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N.B.Skachkov, I.L.Solovtsov

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Скачков Н.Б., Соловцов И.Л. E2 - 11569

Спектр масс и траектории Редже векторных мезонов в релятивистской кварковой модели

На основе релятивистского двухчастичного квазипотенциального уравнения с потенциалом, линейно растущим с расстоянием, вычислены спектр масс и траектории Редже векторных мезонов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Skachkov N.B., Solovtsov I.L. E2 - 11569

Mass Spectrum and Regge Trajectories of Vector Mesons in the Relativistic Quark Model

The mass spectrum and Regge trajectories of vector mesons are calculated by using the relativistic two-particle quasipotential equation with a quark confinement potential linearly growing with a distance.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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For the last years, the J/Ψ meson spectrum has been described by using the nonrelativistic quark model based on the Schrödinger equation with a potential linearly growing with distance. However, an analysis performed in ref. /1/ has shown that the model as applied to the description of the spectrum of usual light vector ρ, ω, ϕ mesons is not self-consistent because the contribution of relativistic corrections is compatible with the contribution of the nonrelativistic Hamiltonian, that was taken originally to be a main term. Therefore, the consistent descritpion requires the essentially relativistic model. In this note, we shall use for this purpose the relativistic two-particle Kadyshevsky quasipotential equation, that in the c.m.s. $(\vec{p}_1 = -\vec{p}_2 = \vec{p})$ has the following form /2/

$$(2E_q - 2E_p)\Psi_q(\vec{p}) = \frac{1}{(2\pi)^3}\int \frac{d\vec{k}}{E_k} V(\vec{p},\vec{k};E_q)\Psi_q(\vec{k}),$$

$$E_k = \sqrt{m^2 + \vec{k}^2}. \qquad (1)$$

In this equation all the momenta of the particles belong to the mass hyperboloid $E_p^2 - \vec{p}^2 = m^2 (h=c=1)$, the upper sheet of which serves as a model of the Lobachevsky space. Therefore it is convenient to pass here to the relativistic configuration representation (RCR). RCR was introduced in /4/ by expanding the wave function of two-quark relative motion over the unitary representations of the group of motions of the Lobachevsky space (the Lorentz group),

$$\Psi(\vec{r}) = \int \xi^*(\vec{p},\vec{r})\,\Psi(\vec{p})\,\frac{d\vec{p}}{E_p};\quad \xi(\vec{p},\vec{r}) = \left(\frac{E_p - \vec{p}\vec{n}}{m}\right)^{-1-irm}, \tag{2}$$

$$\vec{r} = r\vec{n};\quad \vec{n}^2 = 1$$

realized by $\xi(\vec{p},\vec{r})$ functions /3/. It was proposed in /4/ to regard the group parameter r in (2) as a relativistic analog of the modulus of the relative coordinate and the functions $\xi(\vec{p},\vec{r})$ as a relativistic analog of the plane waves $\exp[i\vec{p}\vec{r}]$. After passing to the relativistic configurational representation with the help of the transformation (2) the equation (1) for the partial wave function $\Psi_{q\ell}(r)$ takes the form*/1/

$$\left[\mathrm{ch}\, i\bar{\lambda}\frac{\partial}{\partial r} + \frac{i\bar{\lambda}}{r}\,\mathrm{sh}\, i\bar{\lambda}\frac{\partial}{\partial r} + \frac{\bar{\lambda}^2\ell(\ell+1)}{r^2}\exp\left[i\bar{\lambda}\frac{\partial}{\partial r}\right] - X(r)\right]\Psi_{q\ell}(r) = 0,$$

$$X(r) = \frac{W - V(r)}{2mc^2}, \tag{3}$$

$\bar{\lambda} = \frac{\hbar}{mc}$ - being the Compton wave length (in our case, of quark), W - the total energy of the system (i.e., the meson mass M).

Using the quasiclassical approximation proposed in /6/ for the finite-difference equation (3), we get the solution for

$$\Phi_{q\ell}(r) = \frac{\Psi_{q\ell}(r)}{r},$$

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*Another RCR equation derived from the momentum representation equation in terms of rapidities has been used in ref. /5/.

$$\Phi^{\pm}_{q\ell}(r) = \frac{C_{\pm}}{\sqrt[4]{X^2 - 1 - \frac{\lambda^2\ell(\ell+1)}{r^2}}}\exp\left[\frac{i}{\lambda}\int_{r_-}^{r} dr\,\ln\left[X \pm \sqrt{X^2 - 1 - \frac{\lambda^2\ell(\ell+1)}{r^2}}\right]\right], \tag{4}$$

$r_-(r_+)$ being the classical turning point defined as a root branchpoint in (4).

