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A.V.Efremov, A.V.Radyushkin

**FIELD-THEORETIC TREATMENT  
OF HIGH MOMENTUM TRANSFER PROCESSES.  
II. MASSIVE LEPTON-PAIR PRODUCTION**

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Ефремов А.В., Радюшкин А.В.

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Теоретико-полевой подход к процессам с большой передачей импульса. II. Рождение массивных лептонных пар

Развит теоретико-полевой подход к исследованию асимптотики процесса рождения массивных лептонных пар в адронных столкновениях, основанный на использовании альфа-представления фейнмановских диаграмм. Применительно к этому процессу получено операторное разложение на световом конусе.

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Efremov A.V., Radyushkin A.V.

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Field-Theoretic Treatment of High Momentum Transfer Processes. II. Massive Lepton-Pair Production

Field-theoretic approach is developed for the investigation of massive lepton-pair production in hadronic collisions, which is based on the alpha-representation of the Feynman diagrams. The light cone operator product expansion for this process is obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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## INTRODUCTION

The process of massive lepton-pair production in hadronic collisions  $AB \rightarrow \mu^+ \mu^- X$  plays an important role for the verification of parton model ideas. The well-known parton model formula established by Drell and Yan<sup>/1/</sup> expresses the cross-section of massive lepton pair production in terms of quark ( $f_a$ ) and antiquark ( $f_{\bar{a}}$ ) distribution functions inside the colliding hadrons  $A, B$ \*

$$\frac{d\sigma}{dQ^2} \Big|_{AB \rightarrow \mu^+ \mu^- X} = \frac{4\pi\alpha^2}{3Q^4} \tau \frac{1}{N_c} \sum_a e_a^2 \int_0^1 dx \int_0^1 dy \quad (0.1)$$

$$\delta(xy - \tau) \{ f_{a/A}(x) f_{\bar{a}/B}(y) + [A \leftrightarrow B] \}.$$

In principle, one can obtain the distribution functions from the data on deep inelastic scattering, hence the parton model leads to the scaling law  $d\sigma/dQ^2 = c/Q^4$  with  $c$  being a determinable constant. Recent experimental data, however, point out that the Bjorken scaling law in deep inelastic scattering is violated (see, e.g., ref.<sup>/2/</sup>): the parton distribution functions have indeed an appreciable dependence on the virtuality of the photon

\* The sum here is taken over quark flavours ( $a=u, d, s, c, \dots$ ), the factor  $1/N_c$  is due to colour averaging ( $N_c=3$ ),  $e_a$  is the quark electric charge divided by that of electron.

that probes the structure of the hadron:  $f(x) \rightarrow f(x, Q^2)$ . It was argued<sup>/3-12/</sup> that in the Drell-Yang formula (0.1) one must also use  $f(x, Q^2)$  in place of  $f(x)$ . This argumentation is based on various parton model modifications which take into account the violation of the scaling laws by the logarithmic corrections inherent to renormalizable theories. A further progress is however complicated due to the absence of a general field-theoretic approach to the process  $AB \rightarrow \mu^+ \mu^- X$ , i.e., the approach which can provide a basis for the parton model rather than that taking the parton model as a starting point.

In this paper, which is a sequel to ref.<sup>/13/</sup>, we apply the approach developed there to a study of massive lepton-pair production within the framework of nongauge field theories. Using the  $\alpha$ -representation analysis leads, in a superrenormalizable  $\phi_{(4)}^3$ -theory, to the Drell-Yan formula. In this case the ordinary "naive" parton model is valid as expected. In the  $\phi_{(6)}^3$ -theory one must take into account effects due to the renormalizability of the theory. This results in a modified parton description which uses parton distribution functions  $f(x, \mu^2)$  depending on an additional renormalization parameter related to a subtraction procedure analogous to that used in ref.<sup>/13/</sup>. This subtraction procedure is in essence a reordering of perturbation series terms which gives an expression more suitable for the analysis of asymptotical properties.

The resulting expression is analogous to operator product expansion on the light cone. This allows us to calculate asymptotical behaviour of two characteristics for the  $AB \rightarrow \mu^+ \mu^- X$  process: a) the total cross-section for production of a pair with mass  $Q$ , i.e.,  $d\sigma/dQ^2$ , and b) the differential cross section for production of a pair with large transverse momentum, i.e.,  $d\sigma/dQ^2 dQ_{\perp}^2$  in the region  $Q_{\perp}^2 \gg M^2$ .

#### 1. ANALYSIS OF MASSIVE LEPTON-PAIR PRODUCTION IN THE $\alpha$ -REPRESENTATION

Let us consider first the structure function

$$W(P_A, P_B, Q) = \int e^{iQx} \langle P_A P_B | j(x) j(0) | P_A P_B \rangle d^4x, \quad (1)$$

which is a scalar analog of the function describing the production of a lepton pair having mass  $Q$ . Scalar currents  $j(x)$  are defined as in ref.<sup>/13/</sup> (eq. (1.3))\* . In fig. 1 the subgraphs are shown the contraction of which into point eliminates the dependence of the diagram on the large variables  $s, t, u, Q^2$  ( $s=2(P_A P_B)$ ,  $t=-2(P_A Q)$ ,  $u=-2(P_B Q)$ ). In addition, these subgraphs have a minimal number of external lines. Fig. 1a corresponds to the Drell-Yan configuration. (The configuration shown in fig. 1b does not contribute to the cross-section for kinematical constraints (see part 4 of this paper)).

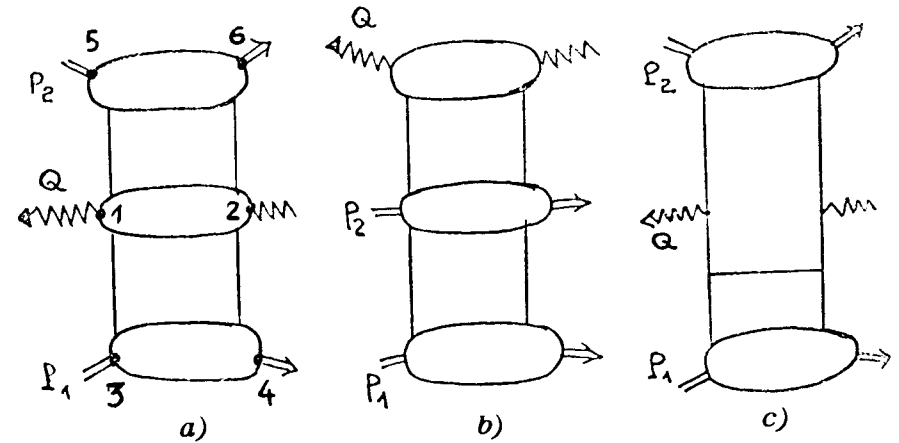


Fig. 1

A minimal  $s, t, u, Q^2$ -subgraph in this case is of an order of  $O(g^2)$  (fig. 1c) and describes a parton subprocess of  $2 \rightarrow 2$  type, for instance  $\psi\psi^* \rightarrow \gamma\phi$ , rather than the Drell-Yan annihilation  $\psi\psi^* \rightarrow \gamma$ .

