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A COVARIANT FORMULATION OF THE RELATIVISTIC HAMILTONIAN THEORY ON THE LIGHT CONE (FIELDS WITH SPIN)

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A COVARIANT FORMULATION OF THE RELATIVISTIC HAMILTONIAN THEORY ON THE LIGHT CONE (FIELDS WITH SPIN)

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Ковариантная формулировка релятивистской гамильтоновой теории на световом конусе. (Поля со спином)

Развитая ранее ковариантная гамильтонова формулировка квантовой теории поля на световом конусе распространяется на случай частиц со спином. Особенности, сопутствующие любой теории поля в переменных светового фронта, устраняются введением в теорию бесконечного числа контрчленов нового типа, которые могут быть включены в гамильтониан взаимодействия. Сформулирована трехмерная диаграммная техника, с помощью которой вычислена собственная энергия фермиона в низшем порядке теории возмущений.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Atakishiyev N.M., Mir-Kasimov R.M., Nagiyev Sh.M. E2 - 11780

A Covariant Formulation of the Relativistic Hamiltonian Theory on the Light Cone (Fields with Spin)

A Hamiltonian formulation of quantum field theory on the light cone, developed earlier, is extended to the case of particles with spin. The singularities accompanying each field theory in light-front variables are removed by the introduction to the theory of infinite number counterterms of the new type, which can be included into interaction Hamiltonian. A three-dimensional diagram technique is formulated, which is applied for the calculation of the fermion self-energy in the lowest order of perturbation theory.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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I. Introduction

In our preceding paper /1/ a version of the covariant Hamiltonian formulation of quantum field theory (QFT) on the light cone was considered for the case of scalar particles. The resulting scheme, which is an alternative to the Hamiltonian field theory, developed in refs. /2/, is based on the Tomonaga-Schwinger equation for the scattering matrix $S(\sigma,-\infty)$ defined on the hyperplanes of the type

$$\mu x \equiv \mu_0 x_0 - \vec{\mu}\vec{x} = \sigma , \qquad (1.1)$$

where μ is an arbitrary light-like 4-vector: $\mu^2 = 0$, $\mu_0 > 0$. This equation has the form

$$i \frac{\delta S(\sigma,-\infty)}{\delta \sigma(x)} = H(x) S(\sigma,-\infty) , \qquad (1.2)$$

where $H(x)$ is the Hamiltonian in the interaction representation and the total scattering matrix $S(\infty,-\infty)$ is given by the relation $S(\infty,-\infty) \equiv S = \lim_{\sigma\to\infty} S(\sigma,-\infty)$. The transition from the description of the time dependence of the events to the description in terms of σ is equivalent to the transition to light-front variables.

To formulate the causality condition in terms of the hyperplanes (1.1), the 4-vector μ is treated as a limit of a sequence of the time-like vectors λ_ε , i.e.,

$$\lim_{\delta \to 0} \lambda_\delta = \mu , \qquad (1.3)$$
$$\lambda_\delta^2 = \delta^2 , \quad \lambda_\delta^0 > 0 .$$

The limiting procedure (1.3) removes at the same time all additional singularities which accompany each scalar field theory in the light-front variables.

The characteristic feature of this approach is the presence in the theory of spurions (quasiparticles), carrying 4-momentum $\mu æ$, where $æ$ is the scalar parameter[2]. To the virtual quasiparticles in the intermediate states there corresponds the propagator $\frac{1}{2\pi}\frac{1}{æ-i\varepsilon}$ and to the physical particles the function $\overset{(+)}{D}(k,m)=\Theta(k^0)\delta(k^2-m^2)$. Thus, physical particles in the intermediate states are on the mass shell and therefore a natural way for extending off the energy shell arises. The projecting properties of the wave functions and other quantities follow directly from the diagrammatic rules[1], which were applied to derive quasipotential type equations and a generalized eikonal representation for the scattering amplitude at high energies[3],*)

In the present paper, which is the continuation of ref.[1], a covariant Hamiltonian formulation of QFT on the light cone is extended to the case of particles with spin and as before our approach is based on the spurion technique[2,5]. Some difficulties, which arise in quantizing field thories on the light front in the case of particles with spin, have been analyzed in refs. [6,7].**) It turns out that in this case the theory is accompanied

*)This diagram technique was also applied for the study of the structure of relativistic wave functions and their connection with the relativistic amplitudes in refs.[4].

