большой передачей импульса. III. Калибровочные теории

Развитый в первых двух частях подход обобщается на калибровочные теории, включая квантовую хромодинамику. Доказана справедливость модифицированной партонной модели для процесса рождения массивных лептонных пар во всех порядках теории возмущений.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Field-Theoretic Treatment of High Momentum Transfer Processes. III. Gauge Theories

The approach developed in the first two parts of the work is generalized to include gauge theories, especially quantum chromodynamics. The validity of the modified parton model for massive lepton-pair production process is proven to all orders in perturbation theory.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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In two previous papers [1,2] we have developed methods of the unified field-theoretic approach to deep inelastic scattering and to massive lepton-pair production. Our starting point was the analysis of simple scalar theories in the alpha-representation. By explicitly using the coordinate representation, the results obtained can be easily generalized to include more complicated nongauge theories describing spin -1/2 particles. The generalization for gauge theories, which is a subject of this paper, is less trivial.

1. DEEP INELASTIC SCATTERING

The analysis of gauge theories is complicated, in particular, by the fact that the field A^{μ} (vector potential) has zero twist. Hence the upper bound on the asymptotic contribution of the subgraph V, related to a parton subprocess, into the structure function $W(\omega, Q^2) \equiv -W^{\mu}_{\mu}(\omega, Q^2)$ of the deep inelastic scattering

$$W_{(V)}(\omega, Q^2) \leq Q^{2-\ell_q} \tag{1}$$

(where ℓ_q is the number of external quark lines of the subgraph V, see eq. (1.A.12)*) is independent of the number of gluon external lines of the subgraph V (*fig. 1a*).

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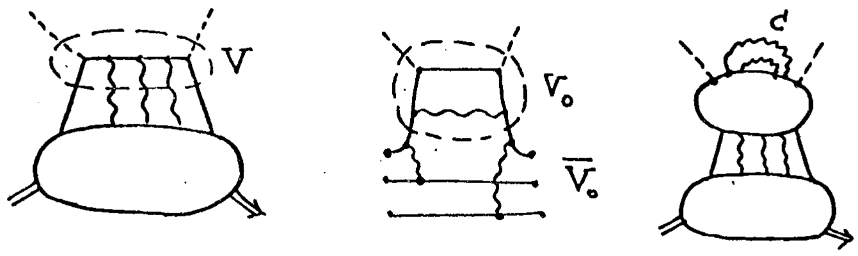
*Henceforth eq. (1.N) (or (2.N)) means eq. (N) from ref. [1] (or ref. [2]).



a) *b)* *c)*

Fig. 1

To sum over gluon lines, let us fix the form of the initial subgraph v_0 (*fig. 1b*). To get all allowable combinations, one must join the lines of the subgraph v_0 with those of the subgraph $\bar{v}_0$ in all possible ways. Every gluon line adds the field $A_\mu(z)$ into the matrix element $\langle P|\psi\dots\psi|P\rangle$. Furthermore, the propagator corresponding to some line of the subgraph v_0 is modified:

$$S^c(x_\alpha - x_\beta) \to g\int d^4z A^a_\mu(z)\, S^c(x_\alpha - z)\gamma^\mu \tau_a\, S^c(z - x_\beta) + \dots , \qquad (2)$$

where $(\tau^a)_{AB}$ is the matrix of the gauge group in the quark (fundamental) representation. It is easy to note that the sum over gluon lines inserted into the (x_α, x_β) - line (*fig. 2a*) gives $\mathcal{S}^c$, the propagator of a spinor particle in an external gluon field, i.e., the perturbative solution to the equation

$$(i\hat{D}_\mu\gamma^\mu - m)\mathcal{S}^c(x_\alpha, x_\beta) = -\delta^4(x_\alpha - x_\beta), \qquad (3)$$

where $\hat{D}_\mu = \frac{\partial}{\partial x^\mu_\alpha} - ig\hat{A}_\mu$ is the covariant derivative acting on the quark field, and $\hat{A}_\mu = A^a_\mu \tau_a$. The solution to this equation can be also written in the following form

$$\mathcal{S}^c(x_\alpha, x_\beta) = \hat{E}_{AB}(x_\alpha, x_\beta)\{S^c(x_\alpha - x_\beta) + \mathcal{O}(G)\}. \qquad (4)$$

a)

b)

Fig. 2

We use the notation

$$\hat{E}_{AB}(x,y) = \left(T_c \exp ig\int_y^x \hat{A}_\mu(z)\,dz^\mu\right)_{AB}, \qquad (5)$$

where T_c means that the integral must be path-ordered along the contour of integration, which is straight line connecting x and y. An analogous problem was treated in an Abelian theory by Gross and Treiman /3/. $\mathcal{O}(G)$ denotes the contribution of operators containing the gluon field strength $G_{\mu\nu} = \frac{i}{g}[\hat{D}_\mu, \hat{D}_\nu]$, for insrance, in an Abelian theory $\mathcal{O}(G) \equiv R(x,y)$ is the solution to the equation

$$i\gamma^\mu \frac{\partial}{\partial x^\mu} R(x,y) + g\gamma^\mu(x^\mu - y^\mu)(R(x,y) + S^c(x-y)) \times$$
$$\times \int_0^1 t\,dt\, G_{\mu\nu}(y + (x-y)t) = 0. \qquad (6)$$

Any operator of the $OG\dots G$ type has twist higher than that of O, because the tensor $G_{\mu\nu}$ is antisymmetric. Hence the terms entering into $\mathcal{O}(G)$ give the power corrections $(M^2/Q^2)^k$ into the asymptotical form of the structure function $W(\omega, Q^2)$.

In an Abelian theory the exponential factors E are easily summed up: $E(x,y)\,E(y,z) = E(x,z)\,[\,1 + O(G)\,]$ (the term $O(G)$ is due to the change of the integration contour from the broken line xyz to the straight line xz). As a result, we obtain the gauge invariant bilocal operator

$$\mathcal{O}_\nu(\xi,\eta;\mu^2) = N_{\mu^2}\,\bar\psi(\xi)\,\gamma_\nu\,(\exp ig\int_\eta^\xi A_\mu(z)\,dz^\mu)\,\psi(\eta) \qquad (7)$$

for a subgraph with quark external lines, whereas the coefficient function $C_{v_0}(x,\xi,\eta)$ remains unchanged. Using the Baker-Hausdorf theorem /3,4/ one can expand $\mathcal{O}(\xi,\eta)$ over gauge-invariant local operators $O_{\mu_1 \dots \mu_2} = \bar\psi\gamma_{\mu_1} D_{\mu_2} \dots D_{\mu_n}\psi$:

$$\mathcal{O}_\nu(\xi,\eta;\mu^2) = \sum_{m=0}^{\infty} \frac{1}{m!}(\xi-\eta)^{\nu_1} \dots (\xi-\eta)^{\nu_m}\, O_{\nu\nu_1\dots\nu_m}\left(\frac{\xi+\eta}{2};\mu^2\right). \qquad (8)$$

For a subgraph having gluonic external lines, the exponentials are summed into 1, and only $O(G)$ terms do remain. The corresponding contribution is $C^{\mu\nu}_{(g)}\,G_{\mu\lambda}(\xi)\,G^{\lambda}_{\nu}(\eta)$. Local operators in this case have twist equal to or greater than two. This gives a more refined estimate $W_{(g)} \le \text{const}$ valid for a gauge-invariant sum of subgraphs in place of a rough estimate $W_{(g)} \le G^2$ valid for a separate subgraph.

