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Структурные константы мезонов в модели кварковых диаграмм

На основе предположения о глобальной дуальности между кварковыми петлевыми диаграммами и вкладом суммы адронных резонансов в правила сумм для структурных параметров мезонов, вытекающие из алгебры токов, вычислены электромагнитные радиусы пионов и каонов, параметры формфакторов $K_{\ell 3}$ -распада и структурные константы $\pi \to \ell \nu \gamma$ -распада.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Meson Structure Constants in a Model of the Quark Diagrams

The electromagnetic radii of pions and kaons, the $K_{\ell 3}$-decay form factors and the $\pi \to \ell \nu \gamma$ decay structure constants are calculated by assuming the global duality between the contributions of the quark loop diagrams and sum of the hadron resonances in the current algebra sum rules for meson structure parameters.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. *INTRODUCTION*

The limitation to lowest order perturbation theory in hadronic coupling constants is widely used in the theory of the deep-inelastic lepton-hadron interactions. The validity of this approximation is linked to the asymptotic freedom property of the fundamental gauge theory of the quark-gluon interactions at small distances. The use of the lowest diagrams in calculation of the observables which are sensitive to the long-range behaviour of the interactions may be justified, generally speaking, in the exceptional cases only. The successful calculation of the $\pi^{\circ} \to 2\gamma$ decay amplitude via the triangle, fermion-loop diagram is the well-known example. The absence of the radiative corrections to the loop diagram representing the $\pi^{\circ} \to 2\gamma$ decay as $\mu_{\pi} \to 0$ (μ_{π} being the pion mass) is related to its "anomalous" nature [1].

This paper presents the results for the structure parameters, defining the pion and kaon weak and electromagnetic form factors at small momentum transfers, via the lowest order, quark-loop diagrams. The arguments presented are heuristic and do not look to be a proof. However, since the results are ancouraging, a more adequate argumentation may, hopefully, be developed some time. Our consideration can be outlined as the "hydrid" one, because the use was simultaneously made of the concepts pertient to the "current" (or field-theoretic) and "spectroscopic" (quantum-mechanical) description of the quark dynamics. We start with the integral sum rules following from the current algebra and

PCAC. We assume these sum rules to be saturated by the infinite number of the hadron resonances viewed as built of minimal number of the constituent quarks which interact with each other through some confining potential. A more detailed dynamics should not be specified. Our most essential assumption is the application of PCAC and the Goldberger-Treiman (GT) relation to the local meson-quark vertices to fix the quark-meson coupling constants. The local approximation of the vertex has analogy to the corresponding quantum-mechanical problems, when one uses the asymptotic form of the wave function in calculating a radius of the bound system. For loosly-bound, nonrelativistic system, like the deuteron, this approximation is reasonably justified because the constituents spend most time beyond the potential range. Probably, the dynamics of pion should be relativistic and rather complicated, bearing in mind the features referring it to the state of the "collective" nature. Because of the absence of the unified description of both PCAC and the composite nature of pion, it is not unreasonable to try the local $\pi q \bar{q}$ -vertex, at least as a first approximation.

The paper is organized as follows. The second part treats the electromagnetic radii of pions and kaons. The K -meson structure parameters require additional assumptions on the SU(3)-symmetry breaking. The $K_{\ell 3}$ -decay form factors, which are especially sensitive to the symmetry breaking, are discussed in the third part. In the 4-th section the spectral function sum rules for the meson leptonic-decay couplings and the $\pi \to \ell \nu \gamma$ -decay parameters are considered and compared with new experimental data. Here, some results of the author's previous paper /2/ are partially reproduced and corrected. The summary and some final remarks conclude the consideration.

2. *ELECTROMAGNETIC RADII OF PIONS AND KAONS*

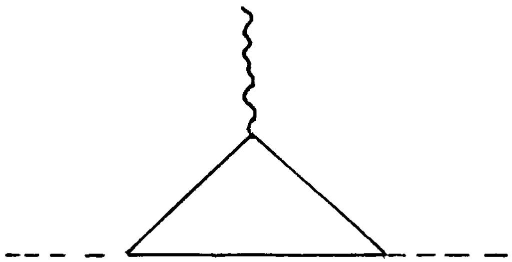
We assume validity of the Cabibbo-Radicati sum rule (s.r.) which relates the pion radius (generally, the hadron isovector radius) to the integral of the radiative cross-sections, induced by the isovector part of the electromagnetic current

$$\frac{4}{3}\pi^2 \alpha < r^2 >_\pi = \int \frac{dE}{E} [\sigma(\gamma^- \pi^+) - \sigma(\gamma^+ \pi^+)]. \qquad (1)$$

