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Учет поправок к амплитуде рассеяния на большие углы

В рамках квазипотенциального подхода Логунова-Тавхелидзе развит последовательный метод вычисления поправок первого и второго порядков по $1/\sqrt{s}$ к амплитуде высокоэнергетического рассеяния на большие углы для аналитических квазипотенциалов.

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Corrections to the Amplitude of Large-Angle Scattering

In the framework of the Logunov-Tavkhelidze quasipotential approach, a consistent method is developed for calculating corrections to the amplitude of high-energy large-angle scattering. The corrections of two orders in $1/\sqrt{s}$ are obtained for analytic quasi-potentials.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. An interest to the large transverse momentum processes, as a tool for observation of the strong interaction structure at short distances stimulated the consideration of different aspects of this problem. Thus, the power behaviour

$$\frac{d\sigma}{dt} \sim \frac{1}{s^n} f\left(\frac{t}{s}\right); \qquad (1.1)$$

observed in elastic large-angle hadron scattering [1], was understood within the assumptions on the quark structure of hadrons, automodelity principle and dynamical interpretation of quark diagrams [2,3].

In the present paper the high energy behaviour of the scattering amplitude is treated in the framework of the Logunov-Tavkhelidze quasipotential approach, that enables us to unify different models by equivalent constraints on quasipotential. The structure of the local two-body quasipotential, compatible in the case of high-energy large-angle scattering with the automodel asymptotics (1.1), was investigated in ref. [4], where it was shown that for the leading term this property is satisfied by phenomenological quasipotential given by the integral representation:

$$V(s,t) = \int_0^\infty dx\, \rho(s,x)\, e^{xt} \qquad (1.2)$$

provided the weak limit for the function $\rho(s,x)$ exists:

$$\lim_{s\to\infty} s^N \rho\left(s, x = \frac{\eta}{s}\right) = \psi(\eta); \quad 0 < \eta < \infty; \ N > 0 . \qquad (1.3)$$

At the same time, the data available correspond to the values of $s < 50\ (GeV)^2$, that at best is just the beginning of the asymptotic region, it's boundary being more or less reliably fixed through calculation of corrections only. In our case, the existence of the quasipotential equation for the amplitude enables us to formulate a regular procedure for calculating corrections to the leading asymptotic term. We get also a simple picture of the interaction: in the asymptotic region fixed angle scattering results from single scattering on the hard core of the potential given by function $\psi(\eta)$; accompanied by soft rescatterings of imcoming and outgoing particles. The contribution of double hard scattering is much lesser than of the leading term, and the low order corrections are determined by soft rescatterings. The properties of the corresponding soft component of quasipotential can be reconstructed from the data on high energy small angle scattering. Thus, through calculating the corrections we estbalish certain correlations between asymptotics of the scattering amplitude at large and small scattering angles. The structure of the corrections is analogous to that for the eikonal amplitude, considered in ref./5/.

It is worth mentioning that the theoretical interpretation of the data on large angle elastic scattering can be performed in the framework of numerous models (see, e.g. ref./6/ and references therein).All of them provide good agreement with the experimental data through having different initial assumptions and analytic representations of the amplitude as function of s and t. Moreover, the majority of models fail to estimate corrections required both for determining the boundary of the asymptotic region and the checking the self-consistency of a model, as at energies presently available the corrections are large and can considerably worsen the agreement with experiment.

The paper is organized as follows. In Sec. 2 the representation of the scattering amplitude, suitable for investigating its asymptotic properties, is given for analytic quasipotentials. The corrections of two orders in $1/p$, where p is c.m.s. momentum, are presented in

Sec. 3. In Sec. 4 the general formulae obtained are applied for the case of one practically important quasipotential.

2. Let us consider the scattering of two identical scalar particles. The quasipotential in the presence of exchange forces /1/ is of the form:

$$V(s,\vec{p},\vec{k}) = g(s,\vec{p}-\vec{k}) + h(s,\vec{p}+\vec{k}). \tag{2.1}$$

