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SUPERGROUP OSp(1,4) AND CLASSICAL SOLUTIONS OF THE WESS-ZUMINO MODEL



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Супергруппа 0Sp(1.4) и классические решения модели Весса-Зумино

Изучены трансформационные свойства классических решений безмассовой модели Весса-Зумино относительно суперконформной группы. Показано, что эти свойства определяются двумя подгруппами 0Sp(1,4) суперконформной группы, пересекающимися по подгруппе 0(2,3). Отмечается возможная связь полученных результатов с возникновением 0Sp(1,4) -структуры в спонтанно нарушенной супергравитации.

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Supergroup OSp(1.4) and Classical Solutions of the Wess-Zumino Model

We study the superconformal transformation properties of recently constructed 0(2,3) -invariant classical solutions of the massless Wess-Zumino model. These properties are shown to be completely determined by two graded subgroups OSp(1,4) of the superconformal group with 0(2,3) as the common even subgroup. One of these OSp(1,4)'s is the maximal stability group of the solutions. The other is spontaneously broken down to 0(2,3). Its odd transformations generate the correct. Grassmann parameter dependence of solutions. Our results admit the straightforward extension to theories with the Euclidean supersymmetry.

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1. In recent papers⁽¹⁻³⁾, supersymmetry was successfully used for construction of non-trivial classical solutions to theories where both the fermionic and bosonic sectors are present. For the 4-dimensional case, the simplest possibility along this line was considered by Baaklini⁽¹⁾ who constructed a family of the 0(2,3)-invariant solutions to the massless Wess-Zumino model⁽⁴⁾. This family is degenerated with respect to supertranslations and, as a consequence, exhibits an explicit dependence on the Grassmann parameters (along with the standard dependence on bosonic parameters of translations and dilatations). Its fermionic component is treated as a classical solution for the fermion field in the background field of the bosonic solution.

Bearing in mind that the whole group of invariance of the massless Wess-Zumino model is the superconformal group^{/5,6/} it is natural to ask how many independent parameters are needed in order to specify completely the solutions of the type treated in ref.^{/1/}. The answer can be obtained by examining the transformation properties of the solutions under the superconformal group. The full number of parameters coincides with the number of those generators of the superconformal group which are not in the maximal stationary subgroup of solutions (the stability subgroup). Such an analysis is carried out in the present paper.

We show that superconformal properties of the 0(2,3)invariant solutions to the Wess-Zumino model are completely characterized by two graded orthosimplectic subgroups 0Sp(1,4) of the superconformal group. They contain 0(2,3) as the common even subgroup and have as

a closure the superconformal group itself. One of these OSp(1,4)'s is just the stability subgroup of solutions in question. The other is spontaneously broken by them down to 0(2,3). Its odd transformations generate the correct Grassmann parameter dependence of solutions. The full number of Grassmann parameters is equal to four just as in ref.¹¹, but dependence on them is essentially different. Bosonic degeneracies are connected with translations, dilatations, and chiral $(\gamma_5 -)$ transformations. Our analysis can be readily extended to the more interesting (and more complicated) case of the massless theories with the Euclidean supersymmetry where the instantontype solutions exist $^{/2/}$ We discuss also a possible relation of the OSp(1,4)-structure of the massless Wess-Zumino model to the analogous structure emerging in the spontaneously broken supergravity 7,8/

2. The Wess-Zumino model^{/4/} is the simplest supersymmetric theory. It describes two interacting Hermitean conjugate chiral superfields $\Phi_{\pm}(\mathbf{x},\theta_{\pm})=A_{\pm}(\mathbf{x})+\overline{\theta}_{\pm}\Psi_{\pm}(\mathbf{x})+\frac{1}{2}\overline{\theta}_{\pm}\theta_{\pm}F_{\pm}(\mathbf{x})$ which are equivalent to the set of four Hermitean boson fields A, B, F, G and Majorana spinor Ψ_{\pm}^{**}

$$A_{\pm} = \frac{1}{\sqrt{2}} (A \pm i B), \quad F_{\pm} = \frac{1}{\sqrt{2}} (F \pm i G), \quad \Psi_{\pm} = \frac{1}{2} (1 \pm i \gamma_5) \Psi.$$

The massless version of the model is completely determined by the action:

