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S.M.Bilenky, G.B.Motz

# A DIRECT TEST OF THE WEINBERG-SALAM MODEL BASED ON THE INVESTIGATION OF P-ODD ASYMMETRIES IN THE PROCESSES $\ell^{\mp} + N \to \ell^{\mp} + X$

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# A DIRECT TEST OF THE WEINBERG-SALAM MODEL BASED ON THE INVESTIGATION OF P-ODD ASYMMETRIES IN THE PROCESSES $\ell^{\mp}+N \to \ell^{\mp}+X$

Биленький С.М., Мотц Г.Б. **E2- 11829**

Р-нечетные асимметрии в процессах $\ell^{\mp}+N\to\ell^{\mp}+X$ и проверка модели Вайнберга-Салама

На основе теории Вайнберга-Салама рассмотрен процесс глубоконеупругого рассеяния продольно поляризованных лептонов (антилептонов) на неполяризованных нуклонах. Для случая изоскалярной мишени получено соотношение между Р-нечетными асимметриями и сечениями процессов $\nu_\mu(\bar\nu_\mu)+N\to\mu^{\mp}+X$. Показано, что измерение как $A_-$, так и $A_+$ позволило бы проверить теорию Вайнберга-Салама без предположений о динамике.

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Bilenky S.M., Motz G.B. **E2 - 11829**

A Direct Test of the Weinberg-Salam Model Based on the Investigation of P-Odd Asymmetries in the Processes $\ell^{\mp}+N\to\ell^{\mp}+X$.

In the framework of the Weinberg-Salam model the P-odd asymmetries $A_{\mp}$ of the processes $\ell^{\mp}+N\to\ell^{\mp}+X$ with polarized leptons are shown to be related to the inclusive cross sections $\nu_\mu(\bar\nu_\mu)+N\to\mu^-(\mu^+)+X$ on an isoscalar target. It is shown that measurements of both $A_-$ and $A_+$ would permit one to test the Weinberg-Salam theory without dynamical assumptions.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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*1.* The investigation of the neutral current is very important for elementary particle physics. During the last time there have been obtained [1,2] detailed data on inclusive processes

$$\nu_\mu(\bar\nu_\mu)+N\to\nu_\mu(\bar\nu_\mu)+X. \qquad (1)$$

These data agree with the predictions of the simplest SU(2)xU(1) gauge theory - the standard Weinberg-Salam model [3]. Further, for the single parameter of this theory, $\sin^2\theta$, in refs.[1,2] there have been obtained the following values

$$\sin^2\theta = 0{,}22\pm 0{,}05, \qquad (2)$$

$$\sin^2\theta = 0{,}24\pm 0{,}02,$$

respectively. A recent phenomenological analysis [4-6] of all neutral current data available from neutrino experiments also favours the Weinberg-Salam structure of the hadronic neutral current.

For the theory experiments in which the weak interaction between electrons (muons) and nucleons is searched for play a special role. Such a neutral current interaction leads to parity violation effects in atoms, to P-odd asymmetries in the deep inelastic scattering of longitudinally polarized leptons on nucleons, etc. Relevant experiments have already been carried out at several laboratories. A search for atomic P-odd effects in $^{209}$Bi gave contradictory results. Thus Oxford [7] and Washington [8] experiments clearly disagree with the Weinberg-Salam theory, whereas the Novosibirsk experiment [9] is in good agreement with this theory. Recently, there have been reported the latest results of the SLAC experiment [10] on deep inelastic scattering of polarized electrons on unpolarized nucleons. In this experiment a P-odd asymmetry has been observed. Its value and sign are in agreement with the predictions of the standard Weinberg-Salam model.

In the present note we consider the deep inelastic scattering of longitudinally polarized leptons (antileptons) on unpolarized nucleons in the framework of the Weinberg-Salam theory. Making use only of the transformation properties of the neutral hadronic current we derive first a relation between the P-odd asymmetries $A_{\mp}$ and the parameter $\sin^2\theta$. Second, we obtain a relation connecting $A_-$ with $A_+$ and with other observable quantities. This relation which can be investigated experimentally would provide a direct test of the standard Weinberg-Salam model.

