

ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
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ДУБНА



G-61

E2 - 11633

4426/2-78

S.V.Goloskokov, A.V.Koudinov, S.P.Kuleshov

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IN THE LARGE ANGLE SCATTERING
OF PARTICLES WITH SPIN

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**NONAUTOMODEL CORRECTIONS
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Submitted to ЯФ



Голоскоков С.В., Кулинов А.В., Кулешов С.П.

E2 - 11633

Неавтомодельные поправки в рассеянии частиц со спином
на большие углы

В рамках квазипотенциального подхода Логунова-Тавхелидзе изучены неавтомодельные поправки первого и второго порядка по $1/\sqrt{s}$ к амплитудам мезон-нуклонного и нуклон-нуклонного рассеяния. Произведено сравнение полученных результатов с экспериментальными данными.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

Goloskokov S.V., Koudinov A.V.,
Kuleshov S.P.

E2 - 11633

Nonautomodel Corrections in the Large Angle
Scattering of Particles with Spin

Nonautomodel corrections of two orders in $1/\sqrt{s}$ to the amplitudes of elastic meson-nucleon and nucleon-nucleon scattering are discussed in the framework of the Logunov-Tavkhelidze quasipotential approach. Formulas obtained are used for the description of the experimental data.

The investigation has been performed at the
Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1978

1. INTRODUCTION

One of the most interesting problems in high energy physics is the investigation of large angle hadron scattering. For the last years considerable progress has been made in this field on the basis of the assumptions about the quark structure of hadrons and automodelity principle. These hypotheses predict the power behaviour of differential cross-sections^{/1,2/}:

$$\frac{d\sigma}{dt} \sim \frac{1}{s^N} f\left(\frac{t}{s}\right); \quad \frac{t}{s} - \text{fixed} \quad (1.1)$$

that renders the main features of the experimental data (see, e.g., ref.^{/3/}).

An extensive investigation of this problem in the framework of the Logunov-Tavkhelidze quasipotential approach was carried out by the Dubna group^{/4-7/}. Their result suggests that phenomenological quasipotentials given by the integral representation

$$\hat{g}(s, \vec{\Lambda}) = g(s) \int_0^\infty dx \hat{\rho}(s, x) e^{-x\vec{\Lambda}^2}; \quad t = -\vec{\Lambda}^2 \quad (1.2)$$

might explain the automodel behaviour (1.1) provided that the weak limit for the function $\rho(s, x)$

$$\lim_{s \rightarrow \infty} s^M \hat{\rho}(s, \mathbf{x} = \eta/s) = \hat{\psi}(\eta); \quad 0 < \eta < \infty; M > 0. \quad (1.3)$$

does exist.

On the other hand, the experimental data on large angle elastic scattering are accessible in the energy range of $s_{\max} \sim 10 \div 50$ (GeV)² for different reactions. The first obvious question is whether the corrections to the leading asymptotic term are large and should be taken into account. The corrections to the scattering amplitude of two scalar particles were discussed in ref.^{/8/}. It was pointed out there that even for $s \sim 50$ (GeV)² the contribution of corrections is substantial and should be necessarily taken into account in the description of data.

The purpose of this paper is to extend the method developed in our previous paper to the case of high-energy large angle meson-nucleon and nucleon-nucleon scattering. In solving this problem we shall essentially employ the γ_5 -invariance of interaction at large energies and momentum transfers^{/9/}.

The paper is organized as follows. In Sec. 2 the general scheme of obtaining corrections of two orders in $1/\sqrt{s}$ is discussed. The processes of meson-nucleon and nucleon-nucleon scattering are considered in Secs. 3 and 4, resp. In Sec. 4 the numerical results are analysed in detail and conclusions are given.

2. THE DESCRIPTION OF LARGE ANGLE SCATTERING FOR ANALYTIC QUASIPOTENTIALS

An approach that we shall follow to analyse the large angle scattering is based on

the quasipotential equation for particles with spin. In the momentum space it is of the form:

$$\hat{G}(s, \vec{p}, \vec{k}) = \hat{g}(s, \vec{p}, \vec{k}) + \int d\vec{q} \hat{g}(s, \vec{p}, \vec{q}) \frac{\hat{A}(s, \vec{q})}{E^2(\vec{q}) - E^2 - i0} \hat{G}(s, \vec{q}, \vec{k}), \quad (2.1)$$

where \vec{p} and \vec{k} are the momenta of the particle before and after the collision, $E = \sqrt{s} = \sqrt{m_1^2 + \vec{p}^2} + \sqrt{m_2^2 + \vec{p}^2}$ is the c.m.s. energy of particles, $E(\vec{q}) = \sqrt{m_1^2 + \vec{q}^2} + \sqrt{m_2^2 + \vec{q}^2}$; m_1 and m_2 are masses of the first and second particles, resp.. $\hat{A}(s, \vec{q})$ is the matrix, its form depending on the spin structure of the process and being unessential here.

Provided that the quasipotential is local ($\hat{g}(s, \vec{p}, \vec{k}) = \hat{g}(s, \vec{p} - \vec{k})$) and given by the representation (1.2), the equation (2.1) is solved by iterations:

$$\begin{aligned} \hat{G}(s, \vec{p}, \vec{k}) &= \sum_{n=0}^{\infty} \hat{G}_{n+1}(s, \vec{p}, \vec{k}); \quad \hat{G}_1(s, \vec{p}, \vec{k}) = \hat{g}(s, \vec{p} - \vec{k}); \\ \hat{G}_{n+1}(s, \vec{p}, \vec{k}) &= \int \prod_{i=1}^n d\vec{q}_i \hat{g}(s, \vec{p} - \vec{q}_1) \hat{A}(s, \vec{q}_1) \hat{g}(s, \vec{q}_1 - \vec{q}_2) \times \quad (2.2) \\ &\times \hat{A}(s, \vec{q}_2) \dots \hat{A}(s, \vec{q}_n) \hat{g}(s, \vec{q}_n - \vec{k}) \prod_{i=1}^n \frac{1}{E^2(\vec{q}_i) - E^2 - i0}. \end{aligned}$$