In a non-Abelian theory the gluon propagator is also modified (*fig. 2b*)

$$g_{\mu\nu}\delta_{ab} D^c(x_\alpha - x_\beta) \to \mathfrak{D}^c_{ab,\mu\nu}(x_\alpha, x_\beta) = \qquad (9)$$
$$= \tilde E_{ab}(x_\alpha, x_\beta)\,\{g_{\mu\nu} D^c(x_\alpha - x_\beta) + O(G)\},$$

where $\tilde E_{ab}$ is defined by eq. (5), but one should take there $\tilde A_\mu = A^a_\mu \sigma_a$ rather than $\hat A_\mu$; σ_a is a matrix of the gauge group in the gluonic (adjoint) representation. The ghost field propagator is also modified

$$\delta_{ab} D^c(x_\alpha - x_\beta) \to \tilde E_{ab}(x_\alpha, x_\beta)\,\{D^c(x_\alpha - x_\beta) + O(G)\}. \qquad (10)$$

The factor $\tilde E_{ab}(x,y)$ has the property $\tilde E_{ab}(x,y) = \tilde E_{ba}(y,x)$ which follows from $(\sigma_a)_{bc} = -(\sigma_a)_{cb}$.

To unite the exponentials corresponding to neighbouring spinor lines, one must commute the exponential with the τ-matrix:

$$\hat E_{AB}(x,y)\,(\tau_a)_{BC} = (\tau_b)_{AB}\,\hat E_{BC}(x,y)\,\tilde E_{ba}(x,y). \qquad (11)$$

We have used here the well-known formula $e^A B e^{-A} = B + [A,B] + \frac{1}{2!}[A,[A,B]] + \dots$ and the relation $[\tau_b, \tau_a] = -(\sigma_b)_{ac}\tau_c$.

The commutation results in an additional exponential $\tilde E$ in the gluonic representation appeared in the r.h.s. of eq. (11). The same factor appears after commuting the factor entering into the modified gluon propagator $\mathfrak{D}^c$ with the σ-matrix in the 3-gluon vertex:

$$\tilde E_{ab}(x,y)\,(\sigma_c)_{bd} = (\sigma_f)_{ab}\,\tilde E_{bd}(x,y)\,\tilde E_{fc}(x,y). \qquad (12)$$

One can represent a 4-gluon vertex as a sum of terms containing only Kronecker deltas δ_{ab} and hence there is no need to commute anything. Taking into account all these remarks one can easily see that for a subgraph with quark external lines all exponentials resulting from the commutation (11), (12) are cancelled by those entering into the modified propagators (9), (10) and only the factor $\hat E(\xi,\eta)$ remains. Analogously, there appears the gauge-invariant bilocal operator

$$N_{\mu^2}\, G_{\mu\lambda}(\xi)\,(T_c \exp ig\int_\eta^\xi \tilde A_\mu(z)\,dz^\mu)\,G^{\lambda}_{\nu}(\eta) \equiv \mathcal{O}^{(g)}_{\mu\nu}(\xi,\eta;\mu^2) \qquad (13)$$

for the subgraph with the gluonic external lines.

It is well known that the presence of the T_c-ordering is essential for eq. (8) to be valid for non-commuting fields $\hat A$, $\tilde A$.

Hence we have seen that the well-known statement (see, e.g., ref. /5/), that it is necessary to use the gauge-invariant local operators in the operator product expansion, can be justified in a direct way.

To obtain the modified parton description [6] one has to introduce quark ($f_a(x)$) and antiquark ($f_{\bar{a}}(x)$) distribution functions

$$\frac{i^{n-1}}{2} \langle P|\bar{\psi}_a \{\gamma_\mu \hat{D}_{\mu_1} \ldots \hat{D}_{\mu_n}\}\psi_a|P\rangle =$$

$$= \{P_{\mu_1} \ldots P_{\mu_n}\} \int_0^1 \frac{dx}{x} [f_a(x,\mu^2) + (-1)^n f_{\bar{a}}(x,\mu^2)] \qquad (14)$$

as well as gluon ones

$$\frac{i^n}{2} \langle P|\mathrm{Tr}\{G_{\alpha\mu_1} \tilde{D}_{\mu_2} \ldots \tilde{D}_{\mu_{n-1}} G^{\alpha}_{\mu_n}\}|P\rangle =$$

$$= \{P_{\mu_1} .. P_{\mu_n}\} \frac{1+(-1)^n}{2} \int_0^1 \frac{dx}{x} f_g(x,\mu^2) \qquad (15)$$

(cf. eqs. (1.27), (1.32), (1.33)). Spin average is assumed in eqs. (14), (15).

2. *MASSIVE LEPTON-PAIR PRODUCTION*

To justify the applicability of the modified parton model for the process $AB \to \mu^+\mu^- X$ it is sufficient to demonstrate that for this process also the only difference between the gauge theories and the nongauge ones is the type of the corresponding local operators.

Let us remind shortly the scheme of the analysis used in refs. [1,2]. We use first the α-representation analysis to establish that the asymptotical behaviour of the function investigated is controlled by the end-point singularities (i.e., by the small-α integration). For the $AB \to \mu^+\mu^- X$ process these functions are $d\sigma/dQ^2$ at $Q^2 \gg M^2$ and $d\sigma/d^4Q$ at $Q_\perp^2 \gg M^2$ (where M is the parameter which characterizes the higher twists contribution). For the leading asymptotical behaviour the s,Q^2-subgraphs are responsible having a minimal possible number of external lines. Hence the factorization properties of the amplitude with respect to 2-particle divisions in the t-channel play a highly important role for an analysis in any representation. These properties are almost trivial in the coordinate representation.

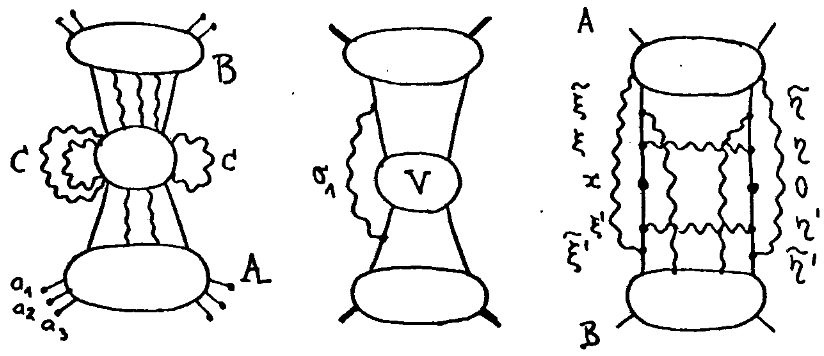
Then we construct a subtraction procedure which provides an infrared regularization ($\lambda_V < 1/\mu^2$) of the contributions due to s,Q^2-subgraphs V and the necessary ultraviolet cut-off for subgraphs lying outside V (e.g., $\lambda_v > 1/\mu^2$ for subgraphs v which become divergent after contraction of V into point). A subtraction procedure of this type does not spoil the factorization properties. The resultant representation

$$W_n = \sum_{a,b} w_n^{(a,b)}(Q^2/\mu^2, g(\mu)) \tilde{f}_n^{(a)}(\mu^2) \tilde{f}_n^{(b)}(\mu^2) + R_n \qquad (16)$$

allows one to obtain the Q-dependence of w_n if the μ-dependence of $\tilde{f}_n$ is known, because W_n is μ-independent. The validity of the representation (16) in gauge theories (with the same $\tilde{f}_n$'s as used for deep inelastic scattering) means, in particular, that the double-logarithmic contributions $(g^2 \ln^2 Q^2/\mu^2)^k$, which appear in some diagrams of gauge theories, are cancelled after summation over all diagrams of a given order.

For nongauge theories treated in ref. [2] our subtraction procedure exhausts all the possibilities to get a leading singularity in the complex J-plane of the Mellin parameter J (say, at $J=0$). Hence the function $R_n(J)$ is regular at $J=0$ and gives only $O(1/Q^2)$ contribution compared to the leading one.