**) References to other works on the quantization of fields on the light front can be found in[8], where the questions of relativistic invariance and covariance of the Hamiltonian formulation on the light cone of dynamical systems with a finite number of degrees of freedom are considered in detail.

by more strong singularities localized on the light cone. In order to remove them we introduce infinite number of counterterms of the new type, which are uniquely defined by the requirement of equivalence of this approach to the standard formulation of the S-matrix. It is demonstrated that these counterterms can be included into an interaction Hamiltonian quite in the same way as the quasilocal terms, which can be added to the S-matrix due to the ambiguity of the T-product of field operators, are usually included into the interaction Hamiltonian[9,10].

In quantizing fields on the light front there arises a very important question whether it is possible to retain the relativistic invariance of the theory. The point is that the use on the light cone the "simple" commutation relations of the type

$$\left[\varphi(x), \varphi(0) \right]_{x^+=0} = -\tfrac{i}{4}\varepsilon(x^-)\delta(\tilde{x}) , \quad x^{\pm} = x^0 \pm x^3 , \quad \tilde{x} = (x^1,x^2) \qquad (1.4)$$

results in the unnecessary surface terms in the commutator of the field operator $\varphi(x)$ and the generators of the Poincaré group[11]. However, as it was shown in ref.[12], these formal difficulties can be eliminated by the consistent modification of the commutation relations on the light cone, leading to the relativistically invariant theory on the light front (see also ref.[13]).

As a simple example of utilization of the covariant Hamiltonian formulation of the field theory on the light cone we consider the interaction Hamiltonian

$$H(x) = -g : \bar{\psi}(x)\Gamma\psi(x)\varphi(x): \quad , \qquad \Gamma = 1 \text{ or } i\gamma^5 , \qquad (1.5)$$

where $\psi(x)$ is a spinor field of the mass M and $\varphi(x)$ is a scalar field of the mass m . We formulate the three-dimensional covariant diagram technique, with the help of which the second-order perturbation theory contribution to the fermion self-energy diagram is calculated.

2. Construction of the S-Matrix on the Light Cone

A formal solution of the Tomonaga-Schwinger equation (1.2) in the case of hyperplanes (1.1) tangent to the light cone along the generatrix

$$x' = x + \rho\mu\,, \quad -\infty < \rho < \infty\,, \tag{2.1}$$

has the form

$$S_\mu = T_\mu \exp\left\{-i\int H(x)d^4x\right\} =$$
$$= 1 + \sum_{n=1}^{\infty}(-i)^n \int \Theta(\mu x_1 - \mu x_2)\cdots\Theta(\mu x_{n-1} - \mu x_n) H(x_1)\cdots H(x_n) d^4x_1 \cdots d^4x_n\,, \tag{2.2}$$

where T_μ is the operator of μx-ordering.

While the usual T-product of the field operators is not defined for the coinciding values of their arguments, the corresponding region for the T_μ-product is somewhat wider. Since, when $(x_1-x_2)^2>0$ and in the case, when $(x_1-x_2)^2=0$ but the difference x_1-x_2 is not parallel to the 4-vector μ (i.e. $x_1-x_2\neq \rho\mu$), the equality $\Theta(\mu x_1-\mu x_2)=\Theta(x_1^0-x_2^0)$ is valid, and when $(x_1-x_2)^2<0$ the $\Theta(\mu x_1-\mu x_2)$ function does not contribute to (2.2) due to the locality condition $[H(x_1),H(x_2)]=0$; therefore the T_μ-product of two field operators in the x-representation is not defined on the generatrix of the type (2.1). Thus, the equality $\Theta(\mu x_1-\mu x_2)=\Theta(x_1^0-x_2^0)$ does not hold in the region $\gamma_\mu(x_1-x_2)$ which can be obtained from (2.1) by excluding the top of the cone $(x_1-x_2)^2=0$, i.e.,

$$\gamma_\mu(x) = \left\{ x\in R^4 \mid x=\rho\mu\,,\ \rho\neq 0\right\}. \tag{2.3}$$