Now inserting the expression (1.2) into (2.2) and changing integration variables

$$\vec{q}_i = \vec{\Delta}_i + \lambda_i; \quad \lambda_i = \frac{\vec{p} + \vec{k}}{2} + [1 - 2 \sum_{\ell=1}^i \frac{1}{x_\ell} / \sum_{\ell=1}^{n+1} \frac{1}{x_\ell}] \frac{\vec{p} - \vec{k}}{2} \quad (2.3)$$

we can rewrite $\hat{G}_{n+1}(s, \vec{p}, \vec{k})$ in the form:

$$\hat{G}_{n+1}(s, \vec{p}, \vec{k}) = (g(s))^{n+1} \int_0^\infty \prod_{i=1}^n dx_i \exp\{t / \sum_{i=1}^{n+1} \frac{1}{x_i}\} \times$$

$$\times \prod_{i=1}^n \frac{d\vec{\Delta}_i}{E^2(\vec{\Delta}_i + \vec{\lambda}_i) - E^2 - i0} \exp\{-C_{ij} \vec{\Delta}_i \vec{\Delta}_j\} \times$$

$$\times \hat{\rho}(s, x_1) \hat{A}(s, \vec{\Delta}_1 + \vec{\lambda}_1) \hat{\rho}(s, x_2) \dots \hat{\rho}(s, x_n) \hat{A}(s, \vec{\Delta}_n + \vec{\lambda}_n) \hat{\rho}(s, x_{n+1}),$$
(2.4)

where

$$c_{ij} \vec{\Delta}_i \vec{\Delta}_j = \sum_{k=1}^{n+1} (\vec{\Delta}_k - \vec{\Delta}_{k-1})^2 x_k; \vec{\Delta}_0 = \vec{\Delta}_{n+1} = 0; t = (p-k)^2.$$

In the limit of high energy large angle scattering the leading contribution to the asymptotics of the integral (2.4) is given by the integration region where $(\sum_{i=1}^n \frac{1}{x_i})^{-1} \sim 0$. It has been proved in ref.^{/4/} that assuming the condition (1.3) the contributions of the regions where one and only one of x_i is near zero, are important only. Considering, for definiteness, x_m to be small, we get for the momenta $\vec{\lambda}_i$:

$$\vec{\lambda}_i \sim \vec{p} - \left(\sum_{j=1}^i \frac{1}{x_j}\right) x_m (\vec{p} - \vec{k}); i < m;$$

$$\vec{\lambda}_i \sim \vec{k} - \left(\sum_{j=i+1}^{n+1} \frac{1}{x_j}\right) x_m (\vec{k} - \vec{p}); i \geq m.$$
(2.5)

Thus, the difference between the momenta $\vec{\lambda}_i$ and corresponding large momenta \vec{p} or \vec{k} is small, and one can replace the dynamical factors $\hat{A}(s, \vec{\Delta}_i + \vec{\lambda}_i)$ and $(E^2(\vec{\Delta}_i + \vec{\lambda}_i) - E^2 - i0)^{-1}$ by their expansions around the points $\vec{\Delta}_i + \vec{\lambda}_i = \vec{p}(\vec{k})$; $i < m$ ($i \geq m$). The higher terms of the

expansions decrease as $1/p$ or $1/p^2$ with growing energy and their consideration is equivalent to the estimation of corrections.

As to the density functions of quasipotentials $\hat{\rho}(s, x_i)$ the following substitutions

$$\hat{\rho}(s, x_m) \rightarrow \hat{\rho}_h(s, x_m);$$
(2.6)

$$\hat{\rho}(s, x_i) \rightarrow \hat{\rho}_s(s, x_i); i \neq m$$

are admitted. Where a "hard" quasipotential with density function $\hat{\rho}_h(s, x)$ is reconstructed according to the function $\hat{\psi}(\eta)$ from (1.3), and a "soft" component with density function $\hat{\rho}_s(s, x)$ is related to small angle scattering.

The formalism presented above is a natural generalization of the ref.^{/8/} treatment for the case of particles with spin. The modifications required are as follows: First $\hat{\rho}_h(s, x)$ and $\hat{\rho}_s(s, x)$ are now matrices unlike the scalar densities of ref.^{/8/}. Second, the presence of $\hat{A}(s, \vec{q})$ and the necessity of transition from the matrix scattering amplitude $\hat{G}(s, \vec{p}, \vec{k})$ to the measurable differential cross-sections and polarizations are additional sources of corrections. But on the whole, the consideration of spin does not bring essential complications as the matrix structure of different amplitudes is easily factorizable and the remaining integrals are exactly the correction integrals of ref.^{/8/}.

3. MESON-NUCLEON SCATTERING

Meson-nucleon scattering amplitude satisfies the quasipotential equation (2.1)

with:

$$\hat{A}(s, \vec{q}) = \frac{1}{\sqrt{m_1^2 + \vec{q}^2}} \{ \gamma_0 E - (1 + \sqrt{\frac{m_1^2 + \vec{q}^2}{m_2^2 + \vec{q}^2}}) (\vec{\gamma} \vec{q} + m_2) \}, \quad (3.1)$$

where m_1 and m_2 are the meson and nucleon masses, resp. In the presence of exchange forces the quasipotential can be written as

$$\hat{V}(s, \vec{p}, \vec{k}) = \hat{g}(s, \vec{p}, \vec{k}) + \hat{h}(s, \vec{p}, \vec{k}) \quad (3.2)$$

with the local component $\hat{g}(s, \vec{p}, \vec{k})$ given by (1.2) and

$$\hat{h}(s, \vec{p}, \vec{k}) = h(s) \int_0^\infty dy \hat{\sigma}(s, y) e^{-y(\vec{p} + \vec{k})^2}. \quad (3.3)$$

