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# ON UNIFIED EXCEPTIONAL GAUGE MODELS WITH STABLE PROTON

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# ON UNIFIED EXCEPTIONAL GAUGE MODELS WITH STABLE PROTON

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О единых исключительных калибровочных моделях со стабильным протоном

Изучаются объединенные $E_7$-калибровочные теории, в которых стабильность протона обеспечивается при помощи механизма Гелл-Манна, Рамона и Сланского, но лептоны могут обладать ненулевым значением нового сохраняющегося квантового числа.

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On Unified Exceptional Gauge Models with Stable Proton

The unified gauge $E_7$-theories are studied with proton stability ensured by a mechanism of the Gell-Mann, Ramond and Slansky type, but nonzero eigenvalues of a new conserved quasi-baryon number are admitted on leptons. The requirement of at least minimal agreement of such theories with phenomenology fixes a restricted class of models. Their basic properties and difficulties are analysed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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## *INTRODUCTION*

The successes of gauge field theory in description of strong (quantum chromodynamics) and weak and electromagnetic interactions have stimulated various attempts to construct the "grand unification" schemes. A brilliant agreement of Weinberg-Salam theory with experiment does not mean, nevertheless, that this unified theory of weak and electromagnetic interactions is the fundamental one. On the one hand it has two independent coupling constants and needs the third one to include the strong interactions. On the other hand, it requires a great number of representations (doublets and singlets) to embrace the known particles. This number is growing year by year and the theory has no internal criteria to set a limit on this number. Furthermore the Weinberg-Salam scheme can not explain the known symmetry between quarks and leptons, which played a key role as a heuristic principle in particle physics last time.

So the idea of grand unification reflects a common aspiration to establish the universal structure in particle physics and to bridge the hadronic and leptonic worlds [1,2,3]. It consists in the following:

1) It is supposed that the grand group $G$ exists and has the rank high enough to contain the groups of weak and electromagnetic interactions. The color group $SU^c(3)$ is considered as the symmetry of strong interactions and the weak group $G_{weak}$ is $SU(2)xU(1)$ or $SU(2)_L xSU(2)_R x$ $xU(1)$, etc.

2) The gauge field theory is built on the basis of the group G: the fundamental fermions, just quarks and leptons, are placed in irreducible representations of this group (the single representation is preferable) the vector gauge fields form the adjoint representation. There is a full symmetry among quarks and leptons originally.

3) A set of scalar fields is introduced to realize the Higgs mechanism. The symmetry G is broken to the symmetry $U(1) x SU^c(3)$ due to nonzero vacuum expectation values (v.e.v. in what follows) of these fields and the remaining U(1)-group is the electric charge group. Hence among the vector fields there are 8 gluons and photon which remain massless. The condition

$$M \gg m_W ,$$

where $m_W$ is a typical mass of gauge field corresponding to $G_{weak}$ and M is a mass of any other vector field, is imposed and is equivalent to the existence of symmetry breaking hierrarchy.

The advantages of such a theory are: renormalizability, a unique gauge coupling constant and common and universal nature of all kinds of interactions* (the differences among interactions are caused by the effects of renormalization and spontaneous symmetry breaking), a possibility of fundamental importance to establish the connections between leptonic and hadronic spectra.

The first theories of this kind were those based on the group SU(5) as a minimal simple Lie group containing $SU(2) x U(1) x SU^c(3)$ and on the group S0(10) as a minimal group containing $SU^c(3) \otimes SU(2)_L x\ SU(2)_R$ x x U(1). However, both these theories have small representations suitable for fermionic assignment and hence it is necessary to introduce several original fermionic multiplets [1,2,11].

Recently Gursey, Sikivie, Ramond, Gell-Mann, Slansky, and others [4-9,14] have shown that exceptional

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*Except gravity.



groups $E_6$ and $E_7$ are suitable for the purposes of choice. First of all, these groups are naturally distinguished from all other simple Lie groups. Second, they admit a natural definition of color while other their subgroups are very convenient for physical applications. Third, having relatively low rank, these groups possess very capacious fundamental representations. When the electric charge operator is determined in the most natural way, the structure of these representations is adequate for unifying quarks and leptons in a single multiplet.

However, like other grand unification schemes, exceptional theories have a trouble with proton stability. Any gauge theory with quark-lepton symmetry inevitably leads to the existence of interactions which break the conservation of baryon number. There are members of the adjoint representation of G which have nontrivial transformational properties with respect to both $SU^c(3)$ and $G_{weak}$. The corresponding vector fields are called lepto-quarks and they enter into the quark-lepton vertices simultaneously (see *Fig. 1*). Hence, if there is

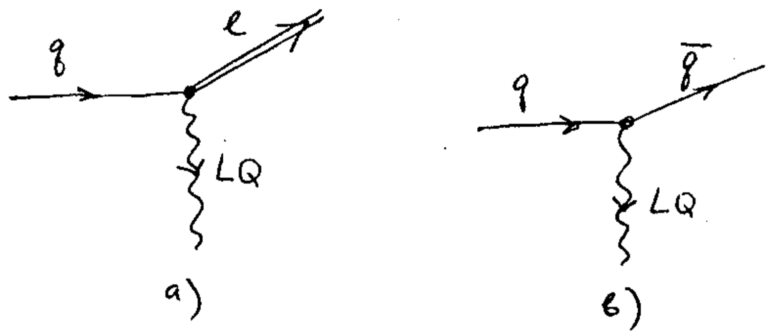


*Fig. 1. The lepto-quark vertices,* q *is a quark,* ℓ *is a lepton,* LQ *is a lepto-quark.*

no some extra selection rule, the proton is decaying as is shown in *Fig. 2.*

The proton lifetime is $> 2 \cdot 10^{30}$ years, according to [10]. To prohibit the proton decay on this level the mass of lepto-quark must be gigantic: $10^{15} - 10^{19}\, GeV/c^2$

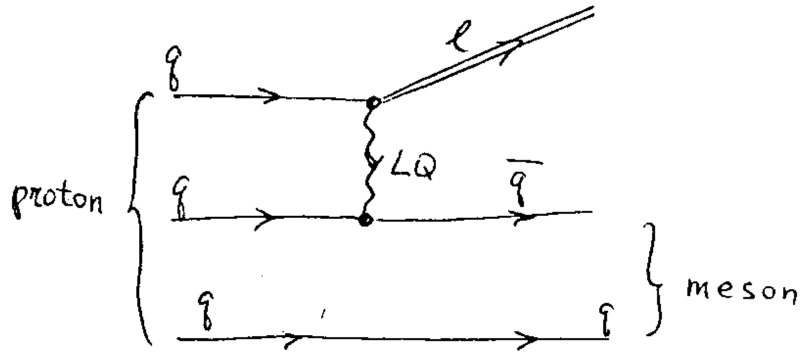

*Fig. 2. The proton-decay diagram.*

(according to various estimates [11,12]) in the theories under consideration. It means that an extremely strong hierarchy of gauge symmetry breaking is needed. However (see [13]) this hierarchy, i.e., the ratio of vector bosons masses which they acquire due to the symmetry breakdown is limited by the value of an order of $g$, where $g$ is the gauge constant, in the framework of perturbation theory *. Thus the problem of proton stability becomes crucial for the theories under consideration.