* We are using the Majorana formalism: $\theta_{\pm} = \frac{1}{2} (1 \pm i\gamma_5) \theta$. Conventions on metric and γ -matrices are the same as in ref. /9/:

$$\begin{split} \eta_{\mu\nu} &= (1\,,-1\,,-1\,,-1\,)\,, \ \{\gamma_{\mu}\,,\gamma_{\nu}\,\} {=} 2\,\eta_{\mu\nu} \ ,\,\gamma_{5}^{2} = -1\,\,, \\ \sigma_{\mu\nu} &= \frac{\mathrm{i}}{2} [\gamma_{\mu}\,,\gamma_{\nu}\,]\,. \end{split}$$

$$S = \int d^{4}x \left[\frac{1}{2} (\partial^{\mu} A \partial_{\mu} A + \partial^{\mu} B \partial_{\mu} B + i \bar{\Psi}_{\gamma}{}^{\mu} \partial_{\mu} \Psi) - \frac{g^{2}}{2} (A^{2} + B^{2})^{2} - g \bar{\Psi} (A - B \gamma_{5}) \Psi \right]$$
(1)

supplemented with the equations for the auxiliary fields F,G:

$$F = -g(A^2 - B^2)$$

 $G = 2 g A B$. (2)

The action (1) in combination with eqs. (2) is invariant under the superconformal graded group. It contains as a subgroup the ordinary conformal group with generators $L_{\mu\nu}$, P_{ρ} , K_{μ} , D and, besides, involves the transformations⁵/₅.

$$\delta A_{\pm} = \overline{\beta} \Psi_{\pm} \pm i\lambda A_{\pm}$$

$$\delta \Psi_{\pm} = \frac{1 \pm i\gamma_{5}}{2} \left[\left(-i\gamma^{\mu} \partial_{\mu} A_{\pm} + F_{\pm} \right) \beta - \frac{i}{2} A_{\pm} \gamma^{\mu} \partial_{\mu} \beta \right] \mp \frac{i}{2} \lambda \Psi_{\pm}$$

$$\delta F_{\pm} = -i \overline{\beta} \gamma^{\mu} \partial_{\mu} \Psi_{\pm} \mp 2 i\lambda F_{\pm} , \qquad (3)$$

where

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$$\beta = a_1 - i x^{\mu} \gamma_{\mu} a_2, \qquad (4)$$

and a_1 , a_2 are constant Grassman spinor parameters associated, respectively, with the supertranslation generator S_a and the generator T_a of special superconformal transformations. Bosonic parameter λ is connected with the chiral transformation generator Π_5 . In what follows, we will need the following (anti) commutation relations from the superconformal algebra^{6/*}:

^{*} Our generators D, K_µ differ from those of ref.^{/10/} by factor -1. The generator Π_5 is related to the generator Π used in ref.^{/6/} as $\Pi_5 = -2 \Pi$.

$$\{S, \overline{S}\} = \gamma^{\mu} P_{\mu} , \{T, \overline{T}\} = -\gamma^{\mu} K_{\mu}$$
(a)

$$\{S, \overline{T}\} = i(D - \gamma_{5} \Pi_{5}) + \frac{1}{2} \sigma^{\mu\nu} L_{\mu\nu}$$
(b)

$$[S, (P_{\mu})] = -(\stackrel{0}{\gamma_{\mu}T}), [T, (P_{\mu})] = (\gamma_{\mu} \stackrel{S}{S})$$
(c)

$$[\Pi_{5}, (\stackrel{S}{T})] = \frac{3}{2} i\gamma_{5} (\stackrel{-S}{T}), [D, (\stackrel{S}{T})] = \frac{i}{2} - (\stackrel{S}{-T}).$$
(d)

The massless Wess-Zumino model is the supersymmetric generalization of the massless ϕ^4 -theory*. Therefore any classical solution of the latter* is simultaneously a particular solution to the equations of motion corresponding to the action (1) (the reverse is not true of course). We are interested here in the solutions $^{/1/}$:

$$A_{0}(x) = \frac{m}{g} a(x) = \frac{m}{g} \frac{2}{1 + m^{2}x^{2}}, \quad G_{0} = B_{0} = \Psi_{0} = 0$$
(6)
$$F_{0}(x) = -\frac{m^{2}}{g} a^{2}(x) \quad [m] = L^{-1}, \quad m \neq 0$$