This s.r. was obtained originally with the help of the algebra of the dipole operators in the infinite momentum reference frame /3/. The simple structure of the dipole moment operator leads to the well-known, in the non-relativistic theory of the atomic and nuclear photoeffect, fact of independence of the value

$$<\Psi_0 | D_i^2 | \Psi_0> = \sum_n <\Psi_0 | D_i | \Psi_n><\Psi_n | D_i | \Psi_0>, \qquad (2)$$

on which the complete set $\{\Psi_n\}$ of the intermediate-state vectors is summed over in Eq. (2): these may be either the eigenfunctions of the total Hamiltonian of the interacting particles or the plane waves corresponding to the free Hamiltonian. The paradoxial feature of this fact is especially spectacular in case of the confining potentials which have no the continuum part of the spectrum at all. Assuming this to be valid in the relativistic domain, we replace the sum over the complete set of solutions of the relativistic $q\bar{q}$ -system with the confinement by the sum over the plane waves which do not describe any real, physical process as far as there are no free quarks (at least the quarks with small masses coinciding numerically with small effective masses of constituent quarks, entering into the bound-state equations). Therefore, this feature of the bremsstrahlung weighed s.r. makes it possible to substitute the summation over the infinite series of the meson resonances by the integration over the states of "non-physical" photodisintegration process $\gamma M \to q \bar{q}$, where the quarks are represented by the plane waves. In turn, the analytic calculation of these integrals with local coupling in the $M q \bar{q}$ -vertex gives exactly the same result as the calculation of simple triangle diagram shown in *Fig. 1* /4/. Thus, under the assumptions made, the calculation of $<r^2>_\pi$ via the quark-loop diagram may be considered just as a technical trick of most short and



Lowest-order Feynman diagram for the meson form factor. The broken lines represent the meson, the wave line, the photon or W*-boson and the solid lines, the quarks.*

simple evaluation of the integral in s.r. (1) over all resonance photoexcitation cross-sections, and we may not pay attention to the fact that the triangle diagram does nor describe the whole form factor and even does not obey the unit normalization at zero momentum transfer. In the limit $\mu_\pi \to 0$ the simple expression

$$\langle r^2 \rangle_\pi = \frac{3}{4\pi^2 F_\pi^2} \tag{3}$$

follows, where the factor 3 is due to the three colour degrees of freedom, $F_\pi = 93$ *MeV* comes from the GT relation applied to the pseudoscalar $\pi q \bar{q}$ -vertex.

Numerically, via Eq. (3), $\langle r^2 \rangle = 0.34$ fm^2 is close to the experimental value $\langle r^2 \rangle_\pi^{exp} = (0.31 \pm 0.04)$ fm^2 found in the high-energy pion scattering on the atomic electrons [5]. This close coincidence constraints possible corrections to Eq. (3). To estimate the mass-extrapolation effect $\mu_\pi \to 0$ one should know the effective quark mass in the fermion propagators. For the purpose of estimation we assume the approximate factorization property of two form factors

$$\frac{1}{12 m_q^2} = \left(\frac{F'(0)}{F(0)}\right)_{\pi^\circ \gamma\gamma^*} \simeq \left(\frac{F'(0)}{F(0)}\right)_{\pi\pi\gamma^*} = \frac{1}{8\pi^2 F_\pi^2} \tag{4}$$

where the left-hand side of Eq. (4) follows from the triangle-diagram evaluation of the $\pi^\circ \gamma \gamma^*$ -vertex with one of photons being taken off-mass-shell: $q^2(\gamma^*) \neq 0$. From Eq. (4) one obtains (putting $m_q = m_u = m_d$ from here on)

$$m_q = \sqrt{\frac{2}{3}}\, \pi F_\pi \simeq 240\,\text{MeV} \tag{5}$$

which is close to, though somewhat less than, the standard value $m_q \simeq m_N / 3$ of the effective quark mass.