The density functions $\hat{\rho}$ and $\hat{\sigma}$ are assumed to satisfy the weak limits (1.8) with equal M . As the backward peak in meson-nucleon scattering is suppressed, then

$$\sigma_s(s, \mathbf{x}) \ll \hat{\rho}_s(s, \mathbf{x})$$

and one can neglect the contribution of the "soft" exchange potential. We retain the contribution of the "hard" exchange forces only, which is estimated by analogy with the local component $\hat{g}(s, \vec{p} - \vec{k})$. The only difference is that the expansions (2,5) are replaced by:

$$\begin{aligned} \vec{\lambda}_i &\approx \vec{p} - \left(\sum_{j=1}^i \frac{1}{x_j} \right) \mathbf{x}_m (\vec{p} + \vec{k}); \quad i < m; \\ \vec{\lambda}_i &\approx \vec{k} - \left(\sum_{j=i+1}^{n+1} \frac{1}{x_j} \right) \mathbf{x}_m (\vec{p} + \vec{k}); \quad i \geq m. \end{aligned} \quad (3.4)$$

The quasipotential (3.2) is a 4x4 matrix, and below we shall discuss its simple γ_5 -invariant form with the density functions:

$$\hat{\rho}(s, \mathbf{x}) = \gamma_0 \rho(s, \mathbf{x}); \quad \hat{\sigma}(s, \mathbf{x}) = \gamma_0 \sigma(s, \mathbf{x}); \quad (3.5)$$

$$g(s) = h(s) = 4ip.$$

Thus, summing up all correction terms we get for helicity spin-non-flip and spin-flip amplitudes to an approximation of $1/p^2$:

$$\begin{aligned} T_{++}(s, t) &= 4ip e^{2i\chi(0)} \sqrt{\frac{1+z}{2}} \int_0^\infty dx \{ \rho_h(s, \mathbf{x}) e^{x^t} f_1(x, z) + \\ &+ \sigma_h(s, \mathbf{x}) e^{x^u} (f_1(x, -z) + 4A_3 x) \}; \end{aligned} \quad (3.6)$$

$$\begin{aligned} T_{+-}(s, t) &= -4ip e^{2i\chi(0)} \sqrt{\frac{1-z}{2}} \int_0^\infty dx \{ \rho_h(s, \mathbf{x}) e^{x^t} f_2(x, z) + \\ &+ \sigma_h(s, \mathbf{x}) e^{x^u} f_2(x, -z) \}, \end{aligned}$$

where $z = \cos \theta$ is the cosine of the c.m.s. scattering angle, $\chi(0)$ is the eikonal phase at zero impact parameter. Also, we have introduced:

$$\begin{aligned} f_1(x, z) &= 1 + \frac{8}{ip} |xp^2(1-z)A_2 - B_1| + \frac{1}{p^2} |x^2p^4(1-z)^2(-4A_3 - 32A_2^2) + \\ &+ xp^2(1-z)(64A_2B_1 - 24A_2^2 - 4A_3) - 4A_2^2 - 32A_2B_1 - 8B_2 + 4F + \\ &+ 32C_1 + (m_1^2 - m_2^2)A_1 + 4A_3xp^2 - 8A_3x^2p^4(1-z)|; \\ f_2(x, z) &= \frac{m_2}{p} + \frac{m_2}{ip^2} |8xp^2(1-z)A_2 + 2A_2 - 8B_1|; \end{aligned} \quad (3.7)$$

and

$$A_1 = \pi^2 \int_0^\infty \frac{dx}{x} \rho_s(s, x) = -\frac{1}{2} \chi(0);$$

$$A_2 = \pi^{3/2} \int_0^\infty \frac{dx}{x^{3/2}} \rho_s(s, x);$$

$$A_3 = \pi^2 \int_0^\infty \frac{dx}{x^2} \rho_s(s, x);$$

$$B_1 = \pi^{7/2} \int_0^\infty \frac{dx_1 dx_2}{x_1 x_2 \sqrt{x_1 + x_2}} \rho_s(s, x_1) \rho_s(s, x_2); \quad (3.8)$$

$$B_2 = \pi^3 \int_0^\infty \frac{dx_1 dx_2}{x_1 \sqrt{x_1 x_2 (x_1 + x_2)}} \rho_s(s, x_1) \rho_s(s, x_2);$$

$$C_1 = \pi^5 \int_0^\infty \frac{dx_1 dx_2 dx_3}{x_1 x_2 x_3 \sqrt{x_1 x_2 + x_2 x_3 + x_3 x_1}} \rho_s(s, x_1) \rho_s(s, x_2) \rho_s(s, x_3);$$

$$F = \pi^3 \int_0^\infty \frac{dx}{x^{5/2}} \frac{dy}{y^{3/2}} \rho_s(s, x) \rho_s(s, y) \int_0^\infty dz_1 dz_2 \theta(z_1 - z_2) \times \\ \times \exp \left\{ -\frac{z_1^2}{4x} - \frac{z_2^2}{4y} \right\}.$$

In order to proceed to numerical estimates, the explicit formulas for density functions are needed. As to the "soft" quasipotential, we can choose it to be the gaussian one with the density function:

$$\rho_s(s, x) = g \delta(x - a) \quad (3.9)$$

and the density functions of the "hard" component will be approximated as follows:

$$\rho_h(s, x) = \frac{Ae^{-2i\chi(0)}}{4ip\Gamma(\kappa+1)s^{N_1-1}} x^\kappa e^{-bx}; \quad (3.10)$$

$$\sigma_h(s, x) = \frac{Be^{-2i\chi(0)}}{4ip\Gamma(\lambda+1)s^{N_2-1}} x^\lambda e^{-bx};$$

$$N_1 + \kappa = N_2 + \lambda = N.$$

Fits of the data on $\pi^\pm p$ small angle scattering give for the values of the eikonal phase and the parameter $a^{1/2}$:

$$i\chi(0) = -0.5; \quad a = 2.5 (\text{GeV}/c)^{-2}. \quad (3.11)$$

Inserting (3.9)-(3.11) into (3.6)-(3.8) and integrating over x we finally get for the amplitudes of $\pi^\pm p$ large angle scattering:

$$T_{++}(s, t) = s^{-N} \sqrt{\frac{1+z}{2}} \left\{ A \left(\frac{s}{|t|+b} \right)^{\kappa+1} \left[\tilde{f}_1(\kappa) - 0.2 \left(\frac{p_0}{p} \right)^2 \frac{(\kappa+1)^2}{1-z} \right] + \right. \\ \left. + B \left(\frac{s}{|u|+b} \right)^{\lambda+1} \left[\tilde{f}_1(\lambda) - 0.2 \left(\frac{p_0}{p} \right)^2 \frac{\lambda(\lambda+1)}{1+z} \right] \right\}; \quad (3.12)$$

