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# FIELD-THEORETIC TREATMENT OF HIGH MOMENTUM TRANSFER PROCESSES.

## I. Deep Inelastic Scattering

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# FIELD-THEORETIC TREATMENT OF HIGH MOMENTUM TRANSFER PROCESSES.

## I. Deep Inelastic Scattering

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Теоретико-полевой подход к процессам с большой передачей импульса. I. Глубоконеупругое рассеяние

Развита методика исследования асимптотики глубоконеупругого рассеяния, основанная на использовании альфа-представления фейнмановских диаграмм. Получены операторные разложения на световом конусе. Исследована связь анализа в альфа-представлении и операторных разложений с модифицированной партонной моделью.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

Efremov A.V., Radyushkin A.V. E2 - 11725

Field-Theoretic Treatment of High Momentum Transfer Processes. I. Deep Inelastic Scattering

Methods are developed for investigation of the asymptotical behaviour of deep inelastic scattering, based on the use of the alpha-representation of Feynman diagrams. Light-cone operator product expansions are obtained. The relation of the alpha-representation analysis and operator product expansion to a modified parton model is investigated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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High momentum transfer processes are now a subject of intensive experimental and theoretical investigations because their study can help to answer the fundamental questions concerning the hadronic structure and the nature of strong interactions. It is also very attractive that in quantum chromodynamics (which is the most probable candidate for a theory of strong interactions) the effective coupling constant is small at large momentum transfers /1/. Thus one may hope to obtain certain results with the help of perturbation theory. Strictly speaking, the constant is small only in the deep Euclidean region. But in any physical process the particles in initial and final states are on their mass shells. Hence it is not correct to say that the high momentum transfer processes are the short distance phenomena. One may only hope that the asymptotical behaviour of the cross-sections considered is determined by the short-distance dynamics, whereas the large distance contribution is described by some dimensionless functions which characterize the probability of the corresponding short-distance subprocess.

This is just the picture dictated by the parton model /2/. The validity of the parton model treatment of two kinematically most simple processes (i.e., $e^+e^-$-annihilation into hadrons and deep inelastic lepton-hadron scattering) has been justified in the QCD framework with the help of the operator product expansion and renormalization group methods, (see, e.g., ref. /1/).

But the parton model remains the only tool for the theoretical analysis of the processes which have a more complicated kinematics such as the massive lepton pair production process $AB \to \mu^+\mu^- X$ [3]. Note that the aforementioned field-theoretical methods have lead to a slightly modified parton picture [4-8] for deep inelastic scattering. In this picture the Bjorken scaling law is violated. Hence one has to modify the parton model also when applying it to other processes. This problem can be solved only within an approach which is not based on the parton model. In our investigation consisting of three papers we try to work out such an approach.

In the present (the first) paper we consider the deep inelastic scattering. This process has been investigated earlier from various view points [1-14]. We give our treatment of this process because, first, in this case it is most easy to formulate the basic statements of our approach. Second, it allows us to demonstrate a full equivalence of our approach (in which different processes are treated on the same footing) and of a standard analysis [1,13]. Unlike the standard approach where the operator product expansion on the light cone [9,10] is postulated, we derive it from an analysis of Feynman diagrams (not claiming for an absolute rigor of such a derivation). This allows also to shed new light on some known facts.

In the second paper we apply the formalism developed in the first paper to a process $AB \to \mu^+\mu^- X$. As a result, we obtain an expansion which justifies the parton model ideas when applied to two characteristics of this process, namely, to total cross-section of producing the pair having mass $Q$ and to differential cross-section of producing a pair having large $Q_\perp \gg M$ transverse momentum $Q_\perp$.

In these two papers we restrict our treatment to nongauge field theories. The final goal of our investigation is the treatment of $AB \to \mu^+\mu^- X$ process in gauge field theories including QCD given in the third paper. The main result is that taking into account the specific features of vector fields does not change essentially the results obtained in the framework of nongauge models. This justifies the efforts spent for a study of these models.



## 1. *DEEP INELASTIC SCATTERING AND α-REPRESENTATION*

Let us consider first a scalar analog of the deep inelastic structure function $W(\omega, Q^2)$ in a theory

$$\mathcal{L}_{int}(x) = g\{\sum_a :\psi_a^*(x)\,\psi_a(x)\,\phi(x):\}_{(N)}$$

in space-time of N dimensions. A complex scalar field $\psi$ describes the "quarks" whereas a real field $\phi$ corresponds to the gluons. By definition

$$W(\omega, Q^2) = \frac{1}{2\pi}\,\mathrm{Disc}_s\, T^+(s, Q^2)\,,$$

$$T^\pm(s, Q^2) = (T(s, Q^2) \pm T(-s, Q^2))/2\,, \tag{1}$$

$$T(s, Q^2) = \frac{1}{4\pi}\int d^N x\, e^{iqx} \langle P|T(j(x)j(0))|P\rangle\,,$$

where

$$s = 2(Pq)\,,\quad Q^2 = -q^2\,,\quad \omega = s/Q^2\,. \tag{2}$$

For simplicity we consider scalar currents

$$j(x) = \sum_a \kappa_a :\psi_a^*(x)\,\psi_a(x): \tag{3}$$

The function $T(s, Q^2)$ can be written in the $\alpha$-representation [15] (the necessary information about the $\alpha$-representation can be found in the Appendix, see also refs. [16-18]) as follows:

$$T(s, Q^2) = \sum_{\substack{\text{diagr.}\\ 1a}} \frac{g^{2z}}{(4\pi)^{Nz/2}} \int_0^{i\infty} \prod_\sigma d\alpha_\sigma\, D^{-N/2}(\alpha) \times$$
$$\times \exp\{(sA_-(\alpha) - Q^2 A(\alpha))/D(\alpha) + I(\alpha, m^2)\} + [s \to -s], \tag{4}$$

where $z$ is the number of loops of the diagram. Due to unequality $|A_-/A| \leq 1$ (see. eq. (A.3)), in the Euclidean region $|s| < Q^2$ one can safely rotate the contours of integration over $\alpha_\sigma$ in eq. (4) to the real positive axis. The expression $Q^2A - sA_-$ can vanish in the region $|s| < Q^2$ only if $A(\alpha) = 0$. This can take place only due to vanishing of some set of parameters $\{\alpha_\sigma\}$ because the function $A(\alpha)$ is nonnegative (see eq. (A.3)). According to a general result, the asymptotical behaviour at large $s, Q^2$ is in this case determined by subgraphs the contraction of which into point eliminates the dependence on $s$ and $Q^2$ (i.e., by $s,Q^2$ - subgraphs). In a superrenormalizable $(\psi^*\psi\phi)_{(4)}$ -theory a leading contribution which behaves like $1/Q^2$ can be obtained only in the "handbag" diagrams by integration over $\alpha_0 \sim 0$ (*fig. 1b*):

$$T(s, Q^2) = \frac{1}{Q^2} \sum_a \kappa_a^2 \left(\frac{g}{4\pi}\right)^{2z} \int_0^\infty \prod_\sigma \frac{d\alpha_\sigma}{D_0^2} e^{I_0(\alpha, m^2)} \times$$

$$\times (1 - \omega^2 |\tilde{A}_-(\alpha)/D_0(\alpha)|^2)^{-1} \{1 + O(1/Q^2)\}. \quad (5)$$