The quantization condition is:

$$\frac{1}{2}\int_{r_-}^{r_+} dr\,\ln\left[\frac{\mathrm{ch}\,\chi(r) + \sqrt{\mathrm{sh}^2\chi(r) - \frac{\lambda^2\ell(\ell+1)}{r^2}}}{\mathrm{ch}\,\chi(r) - \sqrt{\mathrm{sh}^2\chi(r) - \frac{\lambda^2\ell(\ell+1)}{r^2}}}\right] = \lambda\pi\left(n + \frac{1}{2}\right),$$

$$\chi(r) = \mathrm{Ar\,ch}\,X(r). \tag{5}$$

In the nonrelativistic quantum mechanics a procedure is known /7/ which permits for confinement potentials of the type $V(r) = \sigma r^s$, $s > -1$ to extract from the integral part of the quantization condition the dependence on the centrifugal term. By repeating analogous calculations in the relativistic case, the condition (5) changes to the following form

$$\int_0^{r_+} dr\,\chi(r) = \lambda\pi\left(n + \frac{\ell}{2} + \frac{3}{4}\right), \tag{6}$$

where $\chi(r) = \mathrm{Ar\,ch}\,X(r)$ is the rapidity corresponding to the motion of a particle in field $V(r)$ and in the nonrelativistic limit $mc\chi(r)$ turns into the particle momentum $p(r) = \sqrt{m[E - V(r)]}$ in a field $V(r)$.

The modified quantization condition (6) allows us to derive easily the mass spectrum and Regge trajectories of relativistic bound systems. For the potentials $V(r) = \sigma r^s$, $s > 0$ the condition (6) transforms as follows:

$$\sqrt{\frac{\pi}{2}}\left(\frac{2m}{\sigma}\right)^{\frac{1}{s}}\{\mathrm{sh}\,\chi_n\}^{\frac{1}{2}+\frac{1}{s}}\cdot\Gamma\left(1+\frac{1}{s}\right)P^{-\frac{1}{2}-\frac{1}{s}}_{-\frac{1}{2}}(\mathrm{ch}\,\chi_n) =$$

$$= \lambda\pi(n + \frac{\ell}{2} + \frac{3}{4}), \qquad (7)$$

where $\chi_n = \mathrm{Arch}\frac{W_n}{2mc^2}$ and $P_\mu^\nu(\mathrm{ch}\chi_n)$ are the Legendre functions. For the linear potential $s=1$ (7) gives

$$\chi_n \mathrm{ch}\chi_n - \mathrm{sh}\chi_n = \frac{\sigma}{2mc^2}\lambda\pi(n + \frac{\ell}{2} + \frac{3}{4}). \qquad (8)$$

Now let us consider the $\rho(\omega)$-meson family[2]; the first radial excitation of $\rho(0.773)$ is considered to be a vector meson with mass 1.250 *GeV*. The results we have found by using formula (9) practically coincide with the values obtained in ref.[8] for $\ell=0$. However, as yet no unambiguous interpretation does exist for the resonance structure observed in some experiments around 1.250 *GeV*.

Another possibility is to suppose the first radial excitation of ρ-meson to be the vector meson with the mass 1.110 *GeV* discovered experimentally at DESY [9] ** and predicted earlier by using the group-theoretical method for constructing confinement potentials[10]. Relevant calculations by formula (8) are presented in the *Table*.

In our model, parameters are as follows $\sigma = 0.031$ GeV^2; $m_q = 0.18$ *GeV*. The factor $\chi_n/\mathrm{sh}\chi_n$ is a measure of "relativity" of a system. For systems with the nonrelativistic intrinsic motion this factor is near unity.

