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THE COVARIANT THREE-DIMENSIONAL FORMULATION OF THE COMPOSED QUARK MODEL FOR MESONS

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Ковариантная трехмерная формулировка составной кварковой модели мезонов

В рамках квазипотенциального подхода развит трехмерный ковариантный формализм для описания составных систем из двух релятивистских частиц.

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The Covariant Three-Dimensional Formulation of the Composed Quark Model for Mesons

Within the quasipotential approach, a three-dimensional covariant formalism is developed for the description of the two relativistic spinless particle system. As a mathematical tool we have used the Lobachevsky geometry and the harmonic analysis on the Lorentz group (instead of the usual Fourier transformation). It allows us to pass from the integral equation to the finite-difference equation which is a covariant three-dimensional generalization of the Schrödinger equation and whose explicit solutions can be found for some potentials.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. *INTRODUCTION*

The quark model qualitatively describes a lot of elementary particle interactions at high energies. Therefore, it remains actual to develop a self-consistent relativistic description of hadrons as composite objects. For this purpose, it would be desirable to know explicitly covariant wave functions of the quark system and to apply them to find the other important characteristics of particles like form factors, magnetic moments, and so on.

Our aim is to develop such a covariant three-dimensional relativistic formalism which at the same time would be as much as possible close in form to the nonrelativistic one. In the momentum representation, a considerable progress in the three-dimensional covariant description of composite systems was achieved on the basis of the covariant formulation of the Logunov-Tavkhelidze two-particle quasipotential equation [1] in papers [2,3]. However, in p-space the equation for wave function has an integral form that makes it difficult to get solutions in the closed form required for investigations.

In this paper we show that the momentum representation of the covariant equation for the system wave function in the Logunov-Tavkhelidze quasipotential approach can be transformed to a form of a direct geometrical generalization of the Schrodinger equation. Thus the relativistic equation can be obtained from the nonrelativistic one by the change of the Euclidean geometry of relative momentum space to the Lobachevsky one. In order to obtain explicitly the covariant wave functions we apply the method of transition to the relativistic configurational representation proposed earlier within the framework of the Kadyshevsky quasipotential equation /4/. This allows us to retain complete analogy with nonrelativistic expressions. For simplicity we consider here only the case of spinless hadrons composed of spinless quarks.

2. *THE RELATIVISTIC CONFIGURATIONAL REPRESENTATION AND THE LOBACHEVSKY MOMENTUM SPACE*

The relativistic configurational representation (RCR) was first introduced in ref. /5/. As the mathematical tool were taken the expansions over unitary irreducible representations of the Lorentz group /6,8/. Such a transition to the RCR is reasonable since in the Kadyshevsky quasipotential equation /4/, like in the Logunov-Tavkhelidze approach, all the momenta of the particles belong to the mass shell

$$p_0^2 - \vec{p}^{\,2} = m^2. \tag{2.1}$$

Equation (2.1) defines the three-dimensional surface of a hyperboloid whose upper sheet is known to serve as a model of the Lobachevsky space. In the nonrelativistic limit this Lobachevsky space turns into the three-dimensional momentum flat space with the Euclidean geometry.

On the surface (2.1) one can introduce different coordinate systems. As for coordinates in the Lobachevsky space we choose the Cartesian coordinates of vector $\vec{p}$ on hyperplane $p_0 = 0$ onto which the hyperboloid is mapped while projecting from the point $(\infty, \vec{0})$. Therefore, as a model of the Lobachevsky space will be now the whole three-dimensional p-space with the metrics

$$ds^2 = d\vec{p}^{\,2} - \frac{(\vec{p}\cdot d\vec{p})^2}{m^2 + \vec{p}^{\,2}} \equiv g_{ik}\, dp^i\, dp^k ;\ i,k = 1,2,3,$$

and volume element

$$d\Omega_p = \frac{d^{(3)}p}{\sqrt{1+\vec{p}^{\,2}/m^2}} = \frac{d^{(3)}p}{m^{-1}p_0}. \tag{2.2}$$

As is known, all formulae of the relativistic kinematics, including the relativistic law of addition of velocities can be derived by changing the Euclidean geometry of the three-dimensional velocities space to the Lobachevsky geometry /7/. For the momentum space with the Lobachevsky geometry, the vector of "difference" of two vectors $\vec{p}$ and $\vec{k}$ $\vec{\Lambda}_{p,k} \equiv \vec{p}(-)\vec{k}$ /4/ is defined as a spatial component of the 4-vector

$$\Lambda^{\mu}_{p,k} = (L^{-1}_{\lambda_k} \cdot p)^{\mu}, \tag{2.3}$$

i.e.,

$$\vec{\Lambda}_{p,k} \equiv \vec{p}(-)\vec{k} = \overrightarrow{(L^{-1}_{\lambda_k} p)} = \vec{p} - \frac{\vec{k}}{m}\left(p_0 - \frac{\vec{p}\vec{k}}{k_0 + m}\right), \tag{2.4a}$$

$$\Lambda^0_{p,k} = \sqrt{m^2 + \vec{\Lambda}^2_{p,k}} = (L_{\lambda_k} p)^0 = \frac{p_0 k_0 - \vec{p}\vec{k}}{m}, \tag{2.4b}$$

where $L^{-1}_{\lambda_k}$ is the matrix of the pure Lorentz transformation $L^{-1}_{\lambda_k} k = (m, \vec{0})$ *. In the nonrelativistic limit $\vec{\Lambda} \equiv \vec{p}(-)\vec{k} \to \vec{p} - \vec{k}$. The group of motions of the Lobachevsky space is the Lorentz group. Therefore, the matrix elements of its unitary representations compose the complete and orthogonal system on the surface of the mass hyperboloid (2.1) and are well studied in mathematics /6/. In what follows we shall use the functions (notation from ref. /5/)

$$\xi(\vec{\Lambda}, \vec{r}) = \left(\frac{\Lambda_0 - \vec{\Lambda}\vec{n}}{m}\right)^{-1-irm};\ \vec{r} = r\vec{n},\ \vec{n}^2 = 1, \tag{2.5}$$

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*Elements of matrix $L^{-1}_{\lambda_k}$ depend on components of the 4-velocity vector $\lambda^{\mu}_k = k^{\mu}/m$.

which have been derived explicitly in ref. [8] and realize the principal series of unitary irreducible representations of the Lorentz group. Because of the analogy of the three-dimensional r-representation to the nonrelativistic coordinate representation found in ref. [5] it has been proposed to name it "the relativistic configurational representation". The group parameter r which plays the role of the relativistic analog of the modulus of the relative coordinate will be called hereafter "the relativistic relative coordinate".