Recalling that the ultraviolet divergences in QFT are originated by the ambiguity in the definition of the T-product /9,10/, from (2.2) it becomes clear that besides the usual ultraviolet divergences the S_μ-matrix has also the additional ones localized on the light cone.

On introducing now the notation

$$t_\mu\{H(x_1)\cdots H(x_n)\} = T\{H(x_1)\cdots H(x_n)\} - T_\mu\{H(x_1)\cdots H(x_n)\}\,,\quad n\geq 2 \tag{2.4}$$

one can assert that $t_\mu\{H(x_1)\cdots H(x_n)\}$ are in general singular operator functions, nonvanishing only in the case, when the differences x_i-x_j of any two arguments belong to the region $\gamma_\mu(x_i-x_j)$ *). On the other hand, as is known/9,10/ the addition of quasilocal operators to the S-matrix does not violate its unitarity and causality because of the ambiguity of the T-product. Since the operator functions (2.4) are not quasilocal, their presence in (2.2) contradicts the requirement of the causality of the S-matrix. Therefore we will make use of a regularizing procedure, which generalizes the regularization of the S-matrix for removing the ultraviolet divergences in renormalizable theories by inserting counterterms in the interaction Lagrangian (Bogolubov's R-procedure). In consequence of the ambiguity of the T_μ-product one can add to the S-matrix the operator functions (which we shall call counterterms on the light cone), different from zero only in the case when the differences of any two arguments belong to the region (2.3). As the counterterms on the light cone we choose just the operator functions (2.4) so that to every order of perturbation expansion the equality

$$T_\mu\{H(x_1)\cdots H(x_n)\} + t_\mu\{H(x_1)\cdots H(x_n)\} = T\{H(x_1)\cdots H(x_n)\} \tag{2.5}$$

will take place, which is established by means of the limiting procedure (1.3). Hence it follows at once that the matrix

*) In the particular case of two field operators this statement means that the propagator functions, being defined by the use of the T_μ-product, will in general differ from the conventional propagators by the additional covariant terms, which we supply with the symbol Δ . For instance, the fermion field propagator is written as $S^c(x)=S^c_\mu(x)+\Delta S^c_\mu(x)$.

$$\mathrm{reg}\,S_\mu = 1+\sum_{n=1}^{\infty}\frac{(-i)^n}{n!}\int T_\mu\{H(x_1)\cdots H(x_n)\}\,d^4x_1\cdots d^4x_n +$$
$$+\sum_{n=2}^{\infty}\frac{(-i)^n}{n!}\int t_\mu\{H(x_1)\cdots H(x_n)\}\,d^4x_1\cdots d^4x_n \tag{2.6}$$

regularized by the addition of the counterterms on the light cone $t_\mu\{H(x_1)\cdots H(x_n)\}$, does not depend on the 4-vector μ and coincides with the conventional S-matrix, i.e. $\mathrm{reg}\,S_\mu = S$. Thus, in the approach considered the equivalence of $\mathrm{reg}\,S_\mu$ and S follows just from the definition of the counterterms on the light cone, introduced by us. This statement means that to every order of perturbation expansion two scattering matrices $\mathrm{reg}\,S_\mu$ and S (before carrying out the usual renormalization procedure) lead to the same matrix elements owing to the cancellation of the counterterms on the light cone with the additional terms in the propagator functions corresponding to the T_μ-product of field operators.