$$T_{+-}(s, t) = -s^{-N} \sqrt{\frac{1-z}{2}} \left\{ A \left(\frac{s}{|t|+b} \right)^{\kappa+1} \tilde{f}_2(\kappa) + B \left(\frac{s}{|u|+b} \right)^{\lambda+1} \tilde{f}_2(\lambda) \right\};$$

where

$$\tilde{f}_1(\kappa) = 1 - i \left(\frac{p_0}{p} \right) (0.3568\kappa + 0.2307) + \left(\frac{p_0}{p} \right)^2 (0.0363\kappa^2 - \\ - 0.174\kappa - 0.4101); \\ \tilde{f}_2(\kappa) = 0.938 \left(\frac{p_0}{p} \right) - i \left(\frac{p_0}{p} \right)^2 (0.3347\kappa + 0.402); \quad (3.13) \\ p_0 = 1 (\text{GeV}/c).$$

The exponent N in (3.12) should be chosen according to the quark counting rules^{/1/} that gives N=4. We also remark that the consideration of corrections to the leading asymptotic term does not require additional free parameters. From the fit of the data we shall find three real (A,B,b) and two integer (κ, λ) parameters.

4. NUCLEON-NUCLEON SCATTERING

In the case of nucleon-nucleon scattering the matrix $\hat{A}(s, \vec{q})$ is of the form:

$$\hat{A}(s, \vec{q}) = \left[\frac{E^2 - \frac{1}{2} E^2(\vec{q})}{E} + \hat{H}^{(1)}(\vec{q}) + \hat{H}^{(2)}(-\vec{q}) + \frac{2}{E} \hat{H}^{(1)}(\vec{q}) \hat{H}^{(2)}(-\vec{q}) \right], \quad (4.1)$$

where $\hat{H}^{(1,2)}(\vec{q})$ are the energy operators of the first and second particles, resp.:

$$\hat{H}^{(1,2)}(\vec{q}) = m \gamma_0^{(1,2)} + \gamma_0^{(1,2)} \gamma^{(1,2)} \vec{q}. \quad (4.2)$$

Since the scattered particles are identical, the exchange forces should be taken into consideration by antisymmetrization of the amplitude over the states of final particles. Assuming the γ_5 -invariance we can choose the matrix structure of the quasipotential as follows:

$$\begin{aligned} \hat{\rho}_s(s, \mathbf{x}) &= \gamma_0^{(1)} \gamma_0^{(2)} \rho_s(s, \mathbf{x}); \\ \hat{\rho}_h(s, \mathbf{x}) &= \gamma_\mu^{(1)} \gamma^{(2)\mu} \rho_1(s, \mathbf{x}) + \gamma_\mu^{(1)} \gamma_5^{(1)} \gamma^{(2)\mu} \gamma_5^{(2)} \rho_2(s, \mathbf{x}); \end{aligned} \quad (4.3)$$

$$g(s) = 4i.$$

After calculations analogous to that of ref.^{/8/} we get for the helicity amplitudes of pp large angle scattering:

$$\begin{aligned} T_{++,++}(s,t) &= 4i(1+z) e^{2i\chi(0)} \int_0^\infty dx (\rho_1(s, \mathbf{x}) + \rho_2(s, \mathbf{x})) \times \\ &\times \left[F_1(x, z) e^{xt} - \frac{m^2}{2p^2} e^{xu} \right]; \end{aligned}$$

$$\begin{aligned} T_{++,--}(s,t) &= 4i(1-z) e^{2i\chi(0)} \int_0^\infty dx (\rho_1(s, \mathbf{x}) + \rho_2(s, \mathbf{x})) \times \\ &\times \left[F_1(x, -z) e^{xu} - \frac{m^2}{2p^2} e^{xt} \right]; \end{aligned} \quad (4.4)$$

$$\begin{aligned} T_{+-,+}(s,t) &= 8i e^{2i\chi(0)} \int_0^\infty dx (\rho_1(s, \mathbf{x}) - \rho_2(s, \mathbf{x})) \times \\ &\times \left[F_1(x, z) e^{xt} + F_1(x, -z) e^{xu} \right] + \left[(2xA_3 - \frac{m^2}{4p^2}) \rho_1(s, \mathbf{x}) - \right. \\ &\left. - (2xA_3 + \frac{m^2}{4p^2}) \rho_2(s, \mathbf{x}) \right] [(1-z) e^{xt} + (1+z) e^{xu}]; \end{aligned}$$

$$\begin{aligned} T_{++,+-}(s,t) &= -\frac{2im}{p} e^{2i\chi(0)} \sqrt{1-z^2} \int_0^\infty dx (\rho_1(s, \mathbf{x}) + \rho_2(s, \mathbf{x})) \times \\ &\times \left[F_2(x, z) e^{xt} - F_2(x, -z) e^{xu} \right]; \end{aligned}$$

with the notations:

$$\begin{aligned}
F_1(x, z) &= 1 + \frac{8A_2}{ip} (xp^2(1-z) - \frac{1}{2}) + \frac{1}{p^2} [(xp^2(1-z) - \frac{1}{2})^2 \times \\
&\times (4A_3 - 32A_2^2) + xp^2(1-z)(8A_2^2 - 8F) + 8F - 2A_3 - \\
&- m^2(2A_1 + \frac{1}{2}) + 4A_3 p^2 x - 8A_3 x^2 p^4 (1-z)]; \\
F_2(x, z) &= 1 + \frac{4}{ip} A_2 (2xp^2(1-z) - \frac{1}{2}).
\end{aligned} \tag{4.5}$$