Then the scattering amplitude can be represented as:

$$T(s,\vec{p},\vec{k}) = G(s,\vec{p},\vec{k}) + H(s,\vec{p},\vec{k}), \tag{2.2}$$

where

$$g(s,\vec{p}-\vec{k}) = h(s,\vec{p}-\vec{k}),\ G(s,\vec{p},-\vec{k}) = H(s,\vec{p},\vec{k}) \tag{2.3}$$

and hence we get for the total amplitide:

$$T(s,\vec{p},\vec{k}) = (1+\hat{P})\, G(s,\vec{p},\vec{k}), \tag{2.4}$$

where $\hat{P}$ is the reflection operator of the relative coordinate of a particle in the final state. The amplitude $G(s,\vec{p},\vec{k})$ satisfies the Logunov-Tavkhelidze quasipotential equation:

$$G(s,\vec{p},\vec{k}) = g(s,\vec{p}-\vec{k}) + \int \frac{d\vec{q}}{\epsilon(\vec{q})} \frac{g(s,\vec{p}-\vec{q})\, G(s,\vec{q},\vec{k})}{\vec{q}^{\,2}-\vec{p}^{\,2}-i0}$$

$$\epsilon(\vec{q}) = \sqrt{\vec{q}^{\,2}+m^2} \tag{2.5}$$

and on the mass shell we have:

$$G(s,\vec{p},\vec{k})\Big|_{\substack{s=4(p^2+m^2)\\ t=-(\vec{p}-\vec{k})^2}} = G(s,t).$$

Let the quasipotential $g(s, \vec{p}-\vec{k})$ be given by a representation of the type (1.2):

$$g(s,\vec{\Delta}) = 4ip^2 \int_0^\infty dx\,\rho(s,x)\, e^{-x\vec{\Delta}^2} , \qquad (2.6)$$

Equation (2.5) with the quasipotential (2.6) will be solved by iterations.

$$G(s,\vec{p},\vec{k}) = \sum_{n=0}^{\infty} G_{n+1}(s,\vec{p},\vec{k}) \; ; \; G_1(s,\vec{p},\vec{k}) = g(s,\vec{p}-\vec{k}) \; ;$$

$$G_{n+1}(s,\vec{p},\vec{k}) = (4ip^2)^{n+1} \int \prod_{i=1}^{n} \frac{d\vec{q}_i}{\epsilon(\vec{q}_i)(\vec{q}_i^{\,2}-p^2-i0)} \times$$

$$\times \int \prod_{j=1}^{n+1} dx_j\, \rho(s,x_j) \exp\{-x_1(\vec{p}-\vec{q}_1)^2 -$$

$$-\sum_{\ell=1}^{n-1} x_{\ell+1}(\vec{q}_\ell - \vec{q}_{\ell+1})^2 - x_{n+1}(\vec{q}_n - \vec{k})^2\} . \qquad (2.7)$$

To diagonalize the quadratic form in the exponent, let us change the integration variables:

$$\vec{q}_i = \vec{\Delta}_i + \vec{\lambda}_i \; ; \; i = 1, \dots, n;$$

$$\vec{\lambda}_i = \frac{\vec{p}+\vec{k}}{2} + \left(1 - 2\,\frac{\sum_{\ell=1}^{i} \frac{1}{x_\ell}}{\sum_{\ell=1}^{n+1} \frac{1}{x_\ell}}\right) \frac{\vec{p}-\vec{k}}{2} . \qquad (2.8)$$

As a result, we get:

$$G_{n+1}(s,\vec{p},\vec{k}) = (4ip^2)^{n+1} \int \prod_{j=1}^{n+1} dx_j\, \rho(s,x_j) \exp\left\{\frac{t}{\sum_{\ell=1}^{n+1}\frac{1}{x_\ell}}\right\} \times$$

$$\times \int \prod_{i=1}^{n} \frac{d\vec{\Delta}_i \exp(-C_{ij}\vec{\Delta}_i\vec{\Delta}_j)}{\epsilon(\vec{\Delta}_i+\vec{\lambda}_i)[(\vec{\Delta}_i+\vec{\lambda}_i)^2 - p^2 - i0]} \; ; \qquad (2.9)$$

where

$$C_{ij}\vec{\Delta}_i\vec{\Delta}_j = x_1\vec{\Delta}_1^{\,2} + \sum_{\ell=2}^{n} x_\ell(\vec{\Delta}_\ell - \vec{\Delta}_{\ell-1})^2 + x_{n+1}\vec{\Delta}_n^{\,2} .$$

