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**QUARK CONFINEMENT POTENTIAL  
AS THE CONTINUATION OF  
FIELD-THEORETICAL COULOMB POTENTIAL  
AT SMALL DISTANCES AND THE SPECTRUM  
OF NEW VECTOR MESONS BELOW 2 GEV**

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Скачков Н.Б.

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Потенциал записания кварков как продолжение теоретико-полевого потенциала Кулона на малые расстояния и спектр новых векторных мезонов ниже 2 ГэВ

Новый тип потенциала записания кварков, который ранее позволил предсказать существование векторной частицы с массой 1110 МэВ, применяется для вычисления значений масс радиальных возбуждений  $\rho$ ,  $\omega$ ,  $\phi$ -мезонов. Полученные значения масс близки к массам недавно открытых векторных резонансов в области ниже 2 ГэВ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

Skachkov N.B.

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Quark Confinement Potential as the Continuation of Field-Theoretical Coulomb Potential at Small Distances and the Spectrum of New Vector Mesons below 2 GeV

A new type of the quark confinement potential that previously led to the prediction of the 1110 MeV vector particle existence is applied to calculate the masses of the radial excitations of the  $\rho$ ,  $\omega$ ,  $\phi$ -mesons. The values of the masses obtained are close to those of the recently discovered vector resonances below 2 GeV.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research.

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In ref.<sup>/1/</sup> a new type of quark-antiquark potential

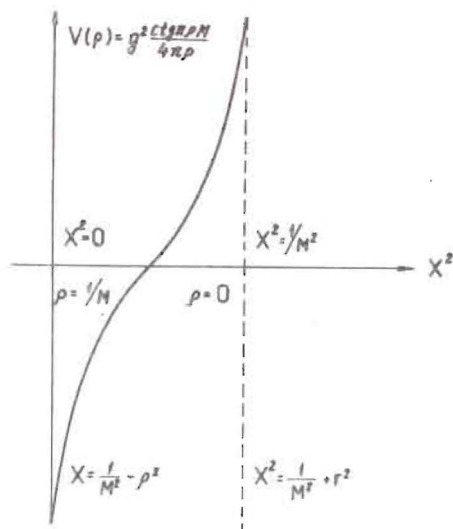
$$V(\rho) = g^2 (4\pi\rho)^{-1} \operatorname{ctg} \pi \rho M_q^{-1}; \quad 0 < \rho \leq M_q^{-1} \quad (1)$$

was introduced (see the figure). This potential confines quarks inside the finite volume with the radius equal to the quark Compton wave length  $M_q^{-1}$  ( $\rho$  is the coordinate reckoned from the boundary of the sphere of the  $R = M_q^{-1}$  radius to the center). It was found<sup>/1/</sup> that the spectrum of the  $\rho$ -meson excited states contains the state with the mass  $M = 1110$  MeV that exactly coincides with the mass of the vector resonance detected shortly after at DESY<sup>/2/</sup> and confined in 1977<sup>/2/</sup>. The potential (1) was constructed from the symmetry considerations by a group-theoretical continuation to the small distances of

the image of the propagator  $-g^2 \frac{1}{(p-k)^2}$  in the relativistic configurational representation.

The relativistic configurational representation (RCR) was introduced in ref.<sup>/3/</sup> in the framework of two-particle equations of quasipotential type<sup>/4/</sup>. In this equations all the momenta of the particles belong to the mass hyperboloid  $E_p^2 - p^2 = M^2$  whose upper sheet serve as a model of the Lobachevsky space. Therefore it was proposed in ref.<sup>/3/</sup> to apply here instead of the conventional Fourier transformation the expansion over the principal series of unitary irreducible representations of the Lorentz





Group-theoretical potential  $V(\rho) = g^2 (4\pi\rho)^{-1} \text{ctg} \pi \rho M_q$  that, confines quarks inside the sphere of the Compton wave length radius  $X^2 = M_q^{-2}$

group<sup>/5,6/</sup> - the group of motions of the Lobachevsky space

$$\Psi(\vec{r}) = (2\pi)^{-3} \int \xi(\vec{p}, \vec{r}) \Psi(\vec{p}) \frac{d^{(3)}p}{E_p} \quad (2)$$

$$\xi(\vec{p}, \vec{r}) = \left( \frac{E_p - \vec{p} \cdot \vec{n}}{M} \right)^{-1 - i r M} ; E_p = \sqrt{\vec{p}^2 + M^2} ; \vec{r} = r \vec{n} ; n^2 = 1$$

