ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2 - 11579

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Submitted to XIX International Conference on High Energy Physics Tokyo, 1978.

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Взаимодействие неодномерных классических Q-солитонов

С помощью ЭВМ исследовано взаимодействие двумерных "заряженных" (x,y,t) Q- солитонов в рамках релятивистски-инвариантного уравнения Клейна-Гордона с насыщением нелинейности.

В результате исследования лобовых столкновений солитоноподобных решений найдены области значений параметров, в которых имеет место квазиупругое взаимодействие солитонов, а также обнаружено наличие связанных состояний двух солитонов. Исследована область устойчивости солитоноподобных решений.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

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Interaction of Non-One-Dimensional Classical Q Solitons

The stability region and the head-on collisions of two-space dimensional "charged" (x,y,t) solitons have been investigated via computer in the framework of the Lorentz-invariant Klein-Gordon equation with the saturable nonlinearity.

The region of parameters has been found where: 1) quasi-elastic soliton interaction occurs and 11) the formation of two soliton bound states takes place. The collision of solitons may lead to their decay.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1978

During the last few years the great interest of elementary particle theorists has been attracted to the models having soliton solutions. It is sufficient to mention some International Conferences [1] and Reviews [2] considering this problem.

It is not surprising, since the solitons, essentially nonlinear extended solutions, have interesting and very peculiar properties, namely, conserved charges which are not connected to Lagrangian symmetry (topological charges), the possibility of constructing relatively light objects from very heavy constituents (the great mass defect, quark confinement) and so on.

We do not discuss in detail the soliton properties considered at greater length in reviews [2]. Notice only that at present time the static soliton properties have been investigated in full for the application in elementary particle theory. We mean the solitons in four-dimensional Minkowski space. Note, that stable nontopological soliton solutions exist only in theories with internal symmetry. It is natural, that the studies have been performed starting with the simplest models, e.g., Q solitons in φ^4 theory ($U(1)$ symmetry). Then higher symmetries have been taken into account (part of which can be spontaneously broken) , i.e. $SU(2) \otimes SU(2)$, $SU(3) \otimes SU(3)$, and $SU(3) \otimes SU(3) \otimes SU(3)$ models.

It is interesting that for a sufficiently complicated model at a proper choice of parameters the authors [3] have succeeded to describe with a good accuracy such properties of nucleons as the ratio of magnetic moments of the proton and the neutron, the ratio of axial and vector constants of the β-decay and the root-mean-square radius of the nucleon.

If the static soliton properties can be successfully investigated by analytical methods even for complicated models, the interaction of non-one-dimensional solitons can be studied in the meanwhile only by a computer. In this case the way from simple models to more complicated ones is natural.

Below we present the results on the interaction dynamics of two two-dimensional Q solitons (U(1) symmetry) in the framework of the Klein-Gordon equation with saturable nonlinearity

$$(\partial_t^2 - \partial_x^2 - \partial_y^2)\varphi + \varphi - \varphi\frac{|\varphi|^2}{1+|\varphi|^2} = 0. \tag{1}$$

We realize that this model is somewhat rough and moreover it is nonrenormalizable x), nevertheless, the qualitative characteristics of the interaction under investigation are undoubtedly interesting from the point of view of a further understanding of more realistic and complicated models.

We find a soliton-like solution of the boundary value problem

$$(\partial_t^2 - \partial_x^2 - \partial_y^2)\varphi + \varphi - \varphi\frac{|\varphi|^2}{1+|\varphi|^2} = 0, \tag{2}$$

$$\frac{\partial\varphi}{\partial x}\Big|_{x=0} = 0, \quad \frac{\partial\varphi}{\partial y}\Big|_{y=0} = 0, \quad \varphi(\infty,\infty,t) = 0,$$

where

$$\varphi(x,y,t) = \psi(x,y)\, e^{i\mu t}. \tag{3}$$

It is known, that a stable solution of problem (2) exists in the region

$$\frac{\mu}{Q}\frac{dQ}{d\mu} < 0, \tag{4}$$

where the "charge" Q is

$$Q = \mu\int\psi^2(x,y,t)\,dx\,dy. \tag{5}$$

To find the existence region of stable solutions to problem (2) we study the dependence $Q(\mu)$. We observe, that for $\mu < 0.2$ the function $Q(\mu)$ has an exponential rising, for greater μ this function decreases and at $\mu \to 1$ it goes to a plateau, that indicates a broad domain of the existence of problem (2) stable solutions.