How one should modify the scheme above to apply it for gauge theories? First, it is necessary to sum over gluons taking part in a parton subprocess (*fig. 3a*). We have seen that for deep inelastic scattering this results only in a natural modification of bilocal operators. Second, we should take into account that the configurations *fig. 3b* (corresponding to the R-function) for individual graphs in gauge theories give $O(1)$ contributions rather than $O(1/Q^2)$ ones. This is due to the fact that in theories involving massless vector particles there appears a new



a) b) c)

Fig. 3

possibility to get the leading pole in J as a result of integration in the region $\alpha_{\sigma_1} \to \infty$ for lines of σ_1-type (*fig. 3b*), which join two parts lying outside the small α region (see Appendix 1). This possibility is not realized if the vector particle has a nonzero mass λ, because there appears a factor $e^{-\lambda\alpha}$. Hence for $\lambda \neq 0$ the diagrams *3b* give the contribution $C(\lambda)\,O(1/Q^2)$. The poles J^{-n} resulting from the region $\alpha \to \infty$ disappear, but they manifest themselves in that $C(\lambda) \to \infty$ as $\lambda \to 0$. The Kinoshita-Lee-Nauenberg (KLN)-theorem [7] asserts that the inclusive cross sections are finite at $\lambda = 0$. If this is true, then the sum of configurations shown in *fig. 3b* gives $O(1/Q^2)$ contribution in theories with $\lambda = 0$ also.

Unfortunately, the general proof of the KLN-theorem for QCD has not been given yet. But a particular configuration depicted in *fig. 3b* was shown [7a] to be finite in the $\lambda \to 0$ limit, if the external lines correspond to colour singlet currents: the singular part of $C(\lambda)$ is given by a sum over colour nonsinglet external lines. In our case, however, we deal with colour singlet clusters of fundamental coloured fields. Note, however, that the only condition needed for transition from the auxiliary Green function $\langle 0|\Phi(a_1)\dots\Phi(a_n)\dots$ (in which the hadron A is

described by the set of fundamental fields $\Phi(a_1)\dots\Phi(a_n)$) to the matrix element $\langle P_A|\dots$ is $\langle 0|\Phi(a_1)\dots\Phi(a_n)|P_A\rangle \neq 0$. Hence we may use the gauge-invariant combination

$$\bar{\psi}(a_1)\, S^c(a_1,a_2)\, \psi(a_2) =$$

$$= \bar{\psi}(a_1)\, T_c \exp\Big(ig \int_{a_2}^{a_1} \hat{A}_\mu(z)\, dz^\mu\Big) [S^c(a_1 - a_2) + O(G)]\, \psi(a_2) \qquad (17)$$

rather than $\bar{\psi}(a_1)\psi(a_2)$ for description of the hadron (which is a meson in this example, the generalization for a three-quark system being straightforward). The change (17) means that the meson is described here by the product of two colour-singlet currents $j(a_1)\,j(a_2)$. Hence the assumption needed to prove the finiteness in the $\lambda \to 0$ limit is fulfilled. Another assumption used in ref. [7a] (that the external momenta p_i are nonexceptional) is necessary to prove the absence of singularities in the limit $m_q \to 0$, $p_i^2 \to 0$. In our case we deal with exceptional momenta, and there really appear terms like $\ln Q^2/p_i^2$ which become infinite as $p_i^2 \to 0$. But this phenomenon is the same we have faced with when treating deep inelastic scattering, it is present, moreover, in nongauge theories also.

Thus, the wee-gluon exchanges which spoil the factorization, give in QCD $O(1/Q^2)$ contribution (as suggested by the pomeron-exchange analysis [8] and parton model ideas [9]) and will be ignored hereafter. We are going now to sum up the contributions of *fig. 3a* configurations to see whether the factorization is of the same type we have obtained in nongauge models (see ref. [2]).

Let us fix the type of an initial subgraph v_0 (which has a minimal possible number of external gluon lines). The initial bilocal operators are $\bar{\psi}(\xi)\psi(\eta)$ and $\bar{\psi}(\xi')\psi(\eta')$. There appears a new possibility: the gluons may be inserted into an external quark line (*fig. 3c*):

$$\psi(\eta) \to \Psi(\eta) = \psi(\eta) + g\int d^4z\, S^c(\eta - z)\, \gamma^\mu \hat{A}_\mu(z)\, \psi(z) + \dots \,. \qquad (18)$$

The function $\Psi(\eta)$ is the field operator of a spinor particle in an external gluonic field, i.e., the solution to the equation $(i\hat{D}_\mu \gamma^\mu - m)\Psi = 0$. Thus the Ψ-operator can be written in the form similar to that of eq. (5):

$$\Psi(\eta) = \hat{E}(0,\eta)\{\psi(\eta) + O(G,\psi)\} \tag{19a}$$

with the boundary condition $\Psi(0) = \psi(0)$ assumed. One may however use another boundary condition. The function

$$\Psi(\eta, z_0) \equiv \hat{T}_c(\hat{E}(z_0, 0)\Psi(\eta)), \tag{19b}$$

where z_0 is a point which can be chosen arbitrarily long away from the origin, is also the solution of the Dirac equation. Physical quantities (e.g., cross sections) should not depend on z_0. It is worthwhile to study the way the cancellation of the z_0-dependence occurs by an explicit use of $\Psi(\eta, z_0)$ rather than $\Psi(\eta)$.

Uniting the factors $E(\xi,\xi')E(\xi',\xi'')$, as we have pointed above, gives additional $O(G(\tilde{\xi}))$ terms, where $\tilde{\xi}$ lies inside the triangle $\xi\xi'\xi''$. Due to definition (19), the triangles having z_0 as a vertex, will never appear, because z_0 can be connected by a straight line with the origin only. In general, one must take different points z_0, z'_0 for Ψ-fields corresponding to partons from A or B, respectively. The gluon field of the hadron A will be denoted as A_μ, whereas that of the B-hadron as B_μ. "Vacuum" gluon field, corresponding to the gluon lines joining only subprocess lines, will be denoted as C_μ. Sometimes we will indicate the type of the field entering into exponential factors, e.g.: $\hat{E}(x, y;\ B)$.

We consider first an Abelian theory. Let us fix the number and type of the A- and B-lines and sum over the C-lines. As a result, we get $\langle 0|1 + O(G)|0\rangle$ (namely, $\langle 0|E(z_0,0)E(0,z'_0)E(z'_0,0)E(0,z_0)[1+O(G)]|0\rangle$ for subgraphs which do not possess gluon divisions in the t-channel, and $\langle 0|E(z_0,z_0)E(z'_0,z'_0)[1+O(G)]|0\rangle$ otherwise). The magnitude of these "vacuum" corrections is proportional to that of the matrix element $\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle$. The authors of ref.[10] investigating the process $e^+e^- \to$ charm have established that the corresponding dimensional parameter M has a characteristic value of an order of 0.3 *GeV*, hence the power corrections $(M^2/Q^2)^k$ are analogous to taking account of transverse momentum of partons. Corrections of the same type appear in analysis of the deep inelastic scattering: summing over the C-lines (*fig. 1c*) gives the matrix element $\langle 0|E(z_0, z_0)[1+O(G)]|0\rangle$.

It makes sense to investigate what is the formal reason for cancellation of the dependence on z_0, z'_0; that is, on large distances. To do this, we will treat the diagram shown in *fig. 3* as a zero-angle scattering amplitude for process $AB \to A'B'$, (it is not assumed here that A, A' and B, B' are identical particles). If the particles A, A' have different charges $(g_A - g_{A'} = g_{B'} - g_B = \Delta g \neq 0)$ then summation over C-fields gives

$$\langle 0|\exp[i\Delta g \int_{z_0}^{0} (Cdz)]\cdot \exp[i\Delta g \int_{0}^{z'_0} (Cdz)](1+O(G))|0\rangle. \tag{20}$$

The dependence on large distances does not disappear as far as $z_0 \neq z'_0$ and $\Delta g \neq 0$.

Now let us sum over the A-lines whereas the B-lines are fixed. As a result, we obtain the matrix element (*fig. 3c*)

$$\langle P_A|\bar{\psi}_a(\xi)\exp[ig_a \int_{0}^{\xi} (Adz)]\exp[ig_a \int_{z_0}^{0} (Adz)] \times$$
$$\times \exp[ig_b \int_{0}^{z_0} (Adz)]\exp[ig_b \int_{\tilde{\eta}}^{0} (Adz)]\psi_b(\tilde{\eta})\{1+O(G)\}|P_A\rangle, \tag{21}$$

where g_a, g_b are parton charges.