Due to (2.5) the functions $t_\mu\{H(x_1)\cdots H(x_n)\}$ whose explicit form depends on the specification of an interaction Hamiltonian and the type of fields are uniquely defined. Let us now discuss the properties of the "t_μ-product" and find a general form of the counterterms on the light cone for the arbitrary interaction Hamiltonians. To this end we first introduce the so-called Δ-operation. If a product $\prod_{k=1}^{n} f_k$ of some functions is given, then by definition

$$\Delta\prod_{k=1}^{n} f_k = \sum_{k=1}^{n} f_1\cdots f_{k-1}\,\Delta f_k\, f_{k+1}\cdots f_n +$$
$$+\sum_{k\neq i=1}^{n} f_1\cdots f_{k-1}\,\Delta f_k\, f_{k+1}\cdots f_{i-1}\,\Delta f_i\, f_{i+1}\cdots f_n + \sum_{k=1}^{n} f_k \prod_{\substack{i=1\\ i\neq k}}^{n}\Delta f_i + \prod_{k=1}^{n}\Delta f_k\,, \tag{2.7}$$

where Δf_k ($k=1,2,\ldots,n$) is a certain new function (which is specified below). Then the t_μ-product of field operators can be defined as their T_μ-product with the subsequent application of the Δ-operation, i.e.,

$$t_\mu\{H(x_1)\cdots H(x_n)\} = \Delta\left[T'_\mu\{H(x_1)\cdots H(x_n)\}\right]. \tag{2.8}$$

Prime over the symbol T_μ means that when calculating T_μ-product by the use of Wick's theorem the term without contractions is to be omitted since owing to the definition (2.5) the t_μ-product of the normal product of operators is equal to zero:

$$t_\mu\{:H(x_1)\cdots H(x_n):\} = 0\,. \tag{2.9}$$

We are now in a position to specify the meaning of the Δ-operation: by acting on the product of the "propagator" functions in (2.8) according to the rule (2.7), it replaces them by the corresponding additional terms (see the footnote on page 5). Thus, the Δ-operation gives an algorithm for calculating the counterterms on the light cone by a given Hamiltonian. We shall find their explicit form for the scalar and fermion fields (in the case of other fields their form can be found quite in the same way).

1. As it was shown in/1/, for a scalar field we have

$$D^c_\mu(x) \equiv \Theta(\mu x)D^{(-)}(x) - \Theta(-\mu x)D^{(+)}(x) = D^c(x)\,,$$

where $D^{(\pm)}(x)$ are positive- and negative-frequency parts of the Pauli-Jordan commutator function $D(x)$. The equality of functions $D^c_\mu(x)$ and $D^c(x)$ implies that $\Delta D^c_\mu(x) = 0$ and therefore in the case when interaction Hamiltonian depends only on the scalar fields (without derivatives) there is no need to introduce the counterterms on the light cone, i.e.,

$$t_\mu\{H(x_1)\cdots H(x_n)\} = 0\,. \tag{2.10}$$

This result is in an agreement with the fact that when formulating a field theory of the spinless particles on the light front the Hamiltonian does not contain additional terms, i.e, up to the sign coincides with the interaction Lagrangian $H(x) = -\mathcal{L}(x)$.

2. In the case of a fermion field the "propagator" function is

$$S^c_\mu(x) = \Theta(\mu x)S^{(-)}(x) - \Theta(-\mu x)S^{(+)}(x),$$

where $S^{(\pm)}(x) = (i\hat\partial + M)D^{(\pm)}(x)$. With the help of the limiting procedure (1.3) it is not hard to show, that

$$S^c_\mu(x) = S^c(x) - \Delta S^c_\mu(x) \ , \qquad \Delta S^c_\mu(x) = i\hat\mu\,\delta(\mu x)D(x) .$$

For the covariant additional term $\Delta S^c_\mu(x)$ we shall also use the representation

$$\Delta S^c_\mu(x) = \frac{1}{(2\pi)^4}\int d^4p\, e^{-ipx}\left(-\frac{\hat\mu}{2p\mu}\right) \tag{2.11}$$

in which the integration is extended only over the values of the momentum p , satisfying the condition $p\mu \neq 0$. From this formula in the particular case when $\mu = (1,0,0,-1)$ there follows the noncovariant additional term in the fermion "propagator" function, obtained in refs./7/.

