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ARE REALLY THE DATA ON THE ELECTROMAGNETIC PION FORM FACTOR AND P-WAVE ISOVECTOR $\pi\pi$-PHASE SHIFT INCONSISTENT?

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Совместимы ли данные по пионному формфактору и P-волновой изовекторной фазе $\pi\pi$-рассеяния?

На основе использования только фундаментальных свойств пионного формфактора анализируется несовместимость данных по пионному формфактору, P-волновой изовекторной фазе $\pi\pi$-рассеяния и эффективному радиусу пиона.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Are Really the Data on the Electromagnetic Pion Form Factor and P-Wave Isovector $\pi\pi$-Phase Shift Inconsistent?

An apparent inconsistency of the data which involve the pion form factor, the P-wave isovector $\pi\pi$ phase shift and the pion charge radius is examined by using only fundamental properties of the pion form factor.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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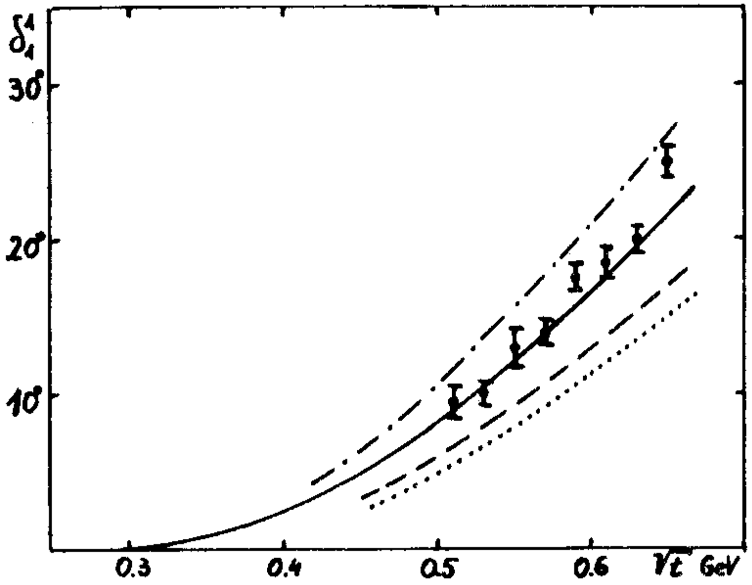
It is well known, that as a consequence of the unitarity condition, the pion form factor phase $\delta_\pi(t)$ for $4m_\pi^2 \le t \le 16\,m_\pi^2$ is exactly equal to the P-wave isovector $\pi\pi$ phase shift $\delta_1^1(t)$. So, it would be natural to expect that the models successfully describing the pion form factor data will give an experimental value of the pion charge radius and the phase $\delta_\pi(t)$ with a right $\pi\pi$ scattering length a_1^1 at the threshold. Moreover, $\delta_\pi(t)$ is expected to be consistent with existing data on $\delta_1^1(t)$ in elastic region.

However, as Hammer et al.[1] have recently (by using the models previously developed by them[2] and other authors[3,4]) shown, not all of the data are consistent. If the model reproduces well the P-wave isovector $\pi\pi$ phase shift data, too small pion charge radius is predicted. Alternatively, if the model fits the form factor data and predicts the reasonable pion charge radius, then the model fails to predict the correct phase shift.

In this paper we repeat the analysis and come to the following conclusion. If we use in the analysis the pion form factor data around the elastic region only, the description of which can be achieved by using exact consequences of basic principles, no discrepancy is found. In such a way we ascertain that inconsistency found by Hammer et al.[1] is due to the model ingredients needful to describe all the pion form factor data. The fact that different models equally well describing the form factor data predict different behaviours of $\delta_1^1(t)$ at low energies (see the *figure*), supports our conclusion.



The P-wave isovector $\pi\pi$ phase shift vs $\sqrt{t}$. The solid curve represents the results of our analysis. The data are from ref. /17/ *. -·-·-·-·-·-·- the phase of the Hammer-Zidell-Reimer-Weber pion form factor* /1/ *; - - - - - - - - the phase of the Gounaris-Sakurai pion form factor* /3/*; . . . . the phase of the Deo-Parida pion form factor* /4/*.*

Now, let us return to the analysis of the data. As is well known, the pion form factor $F_\pi(t)$ as a function of the squared four-momentum transfer t is an analytic function in the entire complex t plane except for a sequence of threshold branch points in which the lowest one is at $t = 4m_\pi^2$, where m_π is the pion mass. The cuts associated with the latter are drawn to $+\infty$. As a consequence of the elastic unitarity and the reality conditions, the elastic threshold at $t = 4m_\pi^2$ is a square root branch point and so, it generates the two-sheeted Riemann surface. By using the conformal mapping (units $h = c = m_\pi = 1$ are used)

$$q = \sqrt{\frac{t-4}{4}} \tag{1}$$

we map these two sheets in t variable onto the q plane and the elastic cut disappears. Then $F_\pi(t)$ in q plane can be expanded around the elastic threshold $q = 0$ into the Taylor series

$$F_\pi(t) = \sum_{n=0}^{\infty} a_n q^n \tag{2}$$

the radius of convergency of which is given by the relation

$$R = | q_1 | , \tag{3}$$

where $q_1 = \pm\sqrt{3}$ corresponds to the first inelastic threshold $t_{4m_\pi} = 16$.