For $\Delta g \neq 0$ there appears the gauge-invariant bilocal operator (7) whereas for $\Delta g \neq 0$ there remains the dependence on large distances due to the factor $\exp[i\Delta g \int_{z_0}^{0} (Adz)]$.

Summing then over the B-lines gives (for $\Delta g = 0$) the second gauge-invariant operator $\bar{\psi}(\xi')E(\xi', \eta';\ B)\psi(\eta')$.

Using formula (8) one can expand the bilocal operators over local ones, and the further analysis proceeds just in the same manner as it was done for nongauge theories (see ref. [2]).

The summation over the gluonic A-, B- and C-lines for non-Abelian gauge theories also results in gauge-invariant bilocal operators $\langle P_A | \mathcal{O}(\xi,\eta;\mu^2) | P_A \rangle$ (details are presented in Appendix 2). It is very essential there that the contribution of the parton subprocess is projected onto the colour singlet states in the t-channel. For hadrons only the projection onto the colour singlet operators gives nonzero matrix elements whereas for coloured objects A, B (e.g., for quarks) the colour octet projection is also nonzero. But in this case the dependence on z_0, z_0' does not disappear, that means that there is no cancellation of double logarithmic terms. To eliminate the large-distance dependence, one should take the colour singlet projection, i.e., to perform something like colour averaging. For quarks inside the hadrons this averaging holds automatically.

3. *PARTON INTERPRETATION*

The results obtained above have a simple interpretation in the parton language. The deep inelastic structure function $W(\omega, Q^2)$ is given in the tree approximation by the diagrams *fig. 1a*. This corresponds to the expansion of the gauge-invariant operator

$$O_{\mu_1 \ldots \mu_n} = i^{n-1} \bar{\psi} \gamma_{\mu_1} (\vec{\partial}_{\mu_2} - igA_{\mu_2}) \ldots (\vec{\partial}_{\mu_n} - igA_{\mu_n}) \psi \qquad (22)$$

(we take here for simplicity the derivative $\vec{\partial}$ rather than $\overleftrightarrow{\partial}$) over gauge-dependent operators

$$O_{\mu_1 \ldots \mu_n} = \bar{\psi} \gamma \nabla^{n-1} \psi + g \sum_{m=0}^{n-2} (\bar{\psi} \gamma \nabla^{n-m-2} \psi)(\nabla^m A) C^{m+1}_{n-1} + \ldots , \qquad (23)$$

where $\nabla_\mu \equiv i\vec{\partial}_\mu$. It is possible to introduce the gauge-dependent parton distribution functions related to these

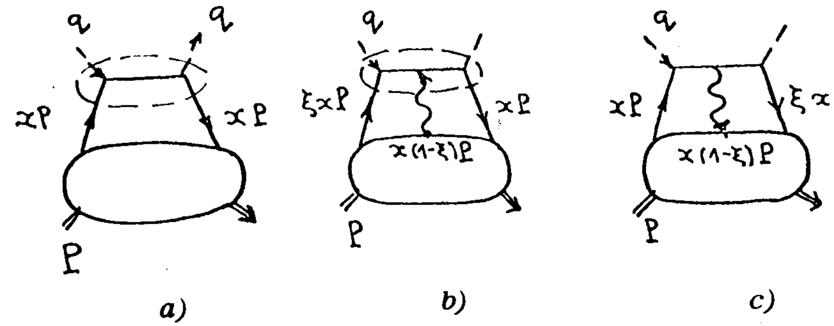


Fig. 4

operators. The function $f_0(x;-x)$ (*fig. 4a*) describes a quark (or an antiquark) with momentum xP. The function $f_1(x\xi, x(1-\xi); -x)$ (*fig. 4b*) describes an outgoing quark with momentum $x\xi P$, an outgoing gluon with momentum $x(1-\xi)P$ and an incoming quark with momentum xP. *Figure 4c* describes a similar process with an incoming gluon. At the first sight, there exists the evident relation

$$f_1(x\xi, x(1-\xi); -x) = f_1(x; -x(1-\xi), -x\xi) \qquad (24)$$

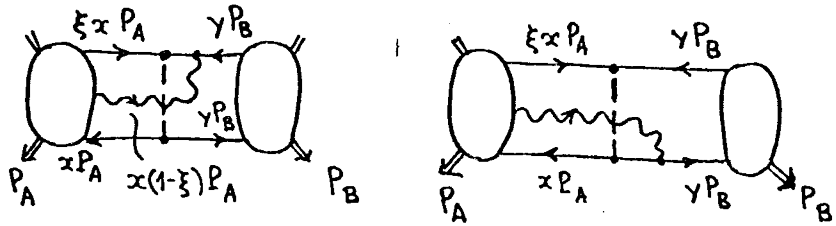
But it holds only as far as the brackets $\langle P|$ and $|P\rangle$ describe identical particles. Suppose that they really have different charges (in an Abelian theory) or different colours (in a non-Abelian one). Then there would be no reason for such a relation to hold, because the amplitude for a quark to be accompanied by a gluon depends both on the quark charge and on that of the spectator quarks.

Eq. (22) means that the cross-sections of different subprocesses are connected in such a way that the functions $f_0, f_1, \ldots f_k, \ldots$ (where f_k corresponds to the presence of k gluons) appear in the final expression only in definite gauge-invariant combinations.

The contribution of the diagrams *4b,c* is

$$\int_0^1 dx \int_0^1 d\xi [f_1(x\xi, x(1-\xi); -x) + f_1(x; -x(1-\xi), -x\xi)] \times$$
$$\times \hat{s}^2 (\xi\hat{s} - Q^2)^{-1} (\hat{s} - Q^2)^{-1} , \qquad (25)$$

where $\hat{s} = x \cdot 2(Pq)$, $Q^2 = -q^2$.



a) b)

Fig. 5

The cross section $d\sigma/dQ^2$ for massive lepton-pair production is described in the same approximation by diagram *5a,b*. In an Abelian theory the contribution of *fig. 5a* is as follows

$$\int_0^1 dx \int_0^1 dy \int_0^1 d\xi f_0(y;-y)\, f_1(\xi x,(1-\xi)x;-x)\, \frac{g_a \hat{s}^2}{(1-\xi)\hat{s}(\hat{s}-Q^2)}, \quad (26)$$

where $\hat{s} = xy\cdot 2(P_A P_B)$ and g_a is the quark charge. *Figure 5b* gives the analogous contribution

$$\int_0^1 dx \int_0^1 dy \int_0^1 d\xi f_0(y;-y)\, f_1(\xi x,(1-\xi)x;-x)\, \frac{(-g_b)\hat{s}^2}{(\xi\hat{s}-Q^2)(1-\xi)\hat{s}}, \quad (27)$$

where g_b is the charge of another quark. If $g_A = g_{A'}$, then also $g_a = g_b$, and as a result, the total contribution is proportional to $g_a \hat{s}^2/(\xi\hat{s}-Q^2)(s-Q^2)$. The factor $(1-\xi)^{-1}$ has disappeared, and we have obtained just the same structure as in deep inelastic scattering.

The expansion $(1-\xi)^{-1} = \Sigma\xi^n$ is analogous to eq. (A.7) whereas the expansion

$$\frac{\hat{s}^2}{(\hat{s}-Q^2)(\xi\hat{s}-Q^2)} = \sum_{n=2}^{\infty} (x\omega)^n \sum_{m=0}^{n-2} \xi^m =$$

$$= \sum_{n=2}^{\infty} (x\omega)^n \sum_{m=0}^{n-2} C_{n-1}^{m+1} \xi^{n-m-2} \cdot (1-\xi)^m \quad (28)$$

corresponds to the second term in eq. (23).