According to ref. [5] this parameter enumerates the eigenvalues of the invariant Casimir operator of the Lorentz group $\hat{C}_L = \frac{1}{4} M_{\mu\nu} M^{\mu\nu}$ ($M^{\mu\nu}$ - are the group generators) [6,8]

$$\hat{C}_L \xi(\vec{\Delta},\vec{r}) = (1/m^2 + r^2)\,\xi(\vec{\Delta},\vec{r})\,; \quad 0 \le r < \infty\,. \tag{2.6}$$

An important property of the "relativistic relative coordinate" introduced via functions (2.5) is the relativistic invariance of its modulus r (as a parameter which enumerates the eigenvalues of the invariant Casimir operator $\hat{C}_L$). Therefore the transform of the spherical-symmetric quasipotential in the RCR is an invariant function. In the nonrelativistic limit $\xi(\vec{p},\vec{r}) \to \exp(i\vec{p}\vec{r})$ $(c\to\infty)$.

Our aim is the transition into the RCR in the covariant quasipotential equation for the wave function of relative motion of two relativistic spinless particles. The wave function $\Phi(p_1,p_2)$ of a system of two noninteracting particles with equal masses $m_1 = m_2 = m$ transforms by the law [9,10]

$$[U(a,L)\Phi](p_1,p_2) = [U^{(1)}(a,L) \otimes U^{(2)}(a,L)\Phi](p_1,p_2) =$$

$$= \exp[i(p_1+p_2)a]\,\Phi(L^{-1}p_1, L^{-1}p_2)\,. \tag{2.7}$$

It is convenient to pass to the total 4-momentum $\mathcal{P}^\mu$ and half-difference of two momenta q^μ

$$\mathcal{P}^\mu = (p_1+p_2)^\mu\ ; \ q^\mu = \frac{1}{2}(p_1-p_2)^\mu\ . \tag{2.8}$$

The wave function of the *system as a whole* depends in the spinless case on six variables: the total mass M, total momentum $\vec{\mathcal{P}}$

$$M^2 = \mathcal{P}^\mu \mathcal{P}_\mu = (p_1+p_2)^2\,; \qquad \vec{\mathcal{P}} = \vec{p}_1 + \vec{p}_2\ ; \tag{2.9}$$

total orbital momentum

$$J^k = \frac{1}{2}\epsilon^{k\ell m} M_{\ell m}\ ;\ M_{\ell m} = M^{(1)}_{\ell m} \otimes I^{(2)} + I^{(1)} \otimes M^{(2)}_{\ell m}$$

($M_{\ell m}$ - are generators of $O(3)$; $\mu,\nu = 0,1,2,3$; $k,\ell,m = 1,2,3$)

$$\{\vec{J}\}\Phi(p_1,p_2) = \{-i[\vec{p}_1 \times \frac{\partial}{\partial \vec{p}_1}] - i[\vec{p}_2 \times \frac{\partial}{\partial \vec{p}_2}]\}\Phi(p_1,p_2) \tag{2.10}$$

and its projection onto Z-axis.

For the covariant description of the relative orbital motion the angular variables of the function $\Phi(M,\vec{\mathcal{P}},\theta,\phi)$ in [9] were chosen to be the spherical coordinates of the vector of half-difference of the particle momenta q^μ (2.8) taken in the c.m.s., i.e., the following vector (notation from [3])

$$(\overset{\circ}{\vec{p}})^i = \{\overrightarrow{L^{-1}_{\lambda\mathcal{P}}\, q}\}^i = \{\overrightarrow{L^{-1}_{\lambda\mathcal{P}}(\frac{p_1-p_2}{2})}\}^i\,,\ i = 1,2,3. \tag{2.11}$$

Since in the c.m.s. the particle momenta are equal in magnitude and opposite in direction, the momentum $\overset{\circ}{\vec{p}}$ equals that of the first particle transformed to the c.m.s. by means of $L^{-1}_{\lambda\mathcal{P}}$ [9]

$$\overset{\circ}{\vec{p}} = \overrightarrow{(L^{-1}_{\lambda\mathcal{P}}\, p\,)} = \vec{p}_1 + \frac{\vec{\mathcal{P}}(\vec{\mathcal{P}}\vec{p}_1)}{M(\mathcal{P}_0+M)} - \frac{\vec{\mathcal{P}}}{M} p_{10}\ , \tag{2.12}$$

i.e., it coincides with the spatial component of the 4-vector (2.3) $\Delta^\mu_{p_1,m\lambda\mathcal{P}} = (L^{-1}_{\lambda\mathcal{P}}\, p\,)^\mu$, that belongs to the mass hyperboloid (2.1)*. In accordance with the notations (2.4a) 3-vector (2.12) can be represented as a non-Euclidean difference of the vectors $\vec{p}_1$ and $m\vec{\lambda}_{\mathcal{P}}$, belonging to the same mass hyperboloid

$$\overset{\circ}{p} = \vec{\Delta}_{p_1, m\lambda_{\mathcal{P}}} = \vec{p}_1 (-) m\vec{\lambda}_{\mathcal{P}} . \qquad (2.13)$$

