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ДУБНА

E2 - 11535

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Submitted to the XIX International Conference on High Energy Physics (Tokyo, 1978) and to the International Seminar on High Energy Physics and Field Theory, Serpukhov, 11–17 July 1978.

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Процессы с большой передачей импульса в квантовой хромодинамике

Предложен единый подход к исследованию инклюзивных процессов с большой передачей импульса в квантовой хромодинамике, позволяющий обосновать справедливость модифицированной партонной модели для процесса рождения массивных лептонных пар во всех порядках теории возмущений. Подход применим также к процессам упругого рассеяния на большие углы при высоких энергиях, в которых участвуют адроны - бесцветные связанные состояния кварков. В качестве примера вычислена асимптотика электромагнитного формфактора пиона в КХД.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

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High Momentum Transfer Processes in QCD

A unified approach to the investigation of inclusive high momentum transfer processes in the QCD framework is proposed. A modified parton model (with parton distribution functions depending on an additional renormalization parameter) is shown to be valid in all orders of perturbation theory. The approach is also applicable for studying wide-angle elastic scattering processes of colourless bound states of quarks (the hadrons). The asymptotical behaviour of pion electromagnetic form factor is calculated as an example.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1978

Introduction

Quantum chromodynamics (QCD) is considered to be a leading candidate for the fundamental theory of strong interactions, and the application of QCD to high momentum transfer processes attracts now a considerable attention[1-14]. Just in these processes the unique property of the QCD, asymptotic freedom, reveals itself. The task is to utilize this property for justification of parton model ideas and to work out a systematic way of calculating the higher order corrections to parton model results.

It is well known that for deep inelastic scattering the approach based on the operator product expansions leads to a modified parton model, with the parton distribution functions depending on an additional renormalization parameter (see, e.g.[15]). In this paper we present the results of investigations (they have been published in a short form in refs.[8,14]) which allow us to assert that the use of the modified parton model for investigation of other high momentum transfer processes (massive lepton pair production in hadronic collisions, high-p_T hadron production, etc.) can be justified from the QCD viewpoint. The approach developed to prove the above statement appears to be also applicable to exclusive wide-angle lepton-hadron and hadron-hadron reactions at high energies. The colour neutrality of the hadrons plays a very important role in this consideration. The result is the parton model of a new type, which uses the so-called parton wave functions[14].

Deep Inelastic Scattering

Parton model contribution is given by the well-known "handbag"-diagram (fig. 1a)

$$T^{\mu\nu}(P,q)=\frac{1}{4\pi}\int e^{iqx}\langle P|:\bar{\psi}(-\tfrac{x}{2})\gamma^{\mu}S^{c}(x)\gamma^{\nu}\psi(\tfrac{x}{2}):|P\rangle d^4x \quad (1)$$

Expanding into the Taylor series

$$:\bar{\psi}(-\tfrac{x}{2})\Gamma\psi(\tfrac{x}{2}):=\sum_{n=0}^{\infty}:\bar{\psi}(0)\overleftrightarrow{\partial}_{\mu_1}\ldots\overleftrightarrow{\partial}_{\mu_n}\Gamma\psi(0):\frac{x^{\mu_1}\ldots x^{\mu_n}}{n!} \quad (2)$$

we obtain an analog of the OPE. But the matrix element of the operator $:\bar{\psi}(0)\overleftrightarrow{\partial}^{n}\Gamma\psi(0):$ possesses in renormalizable theories the divergences which cannot be removed by the ordinary R-operation. One must add the recipe of the composite operator renormalization characterized by a parameter μ.

One encounters a similar difficulty in trying to formulate a prescription of calculating the higher order corrections to the parton model results. In principle, one may consider any diagram having a parton and a virtual photon in the initial state to give a higher order contribution to the cross-section of the parton subprocess. On the other hand, there is a risk of double counting the same diagram (one may believe, for instance, that the outlined part of the diagram 1b must be added to the parton distribution function, i.e., that the diagrams 1a and 1b are identical. It is also worth noting that the contribution of the diagram 1c is proportional to $\ln Q^2/K^2$, where K is the parton momentum. In accordance with the parton model ideas, the partons are only slightly virtual: $|K^2|\ll m_p^2$, hence in the calculation of the corrections to the parton subprocess cross section one is faced with the infrared problems[2].

These difficulties are caused by the absence of a distinct definition of the parton distribution functions and of the parton subprocess cross-section. It is implied, however, that the parton distribution functions accumulate the information about the large distance dynamics (that is, they must be regularized in the ultraviolet region) whereas the parton subprocess is a short distance phenomenon and the corresponding cross-section must have an infrared regularization. One can choose the boundary between small and large distances by convenience. It is, of course, necessary for a self-consistency of the whole scheme that at small distances (or at large virtual momenta $-K^2\geq\mu^2$) the effective coupling constant is small: $\alpha_s(-K^2)/\pi\leq\alpha_s(\mu^2)/\pi\ll 1$. In this region the value of the boundary μ is arbitrary, and the physical results must not depend on a particular choice of μ.