Having defined the additional terms in the scalar and fermion "propagator" functions, one can find the explicit form of the counterterms on the light cone for Hamiltonians, dependent on these fields. For the Hamiltonian (1.5) the counterterm on the light cone of the second order in g has the form:

$$t_\mu\{H(x_1)H(x_2)\} = g^2\left[:\varphi(x_1)\varphi(x_2): + \frac{1}{i}\overset{c}{D}(x_1-x_2)\right]\times$$

$$\times\left\{:\bar\psi(x_1)\frac{1}{i}\Delta S^c_\mu(x_1-x_2)\psi(x_2): + Sp\left[\Delta S^c_\mu(x_1-x_2)S^c_\mu(x_2-x_1)\right] + (x_1\leftrightarrow x_2)\right\}. \tag{2.12}$$

In the last formula the term $Sp\left[\Delta S^c_\mu(x_1-x_2)\Delta S^c_\mu(x_2-x_1)\right]$ in the brace is omitted, which vanishes due to $\hat\mu^2 = 0$. Thus, we see that in the case of spin the S-matrix on the light cone (2.6) is defined by the T_μ -exponential of the interaction Hamiltonian $H(x)$ plus infinite number of counterterms on the light cone. However, they all can be included into the "Hamiltonian" (see/9/). Indeed, if we define the interaction "Hamiltonian" as

$$\mathcal{H}(\mu|x) = H(x) + \sum_{m=2}^{\infty}\frac{i^{m-1}}{m!}\int\Lambda_m(\mu|x,x_1,\dots,x_{m-1})d^4x_1\cdots d^4x_{m-1} \tag{2.13}$$

then it is possible to represent the S-matrix in the form of the T_μ -exponential of $\mathcal{H}(\mu|x)$, i.e.,

$$S = T_\mu\exp\left\{-i\int\mathcal{H}(\mu|x)d^4x\right\}. \tag{2.14}$$

Here $\Lambda_m(\mu|x_1,x_2,\dots x_m)$ are the singular operator functions, transforming like scalars and nonvanishing only when $\mu x_1 = \mu x_2 = \dots = \mu x_m$ and the interval between any two points $x_1,x_2,\dots,x_m$ is equal to zero. Such operator functions we will call the quasilocal operator functions on the light cone. To obtain their explicit form there are the following chains of relations:

$$\Lambda_n(\mu|x_1,\dots,x_n) = (-1)^n t_\mu\{H(x_1)\cdots H(x_n)\} -$$

$$-\sum_{\substack{m=2\\(\Sigma\nu_i=n)}}^{\infty}\frac{1}{m!}P(x_1,\dots,x_{\nu_1}|x_{\nu_1+1}\dots|\dots x_n)T_\mu\left\{\Lambda_{\nu_1}(\mu|x_1,\dots,x_{\nu_1})\cdots\Lambda_{\nu_m}(\mu|\dots x_m)\right\} \tag{2.15}$$

where $\Lambda_1(\mu|x) = \Lambda_1(x) = -H(x)$ by definition while P stands for the symmetrization operator, introduced in/9/. The transformation law of the quasilocal operators on the light cone under the hermitian conjugation

$$\overset{+}{\Lambda}_n(\mu|x_1,\dots,x_n) = (-1)^{n-1}\Lambda_n(\mu|x_1,\dots,x_n) \tag{2.16}$$