The angular variables of the unit vector

$$\vec{e}_{\Delta_{p_1,m\lambda_{\mathcal{P}}}} = \frac{\vec{\Delta}_{p_1,m\lambda_{\mathcal{P}}}}{|\vec{\Delta}_{p_1,m\lambda_{\mathcal{P}}}|} = (\sin\theta_e \sin\phi_e , \sin\theta_e \cos\phi_e, \cos\theta_e), \qquad (2.14)$$

taken in the direction of vector $\vec{\Delta}_{p_1,m\lambda_{\mathcal{P}}}$ describe the relative orbital motion. The modulus of the vector $\vec{\Delta}_{p,m\lambda_{\mathcal{P}}}$ and its time component $\Delta^0_{p,m\lambda_{\mathcal{P}}} = \sqrt{m^2 + \vec{\Delta}^2_{p_1,m\lambda_{\mathcal{P}}}} = \frac{p^\mu \mathcal{P}_\mu}{M}$ are relativistic invariants**.

Under the Lorentz transformation L the vector $\vec{\Delta}_{p_1,m\lambda_{\mathcal{P}}}$ is subjected to rotation only (Wigner rotation)

$$\Delta^0_{p', m\lambda_{\mathcal{P}'}} = \Delta^0_{p,m\lambda_{\mathcal{P}}} \quad ; p' = Lp , \ \mathcal{P}' = L\mathcal{P}. \qquad (2.16)$$

$$\Delta^i_{p', m\lambda_{\mathcal{P}'}} = R^{ij} \{V^{-1} (L^{-1}, \mathcal{P})\} \Delta^j_{p,m\lambda_{\mathcal{P}}} \quad ; i,j=1,2,3. \qquad (2.17)$$

The Wigner rotation matrix V can be defined from the equation

$$L \cdot L_{\mathcal{P}} = L_{\mathcal{P}'} \cdot V(L^{-1} , \mathcal{P}), \qquad \mathcal{P}' = L\mathcal{P}. \qquad (2.18)$$

The total orbital moment of the system (2.10) can be splitted into the orbital moment connected with the motion of the c.m.s. and that one describing the relative orbital motion. In the variables (2.9)-(2.14) this splitting of (2.10) has the form

$$(\vec{J}\Phi)(M, \vec{\mathcal{P}}; \vec{e}_\Lambda) =$$

$$= \{-i[\vec{\mathcal{P}} \times \frac{\partial}{\partial\vec{\mathcal{P}}}] - i[\vec{e}_\Lambda \times \frac{\partial}{\partial\vec{e}_\Lambda}]\} \Phi(M, \vec{\mathcal{P}}, \vec{e}_\Lambda) . \qquad (2.19)$$

With (2.17) it can be easily tested that the relative orbital operator (J_{rel}) under the Lorentz transformation L is transformed by the law /10/

$$i\vec{J}'^{k}_{rel} = [\vec{e}_{\Lambda'} \times \frac{\partial}{\partial\vec{e}_{\Lambda'}}]^k = R^{k\ell} \{V^{-1} (L^{-1},\mathcal{P})\}[\vec{e}_\Lambda \times \frac{\partial}{\partial\vec{e}_\Lambda}]^\ell , \qquad (2.20)$$

i.e., its modulus is a relativistic invariant

$$\vec{J}^2_{rel} = \text{inv} ; \quad \vec{J}^2_{rel} Y_{\ell m} (\vec{e}_\Lambda) = \ell(\ell + 1) Y_{\ell m} (\vec{e}_\Delta) . \qquad (2.21)$$

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*Note, that time component of the 4-vector $\{\overset{\circ}{p}\}^\mu = \{L^{-1}_{\lambda\mathcal{P}} (\frac{p_1 - p_2}{2})\}^\mu$ equals zero $\{\overset{\circ}{p}\}_0 = 0$ because of the c.m.s. $\vec{p}_1 = -\vec{p}_2$. Thus the 4-vector p^μ (coinciding up to the normalization with the Wightman-Gording vector) is space-like, while the $\Delta^\mu_{p,m\lambda_{\mathcal{P}}}$ is a time-like vector:

$$\overset{\circ}{p}{}^2 = \overset{\circ}{p}{}^2_0 - \vec{\overset{\circ}{p}}{}^2 < 0 , \quad \Delta^2 = \Delta^2_0 - \vec{\Delta}^2 > 0 .$$

** On the energy shell they are expressed through the invariant total mass $M^2 = \mathcal{P}^2 = (p_1 + p_2)^2$ by the

$$2\Delta^0_{p_1,m\lambda_{\mathcal{P}}} = M; \quad 2|\vec{\Delta}_{p_1,m\lambda_{\mathcal{P}}}| = \sqrt{M^2 - 4m^2}. \qquad (2.15)$$

As a result the expansion of the product of two Poincare group representations in irreducible ones, i.e., the transition from the WF of two particles to the WF, *describing the system as a whole* and corresponding to the relative orbital motion with the fixed ℓ, looks as follows [9]

$$\Phi(p_1,p_2) \equiv \Phi(M, \vec{\mathcal{P}}; \vec{e}_{\Lambda_{p,m\lambda_{\mathcal{P}}}}) =$$

$$= \sum_{\ell m_z} \Phi_{\ell m_z}(M, \vec{\mathcal{P}}) Y_{\ell m_z}(\vec{e}_{\Delta_{p,m\lambda_{\mathcal{P}}}}) . \quad (2.22)$$

3. *FORMULATION OF THE COVARIANT QUASIPOTENTIAL EQUATION IN THE LOBACHEVSKY MOMENTUM SPACE*

The covariant description of a two-particle system within quantum field theory widely uses the Bethe-Salpeter equation derived within the four-dimensional formalism of Feynman and Dyson. In this approach, the wave function (WF) of a bound state(B)(of two particles) with mass M, spin J, its projection m_z and momentum $\vec{K}$ is represented by the following matrix element

$$\Psi^{J,m_z}_{BK}(x_1,x_2) = \langle 0|T\{\phi_1(x_1)\,\phi_2(x_2)\}|M,\vec{K},J,m_z\rangle . \quad (3.1)$$

The state vector $|M,K,J,m_z\rangle$ in our spinless case has the same quantum numbers as the WF $\Phi_{\ell m_z}(M,\mathcal{P})$, separated in (2.22), and, under the Poincare group transformations, is transformed over the conjugate representation. It is known, however, that the WF of the Bethe-Salpeter equation (3.1) has no clear physical interpretation as it contains an additional dependence on the relative time of two particles $x^0 = x^0_1 - x^0_2$.