Recently, an elegant method to avoid the mentioned difficulty has been proposed in the work of Gell-Mann, Ramond and Slansky [14] (GRS in what follows). It introduces the global symmetry between fermionic multiplets in the theory. This symmetry though being itself spontaneously broken combines with some internal U(1)-symmetry (contained in original grand group) to give a new invariance generalizing baryon number conservation and prohibiting with some auxiliary conditions the proton decay. Besides it solves the problem of proton stability, GRS-mechanism introduces in the theory the

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* Besides that, the gravitation must be taken into account, if masses of the order of Planck mass are present.



selection rules with respect to a new absolutely conserved quantum number A and strongly simplifies the fermionic mixings. These latter arise in the procedure of diagonalization of mass-matrix and present otherwise a practically unsolvable problem. At the same time the GRS-mechanism establishes the restrictions on possible choice of model in the framework of a given theory. By the choice of model we mean here the assignment of quantum numbers, the structure of weak interactions, the pattern of symmetry breaking and so on. On the other hand, the phenomenological restrictions give a possibility to fix a definition of new conserved quantum number.

The aim of the present paper is to investigate the general properties of those models, which can be obtained in the exceptional unified gauge theory using the GRS-mechanism. Our approach though based on the GRS' -one differs from this latter since we do not require the leptons to have zero value of quantum number A. We consider just the opposite case and come to the conclusion that it is reliable providing an interesting possibility to have the common and absolutely conserved nonzero charge both for hadrons and leptons. The GRS-version is the particular case of this general situation. In our treatment of the theory we pay a special attention to its Higgs sector trying to keep only the minimal set of Higgs fields. The requirement of at least minimal agreement of the theory with experiment underlines our analysis. It puts severe restrictions on the possible models and allows one to fix the spectrum of the new conserved charge on physical states. As a result, we have 4 species of models with definite structure of weak interactions and assignment. It is one of our goals to elucidate the difficulties of these models.

We shall concentrate on the theory based on the exceptional group $E_7$ as it seems to be most promising and admits the realization of GRS-mechanism with minimal number of fermionic multiplets. The information about $E_6$ scheme can be obtained from $E_7$-theory by simple reduction using the fact that

$$E_6 \subset E_7$$

and that their representations can be also enclosed one into another. However, the Higgs sector of this model may differ from the $E_7$ -one.

## *$E_7$: A REVIEW OF BASIC PROPERTIES*

The group $E_7$ is the simple Lie group of rank 7. Its dimension is 133. The most natural realization of $E_7$ is provided by the octonion formalism. The maximal subgroup of $E_7$ is SU(6)xSU(3), where SU(3) subgroup is identified in what follows with color $SU^c(3)$ and is related with automorphisms of octonions (see ref. /4/ ). Hence, $G_{weak} \subset SU(6)$. The $SU(6)xSU^c(3)$ reductions of lowest dimension representations are

$$\underline{56} = (20.1^c) + (6.3^c) + (\bar{6}.\bar{3}^c), \tag{1}$$

$$\underline{133} = (35.1^c) + (15.\bar{3}^c) + (\overline{15}.3^c) + (1.8^c). \tag{2}$$

The reduction of the direct product of two 56's is:

$$\underline{56} \times \underline{56} = \underline{1}_A + \underline{1539}_A + \underline{133}_S + \underline{1463}_S , \tag{3}$$

where "S" and "A" mean symmetric and antisymmetric parts, respectively.

All the representations of $E_7$ are real. Among representations in (3) 133, 1463 and 1539 ones are real and 56 representation is pseudoreal (cf. /16/).

In a model building the fermions will be placed in fundamental representation 56. It is clear that leptonic states are in $(20.1^c)$, quark states in $(6.3^c)$ and antiquark states in $(\bar{6}.\bar{3}^c)$. The gauge vector fields form the adjoint representation 133, where $(35.1^c)$ contains W-bosons and photon, $(15.\bar{3}^c)$ and $(\overline{15}.3^c)$ consist of leptoquarks, $(1.8^c)$ is the gluons octet. The invariant Lagrangian in such a theory was built in /15/ using the octonion formalism.

We will assume that the electric charges of quarks are 2/3 and -1/3, while leptons and W-bosons have



charges 0,±1. Then the standard definition of electric charge operator /4/

$$Q = \mathrm{diag}\,(\tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, \tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}) \tag{4}$$

satisfies these conditions and corresponds to the reduction of SU(6) to its maximal subgroup SU(3)xSU(3)xU(1).

## *THE GRS-MECHANISM IN $E_7$-MODEL*

Let us impose now the condition of absolute proton stability on the theory. It is possible if some selection rule is present which, in turn, follows from the fact of existence of some exact symmetry generalizing the baryon number conservation. We shall consider for simplicity that transformations of this new symmetry form some I-parameter group. These transformations must satisfy the following conditions:

1) the quarks and leptons must transform in a different way;

2) there is no massless vector field corresponding to the generator of this new exact symmetry;

3) these transformations commute with electric charge operator and generators of $SU^c(3)$.