The attractive feature of that model ρ'(1.110 *GeV*) is that the mass (1.394 *GeV*) of the $\rho(\omega)$ second radial excitation is close to the mass of the observed[11] vector resonance structure at 1384±8 *MeV*. In the usual scheme with ρ' (1.250) this level does not arise and the next

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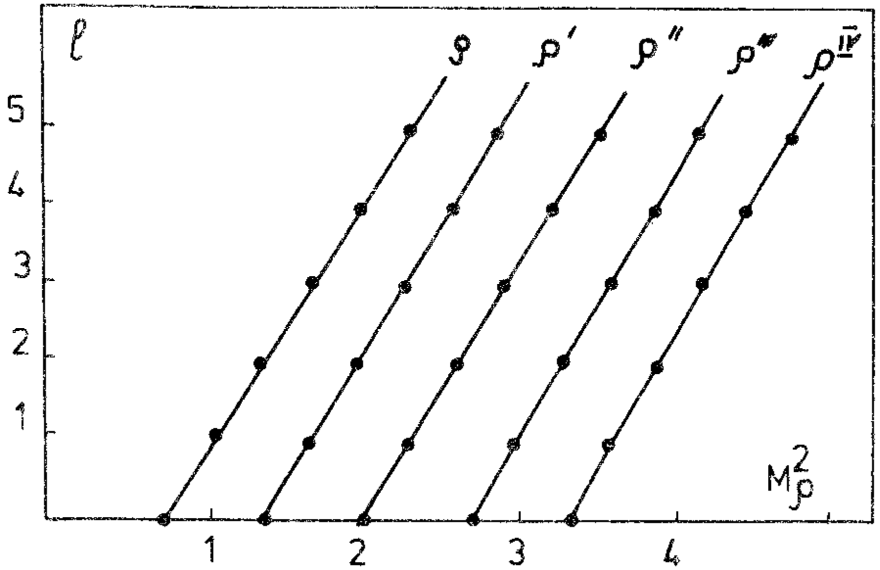
*Since masses of ρ and ω-mesons are close, similar results follow for ω-mesons. They will be listed in a subsequent paper where we will discuss spectra and trajectories of ϕ, Ψ and other mesons and their leptonic widths.

** This meson is narrow, as most of the new vector meson states below 2 *GeV* observed in 1977.

Table

Mass spectrum of vector mesons with the model parameters

$\ell=0$			$\ell=1$		
n	$M^{theor.}$ (GeV)	$M^{exper.}$ (GeV)	n	$M^{theor.}$ (GeV)	$M^{exper.}$ (GeV)
0	0.773	$\rho(\omega)$(0.773)	0	0.95I	δ (9.97)
I	I.II0	[7] (I.II0)	I	I.256	not observed
2	I.394	[11] (I.384)	2	I.525	F_1 (I.54)
3	I.65I	ρ''(I.60) [illegible]	3	I.773	X (I.795)
4	I.89I	not observed	4	2.228	U (2.36)
5	2.II8	ρ (2.I)			



The Regge trajectory of $\rho(\omega)$ *-mesons for the potential* $V(r) = \sigma r$.

to ρ' (1.250) level appears with the mass close to the ρ''-meson mass (1.65). In the scheme under consideration with $\rho'(\omega')$(1.110) the ρ -meson (1.65) arises as a third radial excitation of ρ -meson *.

An important result of the description of light vector mesons within the relativistic two particle equation is the fact that for the potential $V(r) = \sigma r$, the Regge trajectories within a good accuracy are straight-line in the considered mass region (see the *figure*). Recall that in the nonrelativistic limit the straight-line trajectories arise only in the oscillator potential $V(r) = \sigma r^2$.

For the recently discovered γ -mesons /12/ there are known three states (γ(9.4), γ'(10.01) and γ''(10.4)) treated as bound states of heavy $b\bar{b}$ quarks /13/. Our model with $V(r) = \sigma r$ gives the following spectrum for them: for $\ell = 0$, $n = 0,1,2,\ldots$ the masses are: $M_\gamma = 9.40$ *GeV*; 1.010; 10.424; 10.825; 11.193; 11.537; for $\ell = 1$, $n = 0,1,2,\ldots$ the masses are M: 9.712; 10.205; 10.629; 11.013; 11.367; 11.701... (The model parameters here are: $\sigma = 0.364$ GeV^2 $M_b = 4.34$ *GeV*, and factor $\chi_n / \mathrm{sh}\chi_n \approx 0.9$).

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*The description of γ -mesons in the nonrelativistic model with the logarithmic potential is given in ref. /13/

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