Taking further, as usual, the axial-vector constant of quarks to remain non-renormalizable, we have, via the GT-relation, the quark-pseudoscalar coupling constant

$$\frac{g^2_{\pi^\circ q\bar{q}}}{4\pi} = \frac{\pi}{6} \tag{6}$$

which is weaker than the pion-nucleon coupling:

$$g_{\pi^\circ NN} = (G_A/G_V)(m_N/m_q) g_{\pi^\circ qq} \simeq 4{,}9\, g_{\pi^\circ qq} .$$

Using Eq. (5) and expression for $\langle r^2 \rangle_\pi$ derived in Ref. [4] for the arbitrary value of $\eta = \mu_\pi^2 / m_q^2$ we obtain an increases of $\langle r^2 \rangle_\pi$ by 7% as compared to Eq. (3). To estimate the non-resonance contributions to s.r. (1) one can, following [4], use the cross-sections of reaction $\gamma^{\pm} \pi^{+} \to \pi^\circ X$ proceeding through the one-pion-exchange mechanism and computed approximately to keep only the terms most singular as $\mu_\pi \to 0$:

$$\langle r^2 \rangle_\pi^{OPE} = \frac{1}{8\pi^2 F_\pi^2} \ln \frac{\lambda}{\mu_\pi} . \tag{7}$$

The unknown cut-off value λ seems reasonable to put equal to $\lambda = 1/R$, where $R \simeq 1$ *fm* is the absorption radius used in various peripheral-collision models to take absorptive corrections into account. With such an λ, Eq. (7) is about 6% comparing with Eq. (3). We see that sum of these corrections results in 10-15% increase of $\langle r^2 \rangle_\pi$ as given by Eq. (3), the value not exceeding the experimental uncertainties.

The radius of the charged kaon equals that of pion in the limit of SU(3)-symmetry and, under the formulated assumptions, may be calculated through the triangle diagram of *Fig. 1*. We consider the simple model of the SU(3)-symmetry breaking taking $m_s \neq m_u = m_d$ and $\mu_K \neq \mu_\pi$ but keeping the unbroken symmetry relation $g_{Kus} = g_{\pi ud}$ for dimensionless pseudoscalar coupling constants. From this assumption and the GT-relation we have then

$$\frac{F_K}{F_\pi} = \frac{1+y}{2}, \quad y \equiv \frac{m_s}{m_u}. \tag{8}$$

The numerical value $y \simeq 1.4 \simeq \sqrt{2}$ can be chosen to bring the Λ-hyperon magnetic moment calculated within the nonrelativistic quark model to an agreement with experiment[6]. The same value of y leads, via Eq. (8), to $F_K \simeq 1.2 F_\pi$ in fair agreement with an analysis of the leptonic decays[7]: $F_K/F_\pi\, f_+(0) = 1.27 \pm 0.03$, $f_+(0)$ being the vector form factor of the $K_{\ell 3}$-decay, which, up to the second order terms in the SU(3)-breaking interaction can be taken to be unity (the Ademollo-Gatto theorem). With all parameters fixed, the triangle diagram calculation gives

$$\langle r^2 \rangle_{K^+} = 0.4 \ \mathrm{fm}^2, \tag{9a}$$

$$\langle r^2 \rangle_{K^\circ} = -0.05 \ \mathrm{fm}^2. \tag{9b}$$

The relative values of the charged and neutral K-meson radii were discussed previously in the nonrelativistic quark model[6]. A simple account of exclusion of the centre-of-mass motion results in

$$\langle r^2 \rangle_{K^+} = \frac{1+2y^2}{3(1+y^2)} \langle \vec{r}^{\,2}_{12} \rangle, \tag{10a}$$

$$\langle r^2 \rangle_{K^\circ} = \frac{1-y^2}{3(1+y^2)} \langle \vec{r}^{\,2}_{12} \rangle, \tag{10b}$$

where $|\vec{r}_{12}|$ is a distance between two quarks, and averaging over the ground state wave functions is understood. In the nonrelativistic model, the ratio $\langle r^2 \rangle_{K^\circ} / \langle r^2 \rangle_{K^+}$ equals -0.2, and it exceeds by 60% absolute value of this ratio found from Eqs. (9a,b) corresponding to the relativistic Feynman diagram model. This result agrees qualitatively with the conclusions of Ref.[8] where an attempt was also undertaken to estimate the influence of the relativistic effects on the ratio of K-meson radii given by nonrelativistic theory.