A consistent elimination of this dependence is achieved in the framework of the Logunov-Tavkhelidze quasipotential approach[1]. A covariant generalization of the procedure of transition to the single-time WF was proposed in [2,3]. It can be realized in the following way [3]

$$\tilde{\Psi}_{BK}(p_1,p_2) = \int d^4x_1 d^4x_2 \exp(ip_1x_1 + ip_2x_2)\,\delta[\lambda_{\mathcal{P}}(x_1-x_2)]\times$$

$$\times\langle 0|T\{\phi_1(x_1)\phi_2(x_2)\}|M,K,J,m_z\rangle , \quad (3.2)$$

where $\lambda^{\mu}_{\mathcal{P}} = \mathcal{P}^{\mu}/\sqrt{\mathcal{P}^2}$ is the 4-velocity vector of the whole system *. The transition invariance allows us to separate from (3.1) the motion of the c.m.s.

$$\Psi_{BK}(x_1,x_2) = \exp(iK\frac{x_1+x_2}{2})\Psi_{BK}(x) , \quad (3.3)$$

$$x = x_1 - x_2$$

$$\Psi_{BK}(x) = \langle 0|T\{\phi_1(x)\,\phi_2(-x)\}|M,\vec{K}\rangle . \quad (3.4)$$

With (3.3), for the single-time WF (denoted by ~) (3.2) in the momentum space we get

$$\tilde{\Psi}_{BK}(p_1,p_2) = (2\pi)^4\,\delta^{(4)}(\mathcal{P} - K)\tilde{\Psi}_{BK}(q) .$$

$$\mathcal{P} = p_1 + p_2 ; \quad q = \frac{p_1 - p_2}{2} , \quad (3.5)$$

where the WF of the relative motion is defined as follows

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* In general, as 4-vector λ^{μ}, one may take an arbitrary time-like 4-vector. The use of the 4-velocity vector due to λ^{μ} is more convenient and simplifies our consideration. Note also, that in some cases λ^{μ} may be taken to be a light-like vector $\lambda_0^2 - \vec{\lambda}^2 = 0$ [11].

$$\tilde{\Psi}_{BK}(q) = \int d^4x \exp(ix\frac{p_1-p_2}{2})\delta(\lambda_{\mathcal{P}}x)\Psi_{BK}(x) . \qquad (3.6)$$

The argument of the δ-function $\lambda_{\mathcal{P}}x = \frac{\mathcal{P}^\mu \cdot x_\mu}{M}$ has the sense of the proper (relative) time of two particles and $\lambda_{\mathcal{P}}x = x_1^0 - x_2^0 = 0$ in c.m.s.

Consequently, from (3.6) we have excluded in a covariant manner the dependence on the relative time of two particles /3/.

By using the transformation law for field operators of scalar quarks

$$U^{-1}(L)\,\phi(x)\,U(L) = \phi(L^{-1}x) \qquad (3.7)$$

the covariant expression (3.6) can be transformed as follows

$$\tilde{\Psi}_{BK}(q) = \int d^3(L^{-1}_{\lambda_K}x)\,d(L^{-1}_{\lambda_K}x)^0 \exp[\,iL^{-1}_{\lambda_k}(\frac{p_1-p_2}{2})\cdot L^{-1}_{\lambda_k}x]\times$$

$$\times\delta(\lambda_{\mathcal{P}}\cdot x)\langle 0|T\{\phi_1[(L^{-1}_{\lambda_K}x)^0, \overrightarrow{(L^{-1}_{\lambda_k}x)}]\times \qquad (3.8)$$

$$\times\phi_2[-(L^{-1}_{\lambda_k}x)^0, -\overrightarrow{(L^{-1}_{\lambda_k}x)}]|M,J,0\rangle =$$

$$= \int d^3\vec{x}'\exp[\,i(\overrightarrow{L^{-1}_{\lambda_k}q})\cdot\vec{x}'\,]\langle 0|\phi_1(0,\vec{x}')\phi_2(0,-\vec{x}')|M,J,0\rangle .$$

Thus, the WF of relative motion depends on the 3^--vector $\overrightarrow{(L^{-1}_{\lambda_k}q)}$

$$\tilde{\Psi}_{BK}(q) \equiv \tilde{\Psi}_{BK}(\overrightarrow{L^{-1}_{\lambda_k}q}) . \qquad (3.9)$$

In (3.8) we used the fact that due to the conservation law of the 4-velocity of the c.m.s. (which follows from (3.5)).