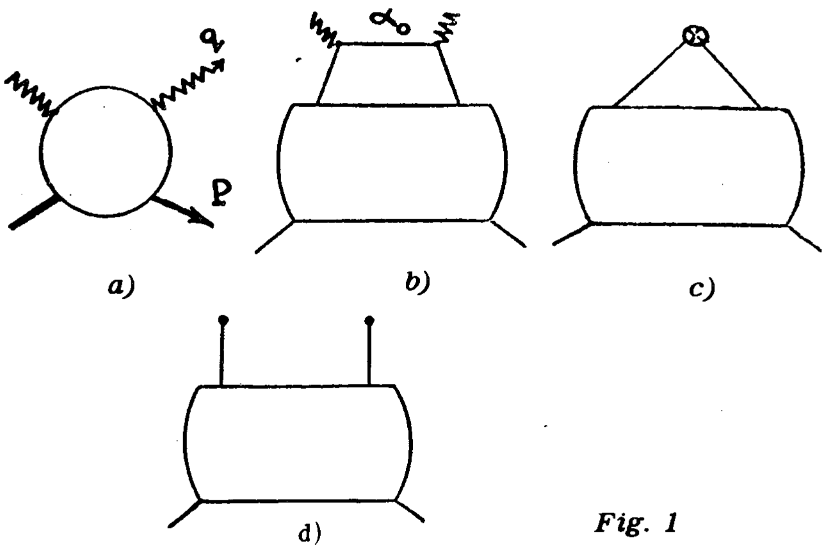



*Fig. 1*



We have taken into account that for such a diagram $A = \alpha D_0$, $A_- = \alpha_0 \tilde{A}_-$; $D = \alpha_0 D_1 + D_0$; $I(\alpha, m^2)|_{\alpha_\sigma = 0} \equiv I_0(\alpha, m^2)$. The function $D_0(\alpha)$ $(D_1(\alpha))$ is a determinant of the diagram shown in *fig. 1c (1d)*. It is clear from the representation (4) that the function $T(\omega, Q^2)$ has a cut for $|\omega| \geq 1$ because $|\tilde{A}_-/D_0| = |A_-/A| \leq 1$. Taking a corresponding discontinuity we obtain the following expression for $W(\omega, Q^2)$:

$$W(\omega, Q^2) = \frac{1}{Q^2} \sum_a \kappa_a^2 [f_a(\frac{1}{\omega}) + f_{\bar{a}}(\frac{1}{\omega})] \{1 + O(1/Q^2)\}, \quad (6)$$

where $f_a(x)$ is the $a$-quark distribution function

$$f_a(x) = \sum_{\text{diagr.1c}} \left(\frac{g}{4\pi}\right)^{2z} \int_0^\infty \prod_\sigma d\alpha_\sigma \frac{e^{I_0(\alpha, m^2)}}{D_0^2(\alpha)} \theta(\tilde{A}_-(\alpha)) \times$$

$$\times \delta(1 - \frac{1}{x} |\tilde{A}_-(\alpha)/D_0(\alpha)|). \quad (7)$$

To get an antiquark function $f_{\bar{a}}(x)$ it is necessary to take $\theta(-\tilde{A}_-)$ in place of $\theta(\tilde{A}_-)$ in the r.h.s. of eq. (7). The expedience of such a definition is clear from the fact that the quantities $e_H$ and $\theta_\psi$

$$e_H = \sum_a e_a \int_0^1 dx \{f_a(x) - f_{\bar{a}}(x)\};$$
$$\theta_\psi = \sum_a \int_0^1 x\,dx \{f_a(x) + f_{\bar{a}}(x)\}, \quad (8)$$

can be interpreted as electric charge of the hadron and momentum carried by quarks because the $\alpha$-representation for $e_H$ and for $\theta_\psi$ coincides with that for matrix elements of the electromagnetic current and the quark part of the energy-momentum tensor respectively:

$$2P_\mu e_H = \langle P | \sum_a e_a : \psi_a^*(x)\, \partial/\partial x^\mu\, \psi_a(x) : | P \rangle,$$

$$2P_\mu P_\nu \theta_\psi = \langle P | \sum_a : (\partial\psi_a^*(x)/\partial x^\mu)(\partial\psi_a(x)/\partial x^\nu) : | P \rangle. \quad (9)$$

## 2. ANALYSIS OF RENORMALIZABLE THEORIES

The theory $g(\psi^*\psi\phi)_{(6)}$ in space-time of 6 dimensions is renormalizable. The coupling constant $g$ is dimensionless, and hence any subgraph $V$ having 4 external lines gives a contribution of $1/Q^2$ order due to integration in the region $\lambda_V \sim 0$ (by definition $\lambda_V = \sum_{\sigma \in V} \alpha_\sigma$). As a result there appear logarithmical factors $(\ln Q^2)^M$. To sum the logarithms it is convenient to use the Mellin transformation

$$T(\omega, Q^2) = \int_{-\delta - i\infty}^{-\delta + i\infty} \Gamma(-j)\Gamma(j-J) \frac{dj\, dJ}{(2\pi i)^2} [\omega^j + (-\omega)^j] \times$$
$$\times (Q^2)^J \Phi^+(J, j, m^2) \ . \qquad (10)$$

The Mellin transform $\Phi^\pm$ has the following $\alpha$-representation

$$\Phi^\pm(J, j, m^2) = \sum_{\text{diagr. 1a}} \kappa_a^2 \left(\frac{g^2}{(4\pi)^3}\right)^z \int_0^\infty \prod_\sigma d\alpha_\sigma \frac{\epsilon_\pm(A_-)}{D^3} \times$$
$$\times \left|\frac{A_-}{D}\right|^j \left(\frac{A}{D}\right)^{J-j} e^{I(\alpha, m^2)} \ , \qquad (11)$$

where $\epsilon_\pm(A_-) = \theta(A_-) \pm \theta(-A_-)$.

We consider first the diagrams without quark loops. Let $T$ be such a diagram, whereas $V$ is the largest $s, Q^2$-subgraph having 4 external lines (in particular, $V$ can coincide with the whole diagram $T$). Integrating over $\lambda_V \sim 0$ gives a pole $(J+1)^{-1}$. We divide the region of integration over $\lambda_V$ into two parts: a) $\lambda_V < 1/\mu^2$, b) $\lambda_V > 1/\mu^2$. Making use of scaling $\alpha_\sigma = \lambda_V \beta_\sigma$ for lines $\sigma \in V$ we obtain

$$\Phi^\pm(J, j, m^2) = \int_0^\infty \lambda_V^J d\lambda_V \prod_\sigma d\beta_\sigma \delta(1 - \Sigma\beta_\sigma) \ ,$$

$$\prod_{\bar\sigma} d\alpha_{\bar\sigma} \phi^\pm(\beta_\sigma, \alpha_{\bar\sigma}, \lambda_V) \equiv \Phi^V_{\text{pole}}(\mu^2) + \Phi^V_{\text{reg}}(\mu^2), \qquad (12)$$

where the lines $\bar\sigma$ belong to subgraph $\bar V \equiv T \setminus V$. Expanding the function $\phi(\beta_\sigma, \alpha_{\bar\sigma}, \lambda_V)$

$$\phi(\beta_\sigma, \alpha_{\bar\sigma}, \lambda_V) = \phi(\beta_\sigma, \alpha_{\bar\sigma}, 0) + \sum_{k=1}^\infty \lambda_V^k \phi_k(\beta_\sigma, \alpha_{\bar\sigma}), \qquad (13)$$

