ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2 - 11557

N.B.Skachkov

QUARK CONFINEMENT POTENTIAL AS THE CONTINUATION OF FIELD-THEORETICAL COULOMB POTENTIAL AT SMALL DISTANCES AND THE SPECTRUM OF NEW VECTOR MESONS BELOW 2 GEV

1978

E2 - 11557

N.B.Skachkov

QUARK CONFINEMENT POTENTIAL AS THE CONTINUATION OF FIELD-THEORETICAL COULOMB POTENTIAL AT SMALL DISTANCES AND THE SPECTRUM OF NEW VECTOR MESONS BELOW 2 GEV

Submitted to Phys.Lett. and to the XIX International Conference on High Energy Physics (Tokyo, 1978)

Скачков Н.Б.

E2 - 11557

Потенциал запирания кварков как продолжение теоретико-полевого потенциала Кулона на малые расстояния и спектр новых векторных мезонов ниже 2 ГэВ

Новый тип потенциала запирания кварков, который ранее позволил предсказать существование векторной частицы с массой 1110 МэВ, применяется для вычисления значений масс радиальных возбуждений ρ, ω, ϕ-мезонов. Полученные значения масс близки к массам недавно открытых векторных резонансов в области ниже 2 ГэВ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

Skachkov N.B.

E2 - 11557

Quark Confinement Potential as the Continuation of Field-Theoretical Coulomb Potential at Small Distances and the Spectrum of New Vector Mesons below 2 GeV

A new type of the quark confinement potential that previously led to the prediction of the 1110 MeV vector particle existence is applied to calculate the masses of the radial excitations of the ρ, ω, ϕ-mesons. The values of the masses obtained are close to those of the recently discovered vector resonances below 2 GeV.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

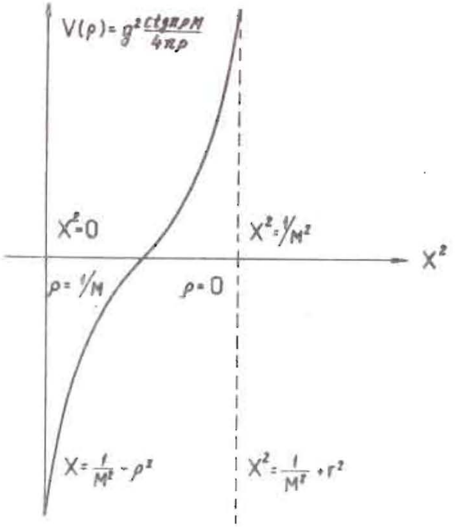
Preprint of the Joint Institute for Nuclear Research. Dubna 1978

In ref.[1] a new type of quark-antiquark potential

$$V(\rho)=g^2(4\pi\rho)^{-1}\mathrm{ctg}\,\pi\rho M_q\,; \qquad 0<\rho\le M_q^{-1} \tag{1}$$

was introduced (see the *figure*). This potential confines quarks inside the finite volume with the radius equal to the quark Compton wave length M_q^{-1} (ρ is the coordinate reconed from the boundary of the sphere of the $R=M_q^{-1}$ radius to the center). It was found[1] that the spectrum of the ρ-meson excited states contains the state with the mass $M=1110$ *MeV* that exactly coincides with the mass of the vector resonance detected shortly after at DESY[2] and confined in 1977[2]. The potential (1) was constructed from the symmetry considerations by a group-theoretical continuation to the small distances of the image of the propagator $-g^2\frac{1}{(p-k)^2}$ in the relativistic configurational representation.

The relativistic configurational representation (RCR) was introduced in ref.[3] in the framework of two-particle equations of quasipotential type[4]. In this equations all the momenta of the particles belong to the mass hyperboloid $E_p^2-p^2=M^2$ whose upper sheet serve as a model of the Lobachevsky space. Therefore it was proposed in ref.[3] to apply here instead of the conventional Fourier transformation the expansion over the principal series of unitary irreducible representations of the Lorentz



Group-theoretical potential $V(\rho)=g^2(4\pi\rho)^{-1}\mathrm{ctg}\,\pi\rho M_q$ *that, confines quarks inside the sphere of the Compton wave length radius* $X^2=M_q^{-2}$

group[5,6] – the group of motions of the Lobachevsky space

$$\Psi(\vec r)=(2\pi)^{-3}\int\xi(\vec p,\vec r)\Psi(\vec p)\frac{d^{(3)}p}{E_p} \qquad (2)$$

$$\xi(\vec p,\vec r)=\left(\frac{E_p-\vec p\vec n}{M}\right)^{-1-irM};\quad E_p=\sqrt{\vec p^2+M^2};\ \vec r=r\vec n;\ \vec n^2=1$$