which generalize solutions found by Fubini $^{/10/}$ in the ϕ^4 -theory. They are invariant under the anti de Sitter subgroup 0(2,3) of the conformal group, with the generators $L_{\mu\nu}$, $R_{\mu} = \frac{1}{2}(P_{\mu} - m^2 K_{\mu})$. At the same time, translations, dilatations, and chiral transformations take the system (6) into the six parameter continuous family:

$$A_{\pm}^{(\lambda,\rho,h)} = \frac{1}{\sqrt{2}} \frac{m}{g} e^{\pm i\lambda} \rho a(\rho x + h)$$

$$F_{\pm}^{(\lambda,\rho,h)} = -\frac{1}{\sqrt{2}} \frac{m^2}{g} e^{\pm 2i\lambda} \rho^2 a^2(\rho x + h)$$
(7)

(solutions (6) correspond to a particular choice of parameters $\rho = 1$, $h_{\mu} = 0$, $\lambda = 0$ $(2\pi n)$ therein). Note that the scale degree of freedom (associated with ρ) merely reflects an arbitrariness in parameter m. The chiral transformation with $\lambda = \pi$ is equivalent to the change $m \rightarrow -m$ in formulas (6).

The main observation by Baaklini^{/1/} is that solutions (6) (or, equally (7)) are not invariant with respect to supertranslations and therefore, acting on (6) by a finite supertransformation with parameter a_1 , one may obtain non-zero solutions for the spinor $\Psi(x)$ which describe a motion of the fermion in the bosonic solution field. The arising set is represented by the superfield

$$\Phi_{\pm}^{a_{1}}(\mathbf{x},\theta_{\pm}) = e^{i\overline{a_{1}}S} \Phi_{\pm}^{0}(\mathbf{x},\theta_{\pm}) = \Phi_{\pm}^{0}(\mathbf{x}+i\overline{a}_{1}\gamma\theta_{\pm}\mp$$
$$\mp \frac{1}{4}\overline{a}_{1}\gamma\gamma_{5}a_{1},\theta_{\pm}+a_{1\pm}), \qquad (8)$$

where $\Phi_{\pm}^{0}(x, \theta_{\pm})$ is the superfield having (6) as components.

We would like to emphasize that the Grassmann parameter dependence given by the formula (8) is not most general. Indeed, one readily verifies that the system (6) is not invariant also under the special superconformal transformations (with parameter a_2). Hence, performing in (6) such a transformation it is possible to set up one more non-trivial family of solutions including, like (8), the fermionic components as well as bosonic ones. Thus we are facing with the problem of how to extract the complete set of independent solutions to the massless Wess-Zumino model. This problem obviously reduces to determining what is the full stability subgroup of the system (6). As the maximal even subgroup leaving (6) invariant is known and it is just 0(2,3), it remains only to examine which combinations of odd generators S_a , T_a annihilate the system (6) (if exist).

We show that such combinations may be really found. Let us pass, in relations (3), (4), to the new set of Grassmann parameters β_{I} , β_{II} :

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^{*} For the standard sign of the coupling constant, i.e., for $\mathcal{L}_{int} = -\lambda \phi^4$, $\lambda > 0$.

$$\beta = \frac{1}{\sqrt{2}} (1 + \ln x^{\mu} \gamma_{\mu}) \beta_{\rm I} + \frac{1}{\sqrt{2}} (1 - \ln x^{\mu} \gamma_{\mu}) \beta_{\rm II}$$
(9)

or, equivalently, to the new spinorial basis in the superconformal algebra:

$$Q_{I} = \frac{1}{\sqrt{2}} (S - mT), \quad Q_{II} = \frac{1}{\sqrt{2}} (S + mT)$$

$$(\bar{a}_{1}S + \bar{a}_{2}T = \bar{\beta}_{I}Q_{I} + \bar{\beta}_{II}Q_{II}).$$
(10)

It is not hard to see that solutions (6) are invariant with respect to supertransformations with parameter $\beta_{\rm I}$. At the same time supertransformations depending on $\beta_{\rm II}$ displace (6) $(\delta \beta_{\rm II} \Psi_0 \neq 0)$. Thus the generator $Q_{\rm I}$ should be included into the stability subgroup whereas $Q_{\rm II}$ serves to introduce Grassmann degrees of freedom. Clearly the full number of independent Grassmann parameters is equal to four.