We elaborate an algorithm of the Lorentz transformation for a soliton-like solution to problem (2) via a computer, which permits to investigate head-on collisions of two solitons.

x) This fact will lead to complicacies in quantization of such objects. However, since we work in the framework of classical field theory, this is not an obstacle.

The soliton velocities are $v_1 = -v_2 = v$, where v varies from 0.2 to 0.9 with a step 0.1. The parameter μ changes from 0.2 to 0.99.

We find, that at small velocities ($v \leq 0.3$) of quasi-solitons (called below as solitons, for simplicity), system (2) is far from an integrable one. In this case a formation of bound bion-type states [4] of two solitons (Figs. 1,2) *) and a decay of solitons by a dissipative instability mode [5] (Figs. 3,4) are possible. For greater soliton velocities, i.e., in the "ultrarelativistic" limit ($v \geq 0.7$), system (2) is near to an integrable one, and soliton interaction becomes practically elastic (Figs. 5,6).

Our numerical calculations show that soliton (2) interaction picture changes essentially with variation of μ. At $\mu = 0.2$ bound states of two solitons arise in a wide region of v's ($0.2 \leq v \leq 0.7$). But in this case the lifetime of the bound state decreases with v. For greater μ's ($0.2 < \mu \leq 0.71$) the region of soliton elastic interaction expands, and the elastic interaction takes place at $v \geq 0.2$. For further increase of μ ($\mu \geq 0.95$), the bound states exist only for $v \leq 0.2$. But at v=0.3 after a collision a dissipation of moving solitons occurs. It is not surprising, since in this region of μ's eq.(1) is close to the Klein-Gordon equation with cubic nonlinearity (KG3). The result on this dissipation coincides with our previous calculations on KG3 [6]. At $v \geq 0.5$ the soliton interaction becomes elastic, moreover for all μ's the inelasticity (the amplitude of perturbations of energy density in the c.m. system of two solitons after collision) decreases with v.

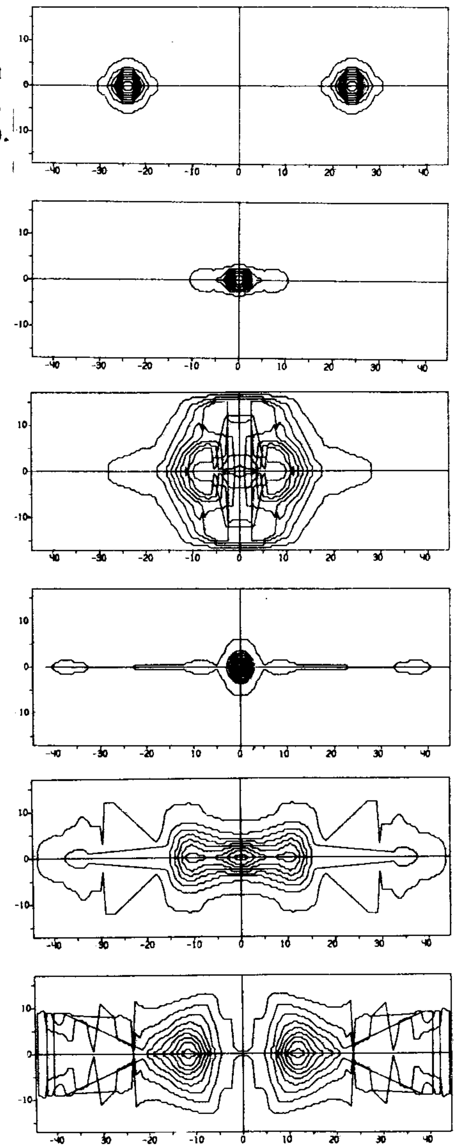
Notice also, that a collapse in the framework of system (2) at $\mu \sim 0.2$ does not go to the end, although a trend to contraction of the soliton arising after collision is observed. That is clear, since at small μ's the value $|\varphi(x,y,0)| \gg 1$, therefore eq.(1) reduces to the linear wave equation, which solutions tend to the dissipation.