( $\Psi(\vec{p})$  - is the wave function of a relative motion of two quarks). In nonrelativistic limit  $\xi(\vec{p}, \vec{r}) \rightarrow e^{i \vec{p} \cdot \vec{r}}$  and (2) turns into three-dimensional Fourier transformation. Thus the group parameter  $r$  in (2) ( $0 \leq r < \infty$ ) was treated in ref.<sup>/3/</sup> as the relativistic generalization of the relative coordinate modulus. The Kadyshevsky equation in the c.m.s. of two quarks ( $\vec{p}_1 = -\vec{p}_2 = \vec{p}$ ;  $E_p = \sqrt{\vec{p}^2 + M^2}$ ) has the form

$$(2E_p - 2E_q) \Psi_q(\vec{p}) = -(2\pi)^{-3} \int V(\vec{p}, \vec{k}; E_q) \Psi_q(\vec{k}) \frac{d^{(3)}k}{E_k} \quad (3)$$

After performing the transformation (2) in (3) it takes in the RCR the local form<sup>/3,4/</sup>

$$(\hat{H}_0 - 2E_q) \Psi_q(\vec{r}) = -V(\vec{r}; E_q) \Psi_q(\vec{r}). \quad (4)$$

The free Hamiltonian  $\hat{H}_0$  of the two-particle system, defined with the help of relativistic "plane waves"  $\xi(\vec{p}, \vec{r})$

as  $\hat{H}_0 \xi(\vec{p}, \vec{r}) = 2E_p \xi(\vec{p}, \vec{r})$ ;  $E_p = \sqrt{\vec{p}^2 + M^2}$ , is a finite-difference operator<sup>/3/</sup>. Using (2) we find that in a new  $r$ -space the transform of the mass-less gluon exchange propagator, taken as a quasipotential  $V(\vec{p}, \vec{k}; E_q) = -g^2 (\vec{p} - \vec{k})^{-2}$ , has a form of the "Coulomb" attractive potential\*\*

$$V(r) = -g^2 (4\pi r)^{-1} \coth \pi r M. \quad (5)$$

In addition to the principal series, used in (2), the Lorentz group has the supplementary series realized by

functions  $\zeta(\vec{p}, \rho) = \left( \frac{E_p - \vec{p} \cdot \vec{n}}{M} \right)^{-1 - \rho M}$  ( $0 < \rho \leq M^{-1}$ ) which can be

obtained formally from  $\xi(\vec{p}, r)$  by the change  $r \rightarrow i\rho$ .

These two series are characterized by different eigenvalues  $X^2$  of the Lorentz group Casimir operator  $\hat{C}_L = \frac{1}{4} M_{\mu\nu} M^{\mu\nu}$ , where  $M_{\mu\nu} = p_\mu \frac{\partial}{\partial p^\nu} - p_\nu \frac{\partial}{\partial p^\mu}$  are the generators of the group<sup>/6/</sup>:

$$\hat{C}_L \xi(\vec{p}, \vec{r}) = X^2 \xi(\vec{p}, \vec{r}); X^2 = M^{-2} r^2 ; 0 \leq r < \infty$$

$$\hat{C}_L \zeta(\vec{p}, \rho) = X^2 \zeta(\vec{p}, \rho); X^2 = M^{-2} \rho^2 ; 0 < \rho \leq M^{-1}. \quad (6)$$

\* The quasipotential  $V(\vec{p}, \vec{k}; E_q)$  is a set of the Feynman propagators<sup>/3,4/</sup>.

\*\* Eq. (4) with the potential (5) and quark confinement potential  $V(r) = \lambda r$  was solved in refs.<sup>/3/</sup> and <sup>/7/</sup>, respectively.

In the nonrelativistic limit  $\hat{C}_L$  transforms to the Casimir operator of the Euclidean group  $\hat{C}_L \rightarrow \hat{C}_E = (i \frac{\partial}{\partial \vec{p}})^2$ ,

whose eigenvalues are the square of the nonrelativistic coordinate  $\hat{C}_E e^{i\vec{p}\vec{r}} = r^2 e^{i\vec{p}\vec{r}}$ .

As is shown in ref.<sup>/8/</sup> the invariant mean-square radius of a system  $\langle r_0^2 \rangle = 6 \frac{\partial F(t)}{\partial t} \Big|_{t=0}$  has also the group-theoretical meaning: it is an eigenvalue of the Casimir operator of the Lorentz group on the invariant form factor  $F(t)$  of the system

$$\langle r_0^2 \rangle = 6 \frac{\partial F(t)}{\partial t} \Big|_{t=0} = \{ \hat{C}_L F(t) \} \Big|_{t=0}.$$

Thus from this expression and (6) it follows that the coordinate  $r$  ( $0 \leq r < \infty$ ) describes the distances larger than the Compton wave length  $M^{-1}$ , and  $\rho$  ( $0 < \rho \leq M^{-1}$ ) is the relative coordinate reckoned from the boundary of the sphere with the  $X = M^{-1}$  radius to its center, so that  $\rho = M^{-1}$  corresponds to the center  $X^2 = 0$ .

