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UNIFIED TREATMENT OF WEAK AND ELECTROMAGNETIC INTERACTIONS IN A SU(4)xSU(4) CHIRAL THEORY OF MESONS

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Эберт Д., Волков М.К. **E2 - 11679**

Единое описание слабых и электромагнитных взаимодействий в SU(4)×SU(4) киральной теории мезонов

Формулируется калибровочная теория слабых и электромагнитных взаимодействий мезонов в рамках кирального Лагранжиана группы SU(4)×SU(4) с использованием калибровочной группы $SU(2)_L \times U(1)$ в схеме Г.И.М. Окончательный эффективный Лагранжиан для слабого взаимодействия имеет структуру ток х ток. Лагранжиан с заряженными токами имеет форму Каббибо, с нейтральными токами - удовлетворяет условию $\Delta S = 0$.

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Ebert D., Volkov M.K. **E2 - 11679**

Unified Treatment of Weak and Electromagnetic Interactions in a SU(4)×SU(4) Chiral Theory of Mesons

Starting from a phenomenological chiral SU(4)×SU(4) Lagrangian for hadrons weak and electromagnetic interactions are introduced by considering nonlinear realizations of the gauge group $SU(2)_L \times U(1)$ of the Weinberg-Salam model.

The final effective Lagrangian for the weak interaction of hadrons has the usual current × current structure. The charmed charged currents are of the Cabibbo type, the neutral current satisfies the condition $\Delta S = 0$ of the G.I.M. scheme.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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Spontaneously broken gauge theories provide a general framework for introducing weak and electromagnetic interactions into a Lagrangian with lepton and quark fields /1/.

Instead of using a quark Lagrangian for describing the hadronic world, there exist also other convenient schemes, at least for interpreting low energy phenomena, which are given in the form of phenomenological Lagrangians written directly in terms of the physical hadron fields. These Langrangians consist of a chiral SU(N)×SU(N) (N=2,3,4..) invariant main part together with some small but important symmetry breaking term /2/. Using a phenomenological chiral SU(4)×SU(4) Lagrangian we want to derive in this note an effective Lagrangian describing the weak and electromagnetic interactions of the pseudoscalar mesons beloging to the 15-plet of SU(4). We use the gauge group $SU(2)_L \times U(1)$ of the standard model /1/.

Let us consider the 15-plet of pseudoscalar (Goldstone) mesons ϕ_i of the group SU(4) described by the matrix

$$M = \exp(-i\gamma_5\ \xi), \qquad \xi = \sum_{i=1}^{15} \lambda_i \frac{\phi_i}{f} \tag{1}$$

where λ_i are the generators of SU(4) and f is a dimensional parameter.

The pseudoscalar mesons which are transformed according to a nonlinear realization of the chiral group $SU(4)\times SU(4)$ are described by the Lagrangian

$$L_{strong} = \frac{f^2}{16}\,\mathrm{Sp}\;\partial_\mu e^{i\gamma_5\xi}\;\partial_\mu e^{-i\gamma_5\xi}$$

$$= -\frac{f^2}{16}\,\mathrm{Sp}\,(e^{-i\gamma_5\xi}\partial_\mu e^{i\gamma_5\xi})(e^{-i\gamma_5\xi}\partial_\mu e^{i\gamma_5\xi})\,. \qquad (2)$$

In the following we have to consider (nonlinear) local gauge transformations of ξ with respect to the group $SU(2)_L\times U(1)$. A convenient realization of the generators of this group is given in terms of 4×4 matrices *

$$\hat{C}_i = \frac{1+\gamma_5}{2}C_i\,, \qquad C_i = \begin{pmatrix} \frac{1}{2}\sigma_i & 0 \\ 0 & \frac{1}{2}\sigma_i' \end{pmatrix} \quad (i=1,2,3)$$

$$\frac{\hat{Y}}{2} = \frac{1+\gamma_5}{2}\frac{Y_w}{2} + \frac{1-\gamma_5}{2}Q\;;\;[\hat{C}_i,\frac{\hat{Y}}{2}] = 0\,, \qquad (3)$$

where σ_i are the usual Pauli matrices ($\sigma_i' = \sigma_1\sigma_i\sigma_1^{-1}$), and Y_w, Q are the weak hypercharge and the