$$\lambda^\mu_{\mathcal{P}} = \frac{\mathcal{P}^\mu}{\sqrt{\mathcal{P}^2}} = \frac{K^\mu}{\sqrt{K^2}} = \lambda^\mu_K \ ; \quad \mathcal{P}^2 \equiv (p_1+p_2)^2 = K^2 = M^2 \qquad (3.10)$$

the time component of vector $(x')^\mu = (L^{-1}_{\lambda_k}x)^\mu$ coincides with the argument of δ-function in (3.8)

$$\delta(\lambda_{\mathcal{P}}x) = \delta(\lambda_K x) \qquad (3.11)$$

and the vector $\overrightarrow{L^{-1}_{\lambda_K}p} = \frac{1}{2}(\overrightarrow{L^{-1}_{\lambda_K}p_1} - \overrightarrow{L^{-1}_{\lambda_K}p_2})$ coincides with the difference of vectors $\vec{p}_1$ and $\vec{p}_2$ in the c.m.s. $\vec{\mathcal{P}} = \vec{p}_1 + \vec{p}_2 = 0$, i.e., with the earlier considered vector (2.9), (2.11) $\vec{\Delta}_{p_1,m\lambda_{\mathcal{P}}}$:

$$\overrightarrow{L^{-1}_{\lambda_K}(\frac{p_1-p_2}{2})} = \overrightarrow{L^{-1}_{\lambda_{\mathcal{P}}}(\frac{p_1-p_2}{2})} = \overrightarrow{\Delta_{p_1,m\lambda_{\mathcal{P}}}} .$$

The vector $\vec{\Delta}_{p_1,m\lambda_{\mathcal{P}}}$ (2.15) (or $\overset{\circ}{\vec{p}}$ in notation of ref. /3/) is a vector from the Lobachevsky space and not an element of three-dimensional Euclidean space, as it was considered in ref. /3/.(Though this statement was not used in ref. /3/ and did not influence the derivation of formulae). In fact, according to Sec.2, just spatial components of the momentum vector are used as coordinates to parametrize the Lobachevsky space. The Lorentz transformation $L^{-1}_{\lambda_K}$ keeps invariant the equation of hyperboloid (2.1). Therefore, the transformed vector $(L^{-1}_{\lambda_K}\cdot p_1)^\mu$ will belong to the same mass hyperboloid (2.1) and its spatial component $L^{-1}_{\lambda_K}\vec{p}_1 = \vec{\Delta}_{p_1,m\lambda_K}$ will be a vector of the same Lobachevsky space as the vector $\vec{p}_1$.

Summarizing, one may say that in the momentum representation the covariant single-time wave function of relative motion (3.6) depends on the three-dimensional vector $\vec{\Delta}_{p_1,m\lambda_{\mathcal{P}}} = -\vec{\Delta}_{p_2,m\lambda_{\mathcal{P}}}$ which is an element of the

Lobachevsky space *, i.e.,

$$\tilde{\Psi}_{BK}(q) = \tilde{\Psi}_{B0}(\vec{\Delta}_{p_1,m\lambda_{\mathcal{P}}}). \qquad (3.12)$$

By using the transformation law of 2-particle WF (2.7) and (3.7) one can obtain from (3.8) the following transformation law of the single-time WF (3.12) under the Lorentz transformation L.

$$U(L)\tilde{\Psi}_{B0}(\vec{\Delta}_{p,m\lambda_{\mathcal{P}}}) = \tilde{\Psi}_{B0}(\vec{\Delta}_{Lp,mL\lambda_{\mathcal{P}}}) =$$

$$= \tilde{\Psi}_{B0}(R^{ij}\{V^{-1}(L^{-1},\mathcal{P})\}\vec{\Delta}^{j}_{p,m\lambda_{\mathcal{P}}}),$$

where the matrix of the Wigner rotation $R\{V^{-1}(L^{-1},\mathcal{P})\}$ is given by (2.16).

Let us now establish the form of the covariant equation for the single-time WF in the three-dimensional momentum space with the Lobachevsky geometry. Performing the same calculations as of paper /3/ for our case of scalar particles with equal masses, the procedure of the covariant equating of relative times of particles of the Green functions

$$G(x_1,x_2;y_1,y_2) = \langle 0|T\{\phi_1(x_1)\phi_2(x_2)\phi_1(y_1)\phi_2(y_2)\}|0\rangle \qquad (3.13)$$

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*An analogous parametrization of the quasipotential WF of the vector of the Lobachevsky space by the time-like vector $\vec{\Delta}_{p,\ m\lambda_{\mathcal{P}}}$ was considered earlier within the Kadyshevsky approach in ref. /12/. However, the used formalism did not allow there a unique solution to the question how to separate the motion of a system as a whole. As a result, the author of ref. /12/ was to postulate that the motion of the c.m.s. as a whole is described by the usual exponential function $\exp[ik(x_1+x_2)/2]$, whereas the use of the WF of the Bethe-Salpeter equation (3.1) automatically leads to this result (3.4).

$$(2\pi)^4\delta^{(4)}(\mathcal{P}-Q)\tilde{G}(\Delta_{p,m\lambda_{\mathcal{P}}};\Delta_{k,m\lambda_Q};\mathcal{P}^2) =$$

$$= \int d^4x_1 d^4x_2 d^4y_1 d^4y_2 \exp(ip_1x_1+ip_2x_2-ik_1y_1-ik_2y_2)\times$$

$$\times\delta[\lambda_{\mathcal{P}}(x_1-x_2)]\delta[\lambda_Q(y_1-y_2)]G(x_1,x_2;y_1,y_2) \qquad (3.14)$$

allows to obtain the free Green function as follows

$$\tilde{G}_0(\Delta_{p,m\lambda};\Delta_{k,m\lambda};\mathcal{P}^2) = \tilde{G}_0^{ret} - \tilde{G}_0^{adv} =$$

$$= i(2\pi)^3\frac{m}{\Delta^0_{p,m\lambda}}\cdot\frac{\delta(\vec{\Delta}_{p,m\lambda}(-)\vec{\Delta}_{k,m\lambda})}{4\Delta^0_{p,m\lambda}\cdot\Delta^0_{k,m\lambda}}\times$$

$$\times\left[\frac{1}{\sqrt{\mathcal{P}^2}-2\Delta^0_{p,m\lambda}+i\epsilon} - \frac{1}{\sqrt{\mathcal{P}^2}+2\Delta^0_{p,m\lambda}+i\epsilon}\right] \qquad (3.15)$$

with splitting into the retarded and advanced parts. A similar spectral representation /3/ easily follows for the total Green function. By using it, we can represent $\tilde{G}$ - function in the neighbourhood of the bound state pole $\mathcal{P}^2 \approx M^2$ in the form

$$\tilde{G}(\Delta_{p,m\lambda};\Delta_{k,m\lambda};\mathcal{P}^2) = i(2\pi)^3\frac{\tilde{\phi}_{BM}(\vec{\Delta}_{p,m\lambda})\otimes\tilde{\phi}^{+}_{BM}(\vec{\Delta}_{k,m\lambda})}{[\sqrt{\mathcal{P}^2}-M+i\epsilon]}. \qquad (3.16)$$