To characterize the virtuality of a momentum going through a given line, it is more convenient to use the α-representation[16] rather than the momentum one. The propagator is given in the α-representation by the formula

$$\frac{1}{m^2-p^2-i\varepsilon}=\int_0^{i\infty}d\alpha\,\exp\{\alpha(p^2-m^2+i\varepsilon)\}, \quad (3)$$

i.e., small α corresponds to large p^2. The contribution of any diagram can be written as an integral over the α-parameters of all lines of the diagram:

$$T(p_1,\ldots,p_n;m)\sim\int_0^{i\infty}\frac{\prod_\sigma d\alpha_\sigma}{D^2(\alpha)}\exp\left\{\frac{Q(\alpha,p)}{D(\alpha)}-\right. \quad (4)$$

$$\left.-\sum_\sigma\alpha_\sigma(m_\sigma^2-i\varepsilon)\right\}\;G(p_1,\ldots,p_n;m),$$

where the functions D, G, Q are determined by the diagram topology. Let us consider an arbitrary diagram (fig. 2). It contributes both to the parton distribution function (large-α contribution) and to parton subprocess cross-section (small-α contribution). Our task is to separate these contributions. We consider first the variant when the incoming large momentum q goes through all the lines of the diagram V, that is $k_i^2 \gtrsim Q^2$ for any line. In the α-parameters language this means that $\lambda_V \equiv \sum_\sigma \alpha_\sigma < 1/\mu^2$ (where $\mu \sim Q$). In this regime the diagram gives the identity contribution into the parton distribution function. All the momenta have an order of Q, hence one can find the asymptotical behaviour of this contribution with the help of dimensional analysis. In a theory with the dimensionless coupling constant

$$F_2(x, Q^2) \sim Q^{2-\Sigma d_i} \, Q^{\Sigma s_i} = Q^{2-\Sigma t_i}, \qquad (5)$$

where t_i are twists (dimension in mass units minus spin) of the i-th external field. The factor Q^{s_i} is due to the fact that the gluon line can add the factor $P_\mu \sim Q$, whereas the quark line gives $u(P) \sim Q$, etc. The particles with spins equal to 0 or 1/2 have $t_i = 1$. That is why the subprocess described by the diagram 2a has the asymptotical behaviour $F_2^{(V)} \sim Q^0$, whereas that described by fig. 2b is damped by the factor $1/Q^2$.

In the remaining part of the region of integration over α-parameters there is a subregion, corresponding to a flow of large momentum through a subdiagram V_1 (fig. 2): $\lambda_{V_1} < 1/\mu^2$, whereas all the momenta corresponding to the lines lying outside V_1 are small: $\lambda_{V \setminus V_1} > 1/\mu^2$. Then the subgraph V_1 contributes to the subprocess and the contribution of the subgraph $V \setminus V_1 \equiv \overline{V}_1$ must be related to the parton distribution functions. Note, that as dictated by eq. (5) the contribution of the subgraph V_1 in the diagram 2b is $F_2^{(V_1)} \sim Q^0$ rather than $1/Q^2$. Hence to calculate asymptotical behaviour, it is necessary to consider only the short-distance contribution of the subgraphs having a minimal possible number of external lines. In vector gluon theories (in the Feynman gauge) the vector field A_μ has zero twist and this produces some complications which we will discuss later on.

Then one must consider the region $\lambda_{V_2} < 1/\mu^2$, but $\lambda_{\overline{V}_2} > 1/\mu^2$, $\lambda_{\overline{V}_2 \setminus \overline{V}_1} > 1/\mu^2$; etc. Very important there is a factorization of large- and small-distance contributions.

After applying the procedure described above, the contribution of any diagram can be written in the coordinate representation in the following form:

$$\int d\xi \, d\eta \sum_v f_v(x, \xi, \eta; \mu^2) \chi_{\bar{v}}(\xi, \eta, a, b; \mu^2) + R_V(x, a, b) \qquad (6)$$

The function f_v is the result of small-α integration ($\lambda_v < 1/\mu^2$) whereas the function $\chi_{\bar{v}}$ is that of large-α integration. The function R_V gives $O(1/Q^2)$ contribution into asymptotical form of F_2.