($\Psi(\vec p)$ – is the wave function of a relative motion of two quarks). In nonrelativistic limit $\xi(\vec p,\vec r)\to e^{i\vec p\vec r}$ and (2) turns into three-dimensional Fourier transformation. Thus the group parameter r in (2) $(0\le r<\infty)$ was treated in ref.[3] as the relativistic generalization of the relative coordinate modulus. The Kadyshevsky equation in the c.m.s. of two quarks ($\vec p_1=-\vec p_2=\vec p$; $E_p=\sqrt{\vec p^2+M^2}$) has the form

$$(2E_p-2E_q)\Psi_q(\vec p)=-(2\pi)^{-3}\int V(\vec p,\vec k;E_q)\Psi_q(\vec k)\frac{d^{(3)}k}{E_k}. \qquad (3)$$

After performing the transformation (2) in (3) it takes in the RCR the local form[3]*

$$(\hat H_0-2E_q)\Psi_q(\vec r)=-V(\vec r;E_q)\Psi_q(\vec r). \qquad (4)$$

The free Hamiltonian $\hat H_0$ of the two-particle system, defined with the help of relativistic "plane waves" $\xi(\vec p,\vec r)$ as $\hat H_0\xi(\vec p,\vec r)=2E_p\xi(\vec p,\vec r)$; $E_p=\sqrt{\vec p^2+M^2}$, is a finite-difference operator[3]. Using (2) we find that in a new r-space the transform of the mass-less gluon exchange propagator, taken as a quasipotential $V(\vec p,\vec k;E_q)=-g^2(p-k)^{-2}$, has a form of the "Coulomb" attractive potential**

$$V(r)=-g^2(4\pi r)^{-1}\coth\pi rM. \qquad (5)$$

In addition to the principal series, used in (2), the Lorentz group has the supplementary series realized by functions $\zeta(\vec p,\vec\rho)=\left(\frac{E_p-\vec p\vec n}{M}\right)^{-1-\rho M}$ $(0<\rho\le M^{-1})$ which can be obtained formally from $\xi(p,r)$ by the change $r\to i\rho$.

These two series are characterized by different eigenvalues X^2 of the Lorentz group Casimir operator $\hat C_L=\frac{1}{4}M_{\mu\nu}M^{\mu\nu}$, where $M_{\mu\nu}=p_\mu\frac{\partial}{\partial p^\nu}-p_\nu\frac{\partial}{\partial p_t^\mu}$ are the generators of the group[6]:

$$\hat C_L\xi(\vec p,\vec r)=X^2\xi(\vec p,\vec r);\quad X^2=M^{-2}+r^2;\quad 0\le r<\infty$$

$$\hat C_L\zeta(\vec p,\vec\rho)=X^2\zeta(\vec p,\vec\rho);\quad X^2=M^{-2}-\rho^2;\quad 0<\rho\le M^{-1}. \qquad (6)$$

---

* The quasipotential $V(\vec p,\vec k;E_q)$ is a set of the Feynman propagators[3,4].

** Eq. (4) with the potential (5) and quark confinement potential $V(r)=\lambda r$ was solved in refs.[3] and [7], respectively.

In the nonrelativistic limit $\hat{C}_L$ transforms to the Casimir operator of the Euclidean group $\hat{C}_L \to \hat{C}_E = (i \frac{\partial}{\partial \vec{p}})^2$, whose eigenvalues are the square of the nonrelativistic coordinate $\hat{C}_E e^{i\vec{p}\vec{r}} = r^2 e^{i\vec{p}\vec{r}}$.

As is shown in ref. [8] the invariant mean-square radius of a system $<r_0^2> \equiv 6 \frac{\partial F(t)}{\partial t}|_{t=0}$ has also the group-theoretical meaning: it is an eigenvalue of the Casimir operator of the Lorentz group on the invariant form factor $F(t)$ of the system

$$<r_0^2> \equiv 6 \frac{\partial F(t)}{\partial t}|_{t=0} = \{\hat{C}_L F(t)\}|_{t=0} .$$

Thus from this expression and (6) it follows that the coordinate r $(0 \le r < \infty)$ describes the distances larger than the Compton wave length M^{-1}, and ρ $(0 < \rho \le M^{-1})$ is the relative coordinate reconed from the boundary of the sphere with the $X = M^{-1}$ radius to its center, so that $\rho = M^{-1}$ corresponds to the center $X^2 = 0$.