shows that only the first term in eq. (13) is responsible for a pole at $J=-1$. Using the factorization properties (eq. (A.4)) of the $A$- and $D$-functions

$$A_\pm(V+\bar V) = A_\pm(V) A_\pm(\bar V) \ ; \ D(V+\bar V) \underset{V}{=} D(V) D_0(\bar V) \ ;$$
$$A(V+\bar V) \underset{V}{=} A(V) D_0(\bar V) \ ; \ \epsilon_\pm(V+\bar V) = \epsilon_\pm(V)\epsilon_\pm(\bar V) \qquad (14)$$

we obtain the very important representation

$$\Phi^\pm(J, j, m^2) \underset{V}{=} \Phi^\pm_{(V)}(J, j, m^2) =$$

$$= \left\{ \left(\frac{g^2}{(4\pi)^3}\right)^{z_V} \int_0^\infty \prod_\sigma d\alpha_\sigma \cdot \left|\frac{A_-}{D}\right|^j \left(\frac{A}{D}\right)^{J-j} \frac{\epsilon_\pm(A_-)}{D^3} \right\} \times \qquad (15)$$
$$\times \left\{ \left(\frac{g^2}{(4\pi)^3}\right)^{z_{\bar V}} \int_0^\infty \prod_{\bar\sigma} d\alpha_{\bar\sigma} \left|\frac{\tilde A_-}{D_0}\right|^j \frac{\epsilon_\pm(A(\bar V))}{D_0^3} e^{I_0(\alpha_{\bar\sigma}, m^2)} \right\}.$$

The sign $\underset{V}{=}$ means that equality holds in the limit $\lambda_V \to 0$ that is, up to terms $O(\lambda_V)$. It follows from eqs. (13) and (15) that the coefficient corresponding to a pole $(J+1)^{-1}$ is a product of two functions, the first one is given by $V$-subgraph and the second one by $\bar V$-subgraph. Integrating the $\alpha$-representation for the function $\Phi_{(V)}$ in the region $0 \le \lambda_V \le 1/\mu^2$ gives a pole contribution

$$\Phi^V_{\text{pole}}(\mu^2) = \left(\frac{1}{\mu^2}\right)^{J+1} C_V(J, j) \tilde f_{\bar V}(j, m^2) \ . \qquad (16)$$

The regular part $\Phi^V_{\text{reg}}(\mu^2)$ is given by subtracting the contribution $\Phi^V_{\text{pole}}(\mu^2)$ from $\Phi$ (but not from $\Phi_{(V)}$) according to eq. (12). Let $V_1 \subset V$ be a maximal s, $Q^2$ -subgraph with 4 external lines. Integrating in the region $\lambda_{V_1} \sim 0$ gives a pole of the function $\Phi^V_{reg}(\mu^2)$ at $J=-1$. Changing $\alpha_\sigma = \lambda_{V_1}\beta_\sigma$ for lines $\sigma \in V_1$ we find that in the limit $\lambda_{V_1} \to 0$ the function $\Phi(J,j,m^2)$ can be represented by eq. (15) (with an obvious change $V \to V_1$). The function $\Phi^V_{pole}(\mu^2)$ can be represented in the following form

$$\Phi^{V}_{pole}(\mu^2) \underset{V_1}{=} \left(\frac{g^2}{(4\pi)^3}\right)^{z_T} \left[\int_0^\infty \prod_{\sigma\in V_1} d\alpha_\sigma \left(\frac{A}{D}\right)^{J-j} \left|\frac{A_-}{D}\right|^j \frac{\epsilon_\pm(A_-)}{D^3}\right] \times$$

$$\times \left[\int_0^\infty \prod_{\sigma\in V\setminus V_1} d\alpha_\sigma \left(\frac{\tilde{A}_-}{D_0}\right)^j \theta(1/\mu^2 - \lambda_{V\setminus V_1}) \frac{\epsilon_\pm(\tilde{A}_-)}{D_0^3}\right] \times$$

$$\times \left[\int_0^\infty \prod_{\sigma\in\bar{V}} d\alpha_\sigma \left|\frac{\tilde{A}_-}{D_0}\right|^j \frac{\epsilon_\pm(\tilde{A}_-)}{D_0^3} e^{I_0(\alpha,m^2)}\right]. \qquad (17)$$

Thus the pole contribution of the subgraph $V_1$ into the function $\Phi^V_{reg}$ can be written as

$$\Phi^{V\ V_1}_{reg\ pole}(\mu^2) = \left(\frac{1}{\mu^2}\right)^{J+1} C_{V_1}(J,j)\{\mathrm{Reg}_{\mu^2}[\tilde{f}_{V_1}(j,m^2)]\}. \qquad (18)$$

The operation $\mathrm{Reg}_{\mu^2}$ is given in this case by the expression

$$\mathrm{Reg}_{\mu^2}[\tilde{f}_{V_1}(j,m^2)] \sim \int_0^\infty \prod_{\sigma\in\bar{V}_1} d\alpha_\sigma \frac{\epsilon_\pm(\tilde{A}_-)}{D_0^3(\bar{V}_1)} e^{I_0(\alpha_\sigma,m^2)} \cdot \left|\frac{\tilde{A}_-}{D_0}\right|^j -$$

$$-\left[\int_0^\infty \prod_{\sigma\in V\setminus V_1} d\alpha_\sigma \left|\frac{\tilde{A}_-}{D_0}\right|^j \frac{\epsilon_\pm(\tilde{A}_-)}{D_0^3(V\setminus V_1)} \theta\left(\frac{1}{\mu^2} - \lambda_{V\setminus V_1}\right)\right] \times$$

$$\times \left[\int_0^\infty \prod_{\sigma\in\bar{V}} d\alpha_\sigma \frac{\epsilon_\pm(\tilde{A}_-)}{D_0^3(\bar{V})} \left|\frac{\tilde{A}_-(\bar{V})}{D_0(\bar{V})}\right|^j e^{I_0(\alpha,m^2)}\right]. \qquad (19)$$



The structure of the subtraction procedure is illustrated in *fig. 2*. The function $\Phi^{V\ V_1}_{reg\ reg}(\mu^2)$ is given by the difference

$$\Phi^{V\ V_1}_{reg\ reg}(\mu^2) = \Phi^V_{reg}(\mu^2) - \Phi^{V\ V_1}_{reg\ pole}(\mu^2). \qquad (20)$$

Then it is necessary to consider the next s, $Q^2$ -subgraph $V_2 \subset V_1$ having 4 external lines, to single out a corresponding pole part, to subtract it, and so on. As