Finally, note that in the region of v and μ, where stable Q solitons interact elastically, there is some shift in their position after the interaction with respect to the initial one, at t=0 (see Fig. 5).

*) Energy density $\mathcal{H}$ is

$$\mathcal{H} = |\varphi_t|^2 + |\varphi_x|^2 + |\varphi_y|^2 + \ln(1+|\varphi|^2).$$

Fig. 1. Bound state of two Q solitons at $v=0.2$, $\mu=0.95$. Figures with isohypses correspond to plane cross sections of Fig. 2. $H=\mathcal{H}/\mathcal{H}_{max}$, $\mathcal{H}_{max}=2,28=\mathcal{H}(t=120)$.



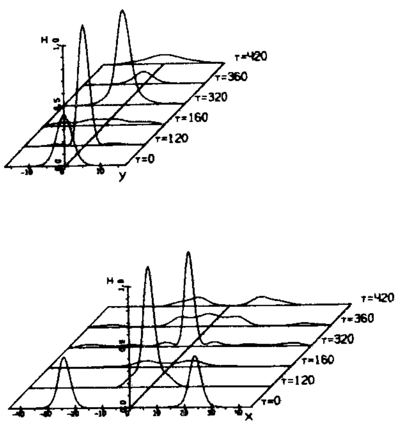


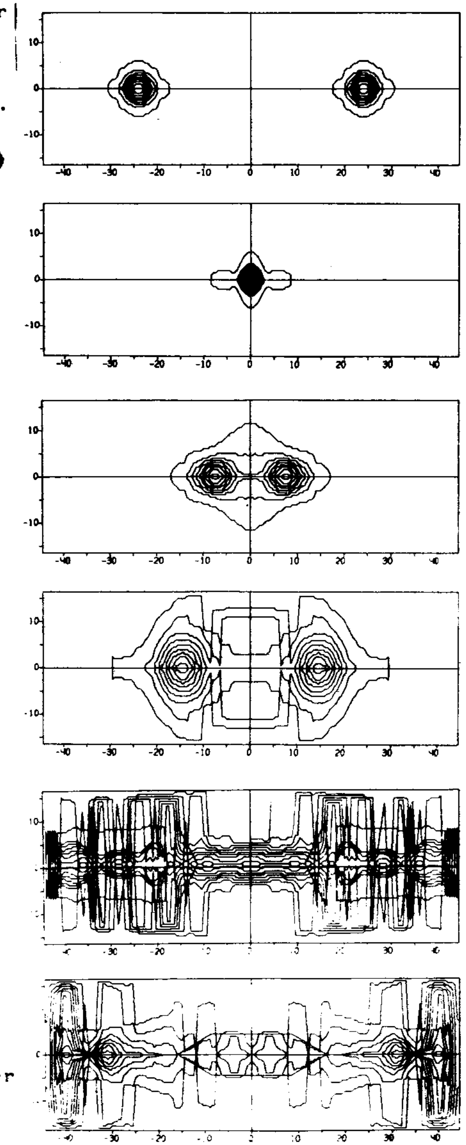
Fig. 2

Bound state of two Q solitons at $v=0.2$, $\mu=0.95$:

a) Cross section in y-axis at the point x, where $H(x,y=0)$ is maxiamal.

b) Cross section in x-axis.