The values $\chi(0)$, A_i , B_i , C_i and F are defined in (3.8). Here we shall choose the same approximations for density functions, as above:

$$\rho_s(s, x) = g\delta(x - a);$$

$$\rho_1(s, x) = \frac{Ce^{-2i\chi(0)}}{4i\Gamma(\nu+1)s^{M_1-1}} x^\nu e^{-dx}; \tag{4.6}$$

$$\rho_2(s, x) = -\frac{De^{-2i\chi(0)}}{4i\Gamma(\gamma+1)s^{M_2-1}} x^\gamma e^{-dx};$$

$$M_1 + \nu = M_2 + \gamma = M.$$

The numerical values of the "soft" quasi-potential parameters are again derived from the fits of the small angle scattering data^{13/}:

$$i\chi(0) = -0.5; \quad a = 2.5(\text{GeV}/c)^{-2}. \tag{4.7}$$

Substituting (4.6) and (4.7) into (4.4) we finally get for the helicity amplitudes of pp-scattering:

$$\begin{aligned}
T_{++,++}(s, t) &= (1+z) \{C(s, t) \tilde{F}_1(\nu, z) + D(s, t) \tilde{F}_1(\gamma, z) - \\
&- 0.44(\frac{p_0}{p})^2 [C(s, u) + D(s, u)]\};
\end{aligned}$$

$$\begin{aligned}
T_{++,--}(s, t) &= (1-z) \{C(s, u) \tilde{F}_1(\nu, -z) + D(s, u) \tilde{F}_1(\gamma, -z) - \\
&- 0.44(\frac{p_0}{p})^2 [C(s, t) + D(s, t)]\}; \tag{4.8}
\end{aligned}$$

$$\begin{aligned}
T_{+-,+-}(s, t) &= 2 \{C(s, t) [\tilde{F}_1(\nu, z) + (0.1(\nu+1) - 0.22(1-z))(\frac{p_0}{p})^2] - \\
&- D(s, t) [\tilde{F}_1(\gamma, z) + (0.1(\gamma+1) + 0.22(1-z))(\frac{p_0}{p})^2] +
\end{aligned}$$

$$\begin{aligned}
&+ C(s, u) [\tilde{F}_1(\nu, -z) + (0.1(\nu+1) - 0.22(1+z))(\frac{p_0}{p})^2] - \\
&- D(s, u) [\tilde{F}_1(\gamma, -z) + (0.1(\gamma+1) + 0.22(1+z))(\frac{p_0}{p})^2]\};
\end{aligned}$$

$$\begin{aligned}
T_{++,+-}(s, t) &= -\sqrt{1-z^2} \{ (C(s, t) - C(s, u)) \tilde{F}_2(\nu) + \\
&+ (D(s, t) - D(s, u)) \tilde{F}_2(\gamma) \};
\end{aligned}$$

where:

$$C(s, t) = \frac{C}{s^M} \left(\frac{s}{|t| + d} \right)^{\nu+1}; \quad C(s, u) = \frac{C}{s^M} \left(\frac{s}{|u| + d} \right)^{\nu+1};$$

$$D(s,t) = -\frac{D}{s^M} \left(\frac{s}{|t|+d}\right)^{\gamma+1}; \quad D(s,u) = -\frac{D}{s^M} \left(\frac{s}{|u|+d}\right)^{\gamma+1};$$

$$\tilde{F}_1(\nu, z) = 1 - i\left(\frac{p_0}{p}\right)0.365\nu + \left(\frac{p_0}{p}\right)^2(0.0363\nu^2 + 0.0182\nu - 0.9617 - 0.2\frac{(\nu+1)^2}{1-z}); \quad (4.9)$$

$$\tilde{F}_2(\nu) = 0.469\left(\frac{p_0}{p}\right) - i\left(\frac{p_0}{p}\right)^2 0.167\left(\nu + \frac{1}{2}\right).$$

The exponent M is again predicted by the quark counting rules: M=5. From the fit of experimental data we shall determine three real (C,D,d) and two integer (ν, γ) parameters.

5. COMPARISON WITH THE EXPERIMENT

Let us proceed now to the analysis of the experimental data on the differential cross sections of $\pi^\pm p$ -scattering in the energy region of $p_L \geq 5(\text{GeV}/c)$ and $|\cos\theta| < 0.8$ ^{14,15/} and pp-scattering for $p_L \geq 7(\text{GeV}/c)$ ^{16/} by using the scattering amplitudes derived above.

The differential cross sections are expressed in terms of helicity amplitudes as follows:

$$\frac{d\sigma_{MN}}{dt} = |T_{++}|^2 + |T_{+-}|^2; \quad (5.1)$$

$$\frac{d\sigma_{NN}}{dt} = |T_{++,++}|^2 + |T_{++,--}|^2 + |T_{+,-,+}|^2 + 4|T_{++,+}|^2. \quad (5.2)$$

Table

| | A | κ | B | λ | b | χ^2 | $\bar{\chi}^2$ | $\chi^2/\bar{\chi}^2$ |
|-----------|------------------|----------|-----------------|-----------|---------------|----------|----------------|-----------------------|
| $\pi^+ p$ | 100.1 ±6.7 | 2 | 10.4 ±1.2 | 3 | 4.34 | 207 | 139 | 1.6 |
| $\pi^- p$ | 201.7 ±8.6 | 1 | 8.0 ±1.1 | 3 | ±0.17 | | | |
| pp | -1083.0 ±68.7 | 3 | 7638.0 ±73.1 | 0 | 4.47 ±0.12 | 223 | 81 | 2.75 |

The results of the fit are presented in the table and illustrated in Figs. 1-3. It is to be noticed that the consideration of corrections despite their rather large value enables us to achieve better description of the data as compared to the fits neglecting corrections^{17/} the number of the fitted parameters being the same.

We also remark that the consideration of corrections results in the deviations from the exact automodelity (1.1), that is, exponent N becomes a function of energy and scattering angle

$$\frac{d\sigma}{dt} \sim \frac{1}{s^{N(s,z)}} f(z) \quad (5.3)$$

and $N(s,z) \rightarrow N$ when $s \rightarrow \infty$.