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* With these generators the covariant derivatives of the usual lepton, quark and Higgs multiplets

$$\ell = \begin{pmatrix} \nu \\ e \\ \mu \\ \nu' \end{pmatrix}, \qquad q_c = \begin{pmatrix} u \\ d_c \\ s_c \\ c \end{pmatrix}, \qquad \Phi_L \equiv \frac{1+\gamma_5}{2}\begin{pmatrix} \phi_+ \\ \phi_0 \\ \phi_0 \\ \phi_+ \end{pmatrix}$$

can be written in the following compact 4-dimensional way

$$\nabla_\mu(\ell;q_c;\Phi_L) = [\partial_\mu + ig\hat{C}_i W_\mu^i + ig'\frac{\hat{Y}}{2}B_\mu](\ell;q_c;\Phi_L)\,,$$

where $Y_w = (-1;\frac{1}{3};1)$; W_μ^i, B_μ are the corresponding gauge bosons.

operator of the electromagnetic charge, respectively. There are the relations $Q = \hat{C}_3 + \frac{\hat{Y}}{2} = C_3 + \frac{Y_w}{2}$ and $Q = I_3 + \frac{Y_s}{2} + \frac{2}{3}C$ (I_3, Y_s, C – (strong) isospin, hypercharge and charm). To get the Cabibbo structure of the weak interactions, it is convenient to rewrite the Lagrangian (2) in terms of the Cabibbo rotated meson fields

$$e^{-i\gamma_5\xi_c} = e^{i2\Theta\frac{\lambda_7}{2}}\,e^{-i\gamma_5\xi}\,e^{-i2\Theta\frac{\lambda_7}{2}}\,. \qquad (4)$$

We may then consider the following (nonlinear) transformations of the rotated meson field ξ_c with respect to $SU(2)_L\times U(1)$* ($\epsilon = \sum_{i=1}^{3} C_i\epsilon_i$):

$$SU(2)_L:\quad e^{-i\gamma_5\xi_c'} = e^{i\frac{1-\gamma_5}{2}\epsilon}\,e^{-i\gamma_5\xi_c}\,e^{-i\frac{1+\gamma_5}{2}\epsilon}$$

$$U(1):\quad e^{-i\gamma_5\xi_c'} = e^{i\eta\frac{\hat{\hat{Y}}}{2}}\,e^{-i\gamma_5\xi_c}\,e^{-i\eta\frac{\hat{Y}}{2}} \qquad (5)$$

where $\hat{\hat{Y}}$ is obtained from $\hat{Y}$ by replacing $\gamma_5 \to -\gamma_5$. A Lagrangian which is invariant under local transformations (5) and simultaneous transformations of gauge bosons W_μ^i ($i=1...3$), B_μ may now be obtained from (2), (4) by means of the following principle of "minimal coupling"

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* These transformation laws may naturally be obtained by requiring the invariance of the following (chiral invariant) meson-quark interaction $\bar{q}e^{-i\gamma_5\xi}q = \bar{q}_c e^{-i\gamma_5\xi_c}q_c$ ($q_c = e^{i2\Theta\frac{\lambda_7}{2}}q$) with respect to the $SU(2)_L\times U(1)$ transformations $q_c \to q_c' = (\exp i\hat{C}_i\epsilon_i)q_c$, $q_c'' = (\exp i\frac{\hat{Y}}{2}\eta)q_c$.