The  $\alpha$ -representation analysis forces us, at the first sight, to made a conclusion that there is no Drell-

\* Henceforth eq. (1.N) means eq. (N) from ref.<sup>/13/</sup>.

Yan mechanism at all. But really there is no contradiction with parton model ideas because the Drell-Yan mechanism corresponds to the so-called "pinch" singularity, given by large  $-a$  integration, rather than to the "end-point" one, given by small  $-a$  integration. Let us write  $W(P_A, P_B, Q)$  in the  $\alpha$ -representation

$$W(P_A, P_B, Q) \sim \int_0^{i\infty} \prod_{\sigma} d\alpha_{\sigma} D^{-N/2}(\alpha) \exp \left\{ \frac{1}{D(\alpha)} (s A_{-}(\alpha) + t A_t(\alpha) + u A_u(\alpha) + Q^2 A_{Q^2}(\alpha)) + I(\alpha, m^2) \right\}, \quad (2)$$

where  $A_{-}(\alpha) = B_{-}(34|56)$ ;  $A_t = B_{-}(12|34)$ ,  $A_u = B_{-}(12|56)$ ,  $A = B(1|2)$  (fig. 1a). Representing  $Q = \lambda P_A + \rho P_B + Q_{\perp}$  and taking into account that for a Drell-Yan diagram  $A_{-} = A_{-}^L A_{-}^R$ ,  $A_t = A_{-}^L D_0^R$ ,  $A_u = D_0^L A_{-}^R$ ,  $A = D_0^L D_0^R$  (where  $A_{-}^L = B(12|34; V^L)$ ,  $A_{-}^R = B_{-}(12; V^R)$ ,  $D_0^L = D_0(V_L)$ ;  $D_0^R = D_0(V_R)$ ) (see ref.<sup>/13/</sup>) we obtain the following representation for the exponential factor entering into eq. (2):

$$\exp \left\{ \frac{s}{D(\alpha)} [(A_{-}^L - \lambda D_0^L)(A_{-}^R - \rho D_0^R) - \tau_{\perp} D_0^L D_0^R] + I(\alpha, m^2) \right\} \equiv \exp \left\{ s \frac{F(\alpha)}{D(\alpha)} + I(\alpha, m^2) \right\}, \quad (3)$$

where  $\tau_{\perp} = Q_{\perp}^2 / s$ .

It was shown by Tiktopoulos<sup>/14/</sup> (see also Appendix) that the pinch singularity, i.e., the vanishing of  $F(\alpha)$  at nonzero  $\alpha$ , gives a leading asymptotical contribution only if  $F(\alpha) = F_1(\alpha) F_2(\alpha)$ , where both the factors  $F_1(\alpha)$ ,  $F_2(\alpha)$  do vanish at nonzero  $\alpha$ . In our case the requirement  $F = F_1 F_2$  can be satisfied only if  $\tau_{\perp} = O(m^2/s) \rightarrow 0$ , i.e., for small transverse momentum of the lepton pair,  $Q_{\perp}^2 = Q(m^2)$ . Then, from eq. (A.2) it follows that

$$W^{+}(P_A, P_B, Q) \sim \frac{1}{Q^2} \int_0^{i\infty} \frac{\prod_{\sigma} d\alpha_{\sigma}}{D^{N/2}(\alpha)} \delta\left(\lambda - \frac{A_{-}^L}{D_0^L}\right) \delta\left(\rho - \frac{A_{-}^R}{D_0^R}\right) \frac{D}{D_0^L D_0^R} \exp \left\{ -Q_{\perp}^2 \frac{D_0^L D_0^R}{D} + I(\alpha, m^2) \right\}. \quad (4)$$

Integrating over transverse momentum  $Q_{\perp}$  gives the expression

$$\int W^{+}(P_A, P_B, Q) d^{N-2} Q_{\perp} \sim \frac{1}{Q^2} [f_{a/A}(\lambda) f_{\bar{a}/B}(\rho) + (A+B)] \quad (5)$$

which has a natural parton interpretation, because the factors  $f(\lambda)$ ,  $f(\rho)$  are distribution functions for partons having momenta  $\lambda$  and  $\rho$ , respectively.

The existence of the Drell-Yan mechanism is essentially connected with the non-Euclidean nature of the space-time. In the Euclidean space there are no pinch singularities, because in that case  $(p_{i_1} + \dots + p_{i_n})^2 > 0$  for any set of momenta, and moreover  $(A_{i_1 \dots i_n}(\alpha)) \geq 0$  (see. eq. (1.A.2)).

When treating deep inelastic scattering (ref.<sup>/13/</sup>) we have considered "Euclidean" region where  $|s| < Q^2$ . This allowed us not to take into account pinch singularities. But it is impossible to represent the expression  $F = \omega A_{-}(\alpha) - A(\alpha)$  as  $F = F_1 F_2$ , hence the pinch singularities do not contribute in the region  $|s| > Q^2$  also. The vanishing of  $F(\alpha)$  at nonzero  $\alpha$  (i.e., when  $\omega = A_{-}(\alpha)/A(\alpha)$ ) gives only a cut for  $|\omega| > 1$ . That is why it is justifiable to investigate  $T(\omega, Q^2)$  at  $|\omega| < 1$  and to perform an analytical continuation of the resulting expression into the essentially non-Euclidean region  $|\text{Re } \omega| > 1$  with the help of the Mellin representation.

Investigation and summation of the pinch contributions is much more complicated problem than that of end-point ones. There is no justification for any appeal to asymptotic freedom, because the region of large  $\alpha$  (or small  $p^2$  in momentum representation) dominates. Hence there is also no reason to expect than higher order cor-

rections will be smaller than the lowest order approximation.

In the large transverse momentum region  $Q_{\perp}^2 = O(s)$  the factorization  $F = F_1 F_2$  does not hold and it is sufficient to consider only small- $a$  singularities. The Born term for production of a pair at high transverse momentum corresponds to a process  $2 \rightarrow 2$ : it is necessary to produce a particle that balances the transverse momentum of a pair.

## 2. ANALYSIS OF THE TOTAL CROSS SECTION $d\sigma/dQ^2$

Although the leading contribution to the form factor  $W(P_A, P_B, Q)$  describing differential cross-section  $d\sigma/d^4Q$  is given by the pinch singularity, it is possible with the help of the well-known transformation<sup>15/</sup> to "euclideaze" the form factor  $W(\tau, Q^2)$  proportional to  $d\sigma/dQ^2$  the total cross-section of producing the pair with mass  $Q$ .