To clarify the meaning of these results we prove the following *Theorem*.

Each of generators Q_{I}, Q_{II} enlarges the algebra of the group O(2,3) to that of the supergroup OSp(1,4). The closure of thus obtained supergroups OSp(1,4) is the superconformal group itself.

Using the anticommutation relations (5a,b) and commutation ones (5c) we find:

$$\{\mathbf{Q}_{\mathrm{II}}, \overline{\mathbf{Q}}_{\mathrm{II}}\} = \gamma^{\mu} \mathbf{R}_{\mu} + \frac{1}{2} \mathbf{m} \sigma^{\mu\nu} \mathbf{L}_{\mu\nu}$$

$$[\mathbf{Q}_{\mathrm{II}}, \mathbf{R}_{\mu}] = -\frac{1}{2} \mathbf{m} \mathbf{Q}_{\mathrm{II}}.$$
(11)

Relations (11) coincide with those of the OSp(1,4)-superalgebra given, for instance, in ref.^{11/} (when comparing with^{11/} it should be remembered that we use different representation for γ -matrices). As $m \rightarrow 0$, the algebra (11) goes into the standard supersymmetry algebra (this can be observed of course directly from the definition (10)). The algebra of generators Q_{I} , R_{μ} , $L_{\mu\nu}$ is also closed and isomorphic to (11). Indeed, we may cast it into the form (11) by passing to the generator $Q_{I}' = \gamma_{5} Q_{I}$. All the remaining generators of the superconformal group are contained in the cross anticommutator:

$$\{\mathbf{Q}_{\mathrm{II}}, \overline{\mathbf{Q}}_{\mathrm{I}}\} = \frac{1}{2} \gamma^{\mu} (\mathbf{P}_{\mu} + \mathrm{m}^{2} \mathbf{K}_{\mu}) - \mathrm{im}(\mathbf{D} - \gamma_{5} \mathbf{\Pi}_{5})$$

that completes the proof of the Theorem.

Thus we arrive at the conclusion that the crucial role in specifying superconformal properties of the 0(2,3)invariant classical solutions to the model under consideration is played by two subgroups $_{\rm 0Sp}(1,4)$ of the superconformal group. One of them , $_{\rm 0Sp}(1,4)$ with generators $Q_{\rm I}$, R_{μ} , $_{\rm L\mu\nu}$ is the stability subgroup of solutions. The other, $_{\rm 0Sp}(1,4)$ generated by $Q_{\rm II}$, R_{μ} , $_{\rm L\mu\nu}$ is broken on these solutions down to $0(2,3) \in (R_{\mu}, L_{\mu\nu})$. Its finite odd transformations project the system (6) onto the quotient (super) space $0{\rm Sp}^{\rm II}(1,4)/0(2,3)$ and fix thereby the Grassmann parameter structure of solutions:

$$\Phi_{\pm}^{\beta_{\mathrm{II}}}(\mathbf{x},\theta_{\pm}) = e^{i\beta_{\mathrm{II}} Q_{\mathrm{II}}} \Phi_{\pm}^{0}(\mathbf{x},\theta_{\pm}).$$
(12)

Acting on the superfield (12) by group elements with generators D, P_{μ} , Π_5 one may include bosonic degrees of freedom and so construct the complete set of solutions.

Write down the components of the superfield $\Phi_{\pm}^{\beta_{I}}(x,\theta_{\pm})$ explicitly:

$$A_{\pm}^{\beta_{\text{II}}}(x) = \frac{m}{\sqrt{2} g} a(x) [1 - ma(x)\bar{\beta} \frac{1 \pm i\gamma_5}{2} \beta + \frac{m^2}{72} a^2(x)(\bar{\beta}\beta)^2]$$

$$\Psi_{\pm}^{\beta_{\text{II}}}(x) = -\frac{\sqrt{2} m}{g} a^2(x) [1 - \frac{m}{3} a(x)\bar{\beta}\beta] \frac{1 \pm i\gamma_5}{2} \beta$$

$$F_{\pm}^{\beta_{\text{II}}}(x) = -\frac{m^2}{\sqrt{2} g} a^2(x) [1 - 2ma(x)\bar{\beta} \frac{1 \mp i\gamma_5}{2} \beta + \frac{m^2}{36} a^2(x)(\bar{\beta}\beta)^2]. \qquad (13)$$