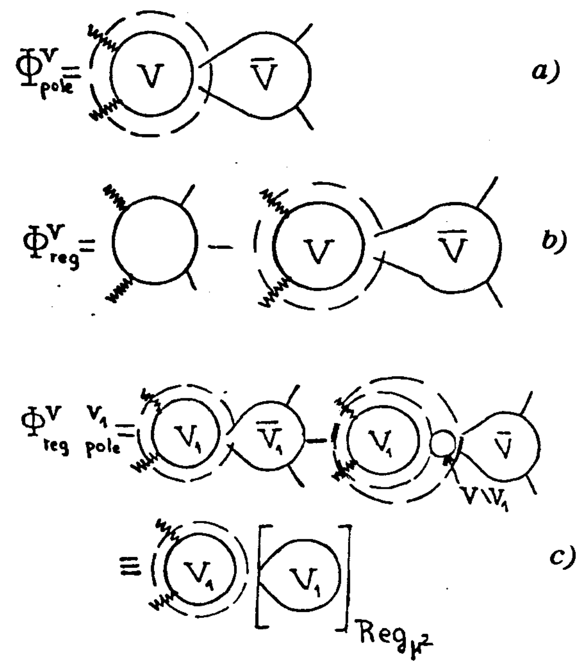



*Fig. 2*

a result, we obtain the following representation for a contribution of the diagram T:

$$\Phi^\pm_T(J,j,m^2) = \left(\frac{1}{\mu^2}\right)^{J+1} \sum_V C_V(J,j)\,\tilde{f}_{\bar{V}}(j,\mu^2) +$$

$$+\tilde{R}_T(J,j,m^2,\mu^2). \qquad (21)$$

When the recipe of the $\mathrm{R}$-operation is fixed, one can single out the leading asymptotic terms

$$T_T(Q^2) = \frac{1}{Q^2}\sum_{n=0}^{z_T} a_n \left[\ln\frac{Q^2}{\mu^2}\right] + O(1/Q^4) \equiv$$

$$\equiv T_T^{(as)}(Q^2) + O(1/Q^4)\,, \tag{22}$$

where $\mu$ is the parameter with dimension of mass. Using another choice of $\mu$ we must use also other coefficients $a_n = a_n(\mu^2)$. But it is evident that the sum of logarithmic terms remains unchanged. The difference $T - T^{(as)} = O(1/Q^4)$ remains unchanged too. It means that although the regular contribution $\tilde{R}$ was $\mu$-dependent by construction, it is possible to make it $\mu$-independent by subtracting from $C_V$ the terms which do not contribute to $T^{(as)}$, i.e., those giving zero after integration around the point $J=-1$ in eq. (10). The function $C_V$ can have a pole $(J+1)^{-n}$ as a senior singularity at $J=-1$, where $n \le z_T$:

$$\frac{\Gamma(j-J)}{\Gamma(j+1)} C_V(J,j) = \frac{1}{(J+1)^n}\left\{\sum_{k=0}^{n} b_k (J+1)^k + \sum_{k=n+1}^{\infty} b_k (J+1)^k\right\} \equiv$$

$$\equiv E_V(J,j) + r_V(J,j)\,. \tag{23}$$

The contribution of $r_V$ is of an order of $O(1/Q^4)$ due to the absence of poles at $J=-1$. We add it to $\tilde{R}$, then

$$\frac{\Gamma(j-J)}{\Gamma(j+1)} \Phi_T^{\pm}(J,j,m^2) = \left(\frac{1}{\mu^2}\right)^{J+1} \sum_V E_V^{\pm}(J,j,g)\, \tilde{f}_V^{\pm}(j,\mu^2,g) +$$

$$+ R_T(J,j,m^2)\,. \tag{24}$$

The function $R_T$ is regular at $J=-1$ and does not depend on $\mu$, whereas the function $E_V(J,j)$ according to



eq. (23) is a sum of poles at $J=-1$. Summing over all the diagrams without quark loops we obtain the representation

$$\frac{\Gamma(j-J)}{\Gamma(j+1)} \Phi^{\pm}(J,j,m^2) = \left(\frac{1}{\mu^2}\right)^{J+1} E^{\pm}(J,j,g(\mu))\tilde{f}^{\pm}(j,\mu^2,g(\mu)) +$$

$$+ R(J,j,m^2)\,. \tag{25}$$

We have added the dependence of the coupling constant on parameter $\mu$. This dependence originates from the ultraviolet divergences. We define the $R$-operation with the help of dimensional regularization [19] $d^N k \to d^{N-2\epsilon} k\,(\mu_R^2)^{\epsilon}$. In the $\alpha$-representation this results in a change $D^{N/2} \to D^{N/2-\epsilon}(\mu_R^2)^{z\epsilon}$. It is sufficient for renormalization, according to 't Hooft, to subtract the poles in $\epsilon$, and then to take limit $\epsilon \to 0$. There appear in particular logarithmical factors $(\ln \mu_R^2/\mu^2)^k$. The asymptotical behaviour of $T(\omega, Q^2)$ does not depend on a particular choice of $\mu_R$ and $\mu$ and to avoid superfluous complications we take $\mu = \mu_R$. From the formal $\alpha$-representation for $\tilde{f}_V(j,g)$ (the first term in eq. (19)) it follows that after the dimensional regularization of the function $\tilde{f}_V(j,g)$ there appear new poles in $\epsilon$ (which are not related to ordinary divergent subgraphs) due to integration over small $\lambda_v$ of subgraphs adjoined to the O-vertex (*fig. 3*). But these poles are removed by the operation $\mathrm{Reg}_{\mu^2}$.

To illustrate this, let us take $v = V \setminus V_1$ (*fig. 2*). Using the factorization properties, eq. (14), it is easy to see that the integrands of both the expressions entering into eq. (19) are equal up to the terms of higher order

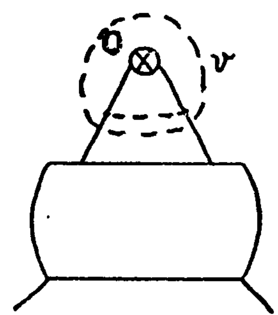



*Fig. 3*

in $\lambda_v$. Hence after integration over the region $\lambda_v \sim 0$ the poles $\epsilon^{-n}$ corresponding to the first and to the second terms in eq. (19) cancel with each other.

### *3. ANALYSIS IN THE $\alpha$-REPRESENTATION AND PARTON MODEL*

The function $\tilde f^{+}(j,\mu^2,g)(\tilde f^{-}(j,\mu^2,g))$ has, according to eq. (A.7), the same $\alpha$-representation

$$\tilde f^{\pm}(j,\mu^2,g(\mu)) = \mathrm{Reg}_{\mu^2} \sum_{\text{diagr.}} \left(\frac{g^2}{(4\pi)^3}\right)^z \times$$

$$\times \int_0^\infty \prod_\sigma d\alpha_\sigma \left|\frac{\tilde A_-}{D_0}\right|^j \frac{\epsilon_\pm(\tilde A_-)}{D_0^3} \exp\{I_0(\alpha, m^2)\}, \qquad (26)$$

as the reduced matrix element of the operator $\psi^*(\overleftrightarrow{\partial})^n\psi$ for even (odd) $n$:

$$i^n \langle P|\psi_a^*\{\overleftrightarrow{\partial}_{\mu_1} \dots \overleftrightarrow{\partial}_{\mu_n}\}\psi_a|P\rangle =$$

$$= 2\{P_{\mu_1}\dots P_{\mu_n}\}[\tilde f_a(n,\mu^2,g) + (-1)^n \tilde f_{\bar a}(n,\mu^2,g)]. \qquad (27)$$

As usual, the braces $\{\ \}$ denote the symmetric traceless part of a tensor. Thus the subtraction procedure formulated above provides a recipe of the composite operator renormalization. We have formulated the recipe of constructing the function $E$ and this uniquely determines the action of the operation $\mathrm{Reg}_{\mu^2}$ on the functions $\tilde f$. For calculational convenience it is preferable to define the operation $\mathrm{Reg}_{\mu^2}$ (for instance, $\mathrm{Reg}_{\mu^2}$ can be understood as the dimensional regularization plus removal of all the poles $\epsilon^{-n}$). Then the structure of the function $E$ is determined by the requirement that the final result for $T^{(as)}$ must be $\mu$-independent. This can



be obtained with the help of the renormgroup equation, i.e., by differentiating eq. (25) with respect to $\mu$:

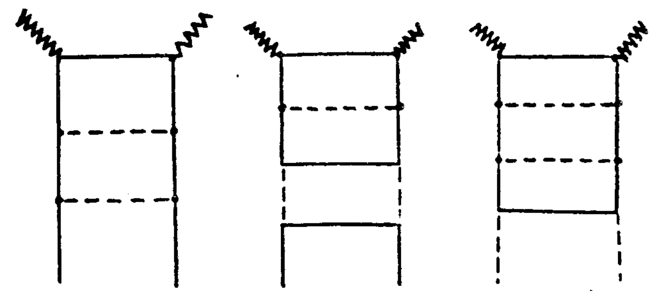
$$[-2(J+1) + \beta(g)\frac{\partial}{\partial g} + \gamma(g,j)]E(J,j,g) = 0, \qquad (28)$$

where

$$\tilde f(j,\mu^2,g)\,\gamma(g,j) = (\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g})\tilde f(j,\mu^2,g);$$

$$\beta(g) = \mu\frac{\partial g}{\partial \mu}. \qquad (29)$$

In the general case when the quark loops are present, there appear three types of singular functions $E_V$ (*fig.4*).



*a)* *b)* *c)*

***Fig. 4***

To the first type there belong subgraphs with quark external lines which do not contain gluon divisions in the $t$-channel. The corresponding contribution $E_V^{NS}$ is proportional to $\kappa_a^2$, where $a$ denotes the type of an external quark line. The subgraphs belonging to the second type have at least 1 gluon division in the $t$-channel. Their contribution is proportional to $\langle\kappa^2\rangle = \sum \kappa_a^2/N_f$, where $a = 1,..,N_f$. Hence [7]

$$\Phi^{\pm}_{\text{quark}}(J,j) = \left(\frac{1}{\mu^2}\right)^{J+1}[E^{NS}(J,j,g)\sum_a \kappa_a^2 \tilde f_a^{\pm}(j,\mu^2,g) +$$

$$+ \langle\kappa^2\rangle E_1(J,j,g)\sum_a \tilde f_a^{\pm}(j,\mu^2,g)]. \qquad (30)$$

The functions $E^{NS}$, $E_1$ do not depend on quark flavours. Let us introduce the function $E^S = E^{NS} + E_1$ which corresponds to the sum of contributions given by all subgraphs having quark external lines. Then the $\Phi$-function can be rewritten as

$$\Phi^{\pm}(J,j) = \left(\frac{1}{\mu^2}\right)^{J+1} \{ E^{NS} \sum_a (\kappa_a^2 - \langle\kappa^2\rangle) f_a^{\pm} +$$

$$+ E^S \langle\kappa^2\rangle \sum_a \tilde{f}_a^{\pm} + E^g \langle\kappa^2\rangle \tilde{f}_g^{\pm} \}, \qquad (31)$$

where $E^g$ is the sum of contributions of subgraphs having gluonic external lines. The function $\tilde{f}_g$ can be connected with the gluon operators by the relation

$$i^n \langle P | \phi \{ \overleftrightarrow{\partial}_{\mu_1} \ldots \overleftrightarrow{\partial}_{\mu_n} \} \phi | P \rangle =$$

$$= 2\{ P_{\mu_1} \ldots P_{\mu_n} \} \tilde{f}\ (n, \mu^2, g) \frac{1 + (-1)^n}{2}. \qquad (32)$$

We identify the functions $\tilde{f}_p(n, \mu^2, g)$ at positive integer $n$ with the moments of parton distribution functions [7]:

$$\tilde{f}_p(n, \mu^2, g) = \int_0^1 \frac{dx}{x} x^n f_p(x, \mu^2, g). \qquad (33)$$

In this normalization we have the sum rule

$$\int_0^1 x\,dx \{ f_g(x, \mu^2, g) + \sum_a [ f_a(x, \mu^2, g) + f_{\bar{a}}(x, \mu^2, g) ] \} = 1 \qquad (34)$$

related to energy-momentum conservation.

Note, that the only assumption used to derive eq. (31) is that all the dependence of the functions E on quark flavours is connected with quark charges only (i.e., we have neglected only the dependence of E on quark masses).



Turning back to the function $T(s, Q^2)$ we get

$$T(s, Q^2) = \frac{1}{Q^2} \int_{-\delta - i\infty}^{-\delta + i\infty} \frac{\omega^j + (-\omega)^j}{2i \sin \pi j} dj [ \tilde{E}(\frac{Q^2}{\mu^2}, j, g(\mu)) \otimes$$

$$\otimes \tilde{f}(j, \mu^2, g(\mu)) + O(1/Q^2) ], \qquad (35)$$

where

$$\tilde{E}(\frac{Q^2}{\mu^2}, j, g(\mu)) = \int_{-\delta - i\infty}^{-\delta + i\infty} \frac{dJ}{2\pi i} \left(\frac{Q^2}{\mu^2}\right)^{J+1} E(J, j, g(\mu)). \qquad (36)$$

Formula (35) is just the Sommerfeld-Watson transformed version of the result given by the standard analysis based on the operator product expansion [1,13]. The terms denoted as $O(1/Q^1)$ in principle, can have a power-like dependence on $j$ for real positive $j$: $O(1/Q^2) \sim$ $\sim (j/Q^2)^M$. Such contributions have been analyzed in ref. [22]. It has been shown there that even in this case one can neglect these terms for $\omega$ not too close to 1 (more precisely, outside the resonance region). In this region eq. (35) in conjunction with eqs. (31), (33) leads to the hard scattering formula

$$T(s, Q^2) = \frac{1}{Q^2} \int_0^1 \frac{dx}{x} t(\frac{Q^2}{\mu^2}, g, \omega x) \otimes f(x, \mu^2, g), \qquad (37)$$

which holds both for the amplitude T and for its discontinuity:

$$W(\omega, Q^2) = \int_0^1 \frac{dx}{x} \{ w^{NS}(Q^2, \mu^2, g(\mu), \omega x) \times$$

$$\times \sum_a (\kappa_a^2 - \langle\kappa^2\rangle) \cdot [ f_a(x, \mu^2, g(\mu)) + f_{\bar{a}}(x, \mu^2, g(\mu)) ] +$$

$$+ w^S(Q^2, \mu^2, g(\mu), \omega x) \cdot \langle\kappa^2\rangle \sum_a [ f(x, \mu^2, g(\mu)) + f_{\bar{a}}(x, \mu^2, g(\mu)) ] +$$

$$+ w^g(Q^2, \mu^2, g(\mu), \omega x) \langle\kappa^2\rangle f_g(x, \mu^2, g(\mu)) \}. \qquad (38)$$

The functions $w(Q^2, \mu^2, g(\mu), \omega x)$ describe the cross-section of a parton subprocess with a proper infrared regularization. Depending on the choice of a subtraction procedure the parameter $\mu$ can be interpreted as a parton virtuality $-\mu^2 = k^2$ ,or as a parton transverse momentum $\mu^2 = k_\perp^2$. An inherent feature of the infrared regularization is a cut-off at $\lambda_V > 1/\mu^2$. It follows from the $\alpha$-representation for a propagator

$$\frac{1}{m^2 - p^2 - i\epsilon} = \int_0^{i\infty} d\alpha \exp(\alpha(p^2 - m^2 + i\epsilon)) ,$$

$$D^c(x, m^2) = \frac{1}{(4\pi)^{N/2}} \int_0^{i\infty} \frac{d\alpha}{\alpha^{N/2}} e^{-\frac{x^2}{4\alpha} - \alpha(m^2 - i\epsilon)} , \qquad (39)$$

that small $\alpha$ corresponds to large momentum $p^2$ (or to small interval $x^2$). Hence the functions E, which are due to small $\alpha$ -integration, correspond to coefficient functions of the OPE and describe the parton subprocess.

