

ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
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ДУБНА



0-35

3320/2-78

14/viii-78

E2 - 11702

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1978

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Submitted to "Physics Letters" B

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E2 - 11702

Структура группы супергравитации

Обнаружено, что группа супергравитации есть прямое произведение общековариантных групп в комплексно-сопряженных правом и левом суперпространствах. Обычная пространственно-временная координата и аксиальное гравитационное суперполе идентифицируются с вещественной и мнимой частями комплексной координаты.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

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E2 - 11702

Structure of Supergravity Group

The supergravity group is found to be the direct product of general covariance groups in complex conjugated left and right handed superspaces. The ordinary space-time coordinate and the axial gravitational superfield are the real and imaginary parts of the complex coordinate respectively.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research.

Dubna 1978

Recently several groups of authors ^{/1,2/} have managed to add to the gravitational supermultiplet $(2, 3/2)$ a minimal set of auxiliary fields and thus to close the supersymmetry algebra. Their set of fields corresponds to the field content of the axial superfield $H^\mu(x, \theta, \bar{\theta})$ which we proposed in 1976 ^{/3/} (see also ref. ^{/4/}) as the most adequate minimal gravitational superfield. However the closing transformation algebra has been found by a cumbersome and vague technique. The lack of geometrical meaning and the used component notations could cause difficulties when generalizing these results for the extended supergravity and investigating the higher-order counter terms. On the other hand the superspace approaches (ref. ^{/5/} and references therein) are far from being clear. They involve the excessively wide general covariance group in the real superspace $\{(x^\mu, \theta^a, \bar{\theta}^{\dot{a}})\}$, and respectively a too great number of superfluous fields.

In the present letter we show that the true supergravity group is much simpler. It is represented by a unification of two complex conjugated general covariance groups in two smaller "chiral" superspaces, just in the left handed one $\{(x^{\mu L}, \theta^a)\}$ and in

the right handed one $\{(x^{\mu R}, \bar{\theta}^{\dot{a}})\}$. The main "metric" object in our approach is the axial superfield $H^\mu(x, \theta, \bar{\theta})$ and it is introduced as the imaginary part of the complex space-time coordinate in our complex superspace while its real part is identified with the true space-time coordinate.

As a starting point we have used the earlier results of our supercurrent approach^{/3,4/}. There we have investigated at the linearized level the interaction of the axial superfield $H^\mu(x, \theta, \bar{\theta})$ with matter chiral superfields $\Phi_L(x, \theta, \bar{\theta})$ and $\Phi_R(x, \theta, \bar{\theta})$ through the supercurrent* and have found two closed transformation groups, one for Φ_L and another for Φ_R (Eqs. (21a) and (21b) in ref.^{/4/}). To understand their geometrical meaning we represented them as transformations of the superspace coordinates. It is well known^{/6/} that the left (right) handed chiral superfield is treated most naturally in the so-called "left" ("right") handed basis of superspace $\{(x^\mu, \theta^a, \bar{\theta}^{\dot{a}})\}$. The left and right bases are connected by a purely imaginary shift $2i\theta\sigma^\mu\bar{\theta}$ of the space-time coordinate x^μ thereby making it complex in essence. Therefore it is tempting to introduce two complex conjugated "chiral" superspaces $\{(x^{\mu L}, \theta^a)\}$ and $\{(x^{\mu R}, \bar{\theta}^{\dot{a}})\}$ instead of the real one $\{(x^\mu, \theta^a, \bar{\theta}^{\dot{a}})\}$, $x^{\mu L}$ and $x^{\mu R}$ being complex conjugated space-time coordinates in the "left" handed and "right" handed superspaces, respectively.

*The concept of supercurrent has been introduced by Ferrara and Zumino^{/7/}. The existence of this object in the general case has recently been proved in our paper^{/8/}.

In such a new framework the transformations (21a) and (21b) in ref.^{/4/} prove simply to form the general covariance group of infinitesimal transformations in the left handed and right handed superspaces respectively

$$\begin{aligned}\delta x^{\mu L} &= a^\mu(x^L, \theta) & \delta x^{\mu R} &= \bar{a}^\mu(x^R, \bar{\theta}) \\ \delta \theta^a &= \epsilon^a(x^L, \theta) & \delta \bar{\theta}^{\dot{a}} &= \bar{\epsilon}^{\dot{a}}(x^R, \bar{\theta})\end{aligned}\quad (1)$$

