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ON THE THEORY OF HIGH-ENERGY NUCLEUS-NUCLEUS SCATTERING IN THE EIKONAL APPROXIMATION

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ON THE THEORY OF HIGH-ENERGY NUCLEUS-NUCLEUS SCATTERING IN THE EIKONAL APPROXIMATION

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К теории ядерно-ядерных взаимодействий при высоких энергиях в эйкональном приближении

В рамках теории многократных столкновений предложен эффективный способ вычисления характеристик широкого класса процессов ядерно-ядерных взаимодействий, не зависящий от вида ядерной плотности.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

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On the Theory of High-Energy Nucleus-Nucleus Scattering in the Eikonal Approximation

A new approach is suggested allowing one to calculate various characteristics of high-energy nucleus-nucleus interactions. It appears to be very useful for practical calculations independently of type of nuclear densities used.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1978

The multiple scattering expansion of the amplitude of nucleus-nucleus scattering F_{AB} (A and B, colliding nuclei) was obtained many years ago. However, the summation of this expansion into the simple and closed expression such as existing for hadron-nucleus scattering has not yet been performed, so that one is forced either to calculate successively all $(2^{A\cdot B}-1)$ terms of F_{AB} under assumption that nuclear densities are gaussians, or to use the rigid projectile approximation which is not symmetrical in respect to the replacement $A \leftrightarrow B$. One can hope that considerable simplifications should arise in the optical limit for the colliding nuclei, i.e., when $A, B \to \infty$.

The purpose of this letter is to present such a closed optical limit expression for F_{AB} in terms of the thickness function T_A, T_B of colliding nuclei, pointing out the main steps in its deriving and to compare its predictions both with experimental data and results following from other theoretical approaches. Moreover, we would like to show here that the knowledge of the functional dependence $F^{opt}_{AB} = F^{opt}_{AB}(T_A, T_B)$ allows one to calculate some other important characteristics of nucleus-nucleus interactions by simple functional differentiation.

The approach suggested is based on separation of all the possible diagrams describing AB scattering into the classes without the closed loops, with one closed loop and so on (Fig. 1), with the following straightforward summation of the totality of diagrams belonging to the class considered. The analysis shows that the relative contribution of this class of diagrams to F_{AB} is of order ϵ^n, where $\epsilon = \frac{\sigma}{16\pi\beta^2}$ ($\sigma = \sigma^{tot}_{NN}$ and β^2 being the slope parameter of nucleon-nucleon differential cross-section) and n is the number of closed loops. Moreover, one can enter through the summation of contributions of diagrams of two first classes that the second class diagrams give only a small correction even for $A, B \sim 40$, so that it is reasonable to assume that contributions of higher order graphs are small enough and may be neglected at high energies.

Before discussing the method, it should be noted that the following approximations, widely employed in many investigations, are used here: i) the Glauber approximation relating AB and nucleon-nucleon (NN) scattering amplitudes, ii) the independent par-

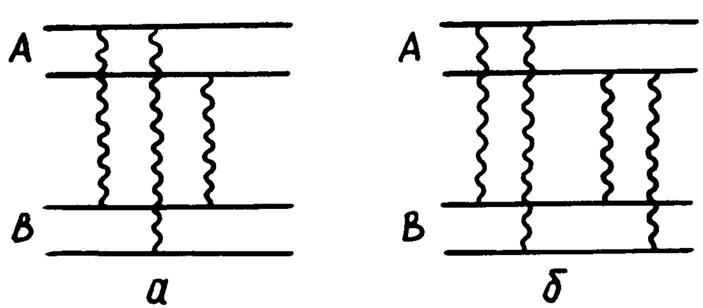

Fig. 1

ticle model for nuclear densities, and iii) the zeroth radius approximation for NN interaction with thickness functions $T_A(b)$, $T_B(b)$ of colliding nuclei A and B, respectively.

For the sake of simplicity the amplitude of NN scattering is taken to be purely imaginary and not depending of spins and isotopic spins.