Let us introduce two fermion multiplets $\Psi_1$, and $\Psi_2$ which are two-component spinors

$$\Psi = \Psi_L = \frac{1+\gamma_5}{2}\Psi .$$

Then it is evident that neither global symmetry between them can satisfy the first condition, while any pure internal symmetry U(1) doesn't satisfy the second. Hence the only possibility is to define (see /14/)

$$A = mU(1) + fZ , \tag{5}$$

where $U(1) \subset SU(6) \subset E_7$, m and f are parameters and the global symmetry $e^{i\alpha Z}$ is determined as follows:

$$\Psi_1 \to e^{i\alpha}\Psi_1 \qquad (\alpha \text{ is a parameter.}) \tag{6}$$
$$\Psi_2 \to e^{-i\alpha}\Psi_2 ,$$

Then we have to require that at least some quarks would have the eigenvalues of $A$ equal to 1/3 that coincides with standard definition of baryon number. Those quark (anti-quark) states which have $A = +\frac{1}{3} (-\frac{1}{3})$ are normal, others are weird. Naturally the number of normal quark states in the theory cannot be less than four, otherwise we won't have a possibility to describe usual $u$, $d$, $s$ and $c$ -quarks. Without fixing the U(1) in $A$ let us consider the hadronic sector of the model. Since we have two 56-plets of two-component spinor fields with $Z = \pm 1$, there are four two-component quark sextets at our disposal:

$$q^i_{1L}, \overline{q_{2R}}^i \subset \Psi_1; \quad q^i_{3L}, \overline{q_{4R}}^i \subset \Psi_2, \quad i = 1 \dots 6,$$

color indices being suppressed.

It is easy to see that if

$$A q^i_1 = a^i_1 q^i_1, \quad A \bar{q}^i_2 = a^i_2 \bar{q}^i_2, \quad A q^i_3 = a^i_3 q^i_3, \quad A \bar{q}^i_4 = a^i_4 q^i_4$$

(no summation over $i$), then

$$a^i_1 = -a^i_4, \quad a^i_3 = -a^i_2 \tag{7}$$

and we conclude that if $q_1$ contains some quark states, then $\bar{q}_4$ contains the corresponding anti-quarks, while $\bar{q}_2$ contains the anti-quarks to $q_3$ provided $A$ does generalize the baryon number. As we have the single gauge multiplet 133 with $Z = 0$ the kinetic term of the theory is:

$$\bar{\Psi}_1 \not{D} \Psi_1 + \bar{\Psi}_2 \not{D} \Psi_2, \tag{8}$$

where $\not{D} = \not{\partial} + \Gamma^A \not{A}^A$, $A = 1, 2 \dots 133$, $\Gamma^A$ are the generators of $E_7$ and $A^A_\mu$ are corresponding gauge fields.

Thus we conclude taking into account (7) that our model is vector-like in hadronic sector and we deduce that

1) the number of normal quarls must be more than 4, otherwise there would be no chance to get rid of right-handed contributions to the usual hadronic weak currents,

2) the spontaneous symmetry breakdown must lead to non-Hermitean mass-matrix, since in this case the mass term

$$\bar{q}_R M q_L + \text{h.c.} \tag{9}$$

after diagonalization of mass-matrix $M$ with the help of the polarization theorem takes the form

$$\bar{q}_R U^{-1} \cdot D \cdot U \cdot V q_L + \text{h.c.}, \tag{10}$$

where $M = H \cdot V$, $V$ is unitary, $H$ is Hermitian, $D = U H U^{-1}$ is diagonal, $U$ is unitary. Thus when making the spinor fields to be the mass-matrix eigenvectors one has to use different transformations for left and right ones. It gives a possibility to describe the left-right asymmetry of weak interactions. However, it is clear, that transformations diagonalizing the mass-matrix, commute with $SU^c(3)$ and the above mentioned asymmetry does not concern the strong interactions.

Now we are going to consider the interactions of fermionic and Higgs fields in view of the second of above conditions. Let $\chi^\circ$ be a scalar field belonging to some irreducible representation in r.h.s. of (3). Then if $Z\chi^\circ = 0$ the interaction of $\chi^\circ$ with fermions may be schematically represented as follows

$$g_0 \Psi_1 \Psi_2 \chi^\circ + \text{h.c.} \tag{11}$$

We require the Lagrangian of the theory to be invariant under the $Z$-transformations, the breakdown of $Z$ is spontaneous and is caused by v.e.v. of scalar fields. The representations 133, 1463 and 1539 are real. Therefore the matrix of v.e.v. $\langle\chi^\circ\rangle$ is Hermitian (recall that $\chi^\circ$ is $Z$-neutral) and it does not satisfy the imposed condition. It is evident that the presence of several representations with $Z = 0$ does not change the situation.

It is convenient to introduce $\chi_{\pm} = \chi_1 \pm i\chi_2$

$Z\chi_{\pm} = \pm 2\chi_{\pm}$.

The possible invariant interactions are

$$g_- \Psi_1 \Psi_1 \chi_- + g_+ \Psi_2 \Psi_2 \chi_+ + \text{h.c.}, \qquad (12)$$

where $g_{\pm}$ are coupling constants. It is clear from Eq.(12) that $\chi_{\pm}$ must belong to a representation which is contained in the symmetrized product $\underline{56} \otimes \underline{56}$, i.e., $\chi_{\pm}$ is $\underline{133}$ or $\underline{1463}$. Each field $\chi_1$ and $\chi_2$ is represented by a Hermitian matrix and $\chi_+ = \chi_-^+$, however, the freedom in choice of coupling constants $g_+$ and $g_-$ provides us with a required asymmetric mass-matrix in a normal hadronic sector provided there are both quark and antiquark normal states in each 56-plet. This requirement is necessary, since otherwise Eq. (12) does not contribute to the mass-matrix. Further to fix the theory, we must define the group $G_{weak}$ of weak and electromagnetic interactions. We choose

$$G_{weak} = SU(2) \times U(1)^n, \quad n < 4.$$

Here all U(1) 's commute with electric charge operator. We shall not fix the value of $n$ since it is related to the problem of neutral currents and the latter is out of our discussion so far *.

Then using the reduction of SU(6) to SU(3)xSU(3)xU(1) we come to the conclusion that taking into account the definition (4) of electric charge we may choose the $SU(2)_{weak}$ either in the form $\lambda_i \times (1 \pm \sigma_3)$, or $\lambda_i \times 1$, $i = 1,2,3$. Here we have used the basis

$$\lambda_m \otimes \sigma_\alpha, \ \lambda_m \otimes 1, \ 1 \otimes \sigma_\alpha$$

for the algebra of SU(6), where $\lambda_m$ are the Gell-Mann matrices, $m = 1,\ldots,8$, $\sigma_\alpha$ are the Pauli matrices, $\alpha = 1,2,3$. $1$ is the unit matrix.

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* It is clear that $G_W$ of type $SU(2)_L xSU(2)_R xU(1)$ is impossible in our vector-like scheme.