The WF $\tilde{\phi}_{BM}(\vec{\Delta}_{p,m\lambda})$ introduced here differs from the earlier WF $\tilde{\Psi}_{B0}(\vec{\Delta}_{p,m\lambda}) = \tilde{\Psi}_{BK}(q)$ by the normalization

$$\tilde{\phi}_{BM}(\vec{\Delta}_{p,m\lambda}) = 2^3\Delta^0_{p,m\lambda}/\sqrt{m}\,\tilde{\Psi}_{B0}(\vec{\Delta}_{p,m\lambda}). \qquad (3.17)$$

Since we work in the three-dimensional momentum space with the Lobachevsky geometry in what follows by the volume element of the momentum space we will mean the volume element $d\Omega = m\frac{d^3\vec{\Lambda}}{\Lambda^0}$ of the Lobachevsky space. The equation of WF of relative motion in terms of the elements of the Lobachevsky space has the form

$$[\mathcal{P}^2 - 4(\Lambda^0_{p,m\lambda_{\mathcal{P}}})^2]\phi_{BM}(\vec{\Lambda}_{p,m\lambda_{\mathcal{P}}}) = \tag{3.18}$$

$$= \frac{1}{(2\pi)^3}\int d\Omega_{\Lambda_{k,m\lambda_{\mathcal{P}}}} V(\vec{\Lambda}_{p,m\lambda_{\mathcal{P}}};\vec{\Lambda}_{k,m\lambda_{\mathcal{P}}};\mathcal{P}^2)\tilde{\phi}_{BM}(\vec{\Lambda}_{k,m\lambda_{\mathcal{P}}}).$$

If the quasipotential in (3.18) does not depend on $\mathcal{P}^2$, the WF $\tilde{\phi}_{BM}(\vec{\Lambda})$ obeys the following normalization condition

$$\int \phi^{+}_{BM}(\vec{\Lambda}_{k,m\lambda})\cdot\tilde{\phi}_{BM}(\vec{\Lambda}_{k,m\lambda})\frac{d^3\vec{\Lambda}_{k,m\lambda}}{m^{-1}\Lambda^0_{k,m\lambda}} = 1. \tag{3.19}$$

In the c.m.s. $\vec{p}_1 = -\vec{p}_2 = \vec{p}$, i.e., in the rest frame of the composite particle $\vec{\lambda}_{\mathcal{P}} = \vec{\mathcal{P}}/M = 0$ and (3.19) takes the form

$$[\mathcal{P}^2_0 - 4(\vec{p}^{\,2} + m^2)]\tilde{\phi}_{BM}(\vec{p}) = \frac{1}{(2\pi)^3}\int \frac{d^3\vec{k}}{m^{-1}k_0} V(\vec{p},\vec{k};\mathcal{P}^2)\phi_{BM}(\vec{k}). \tag{3.20}$$

The definition of the scattering amplitude $G = G_0 + G_0 T G_0$ results in the equation for T

$$T = V + VG_0T. \tag{3.21}$$

If we consider T in (3.21) to be given, e.g., defined by a sum of Feynman diagrams corresponding to a certain interaction Lagrangian in quantum field theory, than equation (3.21) can be treated as an equation for definition of the quasipotential V.

From (3.15) and (3.16) it follows that the single-time Green functions G^{adv} and G^{adv} have no poles at positive $\sqrt{\mathcal{P}^2}$, i.e., the whole information on bound states with positive energy $\mathcal{P}^0 > 0$ is contained in G^{ret}. The states with "negative" energy have no analogs in the nonrelativistic theory. Therefore, if we want to construct an apparatus similar in form to that of quantum mechanics, we have to eliminate from the equation the states with $\mathcal{P}^0 < 0$, i.e., by using the procedure developed in /13/. As a result, we obtain the following equation for the WF

$$2\Delta^0_{p,m\lambda_{\mathcal{P}}}[\sqrt{\mathcal{P}^2} - 2\Delta^0_{p,m\lambda_{\mathcal{P}}}]\tilde{\phi}^{r}_{BM}(\vec{\Delta}_{p,m\lambda_{\mathcal{P}}}) =$$

$$= \frac{1}{(2\pi)^3}\int d\Omega_{\Delta_{k,m\lambda_{\mathcal{P}}}} V^{ret}(\vec{\Delta}_{p,m\lambda_{\mathcal{P}}};\vec{\Lambda}_{k,m\lambda_{\mathcal{P}}};\mathcal{P}^2)\tilde{\phi}^{r}_{BM}(\vec{\Lambda}_{k,m\lambda_{\mathcal{P}}}), \tag{3.22}$$

with the new potentia; V^{ret} (see for details /13/). The WF for quasipotential independent of $\mathcal{P}^2\tilde{\phi}^{r}_{BM}(\vec{\Lambda})$ obey the normalization condition

$$\int\tilde{\phi}^{+r}_{BM}(\vec{\Lambda}_{k,m\lambda})[2\Delta^0_{k,m\lambda}]\tilde{\phi}_{BM}(\vec{\Delta}_{k,m\lambda})d\Omega_{\Delta_{k,m\lambda}} = \sqrt{\mathcal{P}^2}. \tag{3.23}$$

Equation (3.22) coincides with the quasipotential equation found by Kadyshevsky with the help of the covariant Hamilton, formulation of quantum field theory /4/. This equation is analysed in papers /14/.