*2.* Let us first of all derive the P-odd asymmetry in the processes

$$\ell^{\mp} + N \to \ell^{\mp} + X \tag{3}$$

with longitudinally polarized leptons. The relevant part of the effective weak interaction Hamiltonian will be written in the form

$$\mathcal{H} = \frac{G}{\sqrt{2}} 2 j^{\ell}_{\alpha} \cdot j^{h}_{\alpha} . \tag{4}$$

Here

$$j^{\ell}_{\alpha} = \sum_{\ell = e,\mu} \bar{\ell} \gamma_{\alpha} (g_V + g_A \gamma_5) \ell \tag{5}$$

is the neutral current of the charged leptons,

$$j^{h}_{\alpha} = \sum_{q=u,d,\dots} \bar{q} \gamma_{\alpha} (v_q + a_q \gamma_5) q , \tag{6}$$



being the hadronic neutral current. In the case of the Weinberg-Salam theory we have

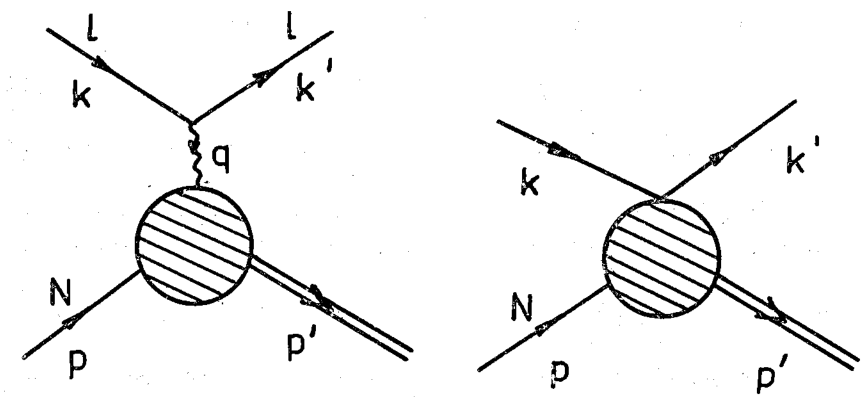
$$g_V = -\frac{1}{2} + 2\sin^2\theta , \quad g_A = -\frac{1}{2} \tag{7}$$

and

$$v_u = \frac{1}{2} - \frac{4}{3}\sin^2\theta , \quad a_u = \frac{1}{2} ,$$

$$v_d = -\frac{1}{2} + \frac{2}{3}\sin^2\theta , \quad a_d = -\frac{1}{2} , \dots \tag{8}$$

In the lowest perturbation order in the electromagnetic and weak interactions the diagrams shown in *Fig. 1* contribute to the process (3).




*1a)* *1b)*

*Diagrams of the process*
$\ell + N \to \ell + X$

The matrix element of the process (3) is given by the expression

$$\langle f\,|\,S\,|\,i\rangle_{\ell^{\mp}} = \pm\, iN_S\, \frac{e^2}{q^2}\,[\bar{u}(k')\gamma_\alpha\, u(k)\langle p'\,|\,J_\alpha^{em}\,|\,p\rangle - \tag{9}$$

$$-\rho\,\bar{u}(k')\gamma_\alpha\,(g_V \pm g_A\gamma_5)u(k)\langle p'|J_\alpha^h\,|p\rangle](2\pi)^4\,\delta(p'-p-q).$$

Here $k(k')$ are the momenta of the initial (final) leptons, $p$ is the momentum of the initial nucleon, $p'$ is the total momentum of the final hadrons, $q = k-k'$, $N_S$ is the product of the standard normalization factors, connected with the lepton lines, and $J_\alpha^{em}$ is the electromagnetic current of the hadrons, and the parameter $\rho$ is

$$\rho = \frac{G}{\sqrt{2}}\;\frac{q^2}{2\pi a} = 1{,}6\cdot 10^{-4}\,\frac{q^2}{M^2} \tag{10}$$

( $M$ is the nucleon mass). The differential cross section of scattering longitudinally polarized leptons (antileptons) off unpolarized nucleons has the form

$$\left(\frac{d\sigma^{\mp}}{dx\,dy}\right)_\lambda = \frac{d\sigma^{em}}{dx\,dy}\,(1+\lambda A_{\mp}), \tag{11}$$

where $\frac{d\sigma^{em}}{dx\,dy}$ is the scattering cross section of the unpolarized particles, $\lambda$ is the longitudinal polarization of the leptons (antileptons) and $A_{\mp}$ is the $P$-odd asymmetry ($x = \frac{q^2}{2M\nu}$ , $y = \frac{\nu}{E}$ being the standard variables). In the approximation considered here, the contribution to the cross section $\frac{d\sigma^{em}}{dx\,dy}$ comes completely from the diagram of *Fig. 1a*). The asymmetry is due to the interference of the two diagrams in *Fig. 1* and reads /11/

$$A_{\mp} = \rho(g_V a_A \pm g_A a_V). \tag{12}$$

The quantities $\rho$ , $g_V$ and $g_A$ are given by formulas (10) and (7), respectively, whereas the quantities $a_A$ and $a_V$ are



$$a_A = \frac{e_{\alpha\beta\rho\sigma}\,k_\rho\,k'_\sigma W^I_{\alpha\beta}}{L_{\alpha\beta}\,(k,k')W^{em}_{\alpha\beta}}, \tag{13}$$

$$a_V = \frac{L_{\alpha\beta}(k,k')W^I_{\alpha\beta}}{L_{\alpha\beta}\,(k,k')W^{em}_{\alpha\beta}}. \tag{14}$$