If the function $f(\xi x,(1-\xi)x;-x)$ does not vanish at $\xi = 1$, then one obtains a logarithmic divergence in eqs. (26), (27) due to the factor $(1-\xi)^{-1}$, which is, consequently, responsible for the presence of the double-logarithmic terms. But when all contributions are summed, the operators $(\bar{\psi}\gamma\nabla^{n-m-2}\psi)(\nabla^m A)$ make up the necessary part of the gauge-invariant operator $O_{\mu_1 \ldots \mu_n}$. The cancellation has taken place between the contributions of two different diagrams - *5a* and *5b*. When one treats the gluon shown in *fig. 5a* as an incoming one, then the corresponding contribution is

$$\int_0^1 dx \int_0^1 dy \int_0^1 d\xi f_0(y_1-y)\, f_1(x;-(1-\xi)x,-\xi x)\, \frac{(-g_a)\hat{s}^2}{(1-\xi)\hat{s}(\xi\hat{s}-Q^2)}. \quad (29)$$

If the relation (24) is satisfied, then assuming (26) and (29) we obtain the factor $g_a\hat{s}^2/(\xi\hat{s}-Q^2)(\hat{s}-Q^2)$ again. This illustrates once more how important is the neutrality of the t-channel for a cancellation of the double logarithmic terms. This note plays an important role for the investigation of exclusive processes (e.g., elastic form factors) involving colourless objects (i.e., hadrons) in QCD [11].

It is clear from the light-cone expansion

$$W \sim \int \sum_n x^{\mu_1} \ldots x^{\mu_n} \langle P|\bar{\psi}\gamma_{\mu_1}(\nabla_{\mu_2} + gA_{\mu_2})\ldots(\nabla_{\mu_n} + gA_{\mu_n}) \times$$

$$\times \psi|P\rangle e^{iqx} E_n(x^2)\, d^4x \quad (30)$$

that in the axial gauge defined by $x_\mu A^\mu = 0$ only the first term contributes into W. As a consequence, in the leading logarithm approximation only the generalized ladder graphs (which include vertex- and self-energy divergent parts) are important in this gauge. That is why this gauge is especially suited for a parton interpretation. This was demonstrated first in an Abelian model by Lipatov [12]. His analysis possesses the main features of the parton language developed by Altarelli and Parisi [13]. The latter is equivalent to the use of such an axial gauge in QCD (see also ref. [14]). The gauge $(xA) = 0$ is essentially identical with any gauge defined by $(qA) + \alpha(PA) = 0$ (where

α is an arbitrary number). Using the gauge $(AP_A)+\alpha(AP_B)=0$ one may consider only the generalized ladders for the investigation of massive lepton-pair production in the leading logarithm approximation [15]. Vector theories in such gauges are closer to nongauge models.

4. *PAIR PRODUCTION AT LARGE TRANSVERSE MOMENTUM*

The summation over gluons participating in the parton subprocess provides a manifest gauge invariance of the corresponding operator $\mathcal{O}(\xi,\eta)$, whereas the coefficient function $C(x,\xi,\eta)$ remains unchanged. Hence, for straightforward calculations it is sufficient to consider only subprocesses with a minimal possible number of participants. In such an approach we use, on the one hand, the axial gauge, but on the other hand we utilize the gauge invariance of the subprocess cross section. The functions $w(1,g(Q_\perp),\tau/xy,\tau_\perp/x)$ and $w(1,g(Q),\tau/xy)$ are series expansions over α_s, whereas all the logarithmic dependence (scaling violation) is accumulated in the functions $f(x,Q_\perp^2)$ or $f(x,Q^2)$.

For instance, the diagrams *fig. 6* give the following contribution into the differential cross section (see Appendix 3):

$$\frac{d\sigma}{dQ^2 d\tau_\perp} = \frac{4\pi\alpha^2}{3Q^4}\,\frac{\tau}{N_c}\,\frac{\alpha_s(Q_\perp)}{2\pi}\int_0^1\frac{dx}{x}\int_0^1\frac{dy}{y}\,\frac{\theta(\sqrt{xy}-\sqrt{\tau_\perp}-\sqrt{\tau+\tau_\perp})}{\sqrt{(xy-\tau)^2-4xy\tau_\perp}}\times$$

$$\times\sum_a e_a^2\Big\{2C_2(R)\Big[\frac{\tau^2+x^2y^2}{xy\tau_\perp}-2\Big] f_{a/A}(x,Q_\perp^2)\, f_{\bar a/B}(y,Q_\perp^2) +$$

$$+T^c(R)\Big[1+\frac{3\tau}{xy}+\frac{xy-\tau}{\tau_\perp}\Big(\Big(\frac{\tau}{xy}\Big)^2+\Big(1-\frac{\tau}{xy}\Big)^2\Big)\Big] f_{g/B}(y,Q^2)\times$$

$$\times\big(f_{a/A}(x,Q_\perp^2)+f_{\bar a/A}(x,Q_\perp^2)\big)+[A\leftrightarrow B]\Big\}\{1+O(\alpha_s(Q_\perp))\}, \quad (31)$$

where $C_2(R)=4/3$, $T^c(R)=1/2$, $N_c=3$.

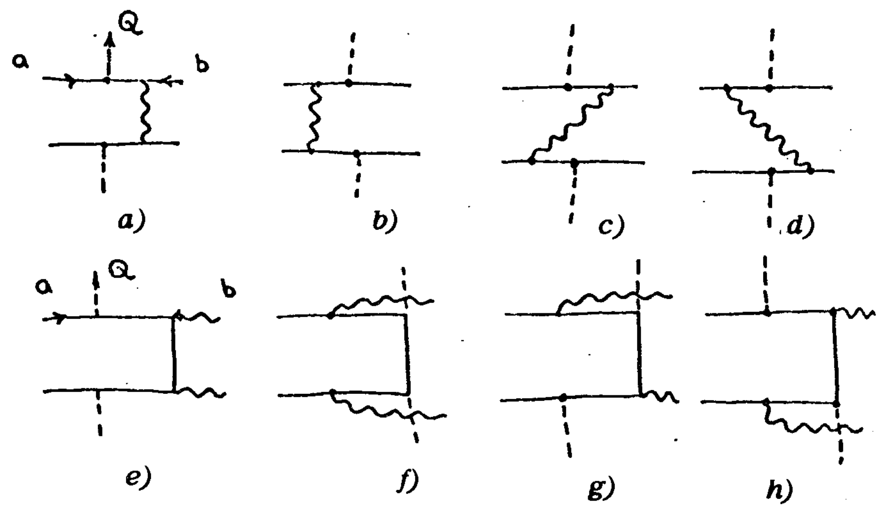


Fig. 6

It is easy to note that the terms containing $1/\tau_\perp$ (i.e., those giving the logarithmic contribution $\sim \ln\mu_0^2$ into the $d\sigma/dQ^2$ after integration over $\tau_\perp$ in the region $\tau_\perp\sim\frac{\mu_0^2}{s}$) with the help of the Altarelli-Parisi equation [13] can be represented as follows

$$\frac{1}{\tau_\perp}\int_{1/\tau}^{1}\frac{dx}{x}\,\frac{d}{d\ln Q_\perp^2}\Big[f_{a/A}(x,Q_\perp^2)\,f_{\bar a/B}\Big(\frac{\tau}{x},Q_\perp^2\Big)+(A\leftrightarrow B)\Big]. \quad (32)$$

From eq. (32) the preference of the choice $\mu=Q_\perp$ in eq. (31) is clear.