4. *COVARIANT QUASIPOTENTIAL WAVE FUNCTIONS IN THE RELATIVISTIC CONFIGURATIONAL REPRESENTATION*

The covariant equations (3.18) and (3.22) describe the relative internal motion of two particles in the coordinate system moving with momentum $\mathcal{P}$. According to the procedure developed in /1-4/ the quasipotential is constructed from the matrix elements of the relativistic scattering amplitude T(p,k) and in the second order in coupling constant coincides with the Feynman matrix elements of the one-boson exchange. We will show that the quasipotential corresponding to the exchange of the massless boson (gluon)

$$V(\vec{p},\vec{k}) = -\frac{g^2}{(p-k)^2}; \quad p^2 = k^2 = m^2 \tag{4.1}$$

can be represented in the local form in the Lobachevsky space. By using notation (2.4) we get [5]

$$V(\vec{p},\vec{k}) = \frac{-g^2}{(p-k)^2} = \frac{-g^2}{2m^2 - 2m\sqrt{m^2+(\vec{p}(-)\vec{k})^2}} = V(\vec{\Delta}^2_{p,k}).$$

The square of vector of difference in the Lobachevsky space of two momentum vectors $\vec{\Delta}_{p,k} = \vec{p}(-)\vec{k}$ is the Lorentz invariant

$$\vec{\Delta}^2_{p,k} = (\vec{p}(-)\vec{k})^2 = \left(\frac{pk}{m}\right)^2 - m^2 = \frac{(L^{-1}_\lambda p \cdot L^{-1}_\lambda \cdot k)^2}{m^2} - m^2 =$$

$$= \frac{\{\Delta^\mu_{p,m\lambda\varphi} \cdot \Delta^\mu_{k,m\lambda\varphi}\}^2}{m^2} - m^2 = \{\vec{\Delta}_{p,m\lambda\varphi}(-)\vec{\Delta}_{k,m\lambda\varphi}\}^2 = \vec{\Delta}^2_{\Delta_{p,m\lambda};\Delta_{k,m\lambda}},$$

therefore

$$V(\vec{\Delta}^2_{p,k}) = V(\vec{\Delta}^2_{\Delta_{p,m\lambda},\Delta_{k,m\lambda}}). \tag{4.2}$$

With this quasipotential, the r.h.s. of (3.18) and (3.22) represents a convolution in the Lobachevsky space what makes applicable the expansion over the matrix elements of group of motions of this space. Let us perform in (3.18) the transformation from the momentum representation to the relativistic configurational representation with the help of the functions (2.5) realizing the unitary irreducible representation of the group of motions of the hyperboloid (2.1):

$$\Psi_{B0}(\vec{r}) = \int d\Omega_{\Delta_{p,m\lambda}} \xi(\vec{\Delta}_{p,m\lambda}, \vec{r}) \Psi_{B0}(\vec{\Delta}_{p,m\lambda}). \tag{4.3}$$

In paper [5] the free Hamilton operator for "plane waves" $\xi(\Delta, r)$ in the Lobachevsky space has been found in the explicit form

$$\hat{H}_0 \xi(\vec{\Delta}_{p,m\lambda\varphi}, \vec{r}) = 2\Delta^0_{p,m\lambda\varphi} \xi(\vec{\Delta}_{p,m\lambda\varphi}, \vec{r}), \tag{4.4}$$

$$\hat{H}_0 = 2m\,\mathrm{ch}\left(\frac{i}{m}\frac{\partial}{\partial r}\right) + \frac{2i}{r}\,\mathrm{sh}\left(\frac{i}{m}\frac{\partial}{\partial r}\right) - \frac{\Delta_{\theta,\phi}}{mr^2}\exp\left(\frac{i}{m}\frac{\partial}{\partial r}\right), \tag{4.5}$$

where $\Delta_{\theta,\phi}$ is the Laplacian on the sphere. By using (4.3), eq. (3.18) with the quasipotential $V\{(\vec{\Delta}_{p,m\lambda}(-)\vec{\Delta}_{k,m\lambda})^2\}$ local in the Lobachevsky space can be represented in the form

$$[\hat{H}^2_0 - \mathcal{P}^2]\Psi_{B0}(\vec{r}) = V(\vec{r}, \mathcal{P}^2)\Psi_B(\vec{r}). \tag{4.6}$$

The transform of the one-gluon exchange propagator (4.2) in r-space is the spherical-symmetric potential

$$V(r) = -\frac{g^2}{4\pi r}\,\mathrm{cth}\,\pi r m. \tag{4.7}$$

The exact solutions of eq. (4.6) with the potential (4.7) coincide with those found in [15] for the c.m.s. case. In the expansion of WF over partial waves

$$\Psi_B(r) = \sum_\ell i^\ell (2\ell+1) \Psi^\ell_B(r) P_\ell(\cos\theta). \tag{4.8}$$

The functions $\Psi^\ell_B(r)$ are solutions of the radial equation with the free Hamiltonian

$$\hat{H}^\ell_0 = 2m\,\mathrm{ch}\left(\frac{i}{m}\frac{\partial}{\partial r}\right) + \frac{2i}{r}\,\mathrm{sh}\left(\frac{i}{m}\frac{\partial}{\partial r}\right) + \frac{\ell(\ell+1)}{mr^2}\exp\left(\frac{i}{m}\frac{\partial}{\partial r}\right). \tag{4.9}$$

Now let us discuss the covariant properties of the free Hamiltonian operator $\hat{H}_0$ (4.5) and of eq. (4.6). The momentum operator in the RCR [15]

$$\hat{\vec{P}}\xi(\vec{p},\vec{r}) = \vec{p}\,\xi(\vec{p},\vec{r}), \tag{4.10}$$

as shown in [15,17], in the spherical coordinates has the form

$$\hat{\vec{P}}_j = -n_j\,(m e^{\frac{i}{m}\frac{\partial}{\partial r}} - \hat{H}_0) + [(\hat{\vec{P}}_{nonrel})_j + i n_j \frac{\partial}{\partial r}]\, e^{\frac{i}{m}\frac{\partial}{\partial r}}, \tag{4.11}$$

where $\hat{\vec{P}}_j$ is the j-th component $(j = x,y,z)$ of the nonrelativistic momentum operator

$$(\hat{\vec{P}}_{nonrel})_j \exp(i\vec{p}\vec{r}) = (\vec{p})_j \exp(i\vec{p}\vec{r}), \tag{4.12}$$

and n_j is the j-th component of the unit vector $\vec{n}$ directed along the radius-vector $\vec{r}: \vec{n} = \vec{r}/|\vec{r}|$.