In these expressions we have

$$L_{\alpha\beta}\,(k,k') = k_\alpha\, k'_\beta - \delta_{\alpha\beta}\,kk' + k'_\alpha\, k_\beta\ , \tag{15}$$

$$W^{em}_{\alpha\beta} = -(2\pi)^6\;\frac{p_0}{M}\;\Sigma\int \langle p'\,|\,J^{em}_\alpha|\,p\rangle\langle p\,|\,J^{em}_\beta\;\;|p'\rangle\,\delta(p'-p-q)d\Gamma \tag{16}$$

and

$$W^I_{\alpha\beta} = -(2\pi)^6\,\frac{p_0}{M}\;\Sigma\int\{\langle p'\,|\,J^{em}_\alpha\;|\,p\rangle\langle p\,|\,J^h_\beta\,|\,p'\rangle +$$

$$+\langle p'|\,J^h_\alpha\;|\,p\rangle\langle p\,|\,J^{em}_\beta\,|\,p'\rangle\}\,\delta(p'-p-q)\,d\Gamma. \tag{17}$$

The quantities $a_A$ and $a_V$ characterize the contributions of the axial-vector and vector parts of the neutral hadronic current, respectively, to the interference of the weak and electromagnetic amplitudes. It is apparent from equation (12) that in the case of the standard Weinberg-Salam theory we have $A_+ \neq A_-$ . Note further, that the order of magnitude of the $P$-odd asymmetries is determined by the parameter $\rho$ and, consequently, $A \sim 10^{-4}\,\frac{q^2}{M^2}$.

*3.* We will now derive the relations which connect $a_V$ and $a_A$ with the parameter $\sin^2\theta$ and observable cross sections. The hadronic neutral current in the standard model has the form

$$j^h_\alpha = \bar{u}_L\gamma_\alpha u_L - \bar{d}_L\gamma_\alpha d_L + \bar{c}_L\gamma_\alpha c_L - \bar{s}_L\gamma_\alpha s_L - 2\sin^2\theta\, j^{em}_\alpha . \quad (18)$$

Here we have used $u_L = \frac{1+\gamma_5}{2}u$, etc. The expression (18) can be written in the following way

$$j^h_\alpha = v^3_\alpha + a^3_\alpha - 2\sin^2\theta\, j^{em}_\alpha + s_\alpha , \quad (19)$$

where

$$v^3_\alpha + a^3_\alpha = \bar{N}\gamma_\alpha(1+\gamma_5)\frac{\tau_3}{2}N \quad (20)$$

is the isovector part of the neutral hadronic current, $(N = \binom{u}{d})$, and $s_\alpha$ is the isoscalar part of the current.

Let us rewrite the expression (19) in the form

$$j^h_\alpha = (1-2\sin^2\theta)j^{em}_\alpha + a^3_\alpha + j^s_\alpha . \quad (21)$$

In eq. (21) we took into account that

$$j^{em}_\alpha = v^3_\alpha + v^s_\alpha , \quad (22)$$

where $v^s_\alpha$ is the isoscalar part of the electromagnetic current and $j^s_\alpha = v^s_\alpha + s_\alpha$.

Let us consider now the deep inelastic lepton scattering off nuclei containing an approximately equal number of protons and neutrons. Then throughout the paper proton and neutron averaged cross sections

$$\frac{d\sigma}{dx\,dy} = \frac{1}{2}\left[\left(\frac{d\sigma}{dx\,dy}\right)_p + \left(\frac{d\sigma}{dx\,dy}\right)_n\right] \quad (23)$$

will be used. Using eq. (21) we obtain the following expression for the tensor $W^I_{\alpha\beta}$ averaged over p and n

$$W^I_{\alpha\beta} = 2(1-2\sin^2\theta)W^{em}_{\alpha\beta} + W^{V;A}_{\alpha\beta} + W^S_{\alpha\beta} . \quad (24)$$