However it is impossible to calculate correctly the $O(\alpha_s)$ contribution into $d\sigma/dQ^2$ from eq. (31) by integration over $\tau_\perp$ because the terms of $p^2/Q_\perp^2$ order which provide the necessary infrared cut-off have been neglected there. One should keep these terms and then integrate over $\tau_\perp$ from zero up to kinematical bound. In the resultant expression the terms proportional to p^2/Q^2 must be omitted. The terms containing $\ln Q^2/p^2$

are absorbed by parton distribution functions (cf. ref./18/) whereas the remaining terms give $O(\alpha_s)$ correction

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{3Q^4}\,\frac{\tau}{N_c}\int_0^1\frac{dx}{x}\int_0^1\frac{dy}{y}\sum_a e_a^2\{f_{a/A}(x,Q^2)\times$$

$$\times f_{\bar a/B}(y,Q^2)\cdot\delta(1-\frac{\tau}{xy}) + [f_{a/A}(x,Q^2)+f_{\bar a/A}(x,Q^2)]\times$$

$$\times f_{g/B}(y,Q^2)\cdot T^c\frac{\alpha_s(Q)}{4\pi}[(1+\frac{3\tau}{xy})(1-\frac{\tau}{xy})-2+$$

$$+2((\frac{\tau}{xy})^2+(1-\frac{\tau}{xy})^2)\ln\frac{xy(xy-\tau)}{\tau^2}]\,\theta(xy-\tau)\}\{1+O(\alpha_s(Q))\}. \quad (33)$$

We have retained only the lowest approximation in $\alpha_s(Q)$ for quark-antiquark and quark-gluon terms. The logarithmic term in eq. (33) tends to transform the distribution function normalized with the help of Q^2 into that normalized with the help of $\tilde Q^2 = Q^2\frac{xy}{\tau}(\frac{xy}{\tau}-1)$. That means that the virtuality of the momentum going through the quark-gluon vertex depends on an average on x, y. One may believe that it is worthwhile to normalize the distribution functions with the help of $\tilde Q$ to get rid of this logarithmic correction. But this procedure cannot be justified from the theoretical viewpoint, because eq. (33) is really a result of inverting the moment relation

$$\int_0^1 W(Q^2,\tau)\tau^{n-1}d\tau = \sum_{a,b}\tilde W_{ab}(\frac{Q^2}{\mu^2}, g(\mu), n)\,\tilde f_a(n,\mu^2)\,\tilde f_b(n,\mu^2)\,. \quad (34)$$

One can choose in eq. (34) the parameter μ equal to any fixed number. But it is clear that μ in eq. (34) cannot depend on the parameters x, y describing the parton subprocess. It is possible, of course, that in some

kinematic region (say $xy \simeq \tau$) the terms like $\alpha_s(Q)\ln(xy-\tau)\,xy/\tau^2$ give large contribution. Then one should either sum up the whole expansion over $\alpha_s\ln(...)$ in this region (which is as a rule very difficult due to calculational complications) or analyse only the region where these factors are small. The formulas with $\mu=\mu(x,y)$ cannot be considered as a rigorous result of QCD. The use of the x, y-dependent parameter μ^2 is also inconvenient from the phenomenological viewpoint, mainly due to a danger to get into trouble with the region of small x, y.

There exists also just a similar problem. Our analysis performed in section 2 shows that all the double logarithmic terms $(g^2\ln^2 Q^2/\mu^2)^p$ cancel each other both for the function $W(Q^2/\mu^2, \tau/xy, g)$ and $W(Q_\perp^2/\mu^2, \tau/xy, \tau_\perp/xy, g)$. The latter function can possess, however, the terms $g^2\ln^2(Q_\perp^2/Q^2) = g^2\ln^2\tau_\perp/\tau$, and the existence of these terms does not contradict the above statement because they do not change with changing μ. But if one takes $\mu = Q_\perp$ from the very beginning, then the terms $g^2\ln^2 Q^2/\mu^2$ and $g^2\ln^2\tau_\perp/\tau$ are mixed together, and this can lead to a confusion. (Say, one can assert that the double logs are not cancelled in this case).

The net result here is the understanding that in the region $\tau_\perp \sim 0$ a very important role in the expansion of the $W(1, \tau/xy, \tau_\perp/xy, g(Q_\perp))$ over $\alpha_s(Q_\perp)$ play the terms $(\alpha_s(Q_\perp)\ln^2\tau_\perp/\tau)^N$ which can be summed up /15/ in an expression similar to the Sudakov exponential. It is evident that the factor like $\exp(-\frac{\alpha_s(Q_\perp)}{2\pi}\ln\frac{\tau_\perp}{\tau})$ suppresses the production of pairs at comparatively small transverse momentum, and hence increases the averaged transverse momentum $\langle Q_\perp^2\rangle$ of a pair. It also leads to the growth of $\langle Q_\perp^2\rangle$ with growing Q^2.

In conclusion we want to express our deep gratitude to D.I.Blokhintsev, V.A.Meshcheryakov and D.V.Shirkov for their interest in this work, and to Y.L.Dokshitzer, D.I.Dyakonov, R.N.Faustov, I.F.Ginzburg, B.L.Ioffe, L.N.Lipatov, S.I.Troyan and V.I.Zakharov for stimulating discussions and helpful remarks.

APPENDIX 1

The treatment of massive lepton-pair production in gauge theories is complicated by double logarithmic terms $(g^2\ln^2 Q^2/\mu^2)^N$ which appear in some types of the diagrams. It was observed via an explicit calculation in the lowest orders of perturbation theory /16–19/, that the double logs cancel when all diagrams of a given order are summed up.

To illustrate the specifics of the double logarithmic situation in the α-representation language, we will consider a one-loop correction for the Dirac part of the quark electromagnetic form factor $f(Q^2)$ (fig. 3a). The contribution we are interested in can be written with the help of the α-representation and Mellin transformation as follows

$$f(Q^2) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \Gamma(1-j)(Q^2)^j\,\phi(j)\,dj\,, \qquad (A.1)$$

$$\phi(j) = -\frac{g^2}{8\pi^2}\int_0^\infty \frac{d\alpha_1\, d\alpha_2 d\alpha_3}{D^2}\left(\frac{\alpha_1\alpha_2}{D}\right)^{j-1}\left(1-\frac{\alpha_1}{D}\right)\left(1-\frac{\alpha_2}{D}\right)\times$$
$$\times \exp\left(p^2\alpha_3(\alpha_1+\alpha_2)/D - \lambda^2\alpha_3\right)\,, \qquad (A.2)$$

where $D=\alpha_1+\alpha_2+\alpha_3$. We consider the behaviour in the Sudakov régime $-p^2=-p'^2 \gg m_g^2$. We treat nevertheless both the massless gluon theory and the theory with a gluon having nonzero mass λ.

The pole $1/j$ results from the integration over three regions 1) $\rho\sim0$, where $\rho=\alpha_1+\alpha_2+\alpha_3$; 2) $\alpha_1\sim0$; 3) $\alpha_2\sim0$. Thus, the maximal singularity is j^{-3}. It corresponds to the well-known contribution $-g^2\ln^2(Q^2/p^2)/16\pi^2$. If the gluons are massless $(\lambda=0)$, there appears the fourth region 4) $\alpha_3\to\infty$ which produces the pole $1/j$ after integration. Combining 2), 3) and 4) we get an extra j^{-3}. This results in

$$f^{(1)}(Q^2)\Big|_{\lambda=0} \cong -\frac{g^2}{8\pi^2}\ln^2\frac{Q^2}{p^2}\,. \qquad (A.3)$$

In massless theory we have the double logarithmic contribution twice as greater as in the massive gluon theory. In the theory where $\lambda\neq0$ there appears the term $\sim\ln\lambda^2$

$$f^{(1)}(Q^2) \cong -\frac{g^2}{16\pi^2}\ln^2\frac{Q^2}{p^2} - \frac{g^2}{16\pi^2}\ln\frac{Q^2}{p^2}\ln\frac{p^2}{\lambda^2} \qquad (A.4)$$

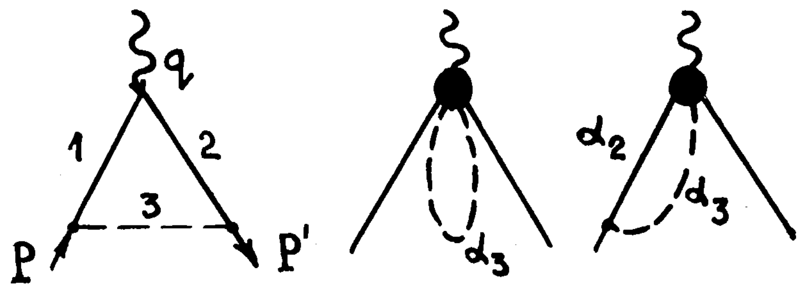
which gives infinity in the limit $\lambda\to0$. This infinity signalizes that for $\lambda=0$ there exists an "infrared" pole j^{-1} which have not been taken into account.