Let us start with the amplitude of elastic AB scattering in the impact parameter representation

$$\Gamma_{A,B\,(A\to\infty)}(b) = \frac{1}{2\pi i p}\int F(q)e^{-iqb}d\vec{q} = \langle (1-\exp - \int \Gamma_B(b-s,\tilde{s}_B)T_A(s)ds\rangle,$$

$$\Gamma_B(\beta,\tilde{s}_B) = 1 - \prod_{i=1}^{B}(1-\gamma(\beta-\tilde{s}_B)),\quad \tilde{s}_B = \{\tilde{s}_{B_i}\} \qquad (1)$$

$$\gamma(b) = \frac{\tilde{\sigma}}{4\pi a}\exp - \frac{b^2}{4a},\quad \tilde{\sigma} = \sigma(1-i\alpha),\quad \alpha = \mathrm{Re}f(0)/\mathrm{Im}f(0).$$

Going up to the optical limit, over the nucleus A, which is equivalent to the substitution $\exp_A x \to \exp x$ and by using the following rule for an exponent:

$$\langle \exp x\rangle = \exp\{\langle x\rangle + \frac{1}{2!}\langle (x-\langle x\rangle)^2\rangle + \frac{1}{3!}\langle (x-\langle x\rangle)^3\rangle + ..\} \qquad (2)$$

we have for the phase shift function

$$-\chi_{opt}(b) = \ln(1-\Gamma^{AB}_{opt}(b))|_{A\to\infty} = \frac{2}{\tilde{\sigma}}\int ds\{x(e^{-y}-1) +$$

$$+\frac{1}{2!}x^2 e^{-2y}(y + \frac{1}{2}\epsilon y^2 + \frac{2}{9}\epsilon^2 y^3 + \dots) +$$

$$+\frac{1}{3!}x^3e^{-3y}(-y+3y^2+\frac{2}{3}\epsilon y^2+\dots)+\dots\}, \qquad (3)$$

$$x=\frac{\tilde\sigma}{2}T_A(s),\quad y=\frac{\tilde\sigma}{2}T_B(b-s),\qquad \epsilon=-\frac{\tilde\sigma}{16\pi a}.$$

In this relation the account of the linear in T_A term alone corresponds to the rigid projectile approximation, whereas the term linear in $T_A T_B$ corresponds to the optical limit of reference /3/.

The detailed analysis of eq. (3) shows that in zeroth radius approximation of NN interactions, it can be rewritten in the form

$$\chi_{opt}(b)=\frac{2}{\tilde\sigma}\int P(x,y)\,\epsilon_i^{(k)}ds, \qquad (4)$$

where dimensionless quantities $\epsilon_i^{(k)}$ are constructed from amplitudes of NN scattering. The simplest quantities of them are given by

$$\epsilon_i^{(1)}=\frac{\tilde\sigma}{\pi^2}\int[f^2(q)/f^2(0)]^2d^2q \qquad i\ge 2. \qquad (5)$$

Expanding the quantity F in a series in ϵ $P(x,y,\epsilon)=\sum_k P^k(x,y)\epsilon^k$ we find in the optical over B limit

$$P^\circ(x,y)=y(e^{-x}-1)+\frac{1}{2\tilde\sigma}e^{-2x}y^2+\frac{1}{3\tilde\sigma}e^{-3x}(y^2+\dots). \qquad (6)$$

Such nonsymmetrical on x and y presentation of actually symmetrical quantities $F^1(x,y)$ allows us to determine the structure of expansion coefficients *

$$P(x,y)=\sum_{m,n}^{\infty}x^my^na_{mn},\quad a_{mn}=a_{nm} \qquad (7)$$

$$a^\circ_{mn}=(-1)^{m+n}\frac{n^m m^n}{n!\,m!} \qquad n,m>1. \qquad (7')$$

It is easy to see that the summation of series (7) with coefficients given by eq. (7') leads to the following result

$$P^\circ=x+y-u-z-uz+\epsilon\Phi(z,u)+o(\epsilon^2)$$

$$u=xe^{-z},\quad z=ye^{-u},\ \Phi(z,u)=-2(uz+ \qquad (8)$$

$$+\int_0^1\frac{dt}{t}\ln(1-uzt)).$$

The quantities F^k, k>2 can be represented by linear superposition of series of the form

$$P^k=\sum_n C_n\left(xy\frac{d}{dx}\,\frac{d}{dy}\right)^nG_n(x,y). \qquad (9)$$

It should be emphasized that the classification of quantities $G_n(x,y)$ itself and the detailed calculation of coefficients of such superposition is independent and rather difficult task, not considered in the present paper. It is obvious only that in the analysis of higher in ϵ contributions to eq. (8) the use of a series expansion in "small" quantities u, z instead of an expansion in "large" quantities x, y has certain advantages.

* Formula (7) has been first obtained by A.V.Andreev in the framework of a somewhat different approach.