The functions  $W(P_A, P_B, Q)$  and  $W(\tau, Q^2)$  are related by

$$W(\tau, Q^2) = Q^2 \int \frac{d^N k}{(2\pi)^N} W(P_A, P_B, k) \delta^+(k^2 - Q^2) \theta(P_A^0 + P_B^0 - k^0), \quad (6)$$

where  $\tau = Q^2/s$ . The function  $W(\tau, Q^2)$  can be treated as an appropriate discontinuity of the function  $T(\tau, Q^2)$ :

$$T(\tau, Q^2) = Q^2 \int d^N x D^c(x, Q^2) \langle P_A P_B | T(j(x)j(0)) | P_A P_B \rangle \quad (7)$$

$$W(\tau, Q^2) = \frac{1}{2\pi} \text{Disc}_{s, (Q^2)} T(\tau, Q^2). \quad (8)$$

The notation  $\text{Disc}_{s, (Q^2)}$  means that the discontinuity corresponds to such a slicing which must cross the line of a fictitious particle with mass  $Q$  (fig. 2a). Hence slicing shown in fig. 2b does not give a contribution to  $\text{Disc}_{s, (Q^2)}$ .

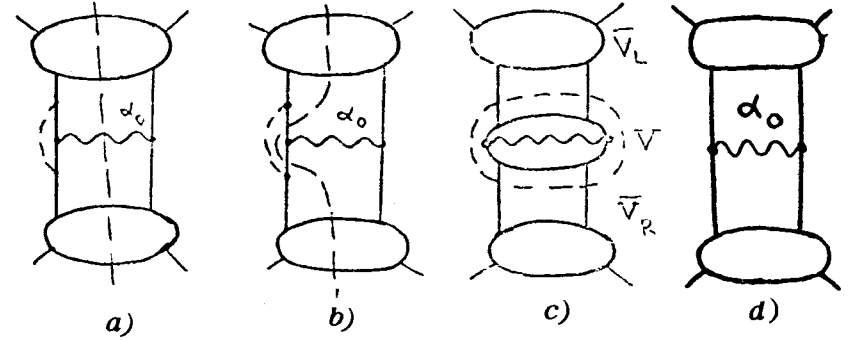


Fig. 2

The function  $T(\tau, Q^2)$  has the  $a$ -representation similar to that of eqs. (2), (3) but for the factor

$$F = \frac{A_-(a)}{D(a)} - a_0 \frac{Q^2}{s}, \text{ which cannot be represented as } F = F_1 F_2,$$

hence the vanishing of  $F$  at nonzero  $a$  results only in the appearance of cuts. In distinction from deep inelastic scattering these cuts are of two types. The first type corresponds to fig. 2a whereas the second one corresponds to fig. 2b.

A general form of the  $s, Q^2$ -subgraph having a minimal possible number of external lines (equal to 4) is shown in fig. 2c. It must contain the  $a_0$ -line. In a superrenormalizable  $\phi_{(4)}^3$ -theory the leading contribution is given by the subgraph shown in fig. 2d. It consists of the  $a_0$ -line only. Integrating over  $a_0 \sim 0$  and taking into account the factorization properties  $D_{a_0} D_0(V_L) D_0(V_R), A_- = a_0 \bar{A}_-(V_L) \times \bar{A}_-(V_R); I(a, m^2) \stackrel{\approx}{=} I_0^R(a, m^2) + I_0^L(a, m^2)$

gives a formula of the Drell-Yan type

$$W(\tau, Q^2) = \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \sum_a \kappa_a^2 \delta(1 - \frac{xy}{\tau}) \{ f_{a/A}(x) f_{a/B}(y) + (A \leftrightarrow B) \}. \quad (9)$$

The functions  $f_a(x)$ ,  $f_{\bar{a}}(y)$  are the same distribution functions as those used in ref. <sup>/13/</sup> given by eq. (1.7).

To treat a renormalizable  $\phi_{(6)}^3$ -theory we write the amplitude  $T$  in the Mellin representation

$$T^\pm(r, Q^2) = Q^2 \int_{-i\infty}^{i\infty} \frac{dj dJ}{(2\pi i)^2} \frac{\pi}{\sin \pi j} \left(\frac{1}{r}\right)^j (Q^2)^J \frac{1 \pm e^{i\pi j}}{2} \times \\ \times \Phi^\pm(J, j, m^2) \quad (10)$$

$$\Phi^\pm(J, j, m^2) = \frac{\Gamma(j-J)}{\Gamma(j+1)} \sum_a \kappa_a^2 \sum_{\text{diagr.}} (g^2/(4\pi)^3)^Z \int_0^\infty \prod_\sigma da_\sigma \times \\ \times a_0^{J-j} \left| \frac{A_-}{D_0} \right|^j \exp \{I(a, m^2)\}. \quad (11)$$

Now we must construct the procedure of singling out the poles in  $J$ . Let  $V$  be a maximal  $s, Q^2$ -subgraph having 4 external lines (fig. 3a). Integrating over the region  $\lambda_V \sim 0$  singles out a pole contribution

$$\Phi(J, j, m^2) = \left(\frac{1}{\mu^2}\right)^{J+1} C_V(J, j) \tilde{f}(j, m^2, \bar{V}_L) \times \\ \times \tilde{f}(j, m^2, \bar{V}_R) + \Phi_{\text{reg.}}^V(J, j, m^2, \mu^2). \quad (12)$$

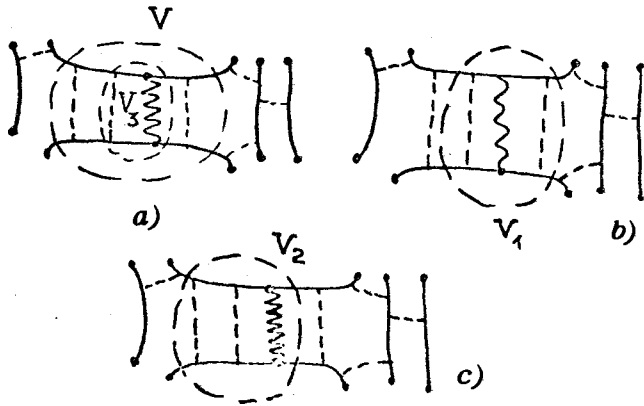


Fig. 3

The factorization of the pole contribution into the 3 factors is due to the following factorization properties of the functions  $A, D$ :

$$A_-(\bar{V}_L + V + \bar{V}_R) = \tilde{A}_-(V_L) A_-(V) \tilde{A}_-(V_R) \\ D(\bar{V}_L + V + \bar{V}_R) = \tilde{D}_0(\bar{V}_L) D(V) \tilde{D}_0(\bar{V}_R) \quad (13) \\ I(a, m^2) = I_0(a^L, m^2) + I_0(a^R, m^2).$$

A regular part  $\Phi_{\text{reg.}}^V(J, j, m^2, \mu^2)$  can possess poles at  $J = -1$  resulting from integration over small  $\lambda_V$  of smaller  $s, Q^2$ -subgraphs  $v \subset V$ . Then we choose such a subgraph  $v$  among these  $s, Q^2$ -subgraphs which is not contained as a whole in any other  $s, Q^2$ -subgraph except  $V$ . There are two such subgraphs  $V_1, V_2$  (fig. 3b, c). Let us single out a pole part of one of them from the function  $\Phi_{\text{reg.}}^V$

$$\Phi_{\text{reg.}}^V \text{ pole}^{V_1} = \left(\frac{1}{\mu^2}\right)^{J+1} C_{V_1}(J, j) \tilde{f}(j, \mu^2, m^2, \bar{V}_1^L) \times \\ \times \tilde{f}(j, \mu^2, m^2, \bar{V}_1^R) + \Phi_{\text{reg.}}^V \text{ pole}^{V_1}(J, j, m^2, \mu^2) \quad (14)$$