$$e^{-i\gamma_5 \xi_c}\partial_\mu e^{i\gamma_5 \xi_c} \Rightarrow$$

$$\{ e^{-i\gamma_5 \xi_c} [\partial_\mu + ig\frac{1+\gamma_5}{2} W_\mu + ig' \frac{\hat{\hat{Y}}}{2} B_\mu] e^{i\gamma_5 \xi_c}$$

$$- ig\frac{1-\gamma_5}{2} W_\mu - ig' \frac{\hat{Y}}{2} B_\mu \} \qquad (6)$$

$$(W_\mu = \sum_{i=1}^{3} C_i W_\mu^i) .$$

Instead of the vector fields W_μ^i, B_μ it is convenient to introduce the charged W-fields $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$ and the fields of the neutral Z-boson and the photon /1/.

$$Z_\mu = \cos\theta \; W_\mu^3 - \sin\theta \; B_\mu$$
$$A_\mu = \sin\theta \; W_\mu^3 + \cos\theta \; B_\mu , \qquad tg\theta = \frac{g'}{g} , \qquad (7)$$

where θ is the Weinberg angle. The new Lagrangian obtained from eq. (2) by inserting eq. (6) then takes the form

$$L_{w+elm.+str.} = L_{strong} - \frac{g}{2\sqrt{2}} (W_\mu^+ (j_\mu)_w + h.c.)$$
$$- \frac{g}{2\cos\theta} Z_\mu (j_\mu)_Z - e A_\mu (j_\mu)_{elm.} +$$
$$+ (\text{bilinear terms in } W_\mu^\pm , Z_\mu , A_\mu), \qquad (8)$$

where the weak charged and neutral currents and the electromagnetic current are given by*

$(C_\pm = C_1 \pm iC_2)$

*For convenience, electromagnetic and weak corrections to the currents have been omitted in eq. (9). They are contained in the bilinear terms of eq. (8).

$$(j_\mu)_w = -\frac{i}{4} f^2 \, Sp \, C_+(\Theta) [h_\mu^{(1)}(\xi) + \gamma_5 h_\mu^{(2)}(\xi)]$$

$$(j_\mu)_Z = -\frac{i}{4} f^2 Sp \{ C_3 [h_\mu^{(1)}(\xi) + \gamma_5 h_\mu^{(2)}(\xi)]$$

$$-2\sin^2\theta \; Q \; h_\mu^{(1)}(\xi) \} \qquad (9)$$

$$(j_\mu)_{elm.} = -\frac{i}{4} f^2 Sp \, Q \, h_\mu^{(1)}(\xi) ; \qquad e = g\sin\theta .$$

The functions $h_\mu^{(1,2)}(\xi)$ are defined by

$$h_\mu^{(1,2)}(\xi) = \frac{1}{2} [e^{i\gamma_5 \xi} \partial_\mu e^{-i\gamma_5 \xi} \pm e^{-i\gamma_5 \xi} \partial_\mu e^{i\gamma_5 \xi}]$$

and

$$C_\pm(\Theta) = e^{-i2\Theta \frac{\lambda_7}{2}} C_\pm e^{i2\Theta\frac{\lambda_7}{2}}$$
$$C_3(\Theta) = C_3 \qquad (Q(\Theta) = Q) \qquad (10)$$

are the Cabibbo rotated generators of $SU(2)_L$.
From eqs. (8,9) the following effective Lagrangian describing processes with W and Z exchange in second order of perturbation theory may be derived

$$L_{weak}^{(ch)} = -\frac{\bar{G}}{\sqrt{2}} \{ j_\mu^{(ch)\,lept.} + \cos\Theta [-\sqrt{2} f \partial_\mu (\pi^- + F^-)$$

$$+ i\sqrt{2}\, \pi^\circ \overleftrightarrow{\partial}_\mu \pi^- + i(\bar{D}^\circ - K^\circ) \overleftrightarrow{\partial}_\mu K^- + i(D^\circ - \bar{K}^\circ) \overleftrightarrow{\partial}_\mu D^-$$

$$+ i\sqrt{\frac{2}{3}} (\eta - \sqrt{2}\,\eta_c) \overleftrightarrow{\partial}_\mu F^-] + \qquad (11)$$

$$+ \sin\Theta [-\sqrt{2} f \partial_\mu (K^- - D^-) - i(\bar{K}^\circ + \bar{D}^\circ) \overleftrightarrow{\partial}_\mu \pi^-$$