In our case, after the transformation (4.3) in the RCR with the function

$$\xi(\vec{\Delta}_{p_1,m\lambda\wp}, \vec{r}) = [\frac{(\Delta_{p_1,m\lambda\wp})^{\mu} \cdot n_{\mu}}{m}]^{-1-irm}, \quad n_{\mu} = (1, \vec{n}). \tag{4.13}$$

"the relativistic relative coordinate" $\vec{r} = r\vec{n}$ is conjugated to a covariantly defined by (2.12-2.13) vector of the relative momentum of a particle in the c.m.s. $\vec{\Delta}_{p_1,m\lambda\wp}$ the components of which change according to (2.16) under the Lorentz transformations. Due to the Lorentz invariance of the scalar product

$$(\Delta_{p_1,m\lambda\wp})^{\mu} \cdot n_{\mu} = \Delta^0_{p_1,m\lambda\wp} - \vec{\Delta}_{p_1,m\lambda\wp}\vec{n} = \text{inv},$$

and of the time component of vector $\Delta^0_{p_1,m\lambda\wp}$ and its modulus $|\vec{\Delta}_{p_1,m\lambda\wp}| = \text{inv}$, it follows from (2.16), (2.17), (4.3) and (4.13) that under the Lorentz transformation L the unit vector $\vec{n}$ directed along the radius-vector of the "relativistic relative coordinate" is transformed by the law

$$n'_j = R_{ji}\{V^{-1}(L^{-1},\wp)\} n_j. \tag{4.14}$$

The modulus of the "relativistic coordinate" is the relativistic invariant (see (2.6)).

From (4.11) it is clear that the analog of the angular momentum operator

$$[\vec{r}\times\hat{\vec{P}}_{relat.}] = [\vec{r}\times\vec{P}_{nonrelat.}]\, e^{\frac{i}{m}\frac{\partial}{\partial r}}, \tag{4.15}$$

introduced in [16,17], differs from the usual one used in quantum mechanics $\vec{L}, \vec{L}^2 = -\Delta_{\theta,\phi}$)

$$\hat{L}_x = i(\sin\phi\frac{\partial}{\partial\theta} + \cos\phi\cdot\mathrm{ctg}\,\theta\frac{\partial}{\partial\phi}),$$

$$\hat{L}_y = i(-\cos\phi\frac{\partial}{\partial\theta} + \sin\phi\cdot\mathrm{ctg}\,\theta\frac{\partial}{\partial\phi}), \tag{4.16}$$

$$\hat{L}_z = -i\frac{\partial}{\partial\phi},$$

(θ and ϕ are spherical coordinates of the vector $\vec{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$) by the factor $\exp(\frac{i}{m}\frac{\partial}{\partial r})$ which disappear in the nonrelativistic limit. Instead of the momentum $\vec{p}$ we use in (4.15) the covariant momentum of a particle in the c.m.s. It can be seen that because of the invariance of the modulus of the "relativistic coordinate", the operator

$$\hat{\vec{\Lambda}} = [\vec{r}\times\hat{\vec{\Delta}}_{p_1,m\lambda\wp}] = \hat{L}_{nonrel.}\exp(\frac{i}{m}\frac{\partial}{\partial r}) \tag{4.17}$$

(components of $\hat{L}_{nonrel}$ are defined by (4.16)) is transformed under the Lorentz transformation by the same law

$$\Lambda'_j = [U^{-1}(L)\Lambda U(L)]_j = R_{ji}\{V^{-1}(L^{-1}, \mathcal{P})\}\Lambda_i \qquad (4.18)$$

as the covariant operator of the internal (relative) orbital momentum (2.20) in the momentum space.
In the c.m.s. (4.17) turns into (4.15).

It should be noted that the free Hamilton operator (4.5) contains the vector squared

$$\vec{\Lambda}^2 = -\Delta_{\theta,\phi}\, e^{\frac{i}{m}\frac{\partial}{\partial r}} = \text{inv}, \qquad (4.19)$$

which is the relativistic invariant as follows from (4.18).

Hence, the free Hamiltonian $\hat{H}_0$ (4.5) is also the relativistic-invariant operator. From (4.19) it follows that the eigenvalues of the operator $\vec{L}^2 = \ell(\ell+1)$ related to $\hat{\vec{\Lambda}}^2$ via (4.17) which enter into the expansion (4.8) are also invariants.

As a result, the radial equations (4.9) at fixed ℓ are invariant equations.

CONCLUSION

We have shown that the covariant three-dimensional quasipotential equation for the single-time wave function of the two-relativistic particle system in the RCR retains its covariance and three-dimensional form. Note that the transition to the RCR through the expansion in harmonic analysis on the Lorentz group was applied earlier in /5/ to the equation in the c.m.s. of two particles.

In this paper we have made use of the invariance of the modulus of the relativistic relative coordinate introduced by the transformation (4.3) to describe the internal relative motion of two particles in a covariant manner. Further applications of this formalism for the invariant description of particle form factors will be considered in subsequent papers. The invariant inclusion, into partial equations, of the dependence on the internal angular momentum provides a possibility of the relativistic description of the mesons Regge trajectories in the quark model.

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