Eq. (6) is valid for any diagram, hence summing over all relevant diagrams, we get

$$\sum_i \int d\xi \, d\eta \, f_i(x, \xi, \eta : \mu^2) \, \chi_i(\xi, \eta, a, b; \mu^2) + R(x, a, b). \qquad (7)$$

The functions f_i, χ_i correspond to the following matrix elements of the Green functions

$$f_i(x,\xi,\eta;\mu^2) = \langle 0|T\left[J(x)J(0):j_i(\xi)j_i(\eta):\right]|0\rangle\Big|_{\mu^2, IR} \quad (8)$$

$$\chi_i(\xi,\eta,a,b;\mu^2) = \langle 0|T\left[:\varphi_i(\xi)\varphi_i(\eta): \Phi(a)\Phi(b)\right]|0\rangle\Big|_{\mu^2, UV}, \quad (9)$$

where φ_i, j_i are the "parton" fields and $\Phi(a)$ are those of external particles. The function f is by construction regularized in the infrared region, whereas the function χ is regularized in the ultraviolet one. The generalization of eqs. (8), (9) for spinor fields is trivial. To consider the particles which are the bound states of n fundamental constituents, we change

$$\chi(\xi,\eta,a,b;\mu^2) \to \chi(\xi,\eta,a_1,\dots,a_n;b_1\dots b_n;\mu^2) =$$

$$= \langle 0|T\left[:\varphi_i(\xi)\varphi_i(\eta): \Phi(a_1)\dots\Phi(a_n)\Phi(b_1)\dots\Phi(b_n)\right]|0\rangle. \quad (10)$$

Applying a standard method[17] (using the expansion $1 = \sum |n\rangle\langle n|$) one obtains matrix elements

$$\mathrm{Reg}_{\mu^2, UV} \langle P|:\varphi_i(\xi)\varphi_i(\eta):|P\rangle. \quad (11)$$

The bilocal operator $\mathrm{Reg}:\varphi(\xi)\varphi(\eta):$ remains finite in the limit $\xi \to \eta$, because the divergences of the matrix element $\langle P|:\varphi(0)\partial_{\mu_1}\dots\partial_{\mu_n}\varphi(0):|P\rangle$ are removed by the subtraction procedure Reg_{μ^2}. A choice of the subtraction procedure is not unique, but it is, of course, necessary that the recipes of the ultraviolet regularization (for the χ -function) and of the infrared one (for the f -function) must be co-ordinated with each other, i.e., the resulting asymptotical behaviour must not depend on μ. In particular, Reg_{μ^2} may be considered as the dimensional regularization $d^4k \to d^{4-2\varepsilon}k\,(\mu^2)^{\varepsilon}$ plus the 't Hooft's renormalization (the removal of poles in ε).

A representation (8) is nothing but the operator product expansion (OPE) on the light cone

$$T\left[J(x)J(0)\right] = \sum_i \int d\xi d\eta\, f_i(x,\xi,\eta;\mu^2)\, O_i(\xi,\eta;\mu^2) + R(x) \quad (12)$$

Expanding the bilocal operator $O_i = :\varphi_i(\xi)\varphi_i(\eta):$ over the local ones, we obtain the OPE in the standard form[18]

$$T\left[J(x)J(0)\right] = \sum_{i,n} F_{i,n}(x^2,\mu^2)\, x^{\mu_1}\dots x^{\mu_n} O^i_{\mu_1\dots\mu_n}(0,\mu^2) + R(x). \quad (13)$$

The transition from the OPE (13) to a modified parton model is achieved by identifying the reduced matrix elements of the local operators with the moments of the parton distribution functions[15]

$$\frac{i^{n-1}}{2}\langle P|\bar\psi_a\{\gamma_{\mu_1}\partial_{\mu_2}\dots\partial_{\mu_n}\}\psi_a|P\rangle = \{P_{\mu_1}\dots P_{\mu_n}\}$$
$$\cdot \int_0^1 \frac{dx}{x}\, x^n \left[f_a(x,\mu^2) + (-1)^n f_{\bar a}(x,\mu^2)\right] \quad (14)$$

Due to a well-known direct relation between the asymptotical behaviour in the Bjorken limit and the light-cone singularities, we may take into account only the contribution of the lowest twist operator. From eqs. (1) and (13) it follows that

$$F_2(x,Q^2) \sim \mathrm{Disc} \sum_{i,n} \int e^{i(qx_{\parallel})} a_{i,n}(\mu^2)(Px_{\parallel})^n F_n^i(x_{\parallel}^2 - x_{\perp}^2, \mu^2)$$

$$\cdot\, d^2x_{\parallel} d^2x_{\perp} \sim$$

$$\sim \sum_{i,n} \int e^{i(qx_{\parallel})} (Px_{\parallel})^n \varphi_n^i(x_{\parallel}^2, \mu^2)\, d^2x_{\parallel}, \qquad (15)$$

where $(x_{\perp}P) = (x_{\perp}q) = 0$. The matrix element of a higher twist operator must contain a dimensional parameter M

$$\langle P|\, O^{(i)}_{\mu_1\ldots\mu_n} |P\rangle = M^{t_i - 2} \{P_{\mu_1}\ldots P_{\mu_n}\}\, b_n^{(i)}(\mu^2) \qquad (16)$$

and its contribution in accordance with (15) is suppressed by a factor $(M/Q)^{t_i-2}$.