(cf. eq. (1.18)). Then we single out the pole part of  $V_2$  from the function  $\Phi_{\text{reg.}}^V \text{ pole}^{V_1}$ . From the resulting re-

gular contribution  $\Phi_{\text{reg.}}^V \text{ pole}^{V_1} \text{ pole}^{V_2}$  we single out the

pole contribution of the subgraph  $V_3$  (fig. 3a), then that of  $V_4$  and so on, just as it was done in analysis of deep inelastic scattering (ref. <sup>/13/</sup>). As a result we obtain a representation

$$\Phi^\pm(J, j, m^2) = \sum_{a,b} \left(\frac{1}{\mu^2}\right)^{J+1} E_{ab}^\pm(J, j, g(\mu)) \tilde{f}_{a/A}^\pm(j, \mu^2, g(\mu)) \\ \tilde{f}_{b/B}^\pm(j, \mu^2, g(\mu)) + R(J, j, m^2), \quad (15)$$

where  $E_{ab}$  is given by a sum over all possible  $s, Q^2$ -subgraphs; furthermore, the contribution of each subgraph is a sum of poles  $\sum c_n (J+1)^{-n}$  (cf. eq. (1.23)). The functions  $\tilde{f}(j, \mu^2, g(\mu))$  are given in the  $\alpha$ -representation by eq. (1.23). We can choose the subtraction procedure in such a way that the recipe of the  $\tilde{f}(j, \mu^2, g)$ -renormalization is that used when analyzing deep inelastic scattering, e.g., we can use 't Hooft's renormalization for  $\tilde{f}(j, \mu^2, g)$ . The functions  $\Phi(J, j, m^2)$  and  $R(J, j)$  are independent of  $\mu$ , hence differentiating eq. (19) in respect to  $\ln \mu$  gives a renormgroup equation

$$[-2(J+1) + \beta(g) \frac{\partial}{\partial g} + 2\gamma(j, g)] E(J, j, g) = 0. \quad (16)$$

The functions  $E$  are given by small- $\alpha$  integration, whereas the functions  $\tilde{f}(j)$  are connected with matrix elements of operators, just as it was the case in ref. '13'. This indicates that the Drell-Yan contribution is given by the expression which has the structure of the operator product expansion. We will turn back to this point later on. Introducing parton distribution functions (eq. (1.34)) gives a hard scattering formula

$$T^\pm(Q^2, \tau) = \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \sum_{a,b} f_{a/A}^\pm(x, \mu^2) f_{b/B}^\pm(y, \mu^2) \times \\ \times t^\pm(Q^2/\mu^2, \tau/xy, g(\mu)). \quad (17)$$

The function  $t^\pm$  is really the function  $T^\pm$  constructed for the parton subprocess  $ab \rightarrow \gamma^* x$  with the infrared regularization characterized by the parameter  $\mu$ .

To calculate the function  $\text{Disc}_{s, (Q^2)} T$  we note that in the r.h.s. of eq. (17) only the function  $t$  is responsible for existence of the discontinuities of the amplitude. Hence one must apply the operation  $\text{Disc}_{s, (Q^2)}$  in the same way both to the full amplitude  $T$  and to the subprocess amplitude  $t$ . This gives  $T \rightarrow W, t \rightarrow w$  in eq. (17).

The function  $w_{ab}(Q^2/\mu^2, \frac{\tau}{xy}, g)$  is proportional to a properly regularized cross section of the parton sub-

process  $ab \rightarrow \gamma^* x$ . The Born term for  $w$  does not depend on  $\mu$ , for example  $w_{a\bar{a}} = \kappa_a^2 \delta(1 - \frac{\tau}{xy})$ , whereas

higher order corrections have logarithms  $(\ln Q^2/\mu^2)^M$ .

Taking  $\mu = Q$  we obtain the final result

$$W(\tau, Q^2) = \sum_{a,b} \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} w_{ab}(1, \bar{g}(Q), \frac{\tau}{xy}) \times \\ \times f_{a/A}(x, Q^2) f_{b/B}(y, Q^2). \quad (18)$$

### 3. PRODUCTION OF PAIRS AND PHOTONS AT HIGH TRANSVERSE MOMENTUM

Let us consider the behaviour of the structure function  $W(P_A, P_B, Q)$  given by eq. (2) in the region of high transverse momenta:  $Q_\perp^2, Q^2, s \rightarrow \infty$ , but  $\tau = Q^2/s, \tau_\perp = Q_\perp^2/s$  fixed. We have established earlier that in this region only end-point singularities contribute (fig. 1d). The necessary factorization relations are ( $T = \bar{V}_L + V + \bar{V}_R$ ):

$$A_-(T) = \bar{A}_-(\bar{V}_L) A_-(V) \bar{A}_-(\bar{V}_R);$$

$$A_t(T) = \bar{A}_-(\bar{V}_L) A_t(V) D_0(\bar{V}_R),$$

$$A_u(T) = \bar{V} D_0(\bar{V}_L) A_u(V) \bar{A}_-(\bar{V}_R); A_{Q^2}(T) = \bar{V} D_0(\bar{V}_L) A_{Q^2}(V) D_0(\bar{V}_R),$$

$$D(T) = \bar{V} D_0(\bar{V}_L) D(V) D_0(\bar{V}_R); I(T) = \bar{V} I_0(\bar{V}_L) + I_0(\bar{V}_R). \quad (19)$$

From eq. (19) it follows that

$$\exp \left\{ \frac{1}{D(\alpha)} [s A_- + t A_t + u A_u + Q^2 A_{Q^2}] + I(\alpha, m^2) \right\}$$



$$= \exp \left\{ 2(P_A P_B) \ell_- r_- \frac{A_-(V)}{D(V)} - 2(P_A Q) \frac{A_t(V)}{D(V)} \ell_- - 2r_- (P_B Q) \frac{A_u(V)}{D(V)} + Q^2 A_{Q^2}/D(V) + I_0(\bar{V}_L) + I_0(\bar{V}_R) \right\}, \quad (20)$$

where  $\ell_- = \bar{A}_-(V_L)/D_0(V_L)$ ;  $r_- = \bar{A}_-(V_R)/D_0(V_R)$ .  
Using the relations like

$$1 = \int_0^1 dx \delta(x - |\frac{\bar{A}_-(\bar{V}_L)}{D_0(\bar{V}_L)}|) [\theta(\bar{A}_-) + \theta(-\bar{A}_-)] \quad (21)$$

gives the hard scattering formula for the amplitude

$$T^{\sigma_1 \sigma_2}(P_A, P_B, Q) = \frac{1}{4} \{ [T(P_A, P_B, Q) + \sigma_1 \{P_A \rightarrow -P_A\}] + \sigma_2 \{P_B \rightarrow -P_B\} \};$$