Note, that the isovector kaon radius deviates from the pion radius

$$\langle r^2 \rangle^{V_3}_K = \langle r^2 \rangle_{K^+} - \langle r^2 \rangle_{K^\circ} = 1.32 \langle r^2 \rangle_\pi, \tag{11}$$

i.e., a sizeable violation of the isovector universality occurs in the considered model contrary to the vector meson dominance model with the universal ρ°-meson-hadron couplings, where the equality $\langle r^2 \rangle^{V_3}_K = \langle r^2 \rangle_\pi$ should hold.

For comparison, we provide also the limiting values of radii as $\mu_K \to 0$:

$$\lim_{\mu_K \to 0} \langle r^2 \rangle_{K^+} = 0.28 \ \mathrm{fm}^2, \tag{12a}$$

$$\lim_{\mu_K \to 0} \langle r^2 \rangle_{K^\circ} = -0.04 \ \mathrm{fm}^2. \tag{12b}$$

It may turn out that due to large K-meson's mass the local coupling approximation for the $Kq\bar{q}$-vertex leads to a too large contribution from integration over the high-momentum part of the quark distribution inside kaon and that correct account of the cut-off form factors in the hadron vertices should result in values more close to Eqs. (12a,b) which are seen to agree much better with the isovector universality hypothesis.

3. *THE* $K_{\ell 3}$ *-DECAY FORM FACTORS*

In spite of a large number of papers devoted to both experimental and theoretical investigations of the $K_{\ell 3}$-

decays, the overall situation looks presently rather unsettled, and it seems still admissible to present consequences of one more theoretical scheme, namely the results following from the simpleminded quark-loop diagram with the local coupling in the meson-quark vertices.

The matrix element of the strangeness changing vector current

$$< \pi^\circ (p_2)| V_\mu^{\Delta S=1}(0)|K^+(p_1)> = \frac{1}{\sqrt{2}}[(p_1+p_2)_\mu f_+(q^2) + + (p_1-p_2)_\mu f_-(q^2)] \tag{13}$$

is parametrized at small values of $q^2 = (p_1-p_2)^2$ as follows

$$f_\pm(q^2) = f_\pm(0)(1+\lambda_\pm q^2/\mu_\pi^2),$$

$$\lambda_0 = \lambda_+ + \xi(0)\mu_\pi^2/(\mu_K^2-\mu_\pi^2), \tag{14}$$

$$\xi(0) = f_-(0)/f_+(0),$$

where λ_0 is the slope of the scalar form factor originating from non-zero divergence $\partial_\mu V_\mu^{\Delta S=1} \neq 0$.

Before reproducing the results of the standard, although lengthy and awkword calculation (some of integrals over the Feynman parameters were computed numerically) we note the following. In the SU(3)-symmetry limit. λ_+ can be related to pion (kaon) radius while $f_-(q^2) = 0$. The duality arguments, expounded earlier, are invoked to justify the loop-diagram model in this case. In the broken SU(3)-symmetry, we propose to consider the model as an attempt of the dynamical realization of simplest consequences of SU(3)-breaking due to the mass-splitting. But as to the $f_-(q^2)$ form factor, the triangle approximation should be taken ad hoc. The favourable thing to note here is just the convergence of all integrals, including $f_-(0)$. Using the physical masses of mesons and effective masses of quarks fixed in Section 2, it was found that

$$\lambda_+ f_+(0) = 0{,}031\ , \tag{15a}$$

$$\lambda_- = 0{,}025\ , \tag{15b}$$

$$f_-(0) = \xi(0)\cdot f_+(0) = -0{,}41. \tag{15c}$$

If, having in mind the Ademollo-Gatto theorem, we put approximately $f_+(0) \approx 1$ then the values of λ_+ and $\xi(0)$ agree reasonably with the data on K_{e3} and $K^+_{\mu 3}$ -decays [9] but tending to disagree with new data on the $K^\circ_{\mu 3}$ -decay [10] which give approximately $\lambda_0 \simeq \lambda_+$ while it follows from Eqs. (15a-c) that $\lambda_0 \approx 0$. Numerically, the results are changing little (about several per cents) in the soft-pion limit $\mu_\pi \to 0$ and, nevertheless, they deviate considerably from the low-energy, soft-pion theorem of the current algebra [11]

$$1 + \lambda_+ \frac{\mu_K^2}{\mu_\pi^2} + \xi(0)\,(1 + \lambda_- \frac{\mu_K^2}{\mu_\pi^2}) = \frac{F_K}{F_\pi f_+(0)} \tag{16}$$