Let us take the generators of $SU(2)_{weak}$ in the form $\lambda_i \ (1 \pm \sigma_3)$. In this case only two left doublets with respect to $SU(2)_{weak}$ exist. One of them belongs to $q_{1L}$ and another, to $q_{3L}$. Clearly, these doublets cannot simultaneously consist of the normal quark states, since their transformational properties under $U(1) \subset A$ are the same and the eigenvalues of $Z$ are opposite. Hence, phenomenology forbids such a choice since it leads to the existence of the only left doublet in the normal hadron sector. Therefore we have to choose

$$SU(2)_{weak} = \lambda_i \otimes 1, \quad i = 1,2,3.$$

It is easy to see that in this case there are four left doublets in the theory and two of them may be normal. The same is valid for the right doublets. It doesn 't contradict the experiment provided the quark assignment is (see [17]):

$$\begin{pmatrix} u \\ d(\theta) \end{pmatrix}_L, \ \begin{pmatrix} c \\ s(\theta) \end{pmatrix}_L, \ b_L$$

$$\begin{pmatrix} u \\ b(\phi) \end{pmatrix}_R \ \begin{pmatrix} c \\ s(\phi) \end{pmatrix}_R, \ d_R$$

($\theta$ is the Cabibbo angle, $\phi$ is the Cabibbo-like angle in right currents). (13)

So, using the very fact of presence of GRS-mechanism with $A$ yet unfixed, we have obtained a lot of information about the structure of the hadronic sector of our model and have fixed the SU(2)-weak.

Now we have to define U(1) entering into the formula (5). To allow the standard weak processes, this U(1)-transformation must commute with $SU(2)_{weak}$. This condition together with the requirement of presence of five or more normal quarks and antiquarks in 56-plet leads to the following possibilities (up to an arbitrary normalization factor and permutation of quark states) in definition of $A$.

I. $U(1) = \mathrm{diag}(1,1,1,b,b,-2b-3)$, $b$ is a parameter

II. $U(1) = \mathrm{diag}(1,1,a,1,1,-a-4)$, $a$ is a parameter

while $A = mU(1) + fZ$

and we have two types of models, respectively.

## THE MODELS OF TYPE I

According to our requirements we set

$$\begin{cases} m+f=\frac{1}{3} \\ -mb+f=-\frac{1}{3} \end{cases} \qquad (14)$$

which guarantees that there are three normal quarks and two normal antiquarks in 56-plet with $Z=+1$, while there are three normal anti-quarks and two normal quarks in 56-plet with $Z=-1$. Since all of these states are two-component spinors, we have totally five normal four-component quarks. It follows from Eq. (14) that

$$m=\frac{2}{3(b+1)}, \qquad f=\frac{b-1}{3(b+1)} \qquad (15)$$

It is convenient to introduce a new parameter

$$n=\frac{-5-b}{3(b+1)} \qquad (16)$$

The eigenvalues of $A$ on quark sextets are:

$$\begin{aligned} Z=+1&: \quad \left(\frac{1}{3},\frac{1}{3},\frac{1}{3},n+\frac{4}{3},n+\frac{4}{3},n-\frac{2}{3}\right) \\ Z=-1&: \quad \left(-n-\frac{2}{3},-n-\frac{2}{3},-n-\frac{2}{3},\frac{1}{3},\frac{1}{3},-\frac{5}{3}\right) \end{aligned} \qquad (17)$$

while on the antiquark ones

$$\begin{aligned} Z=+1&: \quad \left(n+\frac{2}{3},n+\frac{2}{3},n+\frac{2}{3},-\frac{1}{3},-\frac{1}{3},\frac{5}{3}\right) \\ Z=-1&: \quad \left(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},-n-\frac{4}{3},-n-\frac{4}{3},-n+\frac{2}{3}\right). \end{aligned} \qquad (18)$$

It is easy to treat leptons using the fact that 20 of SU(6) is totally antisymmetrized product of three sextets. Hence we have the following eigenvalues of $A$ on leptons:



$$\begin{aligned} Z=+1&: \quad 1^6,\ (-1)^3,\ n^6,\ (-n)^1,\ (n+2)^3,\ (2n+1)^1 \\ Z=-1&: \quad (-1)^6,\ 1^3,\ (-n)^6,\ n^1,\ (-n-2)^3,\ (-2n-1)^1 \end{aligned} \qquad (19)$$

Here $1^6$ means that there are six leptons with $A=1$ and so on. It follows from (19) that using GRS-mechanism we didn't get rid of weird leptons with $A=1$ inhibiting the proton decay in second order in $g$. The masses of weird states must be large enough both in hadronic and leptonic sectors to forbid the proton decay on the kinematic level. It is the necessary condition providing the effectiveness of GRS-mechanism. This condition is much more easy to satisfy than vector boson mass hierarchy.

As to the other leptons, we must know possible values of $n$ before discussing their properties. Let us consider in this view the Higgs sector of the model. To keep Higgs sector minimal, we shall use only those scalar fields which belong to the representations in the direct product 56 56, i.e., 133, 1463 and 1539 giving rise to the fermionic mass term. It was mentioned above that we need the interactions of type (12), so the scalar fields with non-zero $Z$ are present in the Higgs sector as well as with $Z=0$. The charged fields may transform as 133 or 1463. The solution of equations for minimization of the effective potential, a very complicated problem in general, is out of the scope of this paper. Therefore, we shall consider the v.e.v. of a general form, satisfying only the conditions of electric charge, color and new charge $A$-conservation. Without loss of generality, we shall take the matrices of v.e.v. of $Z$-neutral Higgs fields in diagonal form since these matrices are Hermitian and can always be diagonalized through transformations of the symmetry group of the theory. (We will not consider the case of several identical representation with the same $Z$ present in the Higgs sector, as it does not change essentially the situation).

It is evident that matrices of v.e.v. of such fields are invariant under the transformations generated by diagonal generators of SU(6). Therefore these v.e.v.

leave corresponding vector fields massless. It means that v.e.v. of $Z \neq 0$ scalar fields must provide all of these vector fields with mass, except the photon. This requirement turns out to be very restrictive and in conjunction with the condition of invariance under A-transformations fixes the values of the parameter $n$. In what follows we shall assume, guided by considerations of minimality, that there are no Z-charged Higgs fields belonging to <u>1463</u> in the model. Another reason for this assumption is the absence of efficient method to handle with the representations of so high dimensions, while the lowest representations <u>56</u> and <u>133</u> can be described in a very compact and elegant way in the framework of octonion formalism (see ref. /15/) which makes a basis for computations.