Here now $\beta = \frac{1}{12} [1 - im x^{\mu} \gamma_{\mu}]^{\beta} \Pi$ and the function a(x) is defined by eqs $\sqrt{2}(6)$. The dependence of the solutions (13) on the fermionic degrees of freedom essentially differs from that found in ref.¹¹ and does not reduce to it by any change of parameter β_{II} .Note, however, that, acting on (12) successively by chiral and scale transformations with nilpotent parameters in a certain manner composed from the spinor eta_{II} , we may arrive at the superfield (8) (in which α_1 is replaced by β_{II}). To be convinced of this one may proceed as follows: to represent generator Q_{II} as $Q_{II} = \sqrt{2}S - Q_{I}$ then split the exponent $e^{i\vec{\beta}_{II}}Q_{II}$ in eq. (13) by the Baker-Haussdorf formula with making use of relations (5) and finally take into account the condition $Q_I \Phi^0_+(x, \theta_{\pm}) = 0$. The analogous connection can be established between $\Phi^{\beta_{II}}_{\pm}(x, \theta_{\pm})$ and the set of solutions generated from $\Phi^0_{\pm}(\mathbf{x}, \overline{\theta}_+)$ by finite special superconformal transformations (the relevant substitution is $Q_{TI} = \sqrt{2} mT + Q_T$). These arguments indicate that it is possible, in principle, to choose as the basis set the family (8) as well. However, from the group theory point of view it is most natural to do in terms of the set (12) because just this set corresponds to the choice of the basis in the superconformal algebra which is orthonormal, in the sense of the Cartan inner product, with respect to the algebra of the stability subgroup $0S_p^{1}(1,4)$.

It is worth noting that the structure of the subgroup OSp(1,4) leaving solutions invariant and, respectively, the structure of the other subgroup OSp(1,4) generating their Grassmann parameter dependence are fixed up to rotations in the group space of the superconformal group. In other words, the stability subgroup of solutions rotated from (6) by some finite superconformal transformation is rotated with respect to OSp(1,4) by the same "angle"*

* The analogous situation takes place in standard theories of spontaneously broken internal symmetries where to the continuous orbit of vacua continuum of stability subgroups corresponds. To illustrate this point we consider a solution shifted from (6) by the chiral transformation with parameter λ :

$$\begin{split} \Phi_{\pm}^{\lambda}(\mathbf{x}, \theta_{\pm}) &= \mathrm{e}^{\mathrm{i}\Pi_{5}\lambda} \Phi_{\pm}^{0}(\mathbf{x}, \theta_{\pm}) \\ \mathrm{A}_{\pm}^{\lambda}(\mathbf{x}) &= \frac{1}{\sqrt{2}} \mathrm{e}^{\pm \mathrm{i}\lambda} \mathrm{A}_{0}(\mathbf{x}) \\ \mathrm{F}_{\pm}^{\lambda}(\mathbf{x}) &= \frac{1}{\sqrt{2}} \mathrm{e}^{\pm 2\mathrm{i}\lambda} \mathrm{F}_{0}(\mathbf{x}), \quad \Psi_{\pm}^{\lambda} = 0. \end{split}$$

Its stability subgroup is $0S_{\Gamma}^{I(\lambda)}(1,4) \in (Q_{I}^{\lambda}, R_{\mu}, L_{\mu\nu}),$ where $Q_{I}^{\lambda} = e^{i\prod_{5}\lambda} Q_{I} e^{-i\prod_{5}\lambda} = \cos(\frac{3}{2}\lambda)Q_{I} + \sin(\frac{3}{2}\lambda)\gamma_{5}Q_{II}$ (14)

(chiral transformations do not affect the subgroup 0(2,3) because Π_5 commutes with all bosonic generators).Likewise:

$$Q_{II}^{\lambda} = e^{i \prod_{5} \lambda} Q_{II} e^{-i \prod_{5} \lambda} = \cos\left(\frac{3}{2}\lambda\right) Q_{II} + \sin\left(\frac{3}{2}\lambda\right) \gamma_{5} Q_{I}.$$
 (15)