In ref. /4/ it was shown that the dominant contribution to high-energy asymptotics of the amplitude comes from the region where one and only one of the parameters x_j is small, i.e., the integration over it is carried out around zero, $0 < x_m < \epsilon$ and over others from ϵ to ∞, where $\epsilon \sim s^{1-\alpha}$; $0 < \alpha < 1$. The fall-off power of the amplitude with growing $s - 4p^2$ is determined by exponent N in (1.3). Hence, it is obvious that the transformation of the hard component density function $\rho(s, x_m) \to \rho'(s, x_m) = x_m \rho(s, x_m)$ increases the power of fall-off by unity and we have the following equivalence relation for x-parameter of the hard quasipotential:

$$x_m \sim \frac{1}{p^2} . \qquad (2.10)$$

Finally, it is worth mentioning that $1/p$ corrections are pure imaginary in contrast to the leading term, and a correct description of the corrections to the differential cross-section requires the second order corrections to the amplitude.

3. Consider an $n+1$ term of the iterational representation (2.7), $x_m = x$ being the hard quasipotential parameter. Then, to evaluate the leading term and the corrections, we are to expand all quantities of (2.9) in power series in x and $1/p$ neglecting the terms of order higher than the first order in x and the second order in $1/p$.

The corresponding expansions for $\vec\lambda_i$ are of the form:

$$\vec\lambda_i \simeq \vec p - x\Big(\sum_{j=1}^{i}\frac{1}{x_j}\Big)(\vec p-\vec k);\quad i<m;$$
$$\vec\lambda_i \simeq \vec k - x\Big(\sum_{j=i+1}^{n+1}\frac{1}{x_j}\Big)(\vec k-\vec p);\quad i\ge m. \qquad (3.1)$$

In the following we shall employ the notations:

$$\vec\Lambda_i = q_i\vec n_p + \vec\Lambda_{\perp i};\quad i<m;$$
$$\vec\Lambda_i = q_i\vec n_k + \vec\Lambda_{\perp i};\quad i\ge m,$$
$$\alpha_i = \sum_{j=1}^{i}\frac{1}{x_j};\quad \Lambda_{||} = (\vec p-\vec k,\vec n_p) = (\vec k-\vec p,\vec n_k); \qquad (3.2)$$
$$(\vec n_p,\vec n_k) = \cos\theta = z,$$

where $\vec n_p$ and $\vec n_k$ are the unit vectors directed along the momenta $\vec p$ and $\vec k$, resp. Taking into account (3.1) and (3.2) we easily get for $i<m$:

$$\frac{1}{\sqrt{(\vec\Lambda_i+\vec\lambda_i)^2+m^2}} \simeq \frac{1}{p}\Big[1-\frac{q_i}{p}+\frac{1}{p}\Big(x\alpha_i\Lambda_{||}-\frac{\vec\Lambda^2_{\perp i}}{2p}\Big) - \frac{m^2}{2p^2}+\frac{q_i^2}{p^2}\Big], \qquad (3.3)$$

$$\frac{1}{(\vec\Lambda_i+\vec\lambda_i)^2-p^2-i0} \simeq \frac{1}{2p(q_i-i0)}\Big[1+\frac{x\alpha_i\Lambda_{||}-\frac{\vec\Lambda^2_{\perp i}}{2p}}{q_i-i0} + \frac{\Big(x\alpha_i\Lambda_{||}-\frac{\vec\Lambda^2_{\perp i}}{2p}\Big)^2}{(q_i-i0)^2}+\frac{\Lambda^2_{\perp i}}{2p^2}-\frac{q_i}{2p}+\frac{q_i^2}{4p^2}\Big]. \qquad (3.4)$$

The values for $i\ge m$ are obtained by substitution $\alpha_i\to\tilde\alpha_i=\sum_{j=i+1}^{n+1}\frac{1}{x_j}$. Exponential factors, depending on x, lead to the following expansions:

$$\exp(-x(\vec\Lambda_{m-1}-\vec\Lambda_m)^2) \simeq 1 - xq^2_{m-1} - xq^2_m + 2xzq_{m-1}q_m - x\vec\Lambda^2_{\perp m-1} - x\vec\Lambda^2_{\perp m} + 2x(\vec\Lambda_{\perp m-1},\vec\Lambda_{\perp m}), \qquad (3.5)$$

$$\exp\Big(t\Big(\frac{1}{x}+\alpha_{m-1}+\tilde\alpha_m\Big)^{-1}\Big) \simeq \exp(tx)[1-tx^2(\alpha_{m-1}+\tilde\alpha_m)]. \qquad (3.6)$$