The wave functions of two-particle bound systems, the constituents of which can be observed in a free state (like hydrogen atom or positronium), are usually chosen as square - integrable by analogy with the nonrelativistic quantum mechanics. According to [6] such functions of the L_2 class are expanded over the representations of the principle series of $SO(3.1)$ only. The functions $\zeta(p, \rho)$ of the supplementary series are not square-integrable.

Let us extend the class of the used functions to describe the quark dynamics. We shall pass to the space of the non square-integrable functions but its scalar product $(\Psi_1, \Psi_2) = \int \Psi_1(\vec{p}) k_a(\vec{p}, \vec{k}) \Psi_2(\vec{k}) \frac{d\vec{p}}{E_p} \frac{d\vec{k}}{E_k}$ contains the regularization kernal $k_a(\vec{p}, \vec{k})$.

The free Hamiltonian for the supplementary series $\hat{H}_0^\rho$ found in ref. [1] from the relation $\hat{H}_0^\rho \zeta(\vec{p}, \vec{\rho}) = 2E_p \zeta(\vec{p}, \vec{\rho})$ is also the finite difference operator. By analogy with (5) it is possible to write down the equation for the wave function $\Psi_q(\vec{\rho})$ of quark-antiquark relative motion in the region $X^2 < M^{-1}$

$$(\hat{H}_0^\rho - 2E_q)\Psi_q(\vec{\rho}) = V(\rho)\Psi_q(\vec{\rho}). \quad (7)$$

In ref. [1] the quasipotential $V(\rho)$ in (7) was chosen to be the analog of the relativistic Coulomb potential (5) at the distances smaller than the Compton wave length, i.e., the potential (1). In fact, since in (5) the coordinate r has a group-theoretical meaning and the distances smaller than $M^{-1} (X^2 \le M^{-2})$ can be described according to (6) by passing from the principal to supplementary series, then the change $r \to i\rho$ in (5) leads to the potential (1).

Eq. (7) with the potential (1) has been explicitly solved in [1]. The requirement of wave function finiteness at points $\rho = M^{-1}$ and $\rho = 0$ defines the energy spectrum. It has been found that in the field of the potential (1) the quark-antiquark system in the state $\ell = 0$ has *only three energy levels*, i.e., the ground state of a particle and two its radial excitations

$$M^I \equiv 2E_q \approx 1.898 M_q ; \; M^{II} = 2M_q ; \; M^{III} = 2.967 M_q \quad (8)$$

(the quark mass M_q is the model free parameter).

The masses of the radial excitations of ρ, ω and ϕ-mesons in $\ell = 0$ state obtained by using (8) are listed in the *table*. Thus, due to our model in the region of a new $M = 1110$ *MeV* resonance [2] there must exist two particles $\rho'(1106 \pm 4)$ and $\omega'(1120)$. Probably the analog of $\rho - \omega$ -interference effect can be also valid for this ρ', ω'. The $\omega'(1120)$ width may be less than the width of the $\rho'(1106 \pm 4)$, what leads to a relatively small width of the resonance structure observed at $M = 1110$ *MeV*. (The same picture was seen in the ρ, ω photoproduction experiments [9]).

The masses of the second ρ, ω radial excitations $\rho''(1645 \pm 6)$ and $\omega''(1662)$ with account for the experimental errors and ℓ -degeneration of the levels in the Coulomb-like potentials (5) and (1) can be identified with

Table

The masses of the radial excitations of the ordinary ρ, ω, ϕ-vector mesons in the state with $\ell = 0$, obtained in our model.