Fig. 3. Decay of Q solitons after interaction at $v=0.3$, $\mu=0.95$. Figures with isohypses correspond to plane cross sections of Fig. 4. $H=\mathcal{H}/\mathcal{H}_{max}$, $\mathcal{H}_{max}=2,75=\mathcal{H}$ (t=80).



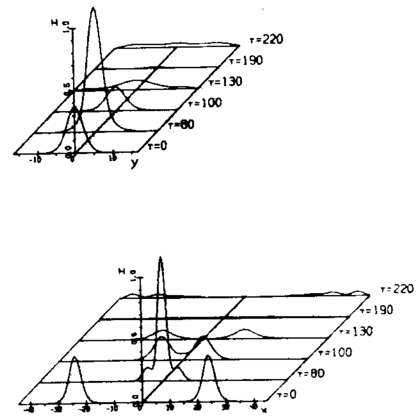
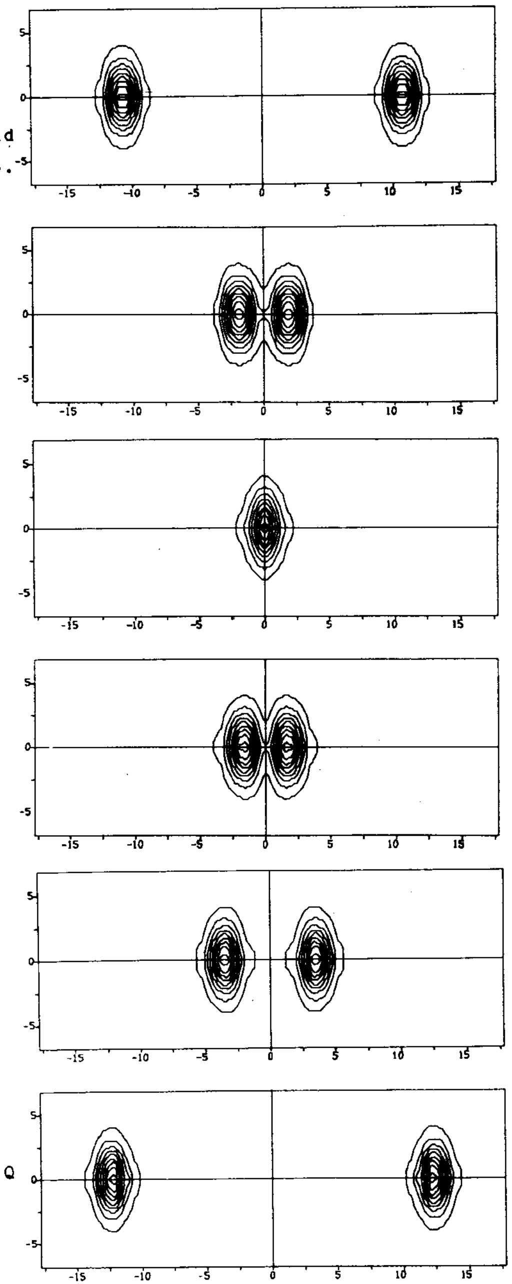


Fig. 4. Decay of Q solitons after interaction at $v=0.3$, $\mu=0.95$.

Fig. 5. Elastic interaction of Q solitons at v = 0.9, μ = 0.7. Figures with isohypses correspond to plane cross sections of Fig.6. H=$\mathcal{H}/\mathcal{H}_{max}$, $\mathcal{H}_{max}$=69,4=$\mathcal{H}$(t=12).