Then, taking into account (3.3)-(3.6) we get for the leading asymptotics of $G_{n+1,m}(s,\vec p,\vec k)$:

$$G^{(0)}_{n+1,m}(s,\vec p,\vec k) = 4ip^2\int_0^\infty dx\,\rho_h(s,x)\,e^{xt}\int dF_{m-1}\int d\tilde F_m, \qquad (3.7)$$

where

$$\int dF_{m-1} = (2i)^{m-1}\int\prod_{j=1}^{m-1}\Big(dx_j\rho_s(s,x_j)\frac{dq_j}{q_j-i0}d^2\vec\Lambda_{\perp j}\Big)\times e^{-\Gamma_m(q)-\Gamma_m(\vec\Lambda_\perp)},$$

$$\Gamma_m(q) = x_1q_1^2 + \sum_{i=2}^{m-1}x_i(q_i-q_{i-1})^2,$$

$$\int d\tilde F_m = (2i)^{n+1-m}\int\prod_{j=m}^{n}\Big(dx_{j+1}\rho_s(s,x_{j+1})\frac{dq_j}{q_j-i0}d^2\vec\Lambda_{\perp j}\Big)\times e^{-\tilde\Gamma_m(q)-\tilde\Gamma_m(\vec\Lambda_\perp)},$$

$$\tilde\Gamma_m(q) = x_{n+1}q_n^2+\sum_{i=m+1}^{n}x_i(q_i-q_{i-1})^2, \qquad (3.8)$$

$\rho_h(s,x)$ and $\rho_s(s,x)$ are the density functions for the hard and soft components of the potential, resp., and, as it is shown in ref. /4/ :

$$\int dF_{m-1} = \frac{1}{(m-1)!}[-2\pi^2 \int_0^\infty \frac{dy}{y}\rho_s(s,y)]^{m-1} = \frac{[i\chi(0)]^{m-1}}{(m-1)!}, \quad (3.9)$$

where $\chi(0)$ is the eikonal phase at zero impact parameter $b=0$.

Substituting (3.7) and (3.9) into (2.7), we finally obtain for the leading term of the amplitude:

$$G^{(0)}(s,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{n+1} G^{(0)}_{n+1,m}(s,t) =$$

$$= 4ip^2 e^{2i\chi(0)} \int_0^\infty dx\ \rho_h(s,x)\, e^{xt} =$$

$$= e^{2i\chi(0)} g_h(s,t). \quad (3.10)$$

The corrections of order $1/p$ and $1/p^2$ to the leading term $G^{(0)}_{n+1,m}(s,\vec{p},\vec{k})$ can be determined if in the expansions (3.3)-(3.6) the leading terms are changed to the corresponding correction terms. The typical integrals are of the form:

$$G^{(1,2)}_{n+1,m}(s,\vec{p},\vec{k}) = 4ip^2 \int_0^\infty dx \rho_h(s,x) e^{xt}\ M_1^{(1,2)} \times$$

$$\times \int dF_{m-1} M_2^{(1,2)} \int d\tilde{F}_m M_3^{(1,2)}, \quad (3.11)$$

where:

$$M_1^{(1,2)} = M_1^{(1,2)}(x,z,p),$$

$$M_2^{(1,2)} = M_2^{(1,2)}(q_i, \vec{\Delta}_{\perp i}, x_i), \quad i = 1,2,\dots,m-1,$$

$$M_3^{(1,2)} = M_3^{(1,2)}(q_i, \vec{\Delta}_{\perp i}, x_j), \quad i = m, m+1,\dots, n;$$

$$j = m+1, m+2, \dots, n+1.$$

The details of calculations and the *table* of correction integrals for all necessary functions $M_2^{(1,2)}$ and $M_3^{(1,2)}$ are presented in the *Appendix*.