$$T^{\sigma_1 \sigma_2}(P_A, P_B, Q) = \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \sum_{a,b} f_{a/A}^{\sigma_1}(x, \mu^2) \times$$

$$\times f^{\sigma_2}(y, \mu^2) t^{\sigma_1 \sigma_2}(x P_A, y P_B, Q, \mu^2, g) + R^{\sigma_1 \sigma_2}(P_A, P_B, Q). \quad (22)$$

where  $\sigma_1, \sigma_2 = \pm$  are the signature factors. A more formal derivation of eq. (22) is based on the use of the Mellin representation

$$T^{\sigma_1 \sigma_2}(P_A, P_B, Q) = \frac{1}{(2\pi i)^4} \int \left( \prod_{i=1}^4 dj_i \Gamma(-j_i) \right) |s|^{j_1} |t|^{j_2}$$

$$|u|^{j_3} (Q^2)^{j_4} \Phi^{\sigma_1 \sigma_2}(j_1, j_2, j_3, j_4; m^2). \quad (23)$$

The Mellin transform  $\Phi^{\sigma_1 \sigma_2}$  is given by

$$\Phi_{(T)}^{\sigma_1 \sigma_2}(j_i, m^2) = \frac{P(c.c)}{(4\pi)^{NZ/2}} \int_0^{i\infty} \prod_{\sigma} d\alpha_{\sigma} \frac{1}{D^{N/2}(\alpha)} \left| \frac{A_-(\alpha)}{D(\alpha)} \right|^{j_1}$$

$$\left| \frac{A_t(\alpha)}{D(\alpha)} \right|^{j_2} \left| \frac{A_u(\alpha)}{D(\alpha)} \right|^{j_3} \left( \frac{A_{Q^2}(\alpha)}{D(\alpha)} \right)^{j_4} e^{I(\alpha, m^2)}$$

$$[\zeta^+(j_1) \zeta^{\sigma_1}(j_2) \zeta^{\sigma_2}(j_3) \epsilon_{\sigma_1}(A_t) \epsilon_{\sigma_2}(A_u) +$$

$$+ \zeta^-(j_1) \zeta^{-\sigma_1}(j_2) \zeta^{-\sigma_2}(j_3) \epsilon_{-\sigma_1}(A_t) \epsilon_{-\sigma_2}(A_u)], \quad (24)$$

where  $\zeta^{\pm}(j) = (1 \pm e^{i\pi j})/2$ ;  $\epsilon_{\pm}(A) = \theta(A) \pm \theta(-A)$ .  
Due to factorization properties (see eq. (19))

$$\Phi_{(T)}^{\sigma_1 \sigma_2}(j_i, m^2) = \tilde{f}_{\bar{V}_L}^{\sigma_1}(j_1 + j_2) \Phi_{(V)}^{\sigma_1 \sigma_2}(j_i, m^2 = 0) \tilde{f}_{\bar{V}_R}^{\sigma_2}(j_1 + j_3). \quad (25)$$

The functions  $\tilde{f}^{\sigma}(j)$  have the ordinary  $\alpha$ -representation (eq. (1.26)). Integrating over the region  $\lambda_V < 1/\mu^2$  gives a pole  $(J+3)^{-1} (1/\mu^2)^{J+3}$ , where  $J = j_1 + j_2 + j_3 + j_4$ . Applying the subtraction procedure which is constructed in the same way as that for  $W(\tau, Q^2)$ , gives

$$\Phi^{\sigma_1 \sigma_2}(j_i, m^2) = \sum_{a,b} f_{a/A}^{\sigma_1}(j_{\lambda}, \mu^2, g(\mu)) \left( \frac{1}{\mu^2} \right)^{J+3}$$

$$\phi_{ab}^{\sigma_1 \sigma_2}(J, j_{\lambda}, j_{\rho}, j_4, g(\mu)) f_{b/B}^{\sigma_2}(j_{\rho}, \mu^2, g(\mu)) +$$

$$+ R^{\sigma_1 \sigma_2}(j_i, m^2), \quad (26)$$

where  $j_{\lambda} = j_1 + j_2$ ;  $j_{\rho} = j_1 + j_3$ . Eq. (22) can be obtained from eq. (26) in the ordinary way. Neglecting the contri-

bution given by R we obtain the hard scattering formula for the structure function  $W(P_A, P_B, Q) = W(Q_\perp^2, \tau, \tau_\perp)$ :

$$W(Q_\perp^2, \tau, \tau_\perp) = \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \sum_{a,b} f_{a/A}(x, \mu^2, g(\mu)) \times \\ \times f_{b/B}(y, \mu^2, g(\mu)) w_{ab}(Q_\perp^2/\mu^2, \tau/xy, \tau_\perp/xy). \quad (27)$$

Taking  $\mu = Q_\perp$  gives a formula of eq. (18) type. The only difference is a change  $w(1, g(Q), \tau/xy) \rightarrow w(1, g(Q_\perp), \tau/xy, \tau_\perp/xy)$ . It is, of course, possible to take  $\mu = Q$  or even  $\mu^2 = s$ . But note that our consideration is meaningful only in the region  $s, Q^2, Q_\perp^2 \gg M^2$ . Hence for  $Q_\perp < Q$  it is the value of  $Q_\perp$  - the smallest of the large variables - which determine the applicability of the arguments based on asymptotic freedom.

The formulas (2), (3), (20) work also in the reversed situation  $Q_\perp > Q^2$ . In this case one can take  $\mu = Q_\perp$  or  $\mu = Q$  (as far as  $Q \gg M$ ). In distinction from the limit  $Q_\perp \rightarrow 0$  in the limit  $Q \rightarrow 0$  the exponential F entering into eq. (3) do not acquire a form  $F = F_1 F_2$ . That is why one can treat the production of a real photon ( $Q^2 = 0$ ) having a high transverse momentum  $Q_\perp \gg M$ . To do this one must take simply  $\tau = 0$  in eq. (27).