The subtraction procedure for $\lambda\neq0$ is constructed in the following way. We integrate first in the region $0\le\rho\le1/\mu^2$ and obtain the pole $\phi^{\rho}_{pole}=(\mu^2)^{-j}j^{-1}\chi(j)$. The function $\chi(j)$ has the poles j^{-1} as a result of integration over $\beta_1\sim0$, $\beta_2\sim0$ ($\beta_i=\alpha_i/\rho$). This gives a contribution $\sim\ln^2 Q^2/\mu^2$. We subtract $\sim\phi^{\rho}_{pole}$ from the function ϕ and then integrate in ϕ^{ρ}_{reg} over $\rho_1\sim0$, $(\rho_1=\alpha_1+\alpha_2)$.

The result is $\phi^{\rho\ \rho_1}_{reg\ pole}=(\mu^2)^{-j}j^{-1}\chi_1(j)$. The coefficient $\chi_1(j)$ has pole $1/j$ from the region $\beta_2\sim0$ (or $\beta_1\sim0$). The "hanging" part (*fig. 7b*) gives a factor $\sim\ln\mu^2/\lambda^2$ which tends to ∞ as $\lambda\to0$. In the regular contribution $\phi^{\rho\ \rho_1}_{reg\ reg}$ we integrate over $\alpha_1\sim0$ (or $\alpha_2\sim0$, respectively). This gives

$$\phi^{\rho\ \ \rho_1\ \ \alpha_1}_{reg\ reg\ pole} \sim j^{-1}(\mu^2)^{-j}\int\limits_{\substack{\alpha_2>1/\mu^2\\ \alpha_2+\alpha_3>1/\mu^2}}^{\infty} d\alpha_2 d\alpha_3(\alpha_2+\alpha_3)^{-2}\times$$
$$\times\left(\frac{\alpha_2}{\alpha_2+\alpha_3}\right)^{-1}\exp\left(p^2\frac{\alpha_2\alpha_3}{\alpha_2+\alpha_3}-\lambda^2\alpha_3\right). \qquad (A.5)$$

If there were no lower bounds on the region of integration in eq. (A.5), there would appear divergences both for $\alpha_2\sim0$ and for $\alpha_2+\alpha_3\sim0$. The divergence at $\alpha_2+\alpha_3\sim0$ is the ordinary ultraviolet divergence resulting from the contraction of the α_1-line into point (*fig. 7c*). It leads to $\ln\mu$-dependence of the regular part (i.e., the matrix element). *Fig. 7c* looks like the well-known diagram



a) b) c)

Fig. 7

contributing to the anomalous dimension of the operator $\bar{\psi}\gamma_{\mu_1}D_{\mu_2}\ldots D_{\mu_n}\psi$. The expansion

$$\left(\frac{\alpha_2}{\alpha_2+\alpha_3}\right)^{-1} = \sum_{k=0}^{\infty}\left(\frac{\alpha_3}{\alpha_2+\alpha_3}\right)^n \qquad \text{(A.6)}$$

in conjunction with the fact that the factor $(\alpha_3/\alpha_2+\alpha_3)^k$ corresponds to the vertex $A_{\mu_1}\partial_{\mu_2}\ldots\partial_{\mu_k}\psi$ · justifies this analogy. The only difference is that for the operator $\bar{\psi}\gamma D^{n-1}\psi$ the sum over k goes only up to $k = n-2$. The divergence at small α_2 converts into the divergence of the sum $\sum^{\infty} 1/k$. The cut-off at $\alpha \sim 1/\mu^2$ corresponds to the cut-off of the sum at $k \sim \ln\mu^2$. As a result, the regular part (A.5) has $\ln^2\mu$ -dependence on μ. Furthermore if one takes $\lambda = 0$, the infrared pole $1/j$ which has not been taken into account reveals itself: the expression (A.5) is divergent at $\lambda = 0$.

In the remaining regular contribution $\phi^{\rho}_{reg}\,{}^{\rho_1}_{reg}\,{}^{\alpha_1}_{reg}$ it is impossible to get pole j^{-1} if one uses a theory with $\lambda \neq 0$, and the corresponding contribution is $O(1/Q^2)\cdot C(\lambda)$. But the factor $C(\lambda)$ approaches infinity as $\lambda \to 0$, which corresponds to the missed infrared pole. In a theory with $\lambda = 0$ one should single out the infrared pole $1/j$ resulting from the region $\alpha_3 \to \infty$ to construct the subtraction procedure. Then all the results obtained will be finite because an infrared regularization is provided by $|p^2| \gg m_q^2$, but the subtraction procedure has a very

complicated form. When investigating the process $AB \to \mu^+\mu^- X$ we use a simplified subtraction procedure (i.e., introduce $\lambda \neq 0$). According to arguments given in Sec. 2, all the singularities in λ (or the infrared poles in j in the theory with $\lambda = 0$) should cancel and the sum of all the contributions like $\phi^{\rho}_{reg}\,{}^{\rho_1}_{reg}\,{}^{\alpha_1}_{reg}$ gives $O(1/Q^2)$ contribution for inclusive cross sections, just like it was in nongauge theories, provided the hadrons are colourless.

APPENDIX 2

In this appendix we will demonstrate how the sum over the gluon A-, B-, C-lines in the diagrams *3a* in a non-Abelian theory leads only to the appearance of the gauge-invariant operators.

First we sum up over the C-lines. In a non-Abelian theory the operator corresponding to a gluon external line is also modified

$$A_\mu(\xi_i) \to \mathfrak{A}_\mu(\xi_i, C) = T_c\tilde{E}(z_0, 0; C)\,\tilde{E}(0, \xi_i; C)A_\mu(\xi_i). \qquad \text{(A.7)}$$

For a B-field one should change $z_0 \to z_0'$, $A \to B$ in eq. (A.7). We commute all the exponentials towards the hadron A, say. This gives the expression (*fig. 3c*)

$$[\bar{\psi}(\tilde{\xi})\hat{E}(z_0', z_0; C)\hat{B}_\mu(\tilde{\xi})\gamma^\mu \ldots (\tau_a A^b_\nu(\tilde{\xi}')\gamma^\nu\tilde{E}_{ba}(z_0', z_0; C))\times$$
$$\times\psi(\xi')][\bar{\psi}(\tilde{\eta}')(\tau_c A^d_\lambda(\tilde{\eta}')\gamma^\lambda\cdot\tilde{E}_{dc}(z_0, z_0'; C))\ldots$$
$$\ldots\hat{B}_\rho(\tilde{\eta})\gamma^\rho\hat{E}(z_0', z_0; C)\,\psi(\tilde{\eta})] \qquad \text{(A.8)}$$

in which all the fields A_μ have a factor $\tilde{E}(z_0', z_0; C)$. Summing then over the gluon A-fields with account of the change $B \to \mathfrak{B}(\xi, \mathfrak{A})$ and commuting the exponentials towards the hadron A, we obtain (*fig. 3c*)

$$\bar{\psi}(\tilde{\xi})\hat{E}(z_0', z_0; C)\hat{E}(\tilde{\xi}, z_0; \tilde{E}(z_0', z_0; C)A)\gamma^\mu B_\mu(\tilde{\xi})\ldots\psi(\xi'),$$
$$\bar{\psi}(\eta')\ldots B_\rho(\tilde{\eta})\gamma^\rho\hat{E}(z_0, \tilde{\eta}; \tilde{E}(z_0, z_0'; C)A)\,\hat{E}(z_0, z_0'; C)\psi(\eta). \qquad \text{(A.9)}$$