Summing up all the contributions we finally get for the first and second order corrections:

$$G^{(1)}(s,t) = 4pe^{2i\chi(0)} \int_0^\infty dx \rho_h(s,x) e^{xt} [4A_2 x p^2(1-z) + A_2 - 6B_1], \quad (3.12)$$

$$G^{(2)}(s,t) = 4ie^{2i\chi(0)} \int_0^\infty dx \rho_h(s,x)\, e^{xt} [4p^4 x^2 (1-z)^2 (A_3 - 8A_2^2) +$$

$$+ 2p^2 x (1-z)(48A_2 B_1 - 24A_2^2 - A_3) -$$

$$- 13A_2^2 + 24A_2 B_1 - 72B_1^2 + 64C_1 - 14B_2 + 4F +$$

$$+ 2m^2 A_1 + 4A_3(x + tx^2)]. \quad (3.13)$$

Here

$$A_1 = \pi^2 \int_0^\infty \frac{dx}{x} \rho_s(s,x) = -\frac{i}{2}\chi(0),$$

$$A_2 = \pi^{3/2} \int_0^\infty \frac{dx}{x^{3/2}} \rho_s(s,x),$$

$$A_3 = \pi^2 \int_0^\infty \frac{dx}{x^2} \rho_s(s,x);$$

$$B_1 = \pi^{7/2} \int_0^\infty \frac{dx_1 dx_2}{x_1 x_2 \sqrt{x_1 + x_2}} \rho_s(s,x_1)\rho_s(s,x_2),$$

$$B_2 = \pi^3 \int_0^\infty \frac{dx_1 dx_2}{x_1 \sqrt{x_1 x_2 (x_1 + x_2)}} \rho_s(s,x_1)\rho_s(s,x_2),$$

$$C_1 = \pi^5 \int_0^\infty \frac{dx_1\, dx_2\, dx_3}{x_1 x_2 x_3 \sqrt{x_1 x_2 + x_2 x_3 + x_3 x_1}} \times$$

$$\times \rho_s(s, x_1)\, \rho_s(s, x_2)\, \rho_s(s, x_3) \ ,$$

$$F = \pi^3 \int_0^\infty \frac{dx}{x^{5/2}} \frac{dy}{y^{3/2}} \rho_s(s,x)\, \rho_s(s,y) \int dz_1 dz_2\, \theta(z_1 - z_2) \times$$

$$\times \exp\left(-\frac{z_1^2}{4x} - \frac{z_2^2}{4y}\right) . \qquad (3.14)$$

The values of the correction coefficients depend on the structure of the soft component of the potential only and are given by equations, analogous to the eikonal phase at zero impact parameter $\chi(0)$, that is in full agreement with the interaction picture proposed in Sec. 1.

4. To conclude with, consider the application of the formulae obtained. As to the quasipotential $g(s, \vec{p}-\vec{k})$ we will choose it to be the Gaussian one with a small additional component, satisfying the condition (1.3), that is given by the density function:

$$\rho(s,x) = g\delta(x-a) + \frac{h}{4ip^2} \frac{1}{s^M} x^\kappa e^{-bx} ; \ \kappa > 0 . \qquad (4.1)$$

Small angle scattering and soft rescatterings, which are the origin of corrections in the case of large angle scattering, are dominated by the region $s \to \infty$; x-fixed, hence:

$$\rho_s(s, x) = g\delta(x - a) \qquad (4.2)$$

and the correction coefficients are given by the equations:

$$A_1 = \pi^2 \frac{g}{a} , \quad A_2 = \pi^{3/2} \frac{g}{a^{3/2}} ,$$

$$A_3 = \pi^2 \frac{g}{a^2} , \quad B_1 = \pi^{7/2} \frac{g^2}{\sqrt{2}\, a^{5/2}} ;$$

$$B_2 = \pi^3 \frac{g^2}{2a^3} , \quad F = \pi^4 \frac{g^2}{2a^3} ,$$

$$C_1 = \pi^5 \frac{g^3}{\sqrt{3}\, a^4} . \qquad (4.3)$$

Hard scattering is dominated by the region $s \to \infty$, $xs = \eta$ -fixed, hence:

$$\rho_h(s,x) = \frac{h}{4ip^2} \frac{1}{s^M} x^\kappa e^{-bx} . \qquad (4.4)$$

Substituting (4.4) into (2.4), (3.12) and (3.13) we finally get:

$$T(s,t) = \frac{h\Gamma(\kappa+1)}{s^M} e^{2i\chi(0)} \left[\frac{1}{(-t+b)^{\kappa+1}} + \frac{1}{(-u+b)^{\kappa+1}} \right] \times$$

$$\times \Big\{ 1 + \frac{1}{ip} [2A_2(2\kappa+3) - 12B_1] + \frac{1}{p^2} [\kappa^2 (A_3 - 8A_2^2) +$$

$$+ \kappa(48A_2 B_1 - 40A_2^2 + A_3) - 45A_2^2 + 75A_2 B_1 -$$

$$- A_3 - 72B_1^2 + 64C_1 - 14B_2 + 4F + 2m^2 A_1] \Big\} -$$

$$- \frac{h\Gamma(\kappa+1)}{s^M} e^{2i\chi(0)} \frac{2A_3(\kappa+1)^2}{p^2} \times$$

$$\times \left[\frac{1}{(1-z)(-t+b)^{\kappa+1}} + \frac{1}{(1+z)(-u+b)^{\kappa+1}} \right] . \qquad (4.5)$$