The smooth approximations of differential cross-sections obtained enable us to determine the effective powers $N(s,z)$. The numerical values of $N(s,z)$ for $\pi^\pm p$ scattering are shown in Figs. 4,5. In the energy region where experimental data on $\pi^\pm p$ large angle scattering are accessible the deviations from the predictions of the quark counting

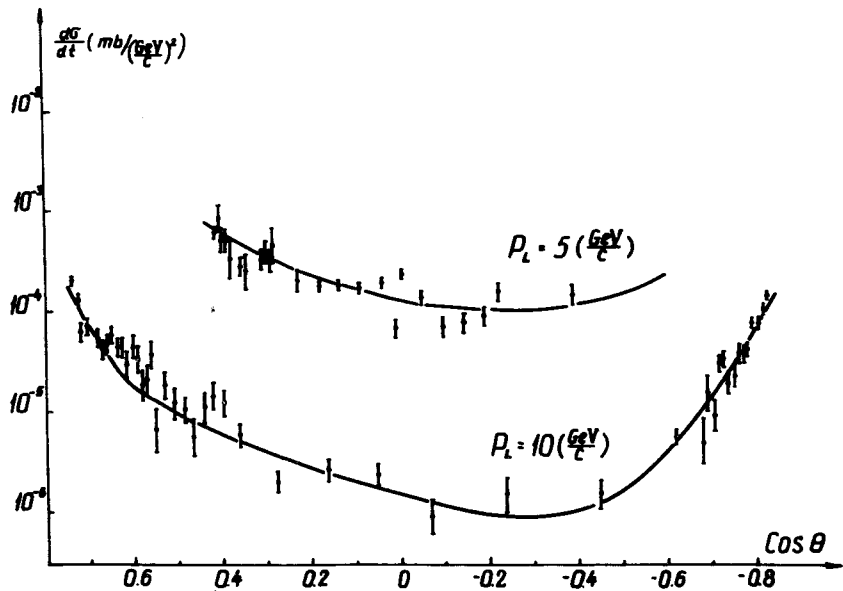


Fig. 1. $\frac{d\sigma}{dt}$ for π^+p scattering.

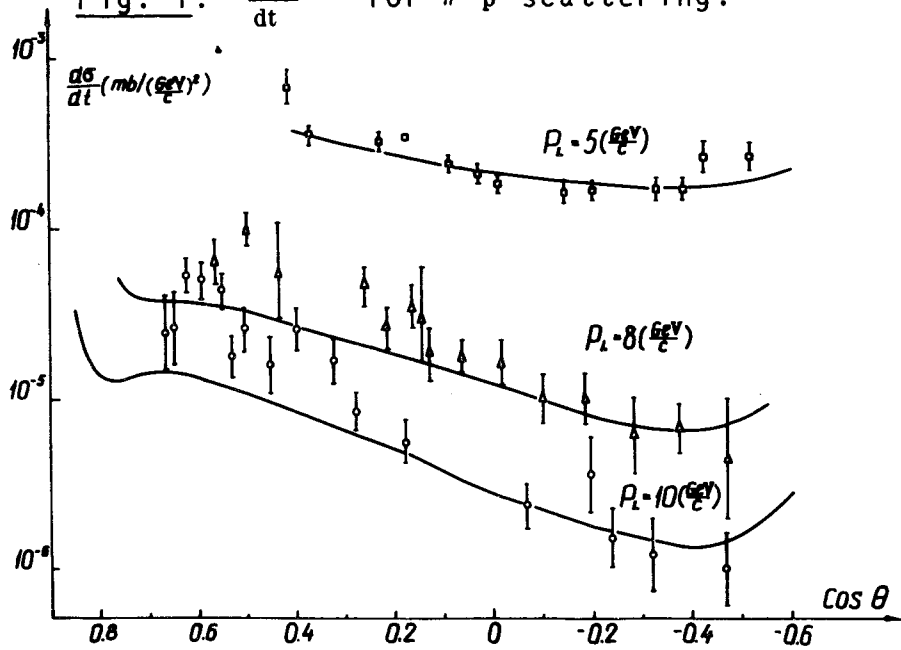


Fig. 2. $\frac{d\sigma}{dt}$ for π^-p scattering.

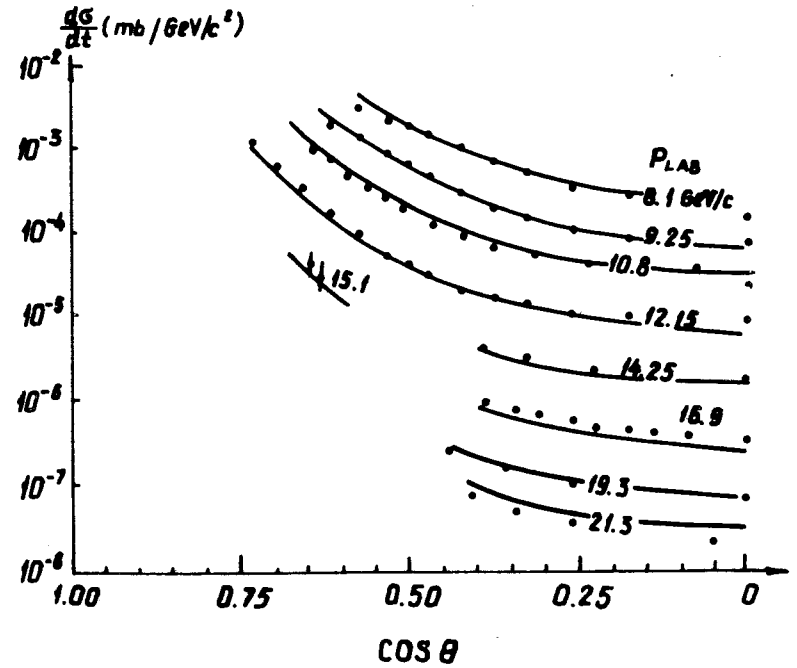


Fig. 3. $\frac{d\sigma}{dt}$ for pp -scattering.