Thus, we have in the Higgs sector $\chi^{\circ}$ and $\chi_{\pm}$ belonging to <u>133</u> and possibly $\tilde{\chi}^{\circ}$ belonging to <u>1539</u> and/or <u>1463</u>. $\tilde{\chi}^{\circ}$ and $'\chi^{\circ}$ are assumed to be diagonal. It is clear that to conserve the color symmetry only the color singlets can acquire nonzero v.e.v., i.e., $(35.1^c)$ in <u>133</u>. The spectrum of electric charge on $(35.1^c)$ is

$$\begin{pmatrix} 0 & +1 & +1 & 0 & +1 & +1 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & +1 & +1 & 0 & +1 & +1 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \end{pmatrix} \qquad (20)$$

where 35-plet of SU(6) is represented by the 6x6 matrix and every member of the multiplet is replaced by its charge.

The spectrum of A on $(35.1^c)$ with $Z=+2$, i.e., on $\chi_+$ is



$$\begin{pmatrix} n+1 & n+1 & n+1 & 0 & 0 & 2 \\ n+1 & n+1 & n+1 & 0 & 0 & 2 \\ n+1 & n+1 & n+1 & 0 & 0 & 2 \\ 2(n+1) & 2(n+1) & 2(n+1) & n+1 & n+1 & n+3 \\ 2(n+1) & 2(n+1) & 2(n+1) & n+1 & n+1 & n+3 \\ 2n & 2n & 2n & n-1 & n-1 & n+1 \end{pmatrix} \qquad (21)$$

while the spectrum on $\chi_-$ can be obtained from (21) by transposing and changing total sign. Only the neutral with respect both to Q and to A scalar fields can possess nonzero v.e.v., i.e., $\chi_{+\,14}$, $\chi_{+\,25}$ and $\chi_{+\,35}$ if $n$ is arbitrary. However, the 6x6 matrix $\langle\chi_+\rangle$ with such nonzero matrix elements commutes with a diagonal traceless matrix of the form

$$\mathrm{diag}(\alpha, \beta, \beta, \gamma, \beta, -\alpha-\gamma-3\beta) \quad .$$

having three arbitrary parameters. Hence in the theory with such a $\langle\chi_+\rangle$ there are two extra massless colourless vector fields apart from the photon. To have the single colourless massless vector field just the photon, some extra restrictions on the value on $n$ are needed which would provide that the only generator commuting with $\langle\chi_+\rangle$ would be the electric charge. It is clear that $n \neq -1$ since otherwise all the leptons in the model are weird. There remain two possibilities *

$$n = 0 \quad \text{and} \quad n = -3$$

both leading to removal of superfluous massless vector fields. We shall emphasize now that our model is vector-like in leptonic sector as it follows from Eq. (19). Hence we conclude that leptons $\nu_\mu$, $\mu$, $\nu_e$, e having well-

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*Note that if $\chi_+$ were 1463 than two other values of $n$ would appear: $n=-1/2$, $n=-2$. The method to obtain them is the same as the one used below in discussion of the models of type II.

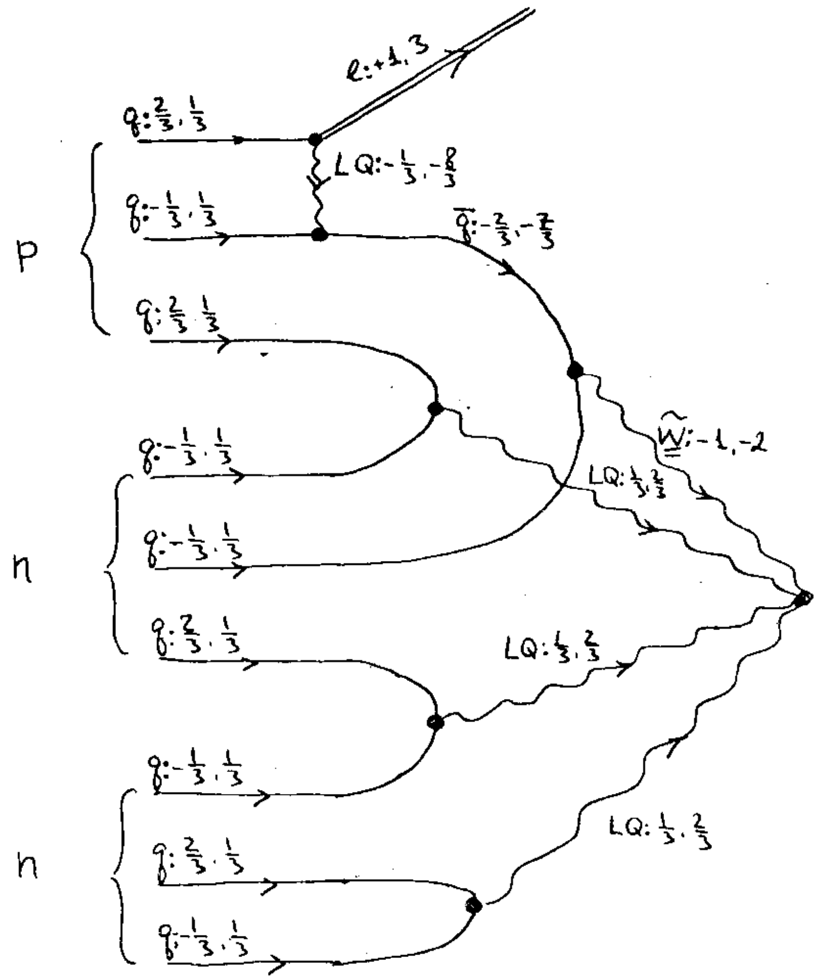


*Fig.3. A virtual transition of tritium to lepton.* $\tilde{W}$ *here denotes vector colourless field corresponding to SU(6)/SU(2)$_{weak}$ while normal leptons have* $A=3$. *The quantum numbers of all particles are shown, the first is electric charge and second is* A.

defined V–A structure of weak interactions must have $A=\pm n$, since all other leptonic states are not mixed via Eq. (12) and, therefore, have pure vector weak couplings. The $SU(2)_{weak}$ structure of lepton multiplet is as follows: one triplet, one doublet and one singlet with $A=n$,



one singlet with $A=-n$, one doublet and one singlet with $A=n+2$ and one singlet with $A=2n+1$. All singlets are electrically neutral.

Let us examine now the models of type I in view of phenomenological applications.