Here the tensor $W^{V;A}_{\alpha\beta}$ is



$$W^{V;A}_{\alpha\beta} = -(2\pi)^6\frac{p_0}{M}\Sigma\int\{<p'|V^3_\alpha|p><p|A^3_\alpha|p'> +$$

$$+<p'|A^3_\alpha|p><p|V^3_\alpha|p'>]\delta(p'-p-q)d\Gamma \quad (25)$$

and $W^S_{\alpha\beta}$ represents the contribution of the isoscalar parts of the electromagnetic and weak neutral currents to $W^I_{\alpha\beta}$. In the high energy region, we are interest in, the contribution from this term is small (this follows from the neutrino data [12]). Omitting $W^S_{\alpha\beta}$ in (13), (14) and (24) we get*

$$a_V = 2(1-2\sin^2\theta), \quad (26)$$

$$a_A = \frac{e_{\alpha\beta\rho\sigma}k_\rho k'_\sigma W^{V;A}_{\alpha\beta}}{L_{\alpha\beta}(k,k')W^{em}_{\alpha\beta}} . \quad (27)$$

The numerator of the expression (27) can easily be related to the inclusive cross sections

$$\nu_\mu + N \to \mu^- + X \quad (28)$$

$$\bar{\nu}_\mu + N \to \mu^+ + X \quad (29)$$

whereas the denominator can be connected with the cross section of deep inelastic scattering of unpolarized leptons on unpolarized nucleons. We have

$$e_{\alpha\beta\rho\sigma}k_\rho k'_\sigma W^{V;A}_{\alpha\beta} = \frac{\pi}{2G^2My}\left[\left(\frac{d\sigma^{cc}}{dx\,dy}\right)_\nu - \left(\frac{d\sigma^{cc}}{dx\,dy}\right)_{\bar{\nu}}\right], \quad (30)$$

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* By using parton model and limiting ourselves by u , d approximation one can easily show that the contribution of the isoscalar term into $a_V$ equals 1/5. Thus, all subsequent relations are valid with accuracy ≈ 20%.

$$L_{\alpha\beta}(k,k')W^{em}_{\alpha\beta} = \frac{q^4}{4\pi\alpha^2 M y} \frac{d\sigma^{em}}{dx\,dy}, \quad (31)$$

where $\left(\frac{d\sigma^{cc}}{dx\,dy}\right)_\nu$ and $\left(\frac{d\sigma^{cc}}{dx\,dy}\right)_{\bar\nu}$, respectively, are the cross sections of the processes (28) and (29). From (27), (30) and (31) the following expression for

$$a_A = \frac{2\pi^2\alpha^2}{G^2 q^4} \frac{\left(\frac{d\sigma^{cc}}{dx\,dy}\right)_\nu - \left(\frac{d\sigma^{cc}}{dx\,dy}\right)_{\bar\nu}}{\frac{d\sigma^{em}}{dx\,dy}} \quad (32)$$

can be obtained.

Finally, from (12) the P-odd asymmetry in the processes (3) is obtained in the form

$$A_- = \rho[-(\tfrac{1}{2}a_A+1)+2\sin^2\theta\,(a_A+1)], \quad (33)$$

$$A_+ = \rho[-(\tfrac{1}{2}a_A-1)+2\sin^2\theta\,(a_A-1)]. \quad (34)$$

Using (33) or (34) with (32) one can determine the parameter $\sin^2\theta$ directly from the experimental data.

*4.* In the framework of the Weinberg-Salam theory, making use of the transformation properties of the neutral hadronic current, one obtained easily (for the case of an isoscalar target) the relations (33) and (34) which connect the P-odd asymmetries in the processes (3) with the cross sections of neutrino processes (28) and (29) and with the cross section of the deep inelastic scattering of unpolarized leptons on unpolarized nucleons. Deriving these formulas we have omitted a contribution proportional to the square of the matrix elements of the isoscalar currents, which is ≲ 20% in the high energy region considered.

If one measures the P-odd asymmetry $A_-$ in the process

$$\ell^- + N \to \ell^- + X$$



with longitudinally polarized leptons, then from (33) we can determine the parameter $\sin^2\theta$ and, consequently, predict the asymmetry $A_+$ in the process

$$\ell^+ + N \to \ell^+ + X.$$

Eliminating the parameter $\sin^2\theta$ from (33) and (34) we obtain:

$$A_+(a_A+1) - A_-(a_A-1) = \rho a_A. \quad (35)$$

Note, that this relation contains only observable quantities. The measurement of both $A_-$ and $A_+$ would permit one consequently, to test the Weinberg-Salam theory. Such a test could appear to be possible in muon experiment prepared at the CERN SPS by a Dubna-CERN collaboration if the required accuracy were attained.

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