Fixing parameter λ specifies the stability subgroup of the solution characterized by given λ . For instance the stability subgroup of the solution corresponding to $\lambda = \pi$ (it is given by formulas (6) up to the change $m \rightarrow -m$) is $OSp^{II}(1,4)$ while OSD(1,4) turns out now to be broken. An interesting feature of the model under consideration having no analogue in the massless ϕ^4 -theory is that the same OSp(1,4) serves to be the stability subgroup simultaneously for several solutions. More specifically, leaves invariant not only solutions given by 0Sb(1.4)formulas (6) ($\lambda = 0$) but also those with $\lambda = \frac{2\pi}{n} n$ (n=1,2). Thus, even after the stability subgroup of a given solution is fixed, there remains some discrete degeneracy associated with chiral invariance. In the quantum case, we may in principle expect a tunneling between sectors characterozed by values of n=0,1,2(if the interpretation of classical solutions as anomalous vacuum averages of corresponding fields $^{10/}$ is acceptable). This could result

in breaking P- and CP-symmetries due to non-zero ground values of pseudoscalar fields B and G in sectors with n=1,2. This point requires of course more detailed treatment. Note that the uncertainty we have mentioned here is specific only for chiral rotations. Solutions generated from $\Phi_{\pm}^{0'}(x, \theta_{\pm})$ by other transformations of the superconformal group are in the one-to-one correspondence with their stability subgroups.

3. We have shown that the structure of the massless Wess-Zumino model after allowing for its 0(2,3)-invariant classical solutions is described most adequately in terms of two supergroups OSp(1,4), one of which being spontaneously broken. On the other hand, Deser and Zumino have pointed out recently $\frac{7,11}{1}$ the particular role of the spontaneously broken OSp(1,4)-supersymmetry in supergravity as providing most suitable framework to describe the spontaneous breakdown of the local supersymmetry. It is tempting to assume that this analogy is not accidental and OSp(1,4)-structures in both theories have a common origin. In other words, local supersymmetry may happen to be broken mainly due to O(2,3)-invariant classical solutions to the supergravity-matter equations. For instance, the system where the supergravity fields couple to massless scalar supermultiplet certainly admits solutions of the type (6). As a preliminary step along this line, it would be interesting to analyse in more detail the structure of the spontaneous breakdown of the OSp(1,4)supersymmetry in the Wess-Zumino model and, particularly, to compare it with the non-linear realization of OSp(1.4) considered recently by Zumino^{/11}/ Such a study is carried out in our forthcoming paper. There we rewrite the action (1) in terms of anti de Sitter space, i.e., pass to manifestly 0(2,3)-invariant notation. In the 0(2,3)formalism, solutions (6) reduce to constants minimizing the related "potential" just as it occurs in the massless ϕ^4 -theory /10/. Thereby the massless Wess-Zumino model gets interpretation as the simplest linear σ -model of spontaneously broken 0Sp(1,4)-supersymmetry (and

simultaneously of the superconformal one). The role of the Goldstone fermion accompanying this breakdown turns out to be played by the spinor $\Psi(x)$. After extracting ground values from boson fields, $\Psi(x)$ acquires a "mass" equal to the inverse radius of anti de Sitter space m which is in agreement with the general result by Zumino^{/11/}

Throughout this paper we were concerned with solutions in Minkowski space. To conclude, we make some comments on how it is possible to continue our study to Euclidean space. A direct "euclidezation" of solutions of the type (8), (13) is not possible as after transition to Euclidean space the action (1) loses its supersymmetry and, consequently, any supersymmetric degeneracy of relevant solutions disappears. The matter here is that Euclidean space possesses no Majorana spinors and therefore there exists no direct Euclidean analogue of the Minkowski supersymmetry ^{/2/} The simplest supergroup in Euclidean space includes Dirac bispinor complex generators 2 and as a result leads to larger supermultiplets in comparison with the conventional case. In particular, the model which is the supersymmetric extension of the Euclidean massless ϕ^4 -theory involves a greater number of independent fields than the Wess-Zumino model. Clearly, to extract a whole set of its classical solutions, a special analysis is needed. It will reduce, of course, as in the case we have considered, to studying the behaviour of some particular solution under relevant superconformal group. Euclidean analogues of solutions (6) respect the group 0(5), therefore it is plausible that their full invariance group is the Euclidean analogue of

the supergroup OSp(1,4), i.e., the minimal enlargement of O(5) by Dirac generators. The Grassmann parameter dependence of solutions will be then given by action of the remaining spinor generator of the Euclidean superconformal group (these parameters comprise complex bispinor, i.e., their number amounts eight).

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