### *THE MODEL OF TYPE I WITH* $n=-3$

In this case the spectrum of A is

$$Z=+1:\ \left(\frac{1}{3},\frac{1}{3},\frac{1}{3},-\frac{5}{3},-\frac{5}{3},-\frac{11}{3}\right)$$
$$Z=-1:\ \left(\frac{7}{3},\frac{7}{3},\frac{7}{3},\frac{1}{3},\frac{1}{3},-\frac{5}{3}\right) \qquad \text{(quarks)} \quad (17')$$

$$Z=+1:\ \left(-\frac{7}{3},-\frac{7}{3},-\frac{7}{3},-\frac{1}{3},-\frac{1}{3},\frac{5}{3}\right)$$
$$Z=-1:\ \left(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{5}{3},\frac{5}{3},-\frac{11}{3}\right) \qquad \text{(antiquarks)} \quad (18')$$

$$Z=+1:\quad 1^6,\ (-1)^6,\ (-3)^6,\ 3^1,\ (-5)^1$$
$$Z=-1:\quad (-1)^6,\ 1^6,\ 3^6,\ (-3)^1,\ 5^1 \qquad \text{(leptons)} \quad (19')$$

All the normal leptons have nonzero "baryon number" $A=\pm 3$. It seems unusual, however, this possibility is quite reliable. Indeed the effects produced by the nonzero "baryon number" of leptons which is larger than 1 can manifest only in decays of many-nucleon systems (nuclei). Really, these processes in the model under consideration take place in high orders in g (see *Fig. 3*) and, what is more, have a powerful damping factor due to the fact that average distance between protons in nucleus is much bigger (in appropriate units) than inverse leptoquark mass determining the effective radius of such an interaction. Therefore, even if $M_{LQ}\sim m_W \sim 10^2 GeV/c^2$ the processes of such a kind are practically unobservable. However, a possibility of existence of a common for leptons and hadrons charge is of a fundamental meaning and may play an important role, e.g., in astrophysics.

Further, in the hadronic sector of the model we have five normal quarks placed into $SU(2)_{weak}$ multiplets as in (13), a big number of weird quarks which must be heavy and can form an anomalous baryons with baryon number 5,7, etc., and weird mesons with nonzero baryon number. The strong interactions are governed by colour $SU^c(3)$.

In the leptonic sector of the model its weak structure causes certain difficulties. The presence of the only $SU(2)_{weak}$ doublet necessitates the triplet assignment for electron. (It is clear that we cannot place $\nu_\mu$ and $\mu$ in triplet, since the difficulties with $\nu_\mu$-neutral current immediately arise). The universality of weak interactions will be preserved if the left multiplets are:

$$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L , \begin{pmatrix} \cdots \\ \frac{1}{\sqrt{2}}\nu_e + \cdots \\ e^- \end{pmatrix}_L \pm \text{ left singlets} \qquad (22)$$

Since the decay channel

$$\tau^+ \to p + \dots$$

is absent it allows us to exclude the possibility that triton is the weird lepton. Hence there must be a place for $\tau$ among the normal leptons. A single vacancy is in the triplet where electron resides. Therefore triton is para-electron with conventional V–A weak couplings. However, such a possibility seems to be ruled out by experiment, see /18/.

*THE MODEL OF TYPE I WITH $n=0$*

The A-spectrum of quarks:

$$Z=+1: \quad (\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{4}{3}, \tfrac{4}{3}, -\tfrac{2}{3})$$
$$Z=-1: \quad (-\tfrac{2}{3}, -\tfrac{2}{3}, -\tfrac{2}{3}, \tfrac{1}{3}, \tfrac{1}{3}, -\tfrac{5}{3}) \qquad (17'')$$



The A-spectrum of antiquarks:

$$Z=+1: \quad (\tfrac{2}{3}, \tfrac{2}{3}, \tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, \tfrac{5}{3})$$
$$Z=-1: \quad (-\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{4}{3}, -\tfrac{4}{3}, \tfrac{2}{3}) \qquad (18'')$$

The A-spectrum of leptons:

$$Z=+1: \quad 1^7, (-1)^3, \ 0^7, 2^3$$
$$Z=-1: \quad (-1)^7, 1^3, \ 0^7, (-2)^3 \qquad (19'')$$

The structure of the model in the hadronic sector is the same as in the previous case. But in the leptonic sector there is an opportunity to avoid the difficulty with $\tau$-lepton by placing it together with its neutrino in a weak doublet with $A=2$. We want to emphasize that in this model the usual leptons possess the zero value of baryon number in a quite orthodox way and the decays of nuclei with the $\tau$-leptons emission are forbidden kinematically as $m_\tau \sim 2m_p$. However, as was pointed above, such an assignment of triton and its neutrino (when $A=2$ ) necessitates pure vector interactions of these leptons. Within the present accuracy of experimental data such a possibility cannot be ruled out, though it seems to be unlikely /18/.

If we choose the left-handed multiplets in this model as (with possible replacement $E \to e$ ):

$$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \begin{pmatrix} E^+ \\ \frac{1}{\sqrt{2}}\nu_e + \frac{1}{\sqrt{2}}N(\theta,\theta') \\ e^- \end{pmatrix}_L , \ N'_L(\theta,\theta'), \ N''_L(\theta,\theta') \qquad (23)$$

(here $N(\theta,\theta')$ means arbitrary mixing of three neutral leptons), then the parameters of spontaneous breakdown must be chosen to provide the following assignment in right multiplets (with possible replacements $\mu \to e$, $E \to \mu$, $E \to e$ )

$$\begin{pmatrix} N(\tilde\theta,\tilde\theta') \\ \mu^- \end{pmatrix}_R \begin{pmatrix} E^+ \\ N'(\theta,\theta') \\ e^- \end{pmatrix}_R , \quad \nu_{e_R}, \nu_{\mu_R}, \quad N''_R(\tilde\theta,\tilde\theta') \tag{24}$$

When $A=2$, we have

$$\begin{pmatrix} \tau^+ \\ \nu_\tau \end{pmatrix}_L \begin{pmatrix} \tau^+ \\ \nu_\tau \end{pmatrix}_R , \quad N_{\tau_L}, \; N_{\tau_R} . \tag{25}$$

Here we assumed that neutrinos are four-component spinors and have a small mass.

It is not our goal to give the detailed analysis of possible applications of such a model for phenomenological purposes. We want to emphasize only that i) all the mixings and masses of particles may be calculated and expressed in terms of v.e.v. The $\chi^\circ$ from 133 is not enough and 1463 and/or 1539 is needed; the agreement with the observed spectrum of masses can be achieved but is hardly interesting, as the number of free parameters becomes very large, ii) such a scheme is extremely non-orthodox due to the triplet assignment for electron, the absence of apparent $\mu - e$ universality, etc. iii) the severe trouble arises, like in other unified models, in description of neutral current phenomena. In this model $\sin^2\theta_W = 3/4$ and this value of Weinberg angle remains unrenormalized due to the absence of superheavy particles. A possible way to avoid the latter difficulty is to introduce several neutral vector bosons with approximately equal masses.