Gauge Theories

The analysis of gauge theories is complicated by the fact that in the simplest gauge (i.e., the Feynman gauge) the vector field A_μ has zero twist. In this case a subprocess can be described by a subgraph v with an arbitrary number of external gluon lines (fig. 3a).

Let us fix the form of initial subgraphs v_0 and $\bar{v}_0$ (fig. 3b). Joining the lines of the subgraph v_0 with the lines of the subgraph $\bar{v}_0$ in all possible ways we obtain the admissible combinations, and then it is necessary to sum over all the possibilities. Every gluon line adds the field $A_\mu(z)$ into matrix element (10) and modifies the propagator $S^c(x_\alpha - x_\beta)$ belonging to the function f_{v_0}:

$$S^c(x_\alpha - x_\beta) \to g\int d^4z\, A^a_\mu(z)\, S^c(x_\alpha - z)\gamma^\mu \tau_a S^c(z - x_\beta) \qquad (17)$$

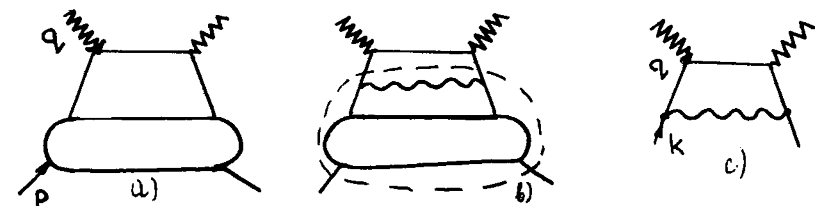


Fig. 1. The diagrams describing higher order corrections to the parton result for deep inelastic scattering.

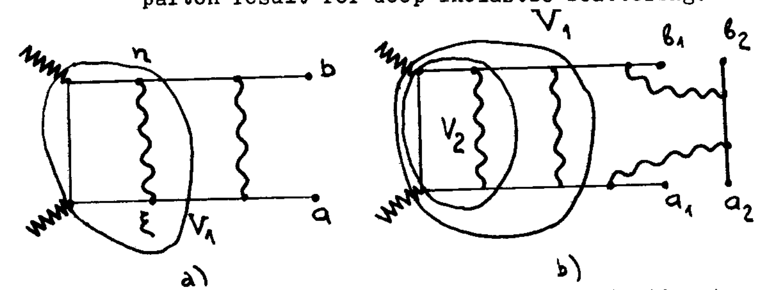


Fig. 2. The structure of leading terms contributing to asymptotical behaviour of the structure functions.

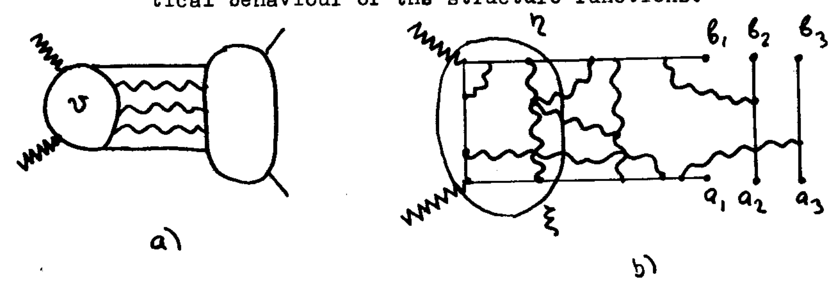


Fig. 3. The structure of leading terms in a gauge theory.

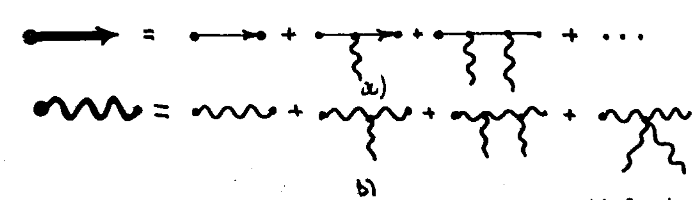


Fig. 4. a) The propagator of a spinor particle in an external gluonic field.

b) The propagator of a vector particle in an external gluonic field.

where τ_a is the matrix of the gauge group in the quark representation. Note that the sum (fig. 4a)

$$\mathcal{S}^c(x_\alpha, x_\beta) = S^c(x_\alpha - x_\beta) + g\int S^c(x_\alpha - z)\gamma^\mu \hat{A}_\mu(z) S^c(z - x_\beta) + \dots \quad (18)$$

is the propagator of a spinor particle in an external gluon field [19], i.e., the solution of the equation

$$(i\hat{\mathcal{D}}_\mu \gamma^\mu - m)\,\mathcal{S}^c(x_\alpha, x_\beta) = -\delta^4(x_\alpha - x_\beta), \quad (19)$$

where $\hat{\mathcal{D}}_\mu = \partial_\mu - ig\hat{A}_\mu$ is the covariant derivative acting on the quark fields, and $\hat{A}_\mu = A^a_\mu \tau_a$. The solution of the eq. (19) can be written as