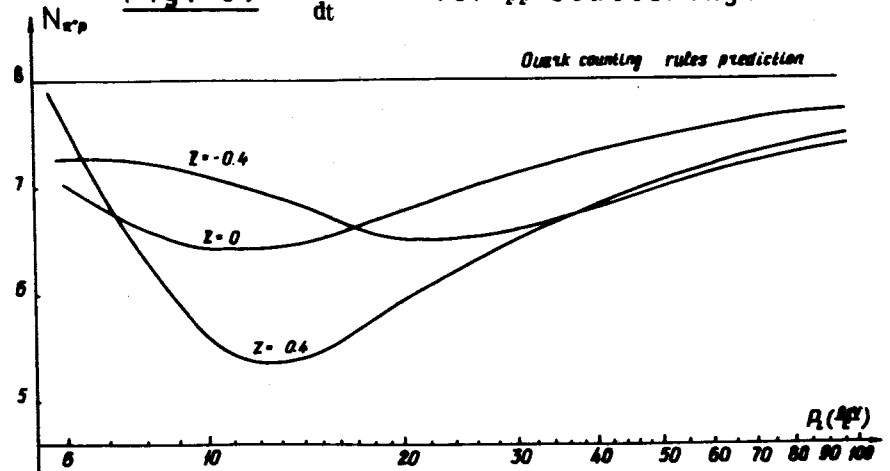


Fig. 4. Effective powers $N(s,z)$ for π^+p scattering.

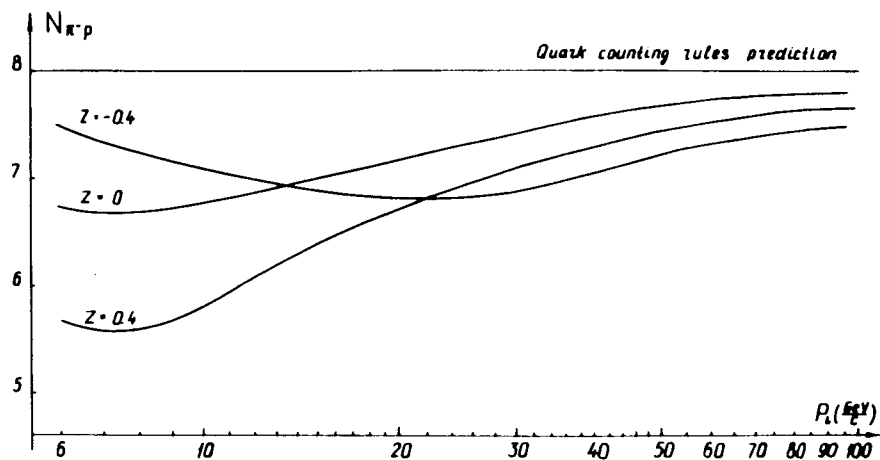


Fig. 5. Effective powers $N(s, z)$ for π^-p scattering.

rules are considerable. This is a manifestation of the fact that in this energy region magnitude of corrections is comparable with that of the leading asymptotic term. In the case of pp -scattering the deviations from the exact automodelity are rather smaller.

It is worth mentioning that for the interval $12 (\text{GeV})^2 < s < 19 (\text{GeV})^2$ our model predicts the following average values for π^-p -scattering:

$$\bar{N}(80^\circ) = 6.2; \bar{N}(90^\circ) = 6.7; \bar{N}(100^\circ) = 7.1 \quad (5.4)$$

that is in good agreement with the results of recent experiments presented in ref. 18.

The consideration of the mass of interacting particles breaks the γ_5 -invariance

of the amplitude that results in nonzero polarization decreasing as s^{-1} with growing energy. The polarizations predicted by the model discussed are plotted in Figs. 6, 7.

We now turn to the description of the exchange process $\pi^-p \rightarrow \pi^0n$. Its amplitude is expressed via the amplitudes of elastic channels by the well-known relation:

$$T_{\pi^-p \rightarrow \pi^0n} = \frac{1}{\sqrt{2}} [T_{\pi^+p} e^{i\delta} T_{\pi^-p}], \quad (5.5)$$

where δ is the relative phase shift of the elastic amplitudes that cannot be found from the analysis of elastic scattering. The consideration of few measured points for the reaction $\pi^-p \rightarrow \pi^0n$ /14/ showed that δ can be chosen to equal $\pi/2$. The resulting predictions for the differential cross sections of the exchange scattering are shown in Fig. 8.

We have investigated in detail the effects of eikonal corrections to the amplitudes of large angle pion-proton and proton-proton scattering. The results obtained may be summarized as follows.

1. The consideration of corrections improves the description of the experimental data that proves self-consistency of the model discussed.

2. The corrections to the amplitudes of $\pi^\pm p$ -scattering are comparable with the leading asymptotic term for energies up to $p_L \sim 40 (\text{GeV}/c)$ that results in considerable deviations from the predictions of quark counting rules. For the case of proton-proton scattering the magnitude of corrections is rather smaller, and the deviations from

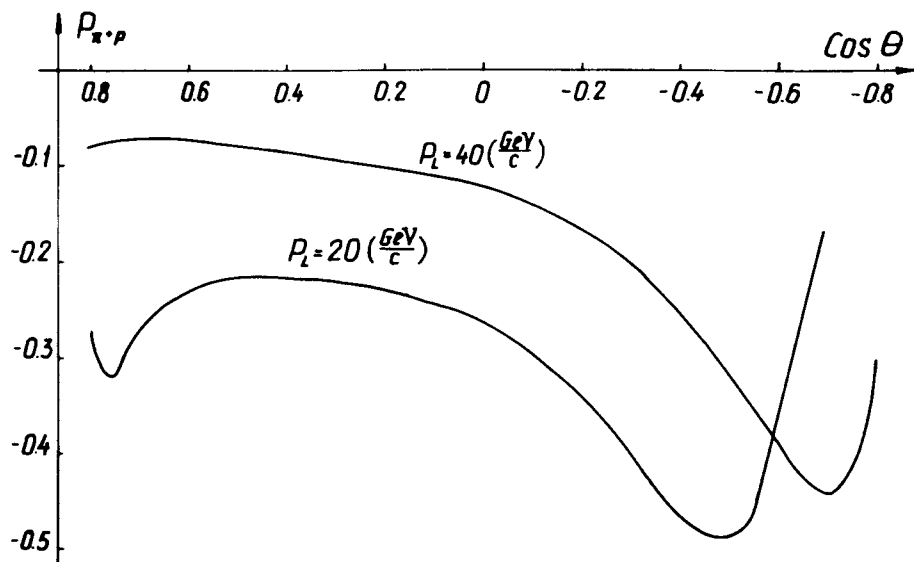


Fig. 6. Predictions for π^+p polarization.