Now let us pass to the models of type II.

## *THE MODEL OF TYPE II*

We again require that

$$\begin{cases} m + f = \frac{1}{3} \\ -ma + f = -\frac{1}{3} \end{cases}$$



and introduce the parameter

$$n = \frac{-5-a}{3(a+1)} .$$

Then we have four normal quarks and one normal antiquark in 56-plet with $Z = +1$.

The spectrum of $A$ is
On quarks:

$$Z = +1: \quad \left(\frac{1}{3}, \frac{1}{3}, \; n+\frac{4}{3}, \; \frac{1}{3}, \; \frac{1}{3}, \; 2n+\frac{1}{3}\right) \tag{26}$$

$$Z = -1: \quad \left(-n-\frac{2}{3}, \; -n-\frac{2}{3}, \frac{1}{3}, -n-\frac{2}{3}, \; -n-\frac{2}{3}, \; n-\frac{2}{3}\right)$$

on antiquarks:

$$Z = +1: \left(n+\frac{2}{3}, n+\frac{2}{3}, -\frac{1}{3}, n+\frac{2}{3}, \; n+\frac{2}{3}, \; -n+\frac{2}{3}\right)$$
$$Z = -1: \left(-\frac{1}{3}, -\frac{1}{3}, -n-\frac{4}{3}, -\frac{1}{3}, -\frac{1}{3}, \; -2n-\frac{1}{3}\right) \tag{27}$$

on leptons:

$$Z = +1: \quad 1^6, \; n^6, \; (-n)^4, \; (2n+1)^4$$
$$Z = -1: \quad (-1)^6, (-n)^6, \; n^4, \; (-2n-1)^4 \tag{28}$$

The theory again is vector-like. The leptonic fields with $A = 2n+1$ have pure vector interactions. Therefore, usual leptons must reside in the set of 10 states with $A = \pm n$. There are two weak doublets, one triplet and three singlets among them.

Considering the Higgs sector of the model we assume as before that only $\chi$'s belonging to 133 have non-zero $Z$-charge. The spectrum of $A$ on such an $\chi_+$ is:

$$\begin{pmatrix} n+1 & n+1 & 0 & n+1 & n+1 & -n+1 \\ n+1 & n+1 & 0 & n+1 & n+1 & -n+1 \\ 2(n+1) & 2(n+1) & n+1 & 2(n+1) & 2(n+1) & 2 \\ n+1 & n+1 & 0 & n+1 & n+1 & -n+1 \\ n+1 & n+1 & 0 & n+1 & n+1 & -n+1 \\ 3n+1 & 3n+1 & 2n & 3n+1 & 3n+1 & n+1 \end{pmatrix} \tag{29}$$

Equation (29) together with Eq. (20) reduces the possible non-zero v.e.v. to two ones: $\langle\chi_+\rangle_{23}$ and $\langle\chi_+\rangle_{53}$. These v.e.v. give, as is easy to see, two massless colourless vector fields, besides photon. So we need to impose some condition on $n$. Since we cannot adopt $n = -1/3$ without destroying the GRS-mechanism, the only possibility is $n = 0$. However, even in this case the superfluous massless field survives. Hence, we come to the conclusion that it is necessary to consider $\chi_\pm$ belonging to $\underline{1463}$ in the models of type II. It considerably complicates the computations, but even when the Higgs is so complicated it is possible to fix the value of $n$ as follows.

Let us define the spectrum of $A$ on the colourless part of symmetrized direct product of two $\underline{56}$'s using the $SU(6)xSU^c(3)$ reduction (1). This part of the direct product is formed by the direct products of the following SU(6) multiplets:

$$\underline{20} \otimes \underline{20}' + \underline{6} \otimes \underline{\bar{6}}' + \underline{\bar{6}} \otimes \underline{6}'.$$

Taking into account that

$$\underline{56} \otimes \underline{56}_S = \underline{133} + \underline{1463}$$

and also (2) and $SU(6)xSU^c(3)$ reduction of 1463-plet:

$$\underline{1463} = (1.1^c) + (35.1^c) + (175.1^c) + (\overline{15}.3^c) + (15.\bar{3}^c) + (105.3^c) +$$
$$+ (\overline{105}.3^c) + (21.6^c) + (\overline{21}.\bar{6}^c) + (35.8^c) \qquad (30)$$

we find that colourless part of $\underline{1463}$ is contained in the direct product $\underline{20} \otimes \underline{20}_S$. Now remembering that $\underline{1463}$ has $Z = 2$ and U(1) of $A$ acts additively on the direct product, we can easily find the spectrum of $A$ with the help of (28).

$$A = 0, \pm 2n, 2, 2(2n+1), \quad 2(n+1), \quad 3n+1, 1 \pm n. \qquad (31)$$

Among the states with $A = 0$ there are 10 electrically neutral ones which again leave the superfluous vector



fields massless. To avoid this, we have two possibilities of fixing (see (31))

$$n = 0 \qquad n = -\frac{1}{2}.$$

Each of them is enough to remove unwanted massless fields. Hence in the models of type II the values of $n$ are also strictly fixed.

There are six (instead of five) normal quarks in the model provided $n = 0$ and it coincides with one proposed by GRS in this case.

The structure of weak interactions of normal leptons in both models of this type (with $n = 0$ and $n = -1/2$, respectively) is the same and being less radical than in the type I models admits the standard assignment of electron and muon together with their neutrinos in weak doublets. The nonzero value of $n$ doesn't lead to difficulties (see *Fig. 4*). There are two charged lepton states entering into the $SU(2)_W$ triplet and in the case $n = -1/2$ still two weak doublets with pure vector interactions. All of them may be used for triton assignment and give us a wide range of possibilities in this direction. However, in all of these cases the weak interactions of $\tau$-lepton are distinct from those of $e$ and $\mu$. The possible lepton assignment with respect to $SU(2)_{weak}$ is ($n = 0$)

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \begin{pmatrix} \tau^+ \\ \nu_\tau \\ T^- \end{pmatrix}_L \quad N_L, N'_L, N''_L \qquad (32)$$

$$\begin{pmatrix} N \\ e^- \end{pmatrix}_R \begin{pmatrix} N' \\ \mu^- \end{pmatrix}_R, \begin{pmatrix} \tau^+ \\ N'' \\ T^- \end{pmatrix}_R, \quad \nu_{\mu_R}, \nu_{e_R}, \nu_{\tau_R} \qquad (33)$$

(There is a possibility of placing $e_R$ or $\mu_R$ or both in triplet in (33). Possible mixings are not shown).