The parameter a is the radius squared of the Gaussian component and, consequently, determines the hadron size, that gives the estimate:

$$a \simeq 5\,(GeV)^{-2}. \tag{4.6}$$

The fits of elastic hadron scattering data give the characteristic values of eikonal phase and parameter κ, as follows

$$2i\chi(0) \simeq -1; \quad \kappa \simeq 2.5. \tag{4.7}$$

For the values of parameters (4.6), (4.7) the expansion of the amplitude for $\theta=90°$ scattering in the center of mass frame is of the form

$$T(s, -\frac{s}{2}+2m^2) = T_0\,(1+\frac{0.875}{ip} - \frac{1.013}{p^2}). \tag{4.8}$$

Here:

$$T_0 = \frac{2h\Gamma(\kappa+1)\,e^{2i\chi(0)}}{s^M(\frac{s}{2}-2m^2+b)^{\kappa+1}}$$

and p is measured in $(GeV/c)^2$.

For the differential cross section we obtain:

$$\frac{d\sigma}{dt} \simeq (\frac{d\sigma}{dt})_0\,[\,1-\delta\,], \tag{4.9}$$

where:

$$\delta \simeq \frac{1.55}{p^2}.$$

The experimental data are now available in the energy region of s up to 50 $(GeV)^2$, consequently

$$\delta \geq 15\%. \tag{4.10}$$

Thus, the corrections to the leading asymptotic term are large and should necessarily be taken into account in the interpretation of data.

So, the investigation of the corrections to the leading asymptotic term of large angle scattering amplitude leads to the following conclusions.

First, the corrections slightly destroy exact automodelity (1.1), that is, the exponent n becomes a function of energy and scattering angle, but with growing energy the deviations from (1.1) become negligible.

Second, the scalar case discussed is of a pure theoretical interest, but in the presence of spin the corrections will result in nontrivial polarization effects, even if up to the leading order the polarization parameter is zero.

Third, it is worth mentioning that the estimation of the corrections requires no additional parameters and allows us to verify the assumptions about their origin from soft rescatterings.

The method proposed can be effectively utilized for the description of real processes: meson-nucleon and nucleon-nucleon scattering. The leading asymptotic terms for these processes were discussed in ref. /7/.

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Appendix

In this appendix, we consider briefly the calculation of the correction integrals:

$$I = \int dF_m M_2$$

Choosing, for example, $M_2 = \frac{a_i}{q_i - i0}$ (in the expansion (3.4) there are terms of this type) we obtain:

$$I_{1,m,i} = \int dF_m \frac{a_i}{q_i - i0} =$$

$$= (2i)^m \int \prod_{j=1}^{m} (dx_j \rho_s(s,x_j) \frac{dq_j}{q_j - i0} d^2\vec{\Lambda}_{\perp j}) \frac{\alpha_i}{q_i - i0} \cdot$$

$$\times e^{-\Gamma_{m+1}(q) - \Gamma_{m+1}(\vec{\Lambda}_\perp)} .$$

The integral over $\vec{\Lambda}_{\perp j}$ is the Gaussian one and is easily calculated

$$\int \prod_{j=1}^{m} d^2\vec{\Lambda}_{\perp j}\, e^{-\Gamma_{m+1}(\vec{\Lambda}_\perp)} =$$

$$= \int \prod_{j=1}^{m} d^2\vec{\Lambda}_{\perp j}\, e^{-\tilde{C}_{ij} \vec{\Lambda}_{\perp i} \vec{\Lambda}_{\perp j}} =$$

$$= \frac{\pi^m}{\det \vec{C}} = \frac{\pi^m}{\prod_{j=1}^{m} x_j} .$$

For integration over q_j let us employ the representation:

$$e^{-x_k(q_k - q_{k-1})^2} =$$

$$= \frac{1}{2\sqrt{\pi x_k}} \int_{-\infty}^{\infty} dz_k\, e^{iz_k(q_k - q_{k-1}) - z_k^2/4x_k} .$$