$$\mathcal{S}^c(x_\alpha, x_\beta) = S^c(x_\alpha - x_\beta)\left[T_c \exp\left\{ig\int_{x_\beta}^{x_\alpha} \hat{A}_\mu(z)\,dz^\mu\right\}\right]\cdot [1 + O(G_{\mu\nu})], \quad (20)$$

where $G_{\mu\nu} = [D_\mu, D_\nu]$ is the tensor of the gluonic field, and T_c means that the integral must be path-ordered. Thus, every propagator S^c entering into $f_{v_0}(\xi, \eta; x)$ must be substituted by $\mathcal{S}^c$. This means that the parton subprocess takes place in the gluonic field of the hadron rather than in the empty space. Any operator of the $:OG\dots G:$ type has a twist higher than that of O. Therefore we can neglect $O(G_{\mu\nu})$ terms and find that in an Abelian theory all the exponentials are either summed up into a factor $\exp\left[ig\int_\eta^\xi \hat{A}_\mu(z)dz^\mu\right]$ for a quark operator or cancelled for a gluon one. Hence the function $f_{v_0}(\xi, \eta; x)$ remains unchanged, but in place of $\bar{\psi}(\xi)\psi(\eta)$ there appears a gauge-invariant bilocal operator

$$\mathcal{O}(\xi, \eta; \mu^2) = \mathrm{Reg}_{\mu^2} :\bar{\psi}(\xi) \exp\left[ig\int_\eta^\xi A_\mu(z)dz^\mu\right]\psi(\eta): \quad (21)$$

Using the Baker-Hausdorf theorem [19] one can expand it over gauge-invariant local operators resulting from the operators (14) by replacement $\partial_\mu \to D_\mu$.

In a non-Abelian theory the gluonic propagator is also modified (fig. 4b):

$$g_{\mu\nu}\delta_{ab}D^c(x_\alpha - x_\beta) \to \mathcal{D}^{ab}_{\mu\nu} = g_{\mu\nu} D^c(x_\alpha - x_\beta)\cdot$$

$$\left[T_c \exp\left\{ig\int_{x_\beta}^{x_\alpha} \tilde{A}_\mu(z)\,dz^\mu\right\}\right]_{ab}\{1 + O(G_{\mu\nu})\}, \quad (22)$$

where $\tilde{A}_\mu \equiv A^a_\mu \sigma_a$, and σ_a are the matrices of the gauge group in the adjont representation. To unite the exponentials corresponding to neighbouring spinor lines one must perform the commutation

$$\left(T_c \exp\left[ig\int_{x_\beta}^{x_\alpha} \hat{A}_\mu(z)\,dz^\mu\right]\right)_{AB} (\tau^a)_{Bc} =$$

$$= (\tau^b)_{AB}\left(T_c \exp\left[ig\int_{x_\beta}^{x_\alpha} \hat{A}_\mu(z)\,dz^\mu\right]\right)_{Bc}\cdot \left(T_c \exp\left[ig\int_{x_\beta}^{x_\alpha} \tilde{A}_\mu(z)\,dz^\mu\right]\right)_{ba} \quad (23)$$

For a subgraph with spinor external lines the additional exponential factors appear after the commutation (23) cancel completely with those entering into the representation (22) for $\mathcal{D}_{\mu\nu}$. For a subgraph with gluon external lines, on the contrary, this gives a gauge-invariant bilocal operator

$$:G_{\mu\nu}(\xi)\left(T_c \exp\left[ig\int_\eta^\xi \tilde{A}_\mu(z)dz^\mu\right]\right) G^\nu_\lambda(\eta): \quad (24)$$

We have obtained a well-known result [20] that one must use gauge-invariant operators in the operator product expansion.

There exists a class of gauges in which a vector theory does not differ essentially from more simpler non-gauge theories. In a gauge theory

$$F_2(x,Q^2) \sim \mathrm{Disc} \sum_n \frac{1}{Q^{2n}} q^{\mu_1}\dots q^{\mu_n} \langle P|\bar\psi\{\gamma_{\mu_1}(\partial_{\mu_2}-igA_{\mu_2})\dots (\partial_{\mu_n}-igA_{\mu_n})\}\psi|P\rangle . \qquad (25)$$

From eq. (25) it follows that in the axial gauge, where $(q,A)=0$, one can use ∂_μ in place of D_μ. That means in this gauge (and in any gauge of the following type: $(q,A)+\alpha(PA)=0$) the contribution of the configuration fig. 3a is $O(1/Q^2)$ /21/. One can use the gauge $(\xi A)-(\eta A)=0$ for a gauge-invariant bilocal operator $O(\xi,\eta)$. So far as one can use the straight line connecting the points ξ and η as the line of integration in eq. (22), in this gauge one also obtains the operator of the same type as in a non-gauge theory.