$$+i\frac{(\pi^0+\sqrt{3}\,\eta)}{\sqrt{2}}\overleftrightarrow{\partial_\mu}\,K^- + i(\bar K^0+\bar D^0)\overleftrightarrow{\partial_\mu}\,F^- + \frac{i}{\sqrt{6}}(2\sqrt{2}\,\eta_c+\eta-\sqrt{3}\,\pi^0)\overleftrightarrow{\partial_\mu}\,D^-\,]\}\times\{\;\ldots\;\}^+$$

and

$$L^{(neutr.)}_{weak} = -\frac{\bar G}{\sqrt{2}}\{\,j_\mu^{(neutr.)lept} + [-f\partial_\mu(\pi^0+\frac{\eta}{\sqrt{3}}-\sqrt{\frac{2}{3}}\,\eta_c) + i(1-2\sin^2\theta)(\pi^-\overleftrightarrow{\partial_\mu}\pi^+ + K^-\overleftrightarrow{\partial_\mu}K^+ + \bar D\overleftrightarrow{\partial_\mu}D^+ + F^-\overleftrightarrow{\partial_\mu}F^+)]\}\times\{\;\ldots\;\} \quad (12)$$

For completeness, we have included the currents arising from the lepton sector. In writing down the expressions (11), (12) we have restricted ourselves, for illustration, to linear and bilinear terms in the meson fields ϕ_i and reexpressed them by the physical fields of the SU(4) 15-plet. Further, we have

$$\frac{\bar G}{\sqrt{2}} = g^2\,(8[(M_W^2)_{Higgs}+\frac{g^2 f^2}{2}])^{-1} \approx \frac{G}{\sqrt{2}},$$

where G is the Fermi constant $\frac{10^{-5}}{m_p^2}$ (There is a correction term to the masses of the W and Z bosons from the meson sector). We find that the structure of the charmed weak charged and neutral currents agrees with the general structure of the Cabibbo currents in the G.I.M. scheme /3/ , where we have, for example,

$$j_\mu^{(ch)}(\Theta) = \cos\Theta\,[\,(j_\mu^1+ij_\mu^2)+(j_\mu^{13}-ij_\mu^{14})\,] + \sin\Theta\,[\,(j_\mu^4+ij_\mu^5)-(j_\mu^{11}-ij_\mu^{12})\,]. \quad (13)$$

In particular, $L^{(ch)}_{weak}$ yields a reasonable description of the leptonic and semileptonic decays of the SU(3) mesons (π, K, η) /2/ in addition to analogous predictions

for charmed meson decays. From eq. (8) one may further derive definite expressions for the weak radiative decays of mesons. Concerning the non-leptonic decays we meet with the same difficulties well-known from the standard Cabibbo Lagrangian of the current - current structure. To get a reasonable agreement with the experimental data one has to introduce additional dynamical assumptions (e.g., octet enhancement, inclusion of renormalization effects /4/), a discussion of which is, however, outside the scope of our note. Finally, we find that the neutral part (12) does not contain strangeness changing terms $-f\partial_\mu \bar K^0$, $K^-\overleftrightarrow{\partial_\mu}\pi^+$, etc., as it is expected for the G.I.M. scheme. Such contributions would lead to decays $K^0\to\mu^+\mu^-$, $K^-\to\pi^- e^- e^+$, etc., that are strongly suppressed in the experiment. From eq. (11) it follows that the dimensional parameter f has just the meaning of an (averaged) pseudoscalar meson decay constant.

Our scheme admits the inclusion of meson mass terms in the standard way by adding the following symmetry breaking term $L_{S.B.}$ to the Lagrangian (8)

$$L_{S.B.} = f^2\,\mathrm{Sp}\,((a\lambda_0+b\lambda_8+c\lambda_{15})\,e^{-i\gamma_5\xi}). \quad (14)$$

In eq. (14) the parameters a, b, c have to be chosen in such a way that the physical meson masses are reproduced /5/.

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