Some corrections to the amplitude of nucleus-nucleus scattering and other quantities could be evaluated easily using formulas (7). So, for instance, the correction of the order $\frac{1}{A}(\frac{1}{B})$ to the $F^{opt}_{AB}(T_A, T_B)$ is given by

$$\delta_{A(B)}F = \frac{1}{2A(B)} \int T_{A(B)}(s_1)\, T_{A(B)}(s_2) \frac{\delta^2 F^{opt}_{AB}(T_A, T_B)}{\delta T_{A(B)}(s_1)\, \delta T_{A(B)}(s_2)} ds_1\, ds_2 . \qquad (10)$$

The correction due to pair correlation effect in the nucleus A has the form

$$\delta F = \frac{1}{2} \int C_2(r_1, r_2) \frac{\delta^2 F^{opt}_{AB}(T_A, T_B)}{\delta T_A(s_1)\, \delta T_A(s_2)} ds_1\, ds_2\, dz_1\, dz_2 \qquad (11)$$

$C_2(r_1, r_2)$ being a pair correlation function.

The result obtained above can be extended to reactions with excitation of colliding nuclei. In the approximation of one step inelastic collision the amplitude of the reaction $B+A_i \to B+A_f$ with excitation of the nucleus A, is given by

$$F_{if} = \int \rho_{if}(r) \frac{\delta F^{opt}_{AB}(T_A, T_B)}{\delta T_A(s)} ds\, dz , \qquad (12)$$

where ρ_{if} is the so-called transition function.

For the quasielastic scattering, when B remains in its ground state while A undergoes all the possible excitations including the breakdown, we find

$$\frac{d\sigma}{d\Omega} = \sum_{k=1}^{A} \frac{1}{k!} \int T_A(s_1) \ldots T_A(s_k) \left| \frac{\delta^{(k)} F^{opt}_{AB}(T_A, T_B)}{\delta T_A(s_1) \ldots \delta T_A(s_k)} \right|^2 ds_1 \ldots ds_k . \qquad (13)$$

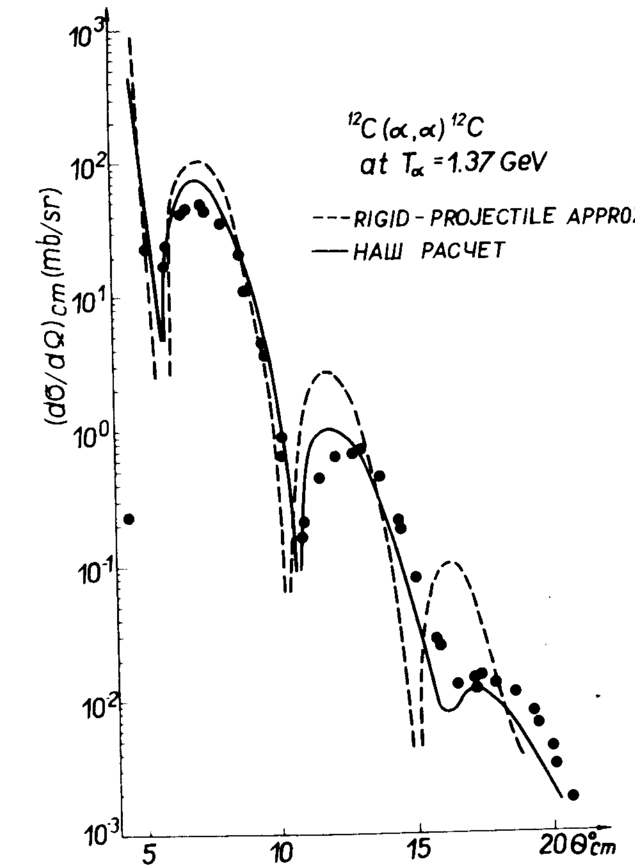

Fig. 2

The results of numerical calculations performed according to formulas (7,12) are examplified in Figs. 2-4 together with the experimental data of ref. [2,4]. The curves —— present our predictions for elastic scattering and the curves —— (fig. 4) those for inelastic scattering with excitation of 2^+(4.43 MeV) and 3^-(3.73 MeV) levels, respectively, in $^{12}C(\alpha,\alpha)^{12}C*$ and $^{40}Ca(\alpha,\alpha)^{40}Ca*$