#### 4. ANALYSIS IN THE COORDINATE REPRESENTATION

Factorization properties in the coordinate representation are expressed by the relation (see fig. 4)

$$\mathcal{F}_W(x, a_i, b_j, c_k, d_l) = \int d\xi d\eta d\alpha d\beta f_{\bar{V}_L}(a_i, b_j; \xi, \eta)$$

$$C_{W \setminus \bar{V}_L \setminus \bar{V}_R}(x, \xi, \eta, \alpha, \beta) f_{\bar{V}_R}(c_k, d_l; \alpha, \beta) \quad (28)$$

which is a starting point of the further analysis. The dashed line in fig. 4a corresponds to a factor 1. The subtraction procedure in the  $\alpha$ -representation was con-

structed only for the functions  $W(\tau, Q^2)$  and  $W(\tau, \tau_\perp, Q_\perp^2)$ . Now we will apply the subtraction procedure to the form factor  $W(P_A, P_B, Q)$ . The functions  $W(\tau, Q^2)$  and  $W(\tau, \tau_\perp, Q_\perp^2)$  can be obtained from  $W(P_A, P_B, Q)$  by integration. It will be evident from the resulting expression that only these two form factors are dominated by small  $x^2$ . Applying the subtraction procedure and summing over all the diagrams gives (cf. eq. (1.47))

$$\langle P_A P_B | T(J(x)J(0)) | P_A P_B \rangle = \int d\xi d\eta d\alpha d\beta \times \\ \times \sum_{i,j} \langle P_A | \mathcal{O}_i(\xi, \eta; \mu^2) | P_A \rangle \langle P_B | \mathcal{O}_j(\alpha, \beta; \mu^2) | P_B \rangle \times \\ \times C_{ij}(x, \xi, \eta; \alpha, \beta; \mu^2) + R(x, P_A, P_B). \quad (29)$$

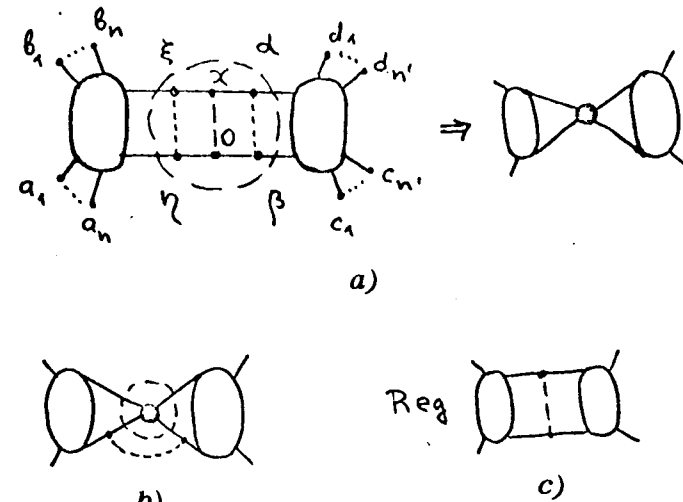


Fig. 4

The contributions denoted as  $R(x, P_A, P_B)$  are shown in fig. 4b,c. Expanding  $\mathcal{O}_{i,j}$  over the local operators gives

$$\mathcal{C}^i(\xi, \eta; \mu^2) \equiv N_{\mu^2}(\phi_i(\eta)\phi_i(\xi)) = \quad (30)$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} (\xi-\eta)^{\mu_1} \dots (\xi-\eta)^{\mu_n} N_{\mu^2}(\phi_i(\eta)\vec{\partial}_{\mu_1} \dots \vec{\partial}_{\mu_n} \phi_i(\eta)).$$

Then we expand  $\phi \partial^n \phi$  over traceless operators

$$\frac{1}{n!} N_{\mu^2}(\phi_i \partial_{\mu_1} \dots \partial_{\mu_n} \phi_i) = \sum_{\ell=0}^n C_{n\ell} S [g_{\mu_1 \mu_2} \dots g_{\mu_{2\ell} \mu_{2\ell}} \times N_{\mu^2}(\phi_i \{ \partial_{\mu_{2\ell+1}} \dots \partial_{\mu_n} \} (\partial^2)^\ell \phi_i)], \quad (31)$$

where  $S$  denotes the symmetrization over  $\mu_1 \dots \mu_n$ . As a result

$$\mathcal{C}(\xi, \eta; \mu^2) = \sum_{m, \ell=0}^{\infty} d_{m\ell} [(\xi-\eta)^2]^\ell O_{\nu_1 \dots \nu_m}^{(2\ell)}(\eta; \mu^2)$$

$$\{ (\xi-\eta)^{\nu_1} \dots (\xi-\eta)^{\nu_m} \}. \quad (32)$$

We must now take an integral

$$\int d\xi d\eta da d\beta C(x; \xi, \eta; a, \beta) \{ (\xi-\eta)^{\nu_1} \dots (\xi-\eta)^{\nu_m} \} \times \{ (a-\beta)_{\mu_1} \dots (a-\beta)_{\mu_n} \} [(\xi-\eta)^2]^k [(a-\beta)^2]^\ell. \quad (33)$$

Performing integration in eq. (33) gives

$$\sum_{r=0}^{\min\{m,n\}} E_{k,\ell}(m,n,r; x^2, \mu^2) \left\{ \begin{array}{c} \mu_1 \dots \mu_r \mu_{r+1} \dots \mu_n \\ \nu_1 \dots \nu_r \end{array} \right\} x^{\mu_{r+1}} \dots x^{\mu_n} \{ x_{\nu_{r+1}} \dots x_{\nu_m} \} (x^2)^{r+k+\ell}. \quad (34)$$

The functions  $E(\dots; x^2, \mu^2)$  in each order of perturbation theory have identical (up to  $(\ln x^2 \mu^2)^p$ ) behaviour on the light cone. Contributions of higher twist operators have additional factors  $(x^2)^{\ell+k}$ . We define matrix elements of operators in the following way

$$\langle P | O_{\mu_1 \dots \mu_n}^{(2k)}(0, \mu^2) | P \rangle = M^{2k} \{ P_{\mu_1} \dots P_{\mu_n} \} b_n^{(k)}(\mu^2), \quad (35)$$

where  $M$  is some parameter with dimension of mass. The coefficients  $b_n^{(k)}$  are dimensionless. As a result we obtain a light cone expansion

$$T(P_A, P_B, Q) = \int e^{iQx} d^4x \sum_{m,n,k,\ell=0}^{\infty} \sum_{r=0}^{\min\{m,n\}} b_m^{(k)}(\mu^2) b_n^{(\ell)}(\mu^2) (M^2)^{k+\ell} (x^2)^{r+k+\ell} \{ P_A P_B \}^r E_{k,\ell}(m,n,r; x^2, \mu^2) \{ x P_A \}^{n-r} \{ x P_B \}^{m-r}. \quad (36)$$

We have introduced a notation

$$\{ AB \}^n \equiv \{ A_{\mu_1} \dots A_{\mu_n} \} \{ B^{\mu_1} \dots B^{\mu_n} \}. \quad (37)$$

To proceed with eq. (36) one must know the relation between the asymptotical properties of  $T$  and the light-cone behaviour of  $E$ -functions.

In the deep inelastic scattering one can write

$$T(P, q) = \int e^{i(qx_{\parallel})} \sum_{k,n=0}^{\infty} b_n^{(k)}(\mu^2) E_k(n, x_{\parallel}^2, -x_{\perp}^2; \mu^2) \{ x_{\parallel} P \}^n (x_{\parallel}^2 - x_{\perp}^2)^k M^{2k} d^2x_{\parallel} d^2x_{\perp} \quad (38)$$

because it is possible to find a frame where  $q_{\perp} = P_{\perp} = 0$ . Integrating over  $x_{\perp}$  gives

$$T(P, q) = \int e^{i(qx_{\parallel})} d^2x_{\parallel} \sum_{k,n} b_n^{(k)}(\mu^2) \phi_k(n, x_{\parallel}^2, \mu^2) \{ x_{\parallel} P \}^n (x_{\parallel}^2)^{k+1} (M^2)^k. \quad (39)$$

It is clear from this representation that the factor  $(x_{\parallel}^2 - x_{\perp}^2)^k$  entering into eq. (38) gives a damping factor  $(M^2/Q^2)^k$  for the higher twist operators.