On the other hand it is well known from the $\pi\pi$ phase shift analysis /5/, that the P-wave isovector inelasticity $\eta_1^1(t)$ starts to be different from one almost at the c.m. energy of 1 *GeV*. This enables us to neglect the four pions cut and to enlarge the radius of convergency of the corresponding Taylor series nearly to the ρ - meson poles /6,7/. In such a way we have 57 experimental points /8/ from the range of momenta $-0.294\ GeV^2 \le t \le 0.490\ GeV^2$ for the analysis.

As a consequence of the reality condition $F_\pi^*(t) = F_\pi(t^*)$ even and odd coefficients of the expansion (2) are the real and purely imaginary ones, respectively. Then the pion form factor may be taken in the following form

$$F_\pi(t) = \sum_{n=0}^{\infty} b_n (i)^n q^n \tag{4}$$

with b_n being real.

Now, imposing on the imaginary part of (4) the threshold conditions

$$\mathrm{Im}\,F_\pi(t)|_{q=0} = 0; \quad \frac{\partial\,\mathrm{Im}\,F_\pi(t)}{\partial q}\Big|_{q=0} = 0; \quad \frac{\partial^2 \mathrm{Im}\,F_\pi(t)}{\partial q^2}\Big|_{q=0} = 0, \tag{5}$$

which are the consequences of the threshold behaviour of the $\pi\pi$-phase shift $\delta_1^1(t) \cdot a_1^1 q^3$ (a_1^1 is the $\pi\pi$ scattering length) and the elastic unitarity condition, one gets

$$b_\pi = 0, \tag{6}$$

and the normalization condition $F_\pi(0)=1$ gives

$$b_0 = 1 - \sum_{n=2}^{\infty} b_n (-1)^n \quad . \tag{7}$$

As a result one gets the following simple parametrization

$$F_\pi(t) = 1 + \sum_{n=2}^{\infty} b_n [(i)^n q^n - (-1)^n] \tag{8}$$

consistent with the basic principles and suitable for the analysis of the data, which involve the pion form factor experimental points from the range of momenta $-0.294\ GeV^2 \le t \le 0.490\ GeV^2$, the P-wave isovector $\pi\pi$ phase shift in the elastic region (including also the value of the scattering length a_1^1) and the pion charge radius.

Further, by using the least-square method and the aforementioned 57 experimental points on the pion form factor, we determine the optimal number of free coefficients b_n in (8). The latter are subsequently used to predict the pion charge radius

$$\langle r_\pi^2 \rangle = 6 \left. \frac{dF_\pi(t)}{dt} \right|_{t=0} , \tag{9}$$

the value of the $\pi\pi$ scattering length

$$a_1^1 = \left. \frac{\dfrac{\partial^3 \mathrm{Im} F_\pi(t)}{\partial q^3}}{6\,|F_\pi(t)|} \right|_{q=0} \tag{10}$$

and the behaviour of the P-wave isovector $\pi\pi$ phase shift

$$\delta_1^1(t) = \mathrm{arc\,tg}\, \frac{\mathrm{Im} F_\pi(t)}{\mathrm{Re}\, F_\pi(t)} \tag{11}$$

in elastic region.

The minimum of χ^2 is achieved already with four free coefficients in (8) and $\chi^2/ndf = 1.02$ (ndf means the number of degrees of freedom). The values of the parameters are as follows:

$$b_2 = -0.197860 \pm 0.016531 \quad b_4 = 0.041677 \pm 0.008659$$
$$b_3 = -0.038254 \pm 0.003991 \quad b_5 = 0.019305 \pm 0.004294 \tag{12}$$

The pion charge radius takes the value $\langle r_\pi^2 \rangle^{1/2} = 0.562 \pm 0.234$ F, which is consistent not only with a generally accepted value [9] but also with two different experimental results [10,11] obtained in the Russian-American collaboration.

For the $\pi\pi$ scattering length we get $m_\pi^3 a_1^1 = 0.034 \pm 0.004$; a number of theoretical estimates [12-16] of the same parameter give $0.027 < m_\pi^3 a_1^1 < 0.045$.

The behaviour of $\delta_1^1(t)$ is graphically presented in the *figure*, where it is compared with the existing experimental data [17] in the elastic region.

As our results show, if we analyse only those data on the pion form factor, which one can describe without any model ingredients, in contradiction to the conclusion of Hammer et al. [1], all data are consistent.

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