Then, taking into account that

$$\int_{-\infty}^{\infty} dq\, e^{iqz} \frac{1}{q - i0} = 2\pi i \theta(z) ,$$

$$\int_{-\infty}^{\infty} dq\, e^{iqz} \frac{1}{(q - i0)^2} = -2\pi z\theta(z) ,$$

we get:

$$I_{1,m,i} = i(-2\pi^{3/2})^m \int_0^\infty \prod_{j=1}^{m} dx_j \rho_s(s,x_j) \times \qquad \text{(A-1)}$$
$$\times \int_{-\infty}^{\infty} \prod_{j=1}^{m} dz_j\, e^{-z_j^2/4x_j} \theta(z_1 - z_2) \ldots \theta(z_{m-1} - z_m) \theta(z_m)(z_i - z_{i+1}) \alpha_i .$$

The corrections to the total amplitude are defined by the sum of integrals (A-1). Thus, taking notice of:

$$\sum_{i=1}^{m-1} \alpha_i (z_i - z_{i+1}) + \alpha_m z_m = \sum_{j=1}^{m} \frac{z_j}{x_j}$$

we finally get:

$$I_1 = \sum_{m=1}^{\infty} \sum_{i=1}^{m} I_{1,m,i} =$$

$$= \sum_{m=1}^{\infty} \sum_{i=1}^{m} (-2\pi^{3/2} i) \int_0^\infty \frac{dy}{y^{5/2}} \rho_s(s,y) \int_0^\infty z dz\, e^{-z^2/4y} \times$$
$$\times \frac{1}{(i-1)!} [-2\pi^{3/2} \int_0^\infty \frac{dx}{x^{3/2}} \rho_s(s,x) \int_z^\infty d\zeta\, e^{-\zeta^2/4x}]^{(i-1)} \times$$
$$\times \frac{1}{(m-i)!} [-2\pi^{3/2} \int_0^\infty \frac{dx}{x^{3/2}} \rho_s(s,x) \int_0^z d\zeta\, e^{-\zeta^2/4x}]^{(m-i)} =$$

$$= -2\pi^{3/2} i \int_0^\infty \frac{dy}{y^{5/2}} \rho_s(s,y) \int_0^\infty z dz\, e^{-z^2/4y} \times$$

$$\times \exp[-2\pi^{3/2} \int_0^\infty \frac{dx}{x^{3/2}} \rho_s(s,x) \int_0^\infty d\zeta\, e^{-\zeta^2/4x}] =$$

$$= -4i\pi^{3/2} e^{i\chi(0)} \int_0^\infty \frac{dx}{x^{3/2}} \rho_s(s,x) = -4ie^{i\chi(0)} A_2 .$$

Other correction integrals are evaluated in the same manner. Their values for different functions $M_{2,m}$ are listed in the *table*.

Table

	$M_{2,m}$	I_1
1.	$\sum_{j=1}^{m} \frac{\alpha_i}{q_i - i0}$	$-4iA_2 e^{i\chi(0)}$
2.	$\sum_{i=1}^{m} q_i$	$i(2A_2 - 4B_1) e^{i\chi(0)}$
3.	$\sum_{1\le i<j\le m} \frac{\alpha_i \alpha_j}{(q_i - i0)(q_j - i0)} + \sum_{j=1}^{m} \frac{\alpha_j^2}{(q_j - i0)}$	$(-8A_2^2 + 2A_3) e^{i\chi(0)}$
4.	$\sum_{1\le i<j\le m} q_i q_j$	$(-4A_2^2 + 8A_2 B_1 + 8C_1 - 8B_1^2) e^{i\chi(0)}$
5.	$\sum_{i=1}^{m} q_i^2$	$(-4A_2^2 - 4B_2 + 8C_1) e^{i\chi(0)}$
6.	$\sum_{i,j=1,\, i\neq j}^{m} \frac{\alpha_i q_j}{q_i - i0}$	$(16A_2^2 - 16A_2 B_1) e^{i\chi(0)}$
7.	$\sum_{i=1}^{m} \alpha_i$	$(-2A_3 + 4F) e^{i\chi(0)}$

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