It is impossible to get rid of  $Q_{\perp}$  for massive lepton pair production, because there are two vectors  $P_A$  and  $P_B$ :

$$T(P_A, P_B, Q) \sim \int e^{i(Q_\perp x_\perp) + i(Q_\parallel x_\parallel)} d^2 x_\parallel d^2 x_\perp$$

$$(M^2)^{k+\ell} \{P_A P_B\}^r \{P_A x_\parallel\}^{n-r} \{P_B x_\parallel\}^{m-r} (x_\parallel^2 - x_\perp^2)^{r+k+\ell}$$

$$E_{k,\ell}(m, n, r; x_\parallel^2 - x_\perp^2; \mu^2). \quad (40)$$

From eq. (40) it follows that in this case one can get dimensionless combinations of two types:  $M^2/Q^2$  or  $M^2/Q_\perp^2$ . Hence it is necessary to take into account higher twist operators in the region  $Q_\perp \sim M$ . The most essential among these operators are known<sup>16</sup> to be connected with parton distributions over transverse momentum. Thus one must know parton distributions over transverse momentum to calculate the distribution of  $\mu$ -pairs over  $Q_\perp$  in the region  $Q_\perp \sim M$ . Less trivial there is the fact that in the region  $Q_\perp \gg M$  it suffices to know only distributions over longitudinal momentum, i.e., in this limit one can take into account only the operators with minimal twist ( $k, \ell = 0$ ). Higher twist contributions will be damped either by the factor  $(M^2/Q^2)^k$  or  $(M^2/Q_\perp^2)^k$ . Another way to avoid the consideration of higher twist operators is to integrate over  $Q_\perp$ :

$$\int T(P_A, P_B, Q) d^2 Q_\perp \sim \int e^{i(Q_\parallel x_\parallel)} \sum s^r (x_\parallel^2)^{r+k+\ell}$$

$$\{x_\parallel P_B\}^{m-r} \{x_\parallel P_A\}^{n-r} (M^2)^{k+\ell} \phi_{k,\ell}(x_\parallel^2, \mu^2; m, n, r) \quad (41)$$

(cf. eq. (46)). In this case the higher twist contributions are suppressed by the factor  $(M^2/Q^2)^k$ . The effect of  $(x_\parallel^2)^r$ -factor is compensated in both cases by the factor  $\{P_A P_B\}^r \sim s^r$ . It is more convenient however to perform the integration over  $Q_\perp$  in another way, that is, by introducing the form factor  $W(\tau, Q^2)$  (see eq. (7)). To treat  $W(\tau, Q^2)$  we introduce a function  $T(r, Q^2)$  (eq. (8)). Then we use the  $\alpha$ -representation for the propagator of a particle with mass  $Q$ :

$$D^c(x, Q^2) = \frac{1}{16\pi^2} \int_0^\infty \frac{da}{a^2} e^{-ix^2/4a - ia(Q^2 - i\epsilon)} \quad (42)$$

Taking into account that the result of the integration over  $x$  can be written as

$$\int \frac{d^4 x}{16\pi^2} e^{-i\frac{x^2}{4a}} E(m, n, r; x^2, \mu^2) (x^2)^r \left\{ \delta_{\nu_1}^{\mu_1} \dots \delta_{\nu_r}^{\mu_r} \right.$$

$$\left. x^{\mu_{r+1}} \dots x^{\mu_n} \right\} x_{\nu_{r+1}} \dots x_{\nu_m} = 2^m \delta_{nm} a^{n+2} \times$$

$$\times \tilde{E}(m, r, a, \mu^2) \left\{ \delta_{\nu_1}^{\mu_1} \dots \delta_{\nu_n}^{\mu_n} \right\} \quad (43)$$

we obtain

$$T(\tau, Q^2) = \sum_{n=0}^\infty b_n^{(0)}(\mu^2) b_n^{(0)}(\mu^2) \frac{2^n \{P_A P_B\}^n}{Q^{2n}} \times$$

$$\times \tilde{t}(n, Q^2/\mu^2, g), \quad (44)$$

where

$$\frac{\tilde{t}(n, Q^2/\mu^2, g)}{Q^{2n+2}} = \sum_{r=0}^n \int_0^\infty \frac{da}{a^2} E(n, r, a, \mu^2, g) a^{n+2} e^{-aQ^2} \quad (45)$$

An easily verified identity

$$\{P_A P_B\}^n = (P_A P_B)^n \frac{1}{R} \left[ \left( \frac{1+R}{2} \right)^{n+1} - \left( \frac{1-R}{2} \right)^{n+1} \right], \quad (46)$$

where  $R = (1 - P_A^2 P_B^2 / (P_A P_B)^2)^{1/2}$ , allows one to take into account the  $\xi$ -scaling<sup>17</sup> type effects. A new variable  $\xi(\mu)$  is

$$\xi(\mu) = \frac{2r}{1 + \sqrt{1 - 4m_B^2/m_A^2/s^2}} \quad (47)$$

But the value of  $m_A^2 m_B^2 / (P_A P_B)^2$ , as a rule, is too small to be considered. For instance, if A and B are the protons and  $P_{lab} = 10 \text{ GeV}$ , then  $4m_p^4/s^2 \approx 10^{-3}$ . Henceforth we take  $\{P_A P_B\}^n = (P_A P_B)^n$ .

We have seen earlier that the function  $T(\tau, Q^2)$  has unnecessary cuts at  $\tau = \pm \delta, \delta \sim 0$ . The function

$$t_{ab}(\tau, Q^2/\mu^2) = \sum_{n=0}^{\infty} \left(\frac{1}{\tau}\right)^n \tilde{t}_{ab}(n, Q^2/\mu^2) \quad (48)$$

which is a function  $T$  constructed for a parton subprocess  $ab \rightarrow \gamma^* x$ , also has these cuts. We represent  $t_{ab}$  as  $t_{ab} = u_{ab} + v_{ab}$ , where  $u_{ab}$  has the imaginary part given by *fig. 2b*, whereas  $v_{ab}$  has the imaginary part given by *fig. 2a*. It is clear that the function

$$T'(\tau, Q^2) = \sum_{n=0}^{\infty} a_n^{(a)}(\mu^2) a_n^{(b)}(\mu^2) \left(\frac{1}{\tau}\right)^n \tilde{v}_{ab}(n, \frac{Q^2}{\mu^2}) \quad (49)$$

has only the cuts at  $\tau = \pm 1$ . The region  $|1/\tau| < 1$  plays for the function  $T'$  the same role as the Euclidean region for deep inelastic structure functions. In this region one can use perturbation theory if the coupling constant  $\bar{g}(Q)$  is small (e.g., in asymptotically free theories). The discontinuity of the function  $T'$  on the r.h.s. cut gives  $W(\tau, Q^2) \equiv W_{AB \rightarrow \gamma^* X} = W_{\bar{A}\bar{B} \rightarrow \gamma^* X}$ , whereas that on

the l.h.s. cut gives  $\bar{W}(\tau, Q^2) \equiv W_{\bar{A}\bar{B} \rightarrow \gamma^* X} = W_{AB \rightarrow \gamma^* X}$ .