Massive Lepton-Pair Production

The procedure we have used above is in essence a reordering of perturbation series terms according to a definite recipe. As a result, we have obtained the representation which has been proved to be very useful for an analysis of the asymptotical behaviour of the process investigated. Now we are going to apply the same approach for an analysis of asymptotical properties of the massive lepton-pair production in hadronic collisions: $AB\to\mu^+\mu^-X$.

An analog of eq. (1) in this case is

$$W \sim \int e^{iQx} \langle P_A|:\varphi(x)\varphi(0):|P_A\rangle\langle P_B|:\varphi(0)\varphi(x):|P_B\rangle d^4x . \qquad (26)$$

In the coordinate representation a dashed line (fig. 5a) denotes a factor $I(x)\equiv 1$. It is evident that one cannot take the limit $x\to 0$ in the matrix elements of eq. (26). Hence we must construct a subtraction procedure for diagrams (fig. 5b).

We consider first zero spin gluons. We assume also that the relevant parton subprocess is described by the subgraphs with 4 external parton lines. As a result, we obtain a representation

$$W\sim\int e^{iQx}d^4x\Big[\int\langle P_A|O_i(\xi,\eta;\mu^2)|P_A\rangle\langle P_B|O_j(\alpha,\beta;\mu^2)|P_B\rangle f_{ij}(x,\xi,\eta,\alpha,\beta;\mu^2)\,d\xi\, d\eta\, d\alpha\, d\beta + R(x)\Big]. \qquad (27)$$

The configurations contributing to R are shown in fig. 6, where the outlined subgraphs are those giving small $-\alpha$ contribution. Expanding the bilocal operators over the local ones we obtain

$$W\sim\int d^4x\, e^{iQx}\sum_{ijmnk}F^{mnk}_{ij}(x^2,\mu^2)(x^2)^k\delta^{\mu_1}_{\nu_1}\dots\delta^{\mu_k}_{\nu_k} x^{\mu_{k+1}}\dots x^{\mu_n} x_{\nu_{k+1}}\dots x_{\nu_m}\langle P_A|O^i_{\mu_1\dots\mu_n}(0)|P_A\rangle\langle P_B|O_j^{\nu_1\dots\nu_m}(0)|P_B\rangle . \qquad (28)$$

The functions F^{mnk}_{ij} in perturbation theory have the same (up to logarithms) singularities on the light cone. The contributions of the diagrams fig. 6 correspond to weaker light-cone singularities of the F -functions. Hence, the utility of the representation (27) depends on the relation between the light-cone singularities and the asymptotical behaviour of the function W. By analogy with eq. (16) we rewrite eq. (28):

The applicability of the light-cone analysis for the process $AB\to\mu^+\mu^-X$ was often called in question /22/, mainly because

$$W \sim \sum_{ij,mnk} \int e^{i(Q_\parallel x_\parallel)+i(Q_\perp x_\perp)} (x_\parallel P_A)^{m-k} (x_\parallel P_B)^{n-k} \cdot$$

$$\cdot (x^2)^k (P_A P_B)^k F^{kmn}_{ij}(x_\parallel^2 - x_\perp^2; \mu^2)\, a^i_m(\mu^2)\, a^j_n(\mu^2)\, d^2x_\parallel d^2x_\perp \qquad (29)$$

Unlike deep inelastic scattering, there is an additional factor $\exp i(Q_\perp x_\perp)$. Hence a higher twist contribution can get as an additional factor either $(M^2/Q_\perp^2)^N$ or $(M^2/Q_\parallel^2)^L$. It is clear that the representation (27) may be useless in the region $Q_\perp \sim M$ because one must take into account operators having arbitrary high twists. One can avoid these complications either by integration over $Q_\perp$, then

$$\tilde{W} \equiv \int W(P_A, P_B, Q)\, d^2Q_\perp \sim \sum_{ij,mnk} \int d^2x_\parallel\, e^{i(Q_\parallel x_\parallel)} \qquad (30)$$

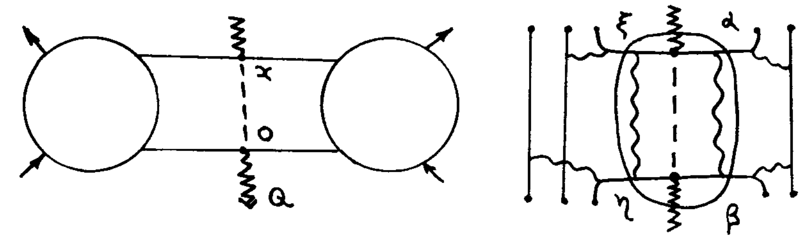
$$F^{mnk}_{ij}(x_\parallel^2, \mu^2)(x_\parallel P_A)^{m-k}(x_\parallel P_B)^{n-k}(x_\parallel^2)^k s^k$$