Meson mass (MeV)	ρ(773 ±2) $I^G = 1^+$	ω (783) $I^G = 0^-$	ϕ(1020) $I^G = 0^-$
First excitation	ρ'(1106 ±4)	ω'(1120)	ϕ'(1460)
Second excitation	ρ''(1645±6)	ω''(1662)	ϕ''(2164)
Quark mass(MeV)	$M_{q\rho}$ =553	$M_{q\omega}$ =560	M_{qs} =730

the resonance structures $M^{exp} \cong 1676 \pm 35$ *MeV*, $\Gamma = 170 \pm 62$ *MeV* ($I^G = 1^+$) and $M^{exp} = 1662 \pm 5$ *MwV*, $\Gamma = 25 \pm 13$ *MeV* ($I^G = 0^-$) being found in 1977 [10].

The masses of ϕ-meson radial excitations, obtained by assuming that ϕ-meson concists of the strange quarks only, are close to those of two new (narrow) resonances $M^{exp} = 1498$ *MeV*, $\Gamma \approx 4$ *MeV* and $M^{exp} = 2130$ *MeV*, ≈ 30 *MeV* [11].

The confining potential can be obtained by the above-mentioned group-theoretical method (changing $r \to i\rho$) from the relativistic analog of the Yukawa potential

$V(r) = [4\pi r \cdot \sinh \pi r M_q]^{-1} \times \cosh[rM \arccos(\frac{\mu^2 - 2M_q^2}{2M_q^2})]$, i.e., the

transformation of the massive gluon exchange propagator $(\mu^2 - t)^{-2}$ in RCR. In the nonrelativistic limit $X^2 = M_q^{-2} \to 0$ and the potential (1) turns into the $\delta(r)$ (or $\delta'(r)$)-type potential.

Two recently observed resonance structures at 1.25 *GeV* (the third ref. of [2]) and 1.82 *GeV* [11] (the G-parity of 1.82 resonance is not yet established [11]) do not appear as radial excitations of ordinary vector meson states. We hope that after including the spin-orbit and spin-spin interactions with the help of the formalism developed in RCR in ref. [12], it will be possible to include these states in our scheme with the potential (1).

The author expresses his sincere gratitude to V.G.Kadyshevsky, P.N.Bogolubov, D.P.Jelobenko, J.Niderle, I.L.Solovtsov and G.Wolf gor useful discussions.

REFERENCES

1. *Skachkov N.B. JETP Letters, 1976, 23, No. 12, p.657; Proc. of 18 Int. Conf. on High Energy Phys., Tbilisi, 1976, JINR, D1,2-10400, Dubna, 1977, v.1., p.C-144.*
2. *Bartalucci S. et al. Proc. of 18 Int. Conf. on High Energy Phys., Tbilisi, 1976. JINR, D1,2-10400, Dubna, 1977, v.2, p.N-28; Nuovo Cim., 1977, 39A, p.374; DESY 77/59, 1977.*
3. *Kadyshevsky V.G., Mir-Kasimov R.M., Skachkov N.B. Nuovo Cim., 1968, 55A, p.233; Soviet Journal of Particles and Nuclei (translation), 1972, vol. 2, No.3, p.69.*
4. *Logunov A.A., Tavkhelidze A.N. Nuovo Cim., 1963, 29, p.38; Kadyshevsky V.G. Nucl.Phys., 1968, B6, p.125.*
5. *Shapiro I.S. Sov.Phys. Doklady, 1956, 1, p.91; Phys. Lett., 1962, 1, p.253.*
6. *Gelfand I.M., Graev M.I., Vilenkin N.Ya. Generalized Functions, vol. 5, Academic Press, New York, 1966.*
7. *Jhung K.S., Ching K.H., Willey R.S. Phys.Rev., 1975, D12, p.1999.*
8. *Skachkov N.B. Soviet Journ. Theor. and Math.Phys., 23, No. 3 (transl. of Theor. i Math. Fiz., 1975, 23, p.313). Proc. of 18 Int. Conf. on High Energy Phys., Tbilisi, 1976, JINR, D1,2-10400, Dubna, 1977, v.1., p.A7-11.*
9. *Alvensleben J. et al. Proc. of the Int. Symp. on Electron and Proton Interactions, Liverpool, 1969.*

10. *Laplanche F. Talk at Int. Symp. on Lepton and Photon Interactions, Hamburg, 1977.*
11. *Bemporad C. Talk at Int. Symp. on Lepton and Photon Interactions, Hamburg, 1977.*
12. *Skachkov N.B., Solovtsov I.L. JINR, E2-9763, Dubna, 1976; Soviet Journ. of Particles and Nuclei (translation), 1978, vol. 9, No. 1, p.5.*

Received by Publishing Department on May 6 1978.