Using the relation $\hat{E}(z_0',z_0;C)\hat{E}(a,b;\hat{E}(z_0',z_0;C)A)=\hat{E}(a,b;A)\hat{E}(z_0',z_0;C)$. and summing over B-fields we obtain the final expression

$$[C_{v_0}(x,\xi,\eta;\xi',\eta')]^{CD}_{AB}\{\bar{\psi}(\xi)\hat{E}(\xi,z_0;A)\hat{E}(z_0',z_0;C)\}_A$$
$$\{\hat{E}(z_0',\xi';B)\psi(\xi')\}^B\{\bar{\psi}(\eta')\hat{E}(\eta',z_0';B\}_C$$
$$\{\hat{E}(z_0,z_0';C)\hat{E}(z_0,\eta;A)\,\psi(\eta)\}^D . \qquad (A.10)$$

Using the analog of the Fierz identity for τ-matrices of the $SU(3)_c$-group

$$\delta^A_{A'}\delta^{C'}_C = \frac{1}{3}\delta^{C'}_{A'}\delta^A_C + 2\sum_a(\tau^a)^A_C(\tau_a)^{C'}_{A'} \qquad (A.11)$$

we see that the dependence on z_0, z_0' disappears for singlet projections, and there appear two gauge invariant operators $<P_A|\mathcal{O}(\xi,\eta;A)|P_A><P_B|\mathcal{O}(\xi',\eta';B)|P_B>$.The generalization for subgraphs having gluonic external lines, is straightforward.

APPENDIX 3

Here we calculate the cross section of producing the massive lepton-pair having large transverse momentum, in the lowest approximation for a parton subprocess (*fig. 6*). Remember that we use the renormalization group improved perturbation theory, hence the approximation used here is equivalent to the leading logarithm approximation of the ordinary perturbation expansion.

The cross section is related to the form factor $W^{\mu\nu}(A,B,Q)$ by the formula

$$d\sigma = \frac{4\pi\alpha^2}{3Q^4}\tau\frac{d^4Q}{(2\pi)^4}(-W^{\mu}_{\mu}(A,B,Q)) . \qquad (A.12)$$

The contributions of the diagrams 6 a)-h) in the Feynman gauge are $g^2/N_c\cdot\delta(\hat{s}+\hat{t}+\hat{u}-Q^2)$ multiplied by the following factors (see also ref. /20/):

$$a)\ 2C_2(R)\frac{\hat{s}Q_\perp^2}{\hat{t}^2}\qquad b)\ 2C_2(R)\frac{\hat{s}Q_\perp^2}{\hat{u}^2}\qquad c)+d)\ 4\frac{Q^2}{Q_\perp^2}C_2(R)$$

$$e)\ 2T^c(R)\frac{\hat{s}}{-\hat{t}}\qquad f)\ 2T^c(R)\frac{-\hat{t}}{\hat{s}}\qquad g)+h)\ 4T^c(R)\frac{Q^2\hat{u}}{-\hat{s}\hat{t}},$$

(A.13)

where $N_c=3$, $C_2(R)=4/3$, $T^c(R)=1/2$, $\hat{u}=(Q-b)^2$, $\hat{t}=(Q-a)^2$, $\hat{s}=(a+b)^2\simeq xys$; $a=xA$, , $b=yB$. The formulas $\hat{u}\hat{t}/\hat{s}=Q_\perp^2$, , $\hat{s}+\hat{t}+\hat{u}=Q^2$ are very helpful to simplify the calculation. As a result we have

$$\frac{d\sigma}{d^4Q} = \frac{4\pi\alpha^2}{3Q^4}\tau\frac{1}{(2\pi)^3}\frac{\bar{g}^2(Q_\perp)}{N_c}\int_0^1\frac{dx}{x}\int_0^1\frac{dy}{y}\delta(\hat{s}+\hat{t}+\hat{u}-Q^2)\,\times$$

$$\times\sum_a e_a^2\{2C_2(R)[\frac{\hat{s}^2+Q^4}{\hat{s}Q_\perp^2}-2]\rho_1^a(x,y,Q_\perp^2) + \qquad (A.14)$$

$$+2T^c(R)[\frac{\hat{u}^2+Q^4}{-\hat{s}\hat{t}}+2]\rho_2^a(x,y,Q_\perp^2)\}\{1+O(\alpha_s(Q_\perp))\},$$

where

$$\rho_1^a(x,y,\mu^2)=\{f_{a/A}(x,\mu^2)f_{\bar{a}/B}(y,\mu^2)+(A\leftrightarrow B)\};$$

$$\rho_2^a(x,y,\mu^2)=\{f_{a/A}(x,\mu^2)+f_{\bar{a}/A}(x,\mu^2)\}f_{g/B}(y,\mu^2)+(A\leftrightarrow B)\}.$$

To get the cross section $d\sigma/dQ^2 d\tau_\perp$ we represent $Q = \lambda A + \rho B + Q_\perp$. Then

$$d^4 Q\, \delta(\hat{u}+\hat{t}+\hat{s}-Q^2) = \frac{1}{s}\delta((x-\lambda)(y-\rho)-\tau_\perp)\, d^4 Q =$$

$$= \frac{dQ^2\, d^2 Q_\perp}{2s} d\rho \frac{\delta(\rho-\rho_-)+\delta(\rho-\rho_+)}{\sqrt{(xy-\tau)^2 - 4xy\tau_\perp}} = \qquad \text{(A.15)}$$

$$= \frac{\pi}{2} dQ^2 d\tau_\perp \frac{1+\{t \leftrightarrow u\}}{\sqrt{(xy-\tau)^2 - 4xy\tau_\perp}}.$$

We have used here that after applying $[1+\{t \leftrightarrow u\}]$ to square brackets in eq. (A.14) one obtains the function which depends only on $\hat{s}$ and Q^2. Substituting eq. (A.15) into eq. (A.14) we obtain eq. (31).

REFERENCES

1. *Efremov A.V., Radyushkin A.V. JINR, E2-11725, Dubna, 1978.*
2. *Efremov A.V., Radyushkin A.V. JINR, E2-11726, Dubna, 1978.*
3. *Gross D.J., Treiman S.B. Phys.Rev., 1971, D4, p.1059.*
4. *Kumar J. Journ. Math.Phys., 1965, 6, pp.1923, 1928,*
5. *Politzer H.D. Phys. Rep., 1974, 14C, p.129.*
6. *Efremov A.V., Radyushkin A.V. Lett. Nuovo Cim., 1977, 19, p.83.*
7. *Kinoshita T. Journ. Math.Phys., 1962, 3, p.650. Lee T.D., Nauenberg M.Phys.Rev., 1964, 133, p.B1549.*

7a. *Sterman G.Phys.Lett., 1978, 73B, p.440.*

8. *Cardy J.L., Winbow G.B. Phys.Lett., 1974, 52B, p.95. De Tar C.E., Ellis S.D., Landshoff P.V. Nucl.Phys., 1975, B87, p.176.*
9. *Drell S.D., Yan T.-M. Ann.Phys. (N.Y.), 1971, 66, p.578.*
10. *Vainshtein A.I., Zakharov V.I., Shirman M.A. Pisma JETF, 1978, 27, p.60.*

11. *Radyushkin A.V. JINR, P2-10717, Dubna, 1977. Efremov A.V., Radyushkin A.V. JINR, E2-11535, Dubna, 1978.*
12. *Lipatov L.N. Yad.Fiz., 1974, 20, p.181.*
13. *Altarelli G., Parisi G. Nucl.Phys., 1977, B126,p.298.*
14. *Dokshitzer Y.L. JETF, 1977, 73, p.1216.*
15. *Dokshitzer Y.L., Dyakonov D.I., Troyan S.I. XIII LINP Winter School, Leningrad, 1978.*
16. *Haliday I.G. Nucl.Phys., 1976, B103, p.343.*
17. *Radyushkin A.V. Phys.Lett., 1977, 69B, p.245.*
18. *Politzer H.D. Nucl.Phys., 1977, B129, p.301.*
19. *Sachrajda C.T. Phys.Lett., 1978, 73B, p.185.*
20. *Kajantee K., Raitio R. Helsinki Univ., HU-TFT-77-21, Helsinki, 1977.*

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