Regular functions $\tilde{f}$ are given by large-$\alpha$ integration and correspond to matrix elements of the composite operators which are properly defined normal products of the fundamental (quark, gluon) fields, or in the parton language, to parton distribution functions. Subtraction procedure provides the ultraviolet regularization for parton distribution functions and the infrared one for parton subprocess cross-sections.

## 4. *SUBTRACTION PROCEDURE IN COORDINATE REPRESENTATION*

An analysis of the theories describing spinor particles is highly complicated in the $\alpha$-representation by preexponential factors due to numerators of spinor propagators. Factorization properties of the preexponential factors can be established only after the very sophisticated treatment. Such an analysis has been performed, however, for 4-leg diagrams /18/ relevant to deep inelastic scat-



tering. It appears that the functional form of the resultant expression does not differ from the corresponding formula (25) of the $\phi^3(6)$-theory. This "coincidence" has a natural explanation from the view-point of the operator product expansion on the light cone /10/

$$T(j(x)\,j(0)) \underset{x^2 \to 0}{=} \sum_{i,n} F_i(x^2, \mu^2)\, x^{\mu_1} \ldots x^{\mu_n} \times$$

$$\times\, O^i_{\mu_1 \ldots \mu_n}(0; \mu^2), \qquad (40)$$

where $\mu$ is the renormalization parameter for local operators $O_{\mu_1 \ldots \mu_n}(x)$. It is well known how to obtain a representation of eq. (35) type from the OPE (eq. (40)) /1,13/,and we will not repeat it here. Let us show that eq. (40) is a coordinate representation version of the subtraction procedure we have formulated above. A derivation of the representation (25) was very essentially based on the factorization properties with respect to 2-particle divisions in the t -channel. These properties are trivial in the coordinate representation (*fig. 5a*)

$$\mathcal{F}_W(x, a_i, b_j) = \int \prod_{k=1}^{n} dx_k T(x, 0; x_1, \ldots, x_n; a_i, b_j) =$$

$$= \int d\xi d\eta \{ \prod_{k=1}^{m} dx_k C_V(x, 0; x_1, \ldots, x_m; \xi, \eta) \} \times$$

$$\times \{ \prod_{k=m+3}^{n} dx_k f_{W \setminus V}(\xi, \eta; x_{m+3}, \ldots, x_n; a_i, b_j) \}, \qquad (41)$$

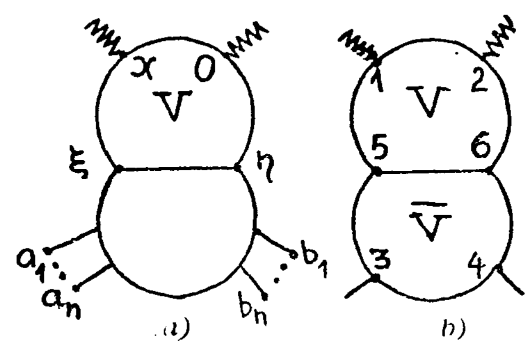



*Fig. 5*

with notation: $\xi = x_{m+1}$ ; $\eta = x_{m+2}$ .To construct the subtraction procedure one must split $W$ into pairs $V, W\setminus V$ and then apply an infrared regularization to $C_V$ (e.g., $\lambda_V < 1/\mu^2$ ) and an ultraviolet one to $f_{W\setminus V}$ ( $\lambda_v > 1/\mu^2$ for all the subgraphs which become divergent after uniting $\xi$ and $\eta$ ). As a result,

$$\mathcal{F}_W(x, a_i, b_j) = \sum_V \int d\xi d\eta\, C_V(x, \xi, \eta; \mu^2) \times$$

$$\times f_{W\setminus V}(\xi, \eta; a_i, b_j; \mu^2) + R_W(x, a_i, b_j) . \tag{42}$$

Summing up over all relevant diagrams we obtain the representation

$$\mathcal{F}(x, a_i, b_j) = \sum_m \int d\xi d\eta\, C_m(x, \xi, \eta; \mu^2) \times$$

$$\times f_m(\xi, \eta; a_i, b_j; \mu^2) + \tilde{R}(x, a_i, b_j) , \tag{43}$$

where $m$ denotes possible two-particle states. The functions $C, f$ are the Green functions

$$C_m(x, \xi, \eta; \mu^2) = \langle 0 | T(J(x)\, J(0)\, j_m(\xi) j_m(\eta)) | 0 \rangle |_{\mu^2, IR} , \tag{44}$$

$$f_m(\xi, \eta; a_i, b_j; \mu^2) = \langle 0 | T[N_{\mu^2}(\phi_m(\xi)\phi_m(\eta)) \times$$

$$\times \Phi(a_1) \ldots \Phi(a_n)\, \tilde{\Phi}(b_1) \ldots \tilde{\Phi}(b_n)] | 0 \rangle , \tag{45}$$

where $\phi_m$ are the "parton" fields, $j_m$ are the corresponding currents, $\Phi, \tilde{\Phi}$ are the fields of external particles.

The symbol $N_{\mu^2}$ denotes the aforementioned ultraviolet regularization procedure characterized by parameter $\mu$. Formally, one can expand $\phi_m(\xi)\phi_m(\eta)$ into the Taylor series

$$\phi_m(\xi)\phi_m(\eta) = \sum_{n=0}^{\infty} (\xi-\eta)^{\mu_1} \ldots (\xi-\eta)^{\mu_n} P^{(m)}_{\mu_1 \ldots \mu_n} \left(\frac{\xi-\eta}{2}\right) \frac{1}{n!} , \tag{46}$$



where $P^{(n)}_{\mu_1 \ldots \mu_n}(x) = \phi_m(x) \overleftrightarrow{\partial}_{\mu_1} \ldots \overleftrightarrow{\partial}_{\mu_n} \phi_m(x)$ . But matrix elements of these operators have divergences which are not removed by the ordinary R-operation*. For bilocal operator $N_{\mu^2}(\phi_m(\xi)\phi_m(\eta))$ one can take limit $\xi \to \eta$ and use eq. (46).

With the help of a standard procedure /21/ (using expansion over a complete set: $1 = \sum |n\rangle\langle n|$) one can get matrix elements $\langle P | P_{\mu_1 \ldots \mu_n} | P \rangle$ in place of auxiliary Green functions.