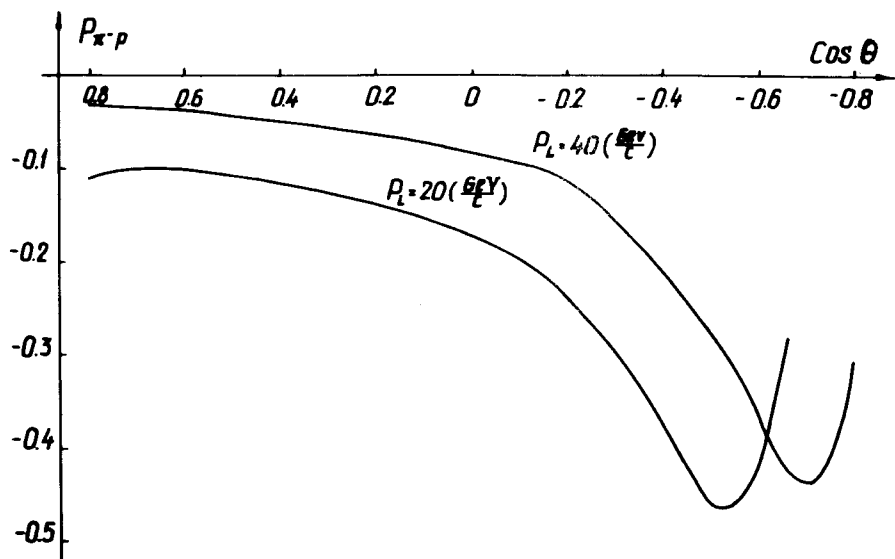


Fig. 7. Predictions for π^-p polarization.

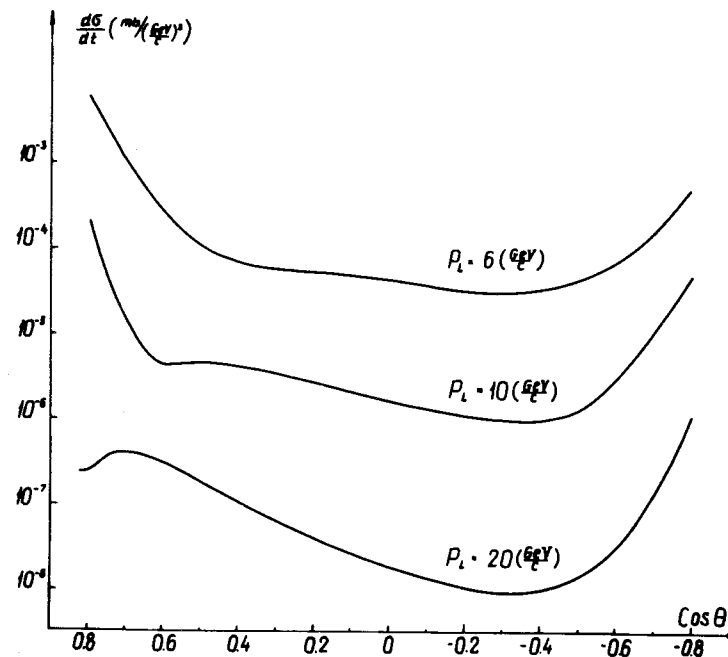


Fig. 8. Predictions for the differential cross section of $\pi^-p \rightarrow \pi^0n$ reaction.

quark counting rules ($N = 10$ in (1.1)) are negligible.

3. The consideration of corrections results in nonzero polarization despite the γ_5 -invariance of interaction.

The authors express their deep gratitude to V.A. Matveev and A.N. Tavkhelidze for interest in the work and useful remarks. We thank also V.K. Mitrjushkin, R.M. Muradyan, L.A. Slepchenko and M.A. Smondryev for fruitful discussions.

REFERENCES

1. Matveev V.A., Muradyan R.M., Tavkhelidze A.N. Lett.Nuovo Cim., 1973, 7, p.719; JINR, E2-8048, Dubna, 1974.
2. Gunion J.F., Brodsky S.J., Blankenbecler R. Phys.Rev., 1973, D8,p.287.
3. Barger V. Rapporteur's Talk at the XVII Int. Conf. on High Energy Phys., London, 1974.
4. Goloskokov S.V. et al. Theoret. i Matemat. Fiz., 1975, 24, p.24.
5. Goloskokov S.V. et al. JINR, P2-8337, Dubna, 1974.
6. Goloskokov S.V. et al. Yad.Fiz., 1976, 24, p.448.
7. Goloskokov S.V. et al. Proc. of the XVIII Int. Conf. on High Energy Physics, Tbilisi, 1976. JINR, D1,2-10400, Dubna, 1977, p.A5-19.
8. Goloskokov S.V., Koudinov A.V., Kuleshov S.P. JINR, E2-11539, Dubna, 1978.
9. Logunov A.A., Meshcheryakov V.A., Tavkhelidze A.N. Doklady Akademii Nauk SSSR, 1962, 142, p.317.
10. Garsevanishvili V.R. et al. Teoret. i Matemat. Fiz., 1972, 12, p.384.
11. Khelashvili A.A. JINR, P2-4327, Dubna, 1969.
12. Dzhgarkava M.I. et al. Nucl.Phys., 1973, B67, p.232; Garsevanishvili V.R. et al. JINR, P2-9947, Dubna, 1976.
13. Garsevanishvili V.R., Matveev V.A., Slepchenko L.A. Particle and Nucleus, I, No. 1, p.91.
14. Bracci E. et al. Compilation of Differential Cross-Sections π -Induced Reactions. CERN preprint, CERN-HERA, 75-2, 1975.
15. Baglin C. et al. Nucl.Phys., 1975, B98, p.365.
16. Benary O. et al. NN and ND Interactions. A Compilation-Berkeley preprint, UCRL-20000,1970.
17. Goloskokov S.V. et al. JINR, P2-10142, Dubna, 1976.
18. Jenkins K.A. et al. Phys.Rev.Lett., 1978, 40, p.425.

Received by Publishing Department
on June 2 1978.