where a^μ, ϵ^a and their complex conjugates $\bar{a}^\mu, \bar{\epsilon}^{\dot{a}}$ are arbitrary vector and spinor superfunctions-parameters, respectively. The group meaning of these transformations is obvious and the group law is

$$\begin{aligned}a^\mu_{\text{bracket}} &= a^\nu(2) \frac{\partial}{\partial x^{\nu L}} a^\mu(1) + \epsilon^\beta(2) \frac{\partial}{\partial \theta^\beta} \bar{a}^\mu(1) - (1 \leftrightarrow 2) \\ \epsilon_a^{\text{bracket}} &= a^\nu(2) \frac{\partial}{\partial x^{\nu L}} \epsilon_a(1) + \epsilon^\beta(2) \frac{\partial}{\partial \theta^\beta} \epsilon_a(1) - (1 \leftrightarrow 2)\end{aligned}\quad (2)$$

and its complex conjugate.

In fact in ref.^{/4/} we have derived the transformations (21a) and (21b) which are rewritten now in the form (1) with the restriction Eq. (11b) in^{/4/} which is equivalent to

$$-\frac{\partial}{\partial \theta^a} \epsilon_a + \frac{\partial}{\partial x^{\mu L}} a^\mu = 0 \quad (3)$$

and its complex conjugate. These constraints are compatible with the group law (2) and

have a simple geometrical meaning. Restrictions (3) mean that the Berezinian of transformations (1) becomes unity or in other words, that the "volume" both in the left and right handed superspaces is conserved. It can be shown that the whole group (1) corresponds to conformal supergravity while its volume-preserving subgroup (Eqs. (1), (3)) describes Einstein supergravity.

After establishing this simple group structure underlying supergravity we have to answer two more questions. First, we need the common real superspace $\{(x^\mu, \theta^a, \bar{\theta}^{\dot{a}})\}$ to deal with real superfields. Second, we have to introduce a "metric" (gauge) object, which will be, as we believe, the axial superfield $H^\mu(x, \theta, \bar{\theta})$. These two problems are solved simultaneously. Let us make a change of variables:

$$x^\mu = \frac{1}{2}(x^{\mu L} + x^{\mu R}), \quad H^\mu = \frac{1}{2i}(x^{\mu L} - x^{\mu R}). \quad (4)$$

Now the real variable x^μ can be identified with the common physical space-time coordinate.

Further, instead of regarding the imaginary part H^μ as an independent coordinate we express it as a function of the remaining variables*.

*Note that Volkov and Akulov have tried to identify the spinor coordinate with a Goldstone neutrino field in their pioneering paper /9/.

$$H^\mu = H^\mu(x^\nu, \theta^a, \bar{\theta}^{\dot{a}}). \quad (5)$$

So we introduced a real superspace $\{(x^\mu, \theta^a, \bar{\theta}^{\dot{a}})\}$ and an axial gauge superfield H^μ in it with transformation laws following directly from Eqs. (1) and (4):

$$H^{\mu'}(x', \theta', \bar{\theta}') = H^\mu(x, \theta, \bar{\theta}) - \frac{i}{2} a^\mu [x^\nu + iH^\nu(x, \theta, \bar{\theta}), \theta^\beta] + \frac{i}{2} \bar{a}^\mu [x^\nu - iH^\nu(x, \theta, \bar{\theta}), \bar{\theta}^{\dot{\beta}}], \quad (6a)$$

$$x^{\mu'} = x^\mu + \frac{1}{2} a^\mu [x^\nu + iH^\nu(x, \theta, \bar{\theta}), \theta^\beta] + \frac{1}{2} \bar{a}^\mu [x^\nu - iH^\nu(x, \theta, \bar{\theta}), \bar{\theta}^{\dot{\beta}}], \quad (6b)$$

$$\theta^{a'} = \theta^a + \epsilon^a [x^\nu + iH^\nu(x, \theta, \bar{\theta}), \theta^\beta], \quad (6c)$$

$$\bar{\theta}^{\dot{a}'} = \bar{\theta}^{\dot{a}} + \bar{\epsilon}^{\dot{a}} [x^\nu - iH^\nu(x, \theta, \bar{\theta}), \bar{\theta}^{\dot{\beta}}].$$

It is not hard to convince oneself that the new transformations (6) obey the same group law (2) as the old ones (1).