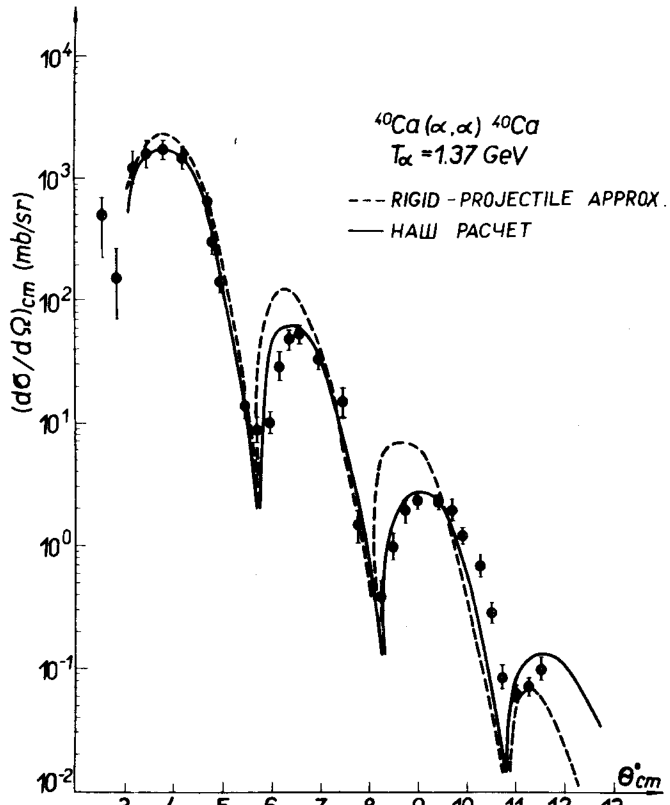

Fig. 3

reactions. For comparison we have plotted there also results of calculations performed in accordance with the rigid projectile approximation (/2/, the curves - - - -) and the optical model by Czyz and Maximon (ref./3/, curves -·-·-·-·-).

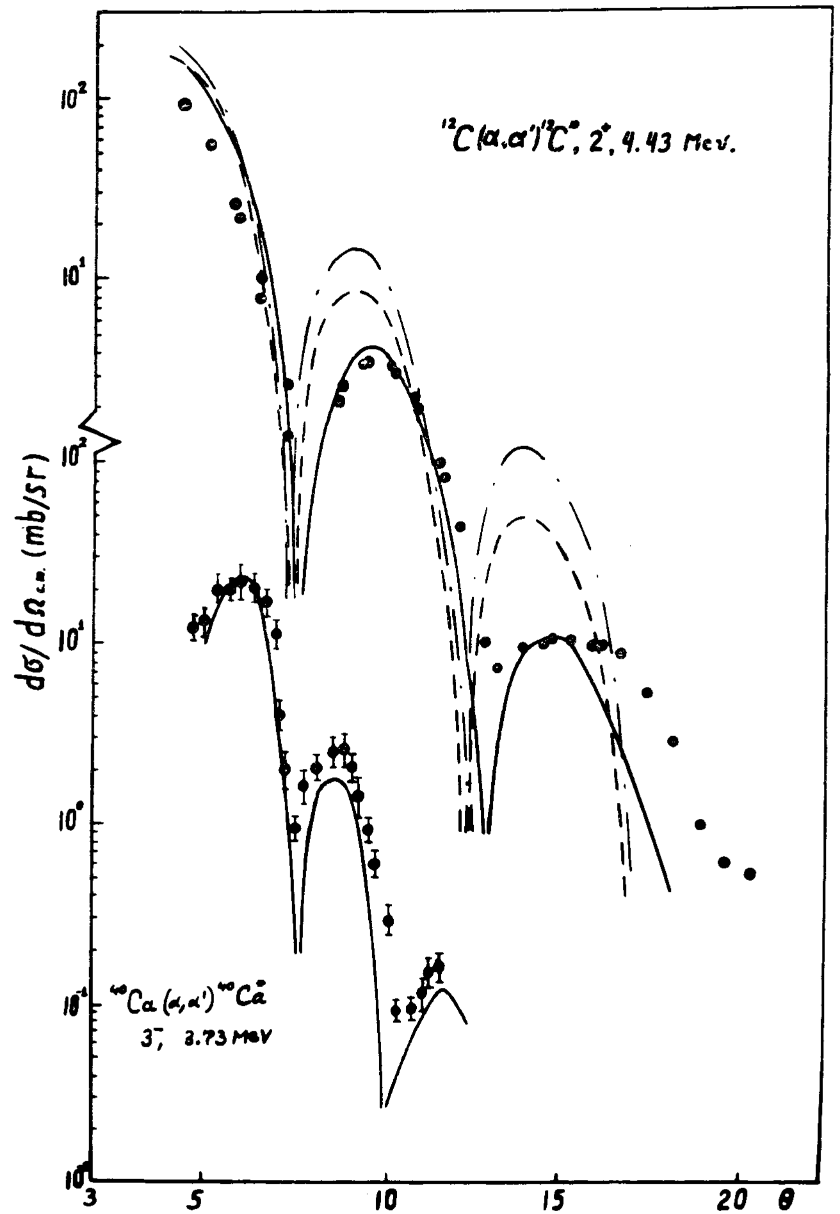

Fig. 4

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