(cf. eq. (16)), or by investigating only the limit $Q_\perp^2 \sim Q_\parallel^2 \sim \sim s \gg M^2$, i.e.,the production of massive pairs at high transverse momentum. It is much more convenient to use a formfactor $W(\tau, Q^2)$ related to the total cross-section $d\sigma/dQ^2$ of producing the pair with mass Q :

$$W(\tau, Q^2) = \int W(P_A, P_B, k)\, \delta^+(k^2 - Q^2)\, \Theta(P_A^0 + P_B^0 - k^0)\, d^4k \qquad (31)$$

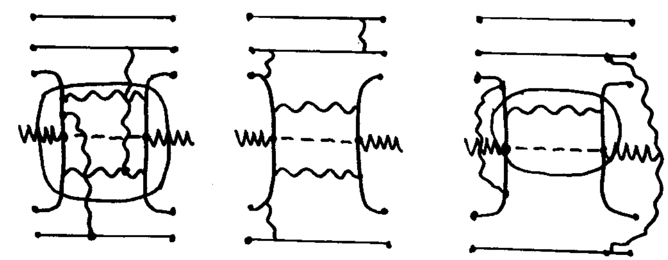
In these cases the contribution of higher twist operators is suppressed by a factor $(M^2/Q^2)^N$. With the help of a more detailed analysis in the α -representation one can show that the contribution from fig. 6c diagramms is also suppressed.*)

*) The same holds for gauge theories.



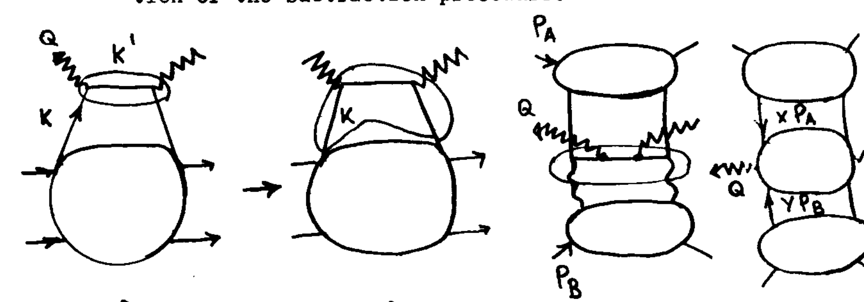
a) b)

Fig. 5. The process $AB \to \mu^+\mu^- X$.



a) b) c)

Fig. 6. The structure of different contributions after application of the subtraction procedure.



a) b) c) d

Fig. 7. a–c) The diagrams describing a bremsstrahlung of a massive virtual photon.

d) A generalized Drell-Yan process.

the functions $F(x^2)$ entering into eq. (28) have in perturbation theory only logarithmical singularities on the light cone which are much weaker than those corresponding to a contribution of twist-2 operators (see (13)). A matrix element $\langle P_A P_B | O_{\mu_1 \dots \mu_n} | P_A P_B \rangle$ is not equal to zero apriori, moreover it can possess an unpredictable dependence on S. It is easy to see that such operators nevertheless do not contribute to the cross-section of massive pair production.

Really, a produced quark must be on its "would be" mass shell, i.e., $(k')^2 = (k-Q)^2 \simeq 0$; $k^0 - Q^0 > 0$ (see fig. 7a). Hence only configurations having $k^2 \gtrsim Q^2$ do contribute to the cross-section. As a consequence, one must attribute the quark line corresponding to the momentum k to the coefficient function $F(x^2)$ rather than to a matrix element (i.e., distribution function). One must answer now the question of how to obtain a very massive virtual quark. We get back the same problem which we have tried to solve. We must obtain now a massive quark rather than a photon - this is just the difference. The only way to break this chain is to assume that at some stage a very large virtual mass is a result of fusion of two particles which have the momenta $x P_A$ and $y P_B$, respectively. A corresponding operator will consist at least of 4 elementary (parton) fields (fig. 7d). It makes sense to call such a configuration a generalized Drell-Yan mechanism. The bremsstrahlung contribution (fig. 7c) is then a possible radiation correction.

In gauge theories (in Feynman gauge) one must sum over gluons taking part in a parton subprocess (fig. 8a). Let us fix the number of gluons related to a particle A taking part

in the subprocess and sum over the gluons related to particle B (the gluon field of the A-particle will be denoted as A_μ, whereas that of B-particle, as B_μ).