$$T'(\tau, Q^2) = \int_1^{\infty} d\sigma \left\{ \frac{W(1/\sigma, Q^2)}{\sigma - \omega} + \frac{\bar{W}(1/\sigma, Q^2)}{\sigma + \omega} \right\}, \quad (50)$$

where  $\omega = 1/\tau$ . Performing Taylor expansion over in the region  $|\omega| < 1$  gives  $\omega/\sigma$

$$\int_1^{\infty} (W(\tau, Q^2) + (-1)^n \bar{W}(\tau, Q^2)) \tau^{n-1} d\tau =$$

$$= \sum_{a,b} a_n^{(a)}(\mu^2) a_n^{(b)}(\mu^2) \tilde{v}_{ab}(n, Q^2/\mu^2, g(\mu)) + \mathcal{O}(1/Q^2). \quad (51)$$

We define

$$a_n^{(a)} = \tilde{f}_a(n) + (-1)^n \tilde{f}_{\bar{a}}(n); \quad \tilde{v}_{ab}(n, Q^2/\mu^2) = \tilde{w}_{ab}(n, Q^2/\mu^2) + (-1)^n w_{ab}(n, Q^2/\mu^2);$$

then

$$\int_0^1 W(\tau, Q^2) \tau^{n-1} d\tau = \sum_{a,b} [w_{ab}(n, Q^2/\mu^2, g) \times \tilde{f}_{a/A}(n, \mu^2, g) \tilde{f}_{b/B}(n, \mu^2, g) + \tilde{w}_{ab}(n, Q^2/\mu^2, g) \times \tilde{f}_{a/A}(n, \mu^2, g) \tilde{f}_{b/B}(n, \mu^2, g) + (A \leftrightarrow B)] + \mathcal{O}\left(\frac{1}{Q^2}\right) \quad (52)$$

(we have used the symmetry relation  $w_{ab} = w_{ba}$ ;  $\tilde{w}_{ba} = \tilde{w}_{ab}$ ). The meaning of the functions  $w, \tilde{w}$  is clear from eq. (52): the function  $w_{ab}$  (or  $\tilde{w}_{ab}$ ) is the moment of the structure function  $W$ , corresponding to the process  $a\bar{b} \rightarrow \gamma^* x$  (or  $ab \rightarrow \gamma^* x$ ). To invert the moments it is necessary to take into account that the corrections  $\mathcal{O}(1/Q^2)$  can give a sizeable contribution at  $\tau$  close to 1. That is why one can use the hard scattering formula (17) only outside the resonance region, i.e., for  $\tau$  not too close to 1.

Note, that the sum rule eq. (52) requires the data at fixed  $Q^2$ . As a rule, the data at fixed  $s$  are analysed. It is possible, of course, to take  $\mu^2 = s$ , but then appear obvious logarithmic factors  $\gamma_n g^2 \ln(Q^2/s)$  in the function  $w(Q^2/\mu^2, n, g(\mu))$  which tend to substitute the distribution function  $f(x, s)$  by  $f(x, Q^2)$ .

Finally, let us consider the configuration shown in *fig. 1b*. Formally, it corresponds to a stronger light-cone singularity than the Drell-Yan diagrams, and the corresponding operator has twist equal to 2. But a produced quark must be on its "would be" mass shell:

$(k')^2 = (k-Q)^2 = 0, \quad k^2 - Q^2 = 0$  (fig. 5a). Hence only configurations with  $k^2 \geq Q^2$  do contribute to cross-section. The line corresponding to the highly virtual momentum  $k$  must be related to the coefficient function  $C(x)$  rather than to a matrix element. We must now explain how to obtain a very massive virtual quark. This is essentially the same problem we have tried to solve: how to get a particle having large virtual mass? The bremsstrahlung mechanism shown in fig. 5b indicates only that such a particle can be obtained from another particle having a large virtual mass. But it cannot answer the question from where this initial particle has appeared, if at the beginning we had only particles having small masses. The only possibility to break this chain is to suppose that the particle having large virtual mass is the result of the fusion of two particles having momenta  $xP_A$  and  $yP_B$ . The corresponding operator consists at least of 4 elementary (parton) fields (fig. 1a). It makes sense to call this configuration the generalized Drell-Yan mechanism. The bremsstrahlung contribution (fig. 5b) is then simply a higher order correction described by eq. (52) rather than a contribution of a new type.

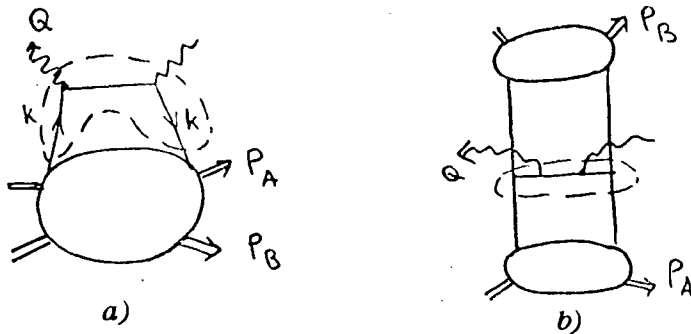


Fig. 5

## APPENDIX

Here we cite the result obtained by Tiktopoulos<sup>14</sup> for pinch singularities which appear due to vanishing of the function  $F$  (see eq. (2), (3)) at nonzero  $a$ .

Let  $F(a, \tau) = A_0(a, \tau) F_1(a, \tau) \dots F_k(a, \tau)$ , where any factor  $F_m$  can vanish at nonzero  $a$ . Then the Mellin transform  $\Phi$  for amplitudes having definite signature:

$$\frac{1}{2} [M(s, \tau, m^2) \pm M(-s, \tau, m^2)] = \int_{-i\infty}^{i\infty} \frac{dj}{2\pi i} \frac{s^j \pm (-s)^j}{2} \Gamma(-j) \Phi^\pm(j, \tau, m^2) \quad (\text{A.1})$$

has the following representation

$$\Phi^\pm(j, \tau, m^2) \sim \left( \frac{1}{j+1} \pm \frac{1}{j+1} \right)^k \int_0^\infty \frac{\sigma^{j+1} d\sigma}{D^{N/2}(\sigma)} \times \left( \frac{A_0(a, \tau)}{D(a)} \right)^j e^{I(a, m^2)k} \prod_{m=1}^k \delta(F_m(a)). \quad (\text{A.2})$$

Thus, the pinch singularities result in a pole  $(j+1)^{-k}$  for amplitudes having positive signature. But the factor  $s^j \pm (-s)^j$  entering into eq. (A.1) compensates one degree of the pole at  $j=-1$ . Hence the pinch singularity contributes only for  $k \geq 2$  and only for positive signature amplitudes.

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