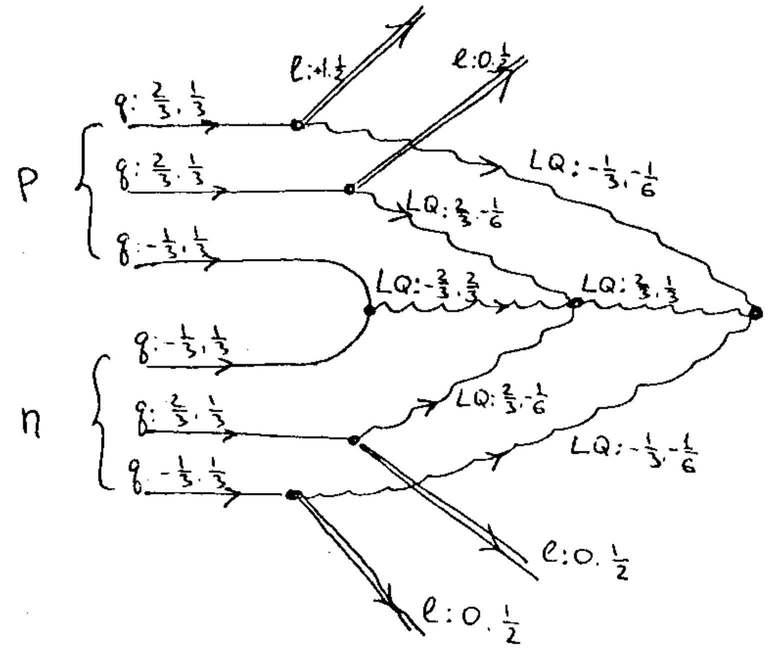


*Fig. 4. The leptonic decay of deuterium in the model with* $n = \frac{1}{2}$.

The hadronic sector of the model is the same as in (13) with additional normal quark with $Q=-1/3$ in the case $n=0$. This quark forms the left weak singlet and gives a chance (provided the v.e.v. of scalar fields have appropriate values) to avoid the appearance of s-quark in the right weak doublets. The mentioned problem of neutral currents remains in the model of this type as well as in the previous ones. The usage of Higgs fields of high dimensions, which have the v.e.v. matrices of non-diagonal form extremely complicates the computations.



*CONCLUSIONS*

Thus imposing the requirements of minimal agreement with experiment, i.e., the single photon existence, the V–A character of weak interactions, the presence of at least four normal quarks, we have determined all possible models in the framework of the exceptional $E_7$-gauge theory with GRS-mechanism. All the basic properties of these models, i.e., the structure of weak interactions, the composition of the weak multiplets, the values of the new charge A, the pattern of symmetry breaking, are fixed by these requirements. The following common features characterize these models: 1) absolute stability of proton, 2) existence of weird particles, namely, leptons with "baryon number" 1, which can decay emitting proton, baryons with baryon number other than 1 and mesons with nonzero baryon number, 3) a possibility to have nonzero quasi-baryon number for standard leptons; it is important that its possible values are fixed, 4) a possibility for lepto-quarks to have a mass comparable with that of intermediate vector bosons, 5) necessity to use the large number of Higgs fields belonging to the high-dimensional representations.

The latter property is related to the fact that in the theories under consideration the pattern of symmetry breakdown is as much important as the choice of an original gauge group. Since the masses of particles, mixing and final structure of interactions appear as a result of symmetry breaking, it is the pattern of symmetry breakdown which in grand total determines the theory. It is clear that in unified schemes the set of requirements to be satisfied via spontaneous breakdown is large due to their universal character and necessitates the usage of cumbersome Higgs multiplets which, in their turn, extremely complicate all the computations and solution of equations for v.e.v. (e.g., even in the case when it is possible to fix the numerical values of coupling constants in scalar field potential, the solution of corresponding equations is a very hard problem, see /9/ ). Besides, the existence of such a vast amount of scalar fields is unsatisfactory. It is quite possible that the Higgs mechanism being used in some semiphenomenological description of more profound effects, e.g., the dynamical symmetry breakdown, see [19]. On this ground, it seems to us that deeper understanding of effects related to the symmetry breaking is needed for progress along the way of building the unified scheme which would be capable to give rigorous quantitative results.

So, there is a lot of difficulties but, nevertheless, the idea of grand unification is very attractive and this fact has stimulated us to analyse the general restrictions on the models in the grand-unified exceptional models.

*REFERENCES*


1. *Georgi H., Glashow S.L. Phys.Rev.Lett., 1974, 32, p.438.*
2. *Fritzch H., Minkowski P. Ann. of Phys. (N.Y.), 1975, 93, p.193.*
3. *Pati J.C., Salam A. Phys. Rev., 1973, D8, p.1240; ibid., 1974, D10, p.275.*
4. *Gursey F. In: Mathematical Problems in Theoretical Physics. Ed. H.Araki, Springer, 1975, p.189.*
5. *Gursey F., Ramond P., Sikivie P. Phys. Lett., 1976, 60B, p.177.*
6. *Gursey F., Sikivie P. Phys. Rev.Lett., 1976, 36,p.775.*
7. *Ramond P. Nucl.Phys., 1976, B110, p.214; ibid., 1977, B126, p.109.*
8. *Sikivie P., Gursey F. Phys. Rev., 1977, D16, p.816.*
9. *Fradkin E.S., Kalashnikov O.K., Konstein S.E. Lett. Nuovo Cim., 1978, 21, p.5.*
10. *Reines F., Crouch M.F. Phys. Rev.Lett., 1974, 32, p.493.*
11. *Buras A.J. et al. CERN Preprint TH-1304, October, 1977.*
12. *Georgi H., Quinn H., Weinberg S. Phys. Rev.Lett., 1974, 33, p.451.*
13. *Gildener E. Phys. Rev., 1976, D14, p.1667.*
14. *Gell-Mann M., Ramond P., Slansky R. Los-Alamos Preprint LA-UR-77-2059.*
15. *Ogievetsky V., Tzeitlin V. J.Phys.A, 1978, 11, p.1419.*
16. *Mehta M.L., Srivastava P.K. Journ. of Math. Phys., 1966, 7, p.1833.*
17. *Harari H. Weizmann preprint WIS-77/56-Ph., November, 1977.*
18. *Perl M.L. Talk at the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, August, 1977.*
19. *Jackiw R., Johnson K. Phys. Rev., 1973, D8, p.2386.*




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