Representation (43) in conjunction with eqs. (45), (46) gives the following operator product expansion on the light cone:

$$T(J(x)\, J(0)) = \sum_i \int d\xi d\eta\, C_i(x, \xi, \eta; \mu^2) \mathcal{O}_i(\xi, \eta; \mu^2) + \tilde{R}(x) , \tag{47}$$

where $\mathcal{O}_i(\xi, \eta) = N_{\mu^2}(\phi_i(\xi)\phi_i(\eta))$. Using eq. (46), expanding $P_{\mu_1 \ldots \mu_n}$ over operators $O^{(n)}_{\{\mu_1 \ldots \mu_\ell\}}$ with definite Lorentz spin

$$P_{\mu_1 \ldots \mu_n} = \sum_{\ell=0}^{n} C_{n\ell} \left[ O^{(n-\ell)}_{\mu_1 \ldots \mu_\ell} g_{\mu_{\ell+1}\mu_{\ell+2}} \ldots g_{\mu_{\ell-1}\mu_n} \right]_{symm} , \tag{48}$$

and integrating over $\xi, \eta$

$$\int d\xi d\eta\, C_i(x, \xi, \eta; \mu^2) \{(\xi-\eta)^{\mu_1} \ldots (\xi-\eta)^{\mu_\ell}\} [(\xi-\eta)^2]^{\frac{n-\ell}{2}} =$$

$$= \{x^{\mu_1} \ldots x^{\mu_\ell}\} h_i^{(n,\ell)}(x^2\mu^2)(x^2)^{(4+d_i+n-\ell)/2} , \tag{49}$$

(where $d_i$ is the dimension of the function $C_i$ in length units), we obtain the operator product expansion in the standard form

$$T(J(x) J(0)) = (x^2)^{4+d_i/2} \{ \sum_i h_i^{(n,n)}(x^2\mu^2) \times$$

$$\times x^{\mu_1} \ldots x^{\mu_n} O^{(0)}_{\mu_1 \ldots \mu_n}(0; \mu^2) + R(x) \} . \tag{50}$$

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*A mathematically rigorous analysis of composite operators in quantim field theory has been given in ref. /20/, where one can also find a complete set of references to earlier investigations.

It was essential for integration in eq. (49) that the matrix element $\langle P|O(x)|P\rangle$ is $x$-independent, hence, strictly speaking, eq. (50) is valid only if it is assumed that all operators are taken in symmetric brackets. We have added higher twist operator contributions to the function $\tilde{R}(x)$, which has a weaker singularity on the light cone than the functions $h_i^{(n,n)}(x^2\mu^2)$.

The contribution of an $s, Q^2$ - subgraph in theories describing particles with nonzero spin is given by the following expression (see. eq. (A.12))

$$F_2^{(V)} \lesssim Q^{2-\Sigma t_i} , \tag{51}$$

where $t_i$ are the twists of fields corresponding to external lines of the subgraph $V$. In a Yukawa type theory (i.e., $\mathcal{L} = g\bar{\psi}\psi\phi$ in the 4-dimensional space-time) the twists of all the particles are equal to 1. Hence it is necessary to consider subgraphs having two (in addition to two photon lines) external lines. The only complication is the use of the Fierz identity for spinor two-particle divisions

$$\delta^{a}_{\beta}\,\delta^{\beta'}_{a'} = \frac{1}{4}\sum_i (\Gamma_i)^{a}_{a'}(\Gamma_i)^{\beta'}_{\beta} \;;\; \Gamma_i = 1,\, i\gamma_5, \gamma_5\gamma_\mu, \sigma_{\mu\nu}/\sqrt{2}, \gamma_\mu . \tag{52}$$

An upper estimate (eq. (51)) is realized for a $\gamma_\mu$-projection. In this case the twist of a composite operator is equal to the sum of twists of constituent fields.

Hence, in scalar or pseudoscalar gluon theories there are no essential complications in a coordinate representation treatment due to nonzero spin of the fermion (quark) field, and the result (25) remains unchanged.

To complete the paper, let us summarize the main results. We have developed a method for investigating the asymptotical behaviour of deep inelastic scattering in scalar field theory models. The method uses the $\alpha$-representation and the Mellin transformation. An algorithm is given for constructing a subtraction procedure which separates the contributions of large and small distances. The relation is investigated of this procedure



to that one of the composite operator renormalization and of the infrared regularization for short-distance contributions. An explicit use of the coordinate representation allows us to demonstrate that the subtraction procedure leads to the light-cone operator product expansion. And finally, the equivalence of the field-theoretic approach and of the modified parton model is shown.

*APPENDIX*

Using $\alpha$-representation /15/ for propagator (eq. (39)) one can represent the contribution of any diagram of a scalar theory as follows

$$M(p_1,\dots,p_n; m^2) =$$

$$= \frac{P(c.c.)}{(4\pi)^{Nz/2}} \int_0^{i\infty} \prod_\sigma d\alpha_\sigma D^{-N/2}(\alpha)\, e^{\frac{Q(\alpha,p)}{D(\alpha)} - \sum_\sigma (m_\sigma^2 - i\epsilon)\alpha_\sigma} , \tag{A.1}$$

where $P(c.c.)$ is a product of coupling constants, $z$ is the number of loops of the diagram. $M$-matrix in eq. (A.1) has a natural normalization: tree graphs do not possess factors like $2\pi$ or $\pi$. The Symanzik functions /22/ $Q(\alpha, p)$ and $D(\alpha)$ are determined by the topological structure of the diagram. They are independent of $N$ - the number of space-time dimensions.

Let us remind that "$k$-tree" of a graph is called its subgraph which contains all the vertices of the initial graph and consists of $k$ components. $k$-tree $V_k$ is uniquely determined by its chords (the lines which must be eliminated from $V$ to get $V_k$). The product of $\alpha$-parameters corresponding to these chords is also called $k$-tree. 1-tree is simply tree. The determinant $D(\alpha)$ of a graph is the sum of all its trees. Let us denote $B(i_1,\dots,i_k | j_1,\dots,j_\ell)$ the sum of such 2-trees of the subgraph $V$ for which the vertices $i_1,\dots,i_k$ belong to one component; $j_1,\dots,j_\ell$, to another whereas nonenumerated vertices can belong to any of them. Then

$$Q(\alpha,p) = \Sigma B(i_{\alpha_1}\dots i_{\alpha_m} \,|\, j_{\alpha_{m+1}}\dots j_{\alpha_n}; V)\times$$

$$\times(p_{\alpha_1}+\dots+p_{\alpha_m})^2, \qquad \text{(A.2)}$$

where $i_{\alpha_1},\dots,i_{\alpha_n}$ are the vertices which the external momenta $p_{\alpha_1},\dots,p_{\alpha_n}$ enter into, and the summation runs over all the possible divisions of these vertices into two connected components. It is clear that

$$(p_{\alpha_1}+\dots+p_{\alpha_m})^2 = (p_{\alpha_{m+1}}+\dots+p_{\alpha_n})^2. \qquad \text{(A.3)}$$

By construction $D(\alpha)\geq 0$; $B(i\dots|j\dots)\geq 0$ for real positive $\alpha$. Determinant is a homogeneous function of $\alpha$-parameters: $D(\lambda\beta)=\lambda^z D(\beta)$. In the same way $Q(\lambda\beta,p) = \lambda^{z+1}Q(\beta,p)$.