Our gravitational superfield $H^\mu(x, \theta, \bar{\theta})$ is introduced in a rather unconventional manner. It is an object of dual nature, playing the role of a coordinate in the complex superspace $\{(x^{\mu L}, x^{\mu R}, \theta^a, \bar{\theta}^{\dot{a}})\}$ and of a "metric" object of the real subspace $\{(x^\mu, \theta^a, \bar{\theta}^{\dot{a}})\}$. In order to understand better its second nature, it is worthwhile to write down the transformation law (6) in more habitual terms of component fields. We shall do it in this letter for the conformal

supergravity case only. The first step is to fix properly the gauge to avoid the complicated nonlinearity of Eq. (6). This is done by means of a trick proposed by Wess and Zumino for the supersymmetric Yang-Mills case^{/10/}. The few first components of H^μ are gauged out and its decomposition is reduced to

$$H^\mu = \partial \sigma_a \bar{\theta} e^{\mu a} + \kappa \theta \bar{\theta} \cdot \bar{\theta} \psi^\mu + \kappa \bar{\theta} \bar{\theta} \cdot \theta \psi^\mu + \\ + \theta \bar{\theta} \cdot \bar{\theta} \bar{\theta} (\kappa A^\mu + \frac{1}{4} \epsilon^{\mu\nu\lambda\rho} e_{a\lambda} \partial_\nu e^a{}_\rho) \quad (7)$$

Here the fields are: $e^{\mu a}(x)$ (and its inverse $e_{a\mu}(x)$) is the vierbein field; $\psi^{\mu a}(x)$, $\bar{\psi}^{\mu \dot{a}}(x)$ is the "gravitino" field; $A^\mu(x)$ is the gauge field for the local chiral invariance. After the gauge has been fixed there remains some class of transformations (6) which preserve the gauge. Omitting all the details, we formulate the final results. The fields in Eq. (7) undergo the following transformations: 1) General covariance transformations (with μ being a contravariant index); 2) Local Lorentz ones; 3) Local scale ones; 4) Local chiral ones; 5) Local supersymmetry (Q- and S-supersymmetries with parameters $\epsilon_a(x)$ and $\eta_a(x)$ in four-component notations):

$$\delta e^{\mu a} = \kappa \bar{\epsilon} \gamma^a \psi^\mu,$$

$$\delta \psi^\mu{}_a = \kappa^{-1} [(\gamma^\mu \eta)_a + 2i \hat{V}^\mu \epsilon_a - \frac{i}{2} (\gamma^\mu \gamma^\nu \hat{V}_\nu \epsilon)_a],$$

$$\delta A^\mu = -i \bar{\eta} \gamma_5 \psi^\mu - \frac{1}{4} \epsilon^{\mu\nu\lambda\rho} \nabla_\nu (\bar{\epsilon} \gamma_\lambda \psi_\rho) - \\ - \nabla_\nu \bar{\epsilon} \gamma_5 \gamma^\mu \psi^\nu + \frac{1}{2} \bar{\epsilon} \gamma_5 \gamma^\nu \hat{V}_\nu \psi^\mu. \quad (8)$$

Here $\hat{V}^\mu = \nabla^\mu - \kappa A^\mu \gamma_5$ and ∇^μ is the standard general covariant derivative. These transformations form a closing algebra because they are derived from transformations (6) having an obvious group character. The simplest way to find the bracket parameters is to use the general formula (2) properly adapted to the gauge-fixing procedure. We shall not discuss this business here. Of course, one can check, if one prefers, the closure of the algebra by direct tedious calculations.

The transformations we have obtained are similar to those of conformal supergravity^{/2/}. The difference in local supersymmetry transformations is probably due to the fact that we combine into supermultiplet (2, 3/2) the contravariant vierbein, while in ^{/2/} there appears the covariant vierbein. Besides, on the component field level there remains a great freedom of redefining of the field variables and also of the parameter function $\eta_a(x)$.

In conclusion we wish to point out that a number of questions concerning the formalism remains open. One has to know how to define superfields with external indices, supercovariant derivatives and invariants of the group, etc. However now we can say with certainty that this extremely simple and clear geometrical picture of the supergravity group will provide an adequate basis

for the supergravity theory. A generalization to extended supergravity is straightforward: one has simply to supply the Grassmann variables $\theta^a, \bar{\theta}^{\dot{a}}$ with internal symmetry indices a and to consider the left handed and right handed extended superspaces

$\{(x^{\mu L}, \theta^{a a})\}, \quad \{(x^{\mu R}, \bar{\theta}^{\dot{a} a})\}$. In any case, our approach to supergravity as the theory of an axial superfield generated by the supercurrent^{4/} turns out to be true.

More details on this subject will be given elsewhere.

It is a pleasure for us to thank E.A.Ivanov for many useful discussions.

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Received by Publishing Department
on June 28 1978.