$$(1+0{,}40-0{,}54) \neq 1{,}27 \pm 0{,}03.$$

However, putting formally $\mu_K \to 0$, while evaluating the integrals, one obtains $\xi(0) \simeq 0$ in a better agreement with Eq. (16) and the $K^\circ_{\mu 3}$ -data. In this connection, we cannot but speculate once again that the increase of the kaon mass from zero to the physical value and simultaneous inclusion of the appropriate form factors in hadronic vertices may partially cancel each other giving $\langle r^2\rangle_{K^+} \lesssim \langle r^2\rangle_\pi$ (see Eq. (9a)) and $\xi(0)$ close to zero, that are the results following from the triangle diagram calculation in the limit $\mu_K \to 0$.

4. *SPECTRAL FUNCTION SUM RULES AND STRUCTURE CONSTANTS IN THE* $\pi \to \ell\nu\gamma$ *DECAY*

Into the $\pi \to \ell\nu\gamma$ decay amplitude, besides the internal bremsstrahlung diagrams, a more interesting process of the "direct" photon emission contributes. This structure-dependent part of the total amplitude begins with the terms linear in the photon momentum and is parametrized by two form factors

$$M^{\mu\nu}_{SD} = \epsilon^{\mu\nu\rho\sigma} k_\rho p_\sigma b(t) - i\,(k^\mu p^\nu - g^{\mu\nu}(k\cdot p))\, a(t), \qquad (17)$$

where $k(p)$ is the photon (pion) 4-momentum, $t=(k-p)^2$. The vector form factor $b(0)$ is expressed, with the help of CVC, through the $\pi^0 \to 2\gamma$ decay amplitude and may be found by calculating the quark-loop diagram

$$b(0) = \frac{1}{4\sqrt{2}\,\pi^2 F_\pi}. \qquad (18)$$

To find the axial-vector form factor $a(0)$, we use the integral s.r. derived in Ref.[12] from the current algebra and PCAC

$$\frac{1}{\sqrt{2}}\, a(0) = \frac{F_\pi}{3} \langle r^2 \rangle_\pi - \frac{1}{F_\pi} \int \frac{dq^2}{q^4} \left(\rho_1^V(q^2) - \rho_1^A(q^2)\right) \equiv$$

$$\equiv \frac{F_\pi}{3} \langle r^2 \rangle_\pi - \frac{1}{F_\pi} \rho_{-2}^{V-A}, \qquad (19)$$

where $\rho_1^{V(A)}(q^2)$ is the spectral function of the (axial) vector current propagator corresponding to transfer of the unit (iso) spin in the intermediate hadronic states. There are no special reasons to expect for good, duality-type correspondence between ρ_{-2}^{V-A}(res), including all hadronic resonances, and ρ_{-2}^{V-A} (loop) described by the lowest order quark loops. Nevertheless, the use can be made of simple, explicit forms of the spectral functions in the loop-approximation

$$\rho_1^V(q^2) = \frac{1}{8\pi^2}\left(q^2 + 2m_q^2\right)\left(1 - \frac{4m_q^2}{q^2}\right)^{1/2}, \qquad (20a)$$

$$\rho_1^A(q^2) = \frac{1}{8\pi^2}\left(q^2 - 4m_q^2\right)\left(1 - \frac{4m_q^2}{q^2}\right)^{1/2} \qquad (20b)$$

to obtain the value

$$\rho_{-2}^{V-A}(\text{loop}) = \frac{1}{8\pi^2}. \qquad (21)$$

This gives from (3) and (19)

$$a(0) = b(0) = \frac{1}{4\sqrt{2}\,\pi^2 F_\pi},$$

$$\gamma = \frac{a}{b} = 1. \qquad (22)$$

The value (22) disagrees with the data

$$\gamma_{exp} = \begin{array}{lll} 0{,}4 & \text{or} & -2{,}18 \pm 0.10 \ [13] \\ 0{,}15 \pm 0{,}11 & \text{or} & -2{,}07 \pm 0{,}11 \ [14] \\ 0{,}44 \pm 0{,}12 & \text{or} & -2{,}36 \pm 0.12 \ [15]. \end{array} \qquad (23)$$