The wave functions of two-particle bound systems, the constituents of which can be observed in a free state (like hydrogen atom or positronium), are usually chosen as square-integrable by analogy with the nonrelativistic quantum mechanics. According to<sup>/6/</sup> such functions of the  $L_2$  class are expanded over the representations of the principle series of  $SO(3,1)$  only. The functions  $\zeta(p, \rho)$  of the supplementary series are not square-integrable.

Let us extend the class of the used functions to describe the quark dynamics. We shall pass to the space of the non square-integrable functions but its scalar product

$$(\Psi_1, \Psi_2) = \int \Psi_1(\vec{p}) k_\alpha(\vec{p}, \vec{k}) \Psi_2(\vec{k}) \frac{d\vec{p}}{E_p} \frac{d\vec{k}}{E_k} \quad \text{contains the}$$

regularization kernel  $k_\alpha(\vec{p}, \vec{k})$ .

The free Hamiltonian for the supplementary series  $\hat{H}_0^\rho$  found in ref.<sup>/1/</sup> from the relation  $\hat{H}_0^\rho \zeta(\vec{p}, \vec{\rho}) = 2E_p \zeta(\vec{p}, \vec{\rho})$  is also the finite difference operator. By analogy with

(5) it is possible to write down the equation for the wave function  $\Psi_q(\vec{\rho})$  of quark-antiquark relative motion in the region  $X^2 < M^{-1}$

$$(\hat{H}_0^\rho - 2E_q) \Psi_q(\vec{\rho}) = V(\rho) \Psi_q(\vec{\rho}). \quad (7)$$

In ref.<sup>/1/</sup> the quasipotential  $V(\rho)$  in (7) was chosen to be the analog of the relativistic Coulomb potential (5) at the distances smaller than the Compton wave length, i.e., the potential (1). In fact, since in (5) the coordinate  $r$  has a group-theoretical meaning and the distances smaller than  $M^{-1}$  ( $X^2 \leq M^{-2}$ ) can be described according to (6) by passing from the principal to supplementary series, then the change  $r \rightarrow i\rho$  in (5) leads to the potential (1).

Eq. (7) with the potential (1) has been explicitly solved in<sup>/1/</sup>. The requirement of wave function finiteness at points  $\rho = M^{-1}$  and  $\rho = 0$  defines the energy spectrum. It has been found that in the field of the potential (1) the quark-antiquark system in the state  $\ell = 0$  has only three energy levels, i.e., the ground state of a particle and two its radial excitations

$$M^I = 2E_q \approx 1.498 M_q; \quad M^{II} = 2M_q; \quad M^{III} = 2.967 M_q \quad (8)$$

(the quark mass  $M_q$  is the model free parameter).

The masses of the radial excitations of  $\rho, \omega$  and  $\phi$ -mesons in  $\ell = 0$  state obtained by using (8) are listed in the table. Thus, due to our model in the region of a new  $M = 1110$  MeV resonance<sup>/2/</sup> there must exist two particles  $\rho'(1106 \pm 4)$  and  $\omega'(1120)$ . Probably the analog of  $\rho$ - $\omega$ -interference effect can be also valid for this  $\rho', \omega'$ . The  $\omega'(1120)$  width may be less than the width of the  $\rho'(1106 \pm 4)$ , what leads to a relatively small width of the resonance structure observed at  $M = 1110$  MeV. (The same picture was seen in the  $\rho, \omega$  photoproduction experiments<sup>/9/</sup>).

The masses of the second  $\rho, \omega$  radial excitations  $\rho''(1645 \pm 6)$  and  $\omega''(1662)$  with account for the experimental errors and  $\ell$ -degeneration of the levels in the Coulomb-like potentials (5) and (1) can be identified with



Table

The masses of the radial excitations of the ordinary  $\rho$ ,  $\omega$ ,  $\phi$ -vector mesons in the state with  $\ell = 0$ , obtained in our model.