ensures the hermiticity of $\mathcal{H}(\mu|x)$, i.e.,

$$\overset{+}{\mathcal{H}}(\mu|x) = \mathcal{H}(\mu|x). \tag{2.17}$$

We illustrate the formulated above rules for constructing counterterms on the light cone by two examples. In the case when $H(x)$ depends only upon the scalar fields from (2.10) and (2.15) it follows that all $\Lambda_n(\mu|x_1,\dots,x_n)$ vanish and therefore according to (2.13) $\mathcal{H}(\mu|x)=H(x)$. In the second case, when $H(x)$ has the form (1.5),

$$\Lambda_2(\mu|x_1,x_2)=t_\mu\{H(x_1)H(x_2)\} \tag{2.18}$$

and all remaining quasilocal operators on the light cone vanish since the product of two or more additional terms $\Delta S^c_\mu(x)$ is equal to zero. Consequently,

$$\mathcal{H}(\mu|x)=H(x)+\tfrac{1}{2}\int t_\mu\{H(x)H(y)\}d^4y, \tag{2.19}$$

where $t_\mu\{H(x)H(y)\}$ is given by formula (2.12). In the particular case, when the vector $\mu=(1,0,0,-1)$, the "Hamiltonian" (2.19) after simple manipulations coincides with that on the light cone used in refs./7/.

3. The Causality and Unitarity Conditions

From the results of the previous section it is clear that in the spin case instead of equation (1.2) one should use the equation

$$i\frac{\delta S(\sigma,-\infty)}{\delta\sigma(x)}=\mathcal{H}(\mu|x)S(\sigma,-\infty). \tag{3.1}$$

The causality and unitarity conditions, formulated in/1/ for a scalar theory, are transferred to the case under consideration without change. They have respectively the form:

$$R(\mu\varkappa)-\overset{+}{\tilde{R}}(-\mu\varkappa)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\varkappa'}{\varkappa'-i\varepsilon}\left[\overset{+}{\tilde{R}}(\mu\varkappa'-\mu\varkappa)R(\mu\varkappa)-\overset{+}{\tilde{R}}(\mu\varkappa')R(\mu\varkappa'+\mu\varkappa)\right], \tag{3.2}$$

$$R(\mu\varkappa)-\overset{+}{R}(-\mu\varkappa)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\varkappa'}{\varkappa'-i\varepsilon}\left[\overset{+}{R}(-\mu\varkappa')R(\mu\varkappa-\mu\varkappa')+\overset{+}{R}(\mu\varkappa'-\mu\varkappa)R(\mu\varkappa')\right]. \tag{3.3}$$

The operator $R(\mu\varkappa)$ is connected with the $S(\sigma,-\infty)$-matrix by the relation

$$S(\sigma,-\infty)=1+\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{R(\mu\varkappa)}{\varkappa-i\varepsilon}e^{i\sigma\varkappa}d\varkappa \tag{3.4}$$

and obeys the equation

$$R(\mu\varkappa)=-\tilde{\mathcal{H}}(\mu\varkappa)-\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{\mathcal{H}}(\mu\varkappa-\mu\varkappa')\frac{d\varkappa'}{\varkappa'-i\varepsilon}R(\mu\varkappa'), \tag{3.5}$$

where $\tilde{\mathcal{H}}(\mu\varkappa)$ is the Fourier transform of the "Hamiltonian", i.e.

$$\tilde{\mathcal{H}}(\mu\varkappa)=\int e^{-i\mu\varkappa x}\mathcal{H}(\mu|x)d^4x. \tag{3.6}$$

The operator $\tilde{R}(\mu\varkappa)$ satisfies the equation of the form (3.5), but with the "propagator" $\frac{1}{2\pi}\frac{1}{\varkappa+i\varepsilon}$. On the mass shell the condition (3.3) is written as $R(0)-\overset{+}{R}(0)=i\overset{+}{R}(0)R(0)$ or $S\overset{+}{S}=1$.