Notice, that the straightforward calculation of the quark diagrams with the local couplings, describing the $\pi \to \ell\nu\gamma$ decay, supplemented with a renormalization-type recipe not to deal with infinities, gives also Eq. (22)[16]. Alternatively, the saturation of ρ_{-2}^{V-A} by the $\rho(770)$ and $A_1(1070)$ -mesons gives

$$\rho_{-2}^{V-A}(\text{VMD}) = g_\rho^{-2} - g_A^{-2} \simeq \frac{3}{4} g_\rho^{-2} = \frac{1}{4\pi^2}(0{,}99 \pm 0{,}07) \qquad (24)$$

which is about two times as much as Eq. (21) and leads to a near complete cancellation of two terms in (19), hence $a(0)$ is close to zero. The numerical value of (24) was obtained through using Weinberg's spectral-function s.r.[17] and the experimental value $g_\rho^2/4\pi = 2.38 \pm 0.18$[18]

It is evident thereupon, that the global duality, in the above used sence, does not fulfill for all spectral s.r. Rather than for ρ_{-2}^{V-A}, one can expect, on the ground of formal analogy between the current matrix elements leading to the bremsstrahlung-weighed photoabsorption s.r.

and spectral-function s.r., respectively, the better fulfillment of the "resonance-loop" duality for another moment of the spectral functions:

$$\rho_{-3/2}^{V-A}(\text{res}) \simeq \rho_{-3/2}^{V-A}(\text{loop}) . \tag{25}$$

Applying the resonance saturation scheme to the left-hand side of Eq. (25) and computing, via Eqs. (20), the right-hand side, we have

$$m_\rho g_\rho^{-2} - m_A g_A^{-2} \simeq 3m_q / 16\pi \simeq \sqrt{6}\, F_\pi /16 , \tag{26}$$

where m_q was finally replaced by Eq. (5). Numerical consistency of (26), obtained with $g_\rho^2/4\pi \simeq g_A^2/16\pi \simeq 2{,}4$ and $m_\rho \simeq m_A/\sqrt{2} \simeq 770$ *MeV*, looks satisfactory (within 15 per cent).

Introducing the quantities δ and ϵ through definitions

$$\langle r^2 \rangle_\pi \equiv (1+\delta) \frac{3}{4\pi^2 F_\pi^2} , \tag{27a}$$

$$\rho_{-2}^{V-A} \equiv (1+\epsilon) \frac{1}{4\pi^2} \tag{27b}$$

we have an estimate for ratio γ

$$\gamma = \frac{a}{b} = 2(\delta-\epsilon) \simeq 0{,}3 \pm 0{,}16 , \tag{28}$$

where $\delta \simeq 0{,}15$ and $\epsilon = \pm 0{,}08$ were taken according to Eq. (24) and our estimation of corrections to Eq. (3) discussed in Section 1.

One should stress that due to strong compensation of two terms in s.r. (19), the axial-vector structure constant $a(0)$ is very sensitive even to a small variation of each term. Therefore, a better accuracy in measuring $\langle r^2 \rangle_\pi$ and $\gamma = a/b$ is very desirable as it provides a more critical verification of a number of important theoretical issues including the asymptotic chiral symmetry and the resonance approximation of the spectral-function sum rules.

5. *DISCUSSION AND CONCLUSIONS*

It is out of the scope of this work to compare in detail results presented with those of plenty other papers devoted to the low-energy structure parameters of hadrons and based on such diverse methods as the algebra of currents and fields /11/, effective Lagrangians and vector-meson-dominance model /11/ the chiral-invariant interaction of meson and baryon fields /19/, various versions of the composite quark model. Amongst them, the quantum chromodynamics looks most promising and ambitious model pretending to be a fundamental field theory of strong interactions. However, the key problem of the quark confinement is not solved yet. The aim of our consideration was to try to demonstrate that, under certain conditions, conventional field-theoretic constructions and lowest order Feynman graphs can be used in description even of such hadronic properties which should, in essence, be defined by nonperturbative effects of binding and confinement. An underlying argument is the approximate global duality (or equivalence) of two complete sets of the state vectors, saturating certain integral sum rules, one of the sets being the solutions of the bound state problem with the colour-confining interactions, while the other describes free particles and fields. The choice of s.r. satisfying the assumed duality condition is suggested by correspondence with the well-known results obtained in the nonrelativistic theory of interaction of the radiation with matter. The message is that sum rules for the dipole moment fluctuation seem to be singled both in the nonrelativistic and relativistic regions. In conclusion we add some final comments:

1. We consider the calculation of $\langle r^2 \rangle_\pi$ to be most reliable among other results of this paper and notice that good agreement with experiment is possible only if the quark colours are taken into account. The success of the local approximation for the $\pi q \bar{q}$ -vertex means that the same approximation should work in calculation of the meson radiative decays giving the dominant contributions to s.r. (1). In this regard, we propose the similar mechanisms of the local annihilation of constituent quarks for the description of the $V \to e^+e^-$ and $V \to \pi\gamma$ decays enabling, e.g., in the case of ω-meson to write:

$$\frac{\Gamma(\omega \to e^+e^-)}{\Gamma(\omega \to \pi^\circ\gamma)} \simeq \frac{\sigma((q\bar{q})_\omega \to e^+e^-)}{\sigma((q\bar{q})_\omega \to \pi^\circ\gamma)}\bigg|_{s=m_\omega^2} \simeq \frac{\alpha}{18}\left(\frac{g^2_{\pi^\circ q\bar{q}}}{4\pi}\right)^{-1} \approx \frac{\alpha}{3\pi}, \tag{29}$$

where we used Eq. (6) and the ideal mixing scheme for the ω/ϕ state vectors. With $\Gamma(\omega \to e^+e^-) = (0.76 \pm 0.17)$ *keV* [9] we obtain from (29): $\Gamma(\omega \to \pi^\circ\gamma) = (980 \pm 220)$ *keV* in accord with $\Gamma_{exp}(\omega \to \pi^\circ\gamma) = (880 \pm 90)$ *keV* [9].

2. In the case of kaons, the calculation of $\langle r^2 \rangle_{K^\circ} = -0.04\ fm^2$ has a preference of being comparatively stable against the variation of μ_K. If, further, the isovector universality holds, then $\langle r^2 \rangle_{K^+} = \langle r^2 \rangle_\pi + \langle r^2 \rangle_{K^\circ}$ and $\langle r^2 \rangle_{K^\pm}$ should be slightly (about $10 \div 15\%$) less than $\langle r^2 \rangle_\pi$. The positive sign of $\langle r^2 \rangle_{K^\circ} = (0.08 \pm 0.05) fm^2$ [20] looks somewhat disturbing and that is why forthcoming measurements of the K-meson radii via the kaon scattering on atomic electrons are of particular interest.

3. The calculation of the $K_{\ell 3}$-decay form factors depends substantially on masses of quarks and mesons. Our choice $m_u = m_d = \sqrt{\frac{2}{3}}\,\pi F_\pi \simeq 240$ *MeV* and

$$m_s = m_u\left(\frac{2F_k}{F_\pi} - 1\right) \simeq \sqrt{2}\, m_u$$

is close to the values used in a number of papers [21] dealing with the composite potential models. Herewith, we are apt to ascribe more credence to the quantities that change minimally with varying meson masses. In this regard, we note, that the linear combination

$$f_+(0)(\lambda_+ + \xi(0)\lambda_-) =$$

$$\frac{3\mu_\pi^2}{4\pi^2 F_\pi^2}\left[\frac{y^4 + 13y^2 + 4}{6(y^2-1)^3} - \frac{2y^2(y^2+2)}{(y^2-1)^3}\ln y\right] \approx 0{,}019. \tag{30}$$

does not depend on μ_K, as $\mu_\pi \to 0$, and may be expected to be most reliable numerically. Yet numerical value (30) is difficult to reconcile with $\xi(0) \simeq 0$ as required by the low-energy theorem (16) and the $K^\circ_{\mu 3}$-data.

If consolidated, this may, possibly, be attributed to the nonnegligible role of the nonresonant contributions to $f_\pm(q^2)$.

4. The loop approximation of ρ_{-2}^{V-A} in s.r. (19) results in a value twice lower in comparison with the usual resonance saturation of Eq. (24) by the ρ and A_1-mesons. This occurs mainly because of a large overestimation of the contribution of the low-energy region in ρ_{-2}^{V-A}. The s.r. (19) develops strong sensitivity of the axial-vector structure constant a(0) even to small variation of ρ_{-2}^{V-A} and $\langle r^2 \rangle_\pi$, thus emphasizing the importance of more accurate measuring of this parameter.

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