Meson mass (MeV)	$\rho(773 \pm 2)$ $I^G = 1^+$	$\omega(783)$ $I^G = 0^-$	$\phi(1020)$ $I^G = 0^-$
First excitation	$\rho'(1106 \pm 4)$	$\omega'(1120)$	$\phi'(1460)$
Second excitation	$\rho''(1645 \pm 6)$	$\omega''(1662)$	$\phi''(2164)$
Quark mass (MeV)	$M_{q\rho} = 553$	$M_{q\omega} = 560$	$M_{q\phi} = 730$

the resonance structures  $M^{\text{exp}} = 1676 \pm 35 \text{ MeV}$ ,  $\Gamma = 170 \pm 62 \text{ MeV}$  ( $I^G = 1^+$ ) and  $M^{\text{exp}} = 1662 \pm 5 \text{ MeV}$ ,  $\Gamma = 25 \pm 13 \text{ MeV}$  ( $I^G = 0^-$ ) being found in 1977<sup>/10/</sup>.

The masses of  $\phi$ -meson radial excitations, obtained by assuming that  $\phi$ -meson consists of the strange quarks only, are close to those of two new (narrow) resonances  $M^{\text{exp}} = 1498 \text{ MeV}$ ,  $\Gamma \approx 4 \text{ MeV}$  and  $M^{\text{exp}} = 2130 \text{ MeV}$ ,  $\Gamma \approx 30 \text{ MeV}$ <sup>/11/</sup>.

The confining potential can be obtained by the above-mentioned group-theoretical method (changing  $r \rightarrow i\rho$ ) from the relativistic analog of the Yukawa potential

$$V(r) = [4\pi r \cdot \sinh \pi r M_q]^{-1} \times \cosh [r M \arccos(\frac{\mu^2 - 2M_q^2}{2M_q^2})], \text{ i.e., the}$$

transformation of the massive gluon exchange propagator  $(\mu^2 - t)^{-2}$  in RCR. In the nonrelativistic limit  $X^2 = M_q^2 \rightarrow 0$  and the potential (1) turns into the  $\delta(r)$  (or  $\delta'(r)$ )-type potential.

Two recently observed resonance structures at 1.25 GeV (the third ref. of<sup>/2/</sup>) and 1.82 GeV<sup>/11/</sup> (the G-parity of 1.82 resonance is not yet established<sup>/11/</sup>) do not appear as radial excitations of ordinary vector meson states. We hope that after including the spin-orbit and spin-spin interactions with the help of the formalism developed in RCR in ref.<sup>/12/</sup>, it will be possible to include these states in our scheme with the potential (1).

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## REFERENCES

1. Skachkov N.B. JETP Letters, 1976, 23, No. 12, p.657; Proc. of 18 Int. Conf. on High Energy Phys., Tbilisi, 1976, JINR, D1,2-10400, Dubna, 1977, v.1., p.C-144.
2. Bartalucci S. et al. Proc. of 18 Int. Conf. on High Energy Phys., Tbilisi, 1976, JINR, D1,2-10400, Dubna, 1977, v.2, p.N-28; Nuovo Cim., 1977, 39A, p.374; DESY 77/59, 1977.
3. Kadyshesky V.G., Mir-Kasimov R.M., Skachkov N.B. Nuovo Cim., 1968, 55A, p.233; Soviet Journal of Particles and Nuclei (translation), 1972, vol. 2, No.3, p.69.
4. Logunov A.A., Tavkhelidze A.N. Nuovo Cim., 1963, 29, p.38; Kadyshesky V.G. Nucl. Phys., 1968, B6, p.125.
5. Shapiro I.S. Sov. Phys. Doklady, 1956, 1, p.91; Phys. Lett., 1962, 1, p.253.
6. Gelfand I.M., Graev M.I., Vilenkin N.Ya. Generalized Functions, vol. 5, Academic Press, New York, 1966.
7. Chung K.S., Ching K.H., Willey R.S. Phys. Rev., 1975, D12, p.1999.
8. Skachkov N.B. Soviet Journ. Theor. and Math. Phys., 23, No. 3 (transl. of Theor. i Math. Fiz., 1975, 23, p.313). Proc. of 18 Int. Conf. on High Energy Phys., Tbilisi, 1976, JINR, D1,2-10400, Dubna, 1977, v.1., p.A7-11.
9. Alvensleben J. et al. Proc. of the Int. Symp. on Electron and Proton Interactions, Liverpool, 1969.

10. *Laplanche F. Talk at Int. Symp. on Lepton and Photon Interactions, Hamburg, 1977.*
11. *Bemporad C. Talk at Int. Symp. on Lepton and Photon Interactions, Hamburg, 1977.*
12. *Skachkov N.B., Solovtsov I.L. JINR, E2-9763, Dubna, 1976; Soviet Journ. of Particles and Nuclei (translation), 1978, vol. 9, No. 1, p.5.*

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