4. The Diagram Technique

On the basis of the representation (2.14) for the S-matrix it is possible to develop a covariant three-dimensional diagram technique, in which the spurion 4-momentum is light-like. To this end let us write (2.14) in the form

$$S=T_\mu\exp\left\{-i\int\mathcal{H}'(\mu|x)d^4x\right\}, \tag{4.1}$$

where $\mathcal{H}'(\mu|x)=H(x)+g^2\int h(x,y)d^4y$, $h(x,y)=:\varphi(x)\overline{\psi}(x)\frac{1}{i}\Delta S^c_\mu(x-y)\psi(y)\varphi(y):$

The operator $\mathcal{H}'$ is obtained from $\mathcal{H}$ by the omission of the following three terms. The first one

$$g^2\,Sp\left[\Delta S^c_\mu(x-y)S^c_\mu(y-x)\right]\frac{1}{i}D^c(x-y)$$

contributes only to vacuum diagrams and does not lead to observable effects. The second

$$g^2:\overline{\psi}(x)\frac{1}{i}\Delta S^c_\mu(x-y)\psi(y):\frac{1}{i}D^c(x-y)$$

and the third

$$g^2:\varphi(x)\varphi(y):Sp\left[\Delta S^c_\mu(x-y)S^c_\mu(y-x)\right]$$

divergent terms contribute only to the fermion and meson self-energy, respectively and can be taken into account by the appropriate renormalization. As a result we have the following rules for calculating the matrix elements.

1. Apply to a given Feynman diagram the Δ-operation, which acts on the internal fermion lines and substitutes them according to the rule (2.7) by the fermion lines with the cross, to which there corresponds a factor $\frac{i}{(2\pi)^4}\frac{\hat{\mu}}{2k\mu}$. Omit from thus obtained diagrams those, in which at least in one vertex occur two lines with the cross, and those, in which the ends of at least one fermion line with the cross are connected between themselves by other lines.

2. Number all vertices of the remaining diagrams in an arbitrary way, ascribing the same number to the each pair of vertices, connected by the fermion lines with the cross, and the vertices of a given diagram. Orient all internal lines along the direction of decreasing vertex number. Without making a change in the orientation, substitute some ordinary internal fermion lines by the double ones, leaving the lines with the cross untouched so that in the result the number of fermions and antifermions to be conserved. Omit all diagrams with vacuum transitions - DVT*).

3. Connect the first vertex with the second, the second with the third and so on, by dotted lines, oriented in the direction of increasing vertex number, and ascribe the 4-momentum $\mu\varkappa_j$ to each of them ($j=1,2,\dots,n-1$ for a given Feynman diagram and $j=1,2,\dots,n-m-1$ for a diagram with the cross, where n is the order of the diagram, m is the number of lines with the cross). Draw the dotted lines with a free end so that they enter into the first vertex and leave the last vertex, and ascribe the four momenta $\mu\varkappa$ and $\mu\varkappa'$, respectively, to each of them.

4. Assign to each internal ordinary line with the 4-momentum k the function $S^{(+)}(k,M)=\theta(k^0)(\hat{k}+M)\delta(k^2-M^2)$, to the double line

*) For details see ref. /1/.

- the function $S^{(+)}(k,-M)$ and to the internal wavy line - the function $D^{(+)}(k,m)=\theta(k^0)\delta(k^2-m^2)$. To each internal dotted line with the 4-momentum $\mu\varkappa_j$ a factor $\frac{1}{2\pi}\frac{1}{\varkappa_j-i\varepsilon}$ is assigned.

5. Assign to each vertex of a diagram a factor $\frac{g}{\sqrt{2\pi}}\Gamma\ \delta$, and to the vertices, connected by the lines with the cross, a factor $(2\pi)^4 g\delta$, where the δ-function takes into account the conservation of the 4-momentum in a given vertex. To each external line (except for dotted ones) it is necessary to assign a factor $(2\pi)^{3/2}(2p_0)^{-1/2}$, where p is the 4-momentum of the given line. Besides, assign the corresponding spinors $u,\bar{u}$ or $v,\bar{v}$ to the external fermion lines.