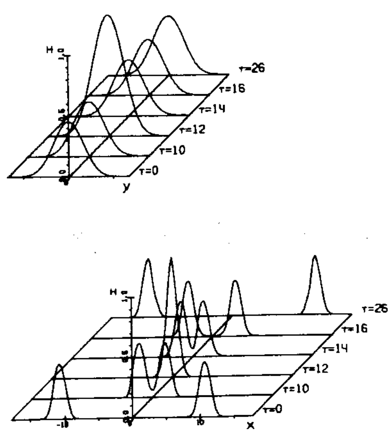
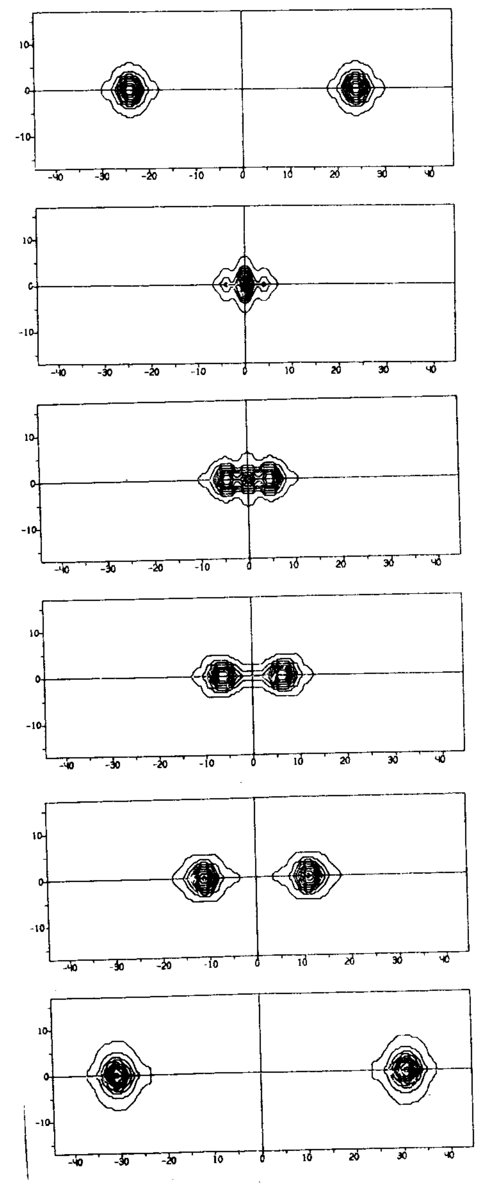


Fig. 6. Elastic interaction of Q solitons at v = 0.9, μ =0.7.

Fig. 7. Elastic interaction of Q solitons at v=0.5, μ=0.95. Figures with isohypses correspond to plane cross sections of Fig.6. H=$\mathcal{H}/\mathcal{H}_{max}$, $\mathcal{H}_{max}$=2,97=$\mathcal{H}$(t=50).



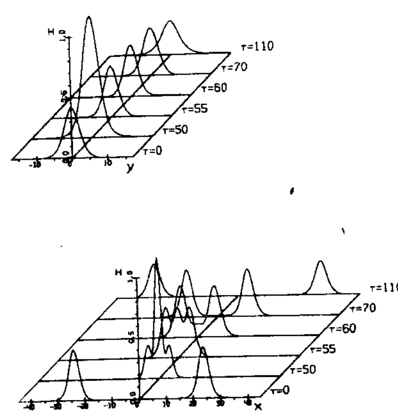


Fig. 8. Elastic interaction of Q solitons at v=0.5, μ =0.95.

The most interesting result, from our point of view, is that the interaction of two solitons in the vicinity of the stability threshold ($\mu \approx 0.2$, $Q = Q_{cr}$) leads, at sufficiently low colliding soliton energies, to formation of a finite-time bound state, which is breaking up into two diverging solitons decaying in their turn into constituents. Finally, the inelasticity of soliton interaction at fixed "charge" Q decreases with growing a relative velocities of solitons $v \to 1$ (this fact has been known for interaction of one-space dimensional Langmuir solitons some years ago [7]).

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Received by Publishing Department
on May 18 1978.