For a 4-leg diagram describing deep inelastic scattering (*fig. 1a*):

$$Q(\alpha,P,q) = q^2B(1|2) + 2(Pq)B_-(12|34) + P^2B(3|4), \qquad \text{(A.4)}$$

where

$$B_\pm(ab|cd) = B(ac|bd)\pm B(ad|bc).$$

By definition $B(1|2) = B(1|234) + B(2|134) + B_+(12|34)$. Hence, $B(1|2)\geq B(12|34)\geq |B_-(12|34)|$ for real positive $\alpha$. It is useful to know the factorization properties of the functions $B, D$ with respect to two-particle divisions in the $t$-channel

$$D(V+\bar V) = D(V)\,B(5|6;\bar V) + B(5|6;V)\,D(\bar V),$$

$$B_\pm(12|34;V+\bar V) = B_\pm(12|56;V)\,B_\pm(56|34;\bar V).$$



The functions $B_\pm(12|56;V)$ $B_\pm(56|34;\bar V)$ are constructed for subgraphs $V,\bar V$ in the same way as the function $B_\pm(12|34, V+\bar V)$ for a subgraph $V+\bar V$. We denote them $A_\pm(V)$, $A_\pm(\bar V)$, $A_\pm(V+\bar V)$, respectively. The function $B_+(5|6;\bar V)\equiv D_0(\bar V)$ is the determinant of the diagram shown in *fig. 1c*. When $\lambda_V\to 0$ we have

$$D(V+\bar V) = \lambda_V^{z_V}\{d(V)D_0(\bar V) + \lambda_V b(5|6;V)D(\bar V)\}, \qquad \text{(A.5)}$$

i.e., $D(V+\bar V) \underset{V}{=} D(V)\,D_0(\bar V)$. The sign $\underset{V}{=}$ means the equality holds in the limit $\lambda_V\to 0$. For the handbag diagram (*fig. 1b*)

$$D = \alpha_0 D(\bar V) + D_0(\bar V);\quad A_- = \alpha_0\tilde A_-;$$

$$B(1|2,\alpha_0+\bar V) = \alpha_0 D_0(\bar V). \qquad \text{(A.6)}$$

The $\alpha$-representation for the diagram *1c* with an operator $i\psi^*\{\overleftrightarrow{\partial}_{\mu_1}\dots\overleftrightarrow{\partial}_{\mu_n}\}\psi$ corresponding to the O-vertex can be written as

$$\{P_{\mu_1}\dots P_{\mu_n}\}\frac{P(c.c.)}{(4\pi)^{Nz/2}}\int_0^{i\infty}\frac{\prod_\sigma d\alpha_\sigma}{D_0^{N/2}}\,e^{I_0(\alpha,m^2)}\left(\frac{\tilde A_-(\alpha)}{D_0(\alpha)}\right)^n. \qquad \text{(A.7)}$$

Expanding the quantities $(p_{\alpha_1}+\dots+p_{\alpha_m})^2$ over independent invariants $s_1,\dots,s_k$, we obtain

$$M(s_1,\dots,s_k;m^2) = P(c.c.)(4\pi)^{-Nz/2}\times$$

$$\times\int_0^{i\infty}\prod_\sigma d\alpha_\sigma D^{-N/2}(\alpha)\exp\{\sum_{i=1}^{k}s_iA_i(\alpha)/D(\alpha) - \sum_\sigma\alpha_\sigma(m_\sigma^2-i\epsilon)\}. \qquad \text{(A.8)}$$

It is clear from this representation that the behaviour of $M(s_1,\dots,s_k;m^2)$ in the region $s_1\sim s_2\sim\dots\sim s_\ell\sim s\gg s_{\ell+1}\sim\dots\sim s_k\sim m^2$ is determined by the region of $\alpha$-parameters, where $\sum_{i=1}^{\ell}(s_i/s)A_i(\alpha)/D(\alpha)\sim 0$. We consider now the following possibility for this expression to vanish: all the functions $A_i/D$; $i=1,\dots,\ell$ vanish when $\alpha_\sigma=0$ for lines $\sigma$ belonging to a subgraph V. Note that the k-trees of a subgraph resulting from an initial graph V after contraction of lines $\sigma_1, \dots, \sigma_n$ into points, can be obtained from the corresponding k-trees of the graph V by putting $\alpha_{\sigma_1} = \dots = \alpha_{\sigma_n} = 0$. Hence the vanishing of parameters $\{\alpha_{\sigma_i}\}^1$ means topologically the contraction of the subgraph v into point. We have supposed that $A_i/D = 0$ when $\alpha_{\sigma_i} = 0$; $\sigma_i \in v$; $i = 1, \dots, \ell$. Hence, the diagram obtained after contraction of the subgraph v into point is independent of large variables $s_1, \dots, s_\ell$. This simplifies the search of such $s_1, \dots, s_\ell$ -subgraphs.

Let V be a connected subgraph with $\ell$ lines, v vertices and z loops ($z = \ell - v + 1$). The scaling

$$\alpha_\sigma = \lambda_V \beta_\sigma;\ \lambda_V = \sum_{\sigma \in V} \alpha_\sigma;\ \prod_\sigma d\alpha_\sigma = \lambda_V^{\ell-1} d\lambda_V \prod_\sigma d\beta_\sigma \delta(1 - \Sigma\beta_\sigma), \tag{A.9}$$

results in the following relation

$$D(\alpha) = \lambda_V^z d(\beta_\sigma, \alpha_{\bar\sigma}; \lambda_V) = \lambda_V^z \{d(\beta_\sigma, \alpha_{\bar\sigma}; 0) + \sum^\infty \lambda_V^k d_k(\beta_\sigma, \alpha_{\bar\sigma})\},$$

$$A_i(\alpha) = \lambda^{z+1} \{a_i(\beta_\sigma, \alpha_{\bar\sigma}; 0) + \sum_{k=1}^\infty \lambda_V^k a_i^{(k)} (\beta_\sigma, \alpha_{\bar\sigma})\}. \tag{A.10}$$

Using the relation $nv = \ell_{ext} + 2\ell$ (where $\ell_{ext}$ is the number of external lines of the dubgraph V) in the scalar $g\phi^n_{(N)}$ -theory (we take $s \sim Q^2$) integrating over $\lambda_V \sim 0$ we obtain

$$M \sim Q^{(N - [\phi]\ell_{ext} - [g]v)}, \tag{A.11}$$

where $[\phi] = (N-2)/2$ is the $\phi$-field dimension, whereas $[g] = N - n(N-2)/2$ is that of the coupling constant (in mass units). If the coupling constant is dimensionless, the asymptotical behaviour of an amplitude depends only on the number of external lines of the subgraph (remember that we consider now only scalar theories), hence the subgraphs with a minimal number of external lines are responsible for the leading asymptotical behaviour. In the superrenormalizable $\phi^3_{(4)}$ -theory we have



$M \sim Q^{(4 - \ell_{ext} - v)}$ : the leading asymptotical contribution in the cases treated in this paper is given by a tree subgraph with a minimal possible number of external lines.

The result (A.11) can be easily obtained by a dimensional analysis. In an arbitrary theory the contribution of a subgraph V has dimension $d_V = N - \Sigma[\phi_i] - \Sigma[g_j]$ (where the summation over i runs over external lines whereas that over j, over vertices of the subgraph V). If the i-th external line describes a particle with spin $s_i$, then there can appear an additional factor $Q^{s_i}$ (for instance, $P_\mu \sim Q$ for a photon line, $u(P) \sim Q^{1/2}$ for a spin -1/2 particle, and so on). This results in

$$M \sim Q^{N - \Sigma t_i - \Sigma[g_j]}, \tag{A.12}$$

where $t_i = [\phi_i] - s_i$ is the twist of field $\phi_i$. In the 4-dimensional space-time we have $t_i = 1$ for particles with $s_i = 0,\ 1/2$; $t_i = 0$ for the vector potential $A_\mu$; tensor $F_{\mu\nu}$ effectively corresponds to $t_i = 1,2$.

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