6. Integrate over all independent 4-momenta k and all $\varkappa_j$ in infinite limits.

7. Sum the coefficient functions, which result from all $n!$ numbering of vertices of the given diagram and $(n-m)!$ numbering of vertices of all diagrams with the cross. Multiply the result by a sign factor and a factor connected with the symmetry properties of the given diagram.

To illustrate these rules some third-order diagrams are drawn on the picture 1.

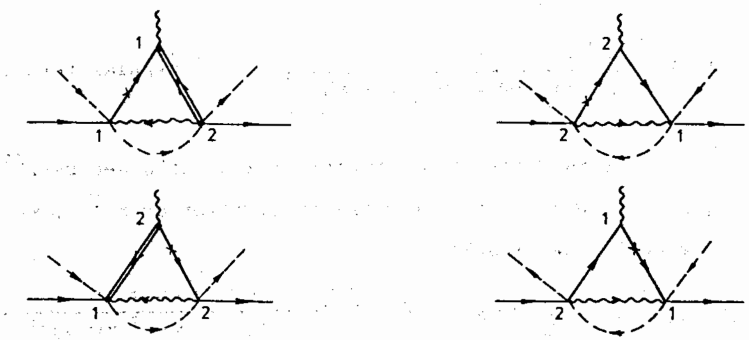

Fig. 1.

As an example, we consider the fermion self-energy $\Sigma(p)$ of second order in perturbation theory (on the energy shell) (Fig. 2). The fact, that there are no lines with the cross is due to the property that according to the first paragraph of the diagrammatic rules the action of the Δ -operation in this case gives zero.

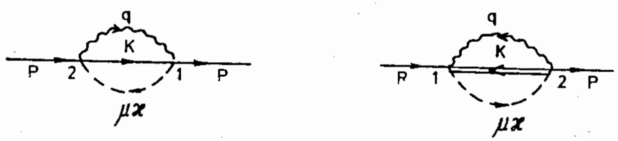

Fig. 2.

The second diagram is a DVT and does not contribute to $\Sigma(p)$. Consequently,

$$\Sigma(p)=\frac{g^2}{(2\pi)^3}\int d^4k\,\frac{d\varkappa}{\varkappa-i\varepsilon}\,\overset{(+)}{S}(k,M)\,\overset{(+)}{D}(p-k+\mu\varkappa) \quad + \text{ regular terms.}$$

The integration over the variable $\varkappa$ gives

$$\Sigma(p)=\frac{g^2}{(2\pi)^3}\int d^4k\,\overset{(+)}{S}(k,M)\,\frac{\Theta(\mu p-\mu k)}{m^2-(p-k)^2-i\varepsilon} \quad + \text{ regular terms}$$

or

$$\Sigma(p)=\frac{g^2}{2(2\pi)^3}\int_0^1 dx\int d\tilde{k}\,\frac{M-(1-x)\hat{p}}{\tilde{k}^2+xM^2+(1-x)m^2-x(1-x)p^2-i\varepsilon}\;+$$

$$+\frac{g^2}{2(2\pi)^3}\,\frac{\hat{\mu}}{2p\mu}\int dx\,\frac{\left(\frac{\tilde{k}^2+M^2}{x}-xp^2\right)d\tilde{k}}{\tilde{k}^2+(1-x)M^2+xm^2-x(1-x)p^2-i\varepsilon} \quad + \text{ regular terms,}$$

where $\tilde{k}$ is the two-dimensional vector (for details see ref. /1/). Performing the regularization and the integration over $\tilde{k}$, we obtain the well-known result

$$\Sigma(p)=c_1\hat{p}+c_2+\frac{g^2}{(4\pi)^2}\int_0^1 dx\,[M-(1-x)\hat{p}]\ln\frac{xM^2+(1-x)m^2}{xM^2+(1-x)m^2-x(1-x)p^2}\,.$$

Here C_1 and C_2 are arbitrary constants.

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