The gluon lines going out of B-particle may be joined either with the internal lines of the initial subprocess or with the external lines going out of B-particle (fig. 8b)*). The insertions into an external spinor line give

$$\psi(\xi) \to \Psi(\xi) = \psi(\xi) + g \int d^4\xi_1 \, S^c(\xi - \xi_1) \hat{B}(\xi_1) \psi(\xi_1) + \dots \tag{32}$$

i.e., ψ turns into the field operator of a spinor particle in an external gluon field of hadron B. We write the solution to the equation $(i \hat{D}_\mu \gamma^\mu - m) \Psi = 0$ in the following way:

$$\Psi(\xi) = \psi(\xi) \left(T_c \exp\left[ig \int_{z_0}^{\xi} \hat{B}_\mu(z) dz^\mu \right] \right) \{ 1 + O(G_{\mu\nu}) \} \tag{33}$$

The point z_0 fixes the normalization condition $\Psi(z_0) = \psi(z_0)$. In the final answer z_0 disappears. Insertions into an external gluon line result in a replacement

$$A^a_\mu(\zeta) \to \mathcal{A}^a_\mu(\zeta) = A^b_\mu(\zeta) \left(T_c \exp\left\{ ig \int_{z_0}^{\zeta} \tilde{B}_\mu(z) dz^\mu \right\} \right)_{ba} \cdot \{ 1 + O(G_{\mu\nu}) \} . \tag{34}$$

After commutations of the exponential factor appearing in eqs. (20), (22), (33), (34) with τ - and σ -matrices, the gluonic factors cancel with each other and after summation over gluon lines going out of A-particle we obtain as a result the representation (27) in terms of the gauge-invariant bilocal operators (21), (24).

---

*) Remember that we consider either a) $d\sigma/dQ^2$, $Q^2 \sim s$ or b) $d\sigma/dQ^2 dQ_\perp^2$, $Q_\perp^2 \sim Q^2 \sim s$.

It is essential that we project on the color singlet operators $\bar{\Psi}\Psi$. If one projects on the color octet operator $\bar{\Psi}\tau^a\Psi$, there remains a factor $T_c \exp \int_{z_0}^{\xi} \tilde{A}_\mu(z)\, dz^\mu$ pointing out that double-logarithmic terms appearing in some diagrams do not cancel after summation over all relevant diagrams. This circumstance plays an important role in consideration of asymptotical behaviour of hadron electromagnetic form factors. The investigation of colourless bound states form factors proves out to be an easier task in some aspects than that of (coloured) quarks.

Thus, just as in deep inelastic scattering, taking into account the specific features of gluon fields results in additional factors $(T_c \exp[ig\int_{\eta}^{\xi} \hat{A}_\mu(z)dz^\mu])$. Hence the contributions of diagrams 6a in the axial gauge $(\xi A)-(\eta A)=0$ are suppressed by a factor $(1/Q^2)^N$ with respect to that of fig. 5b. The investigation of vector gluon theories in a properly chosen gauge does not differ greatly from that of pseudoscalar gluon theory, e.g., in a gauge $(PA)+\alpha(P'A)=0$ it is most easy to show that the diagrams 6c do not contribute in the leading logarithm approximation/7/.

Starting with relations similar to eq. (15) we obtain from eq. (27) hard scattering formulas

$$W(Q^2,\tau)=\int_0^1 \frac{dx}{x}\int_0^1 \frac{dy}{y}\sum_{a,b} f_{a/A}(x,\mu^2) f_{b/B}(y,\mu^2)\, w_{ab}(Q^2,\tau/xy;\mu^2) \tag{35}$$

$$W(Q^2,\tau,\tau_\perp)=\int_0^1 \frac{dx}{x}\int_0^1 \frac{dy}{y}\sum_{a,b} f_{a/A}(x,\mu^2) f_{b/B}(y,\mu^2)\, w_{ab}(Q^2,\tau/xy,\tau_\perp/xy;\mu^2),$$

where $\tau=Q^2/S$, $\tau_\perp=Q_\perp^2/S$. The functions w_{ab} describe a parton subprocess $ab\to\mu^+\mu^- X$. Taking $\mu=Q$ we obtain for the total cross-section

$$\left.\frac{d\sigma}{dQ^2}\right|_{AB\to\mu^+\mu^-X}=\frac{4\pi\alpha^2}{3Q^4N_c}\tau\int_0^1\frac{dx}{x}\int_0^1\frac{dy}{y}\sum_a e_a^2\,\cdot$$

$$\Big\{ f_{a/A}(x,Q^2) f_{\bar{a}/B}(y,Q^2)\,\delta(1-\tau/xy)+$$

$$+\left[f_{a/A}(x,Q^2)+f_{\bar{a}/A}(x,Q^2)\right] f_{g/B}(y,Q^2)\cdot\theta\left(1-\frac{\tau}{xy}\right)\cdot T^c(R)$$

$$\cdot\frac{\alpha_s(Q)}{4\pi}\left[\left(1+\frac{3\tau}{xy}\right)\left(1-\frac{\tau}{xy}\right)+2\left(\left(\frac{\tau}{xy}\right)^2+\left(1-\frac{\tau}{xy}\right)^2\right)\ln\frac{(xy-\tau)^2}{xy\tau}\right]$$

$$+\{A\leftrightarrow B\}\Big\}\ \{1+O(\alpha_s(Q))\}. \tag{36}$$

For the differential cross-section $d\sigma/dQ^2dQ_\perp^2$, in the region $Q_\perp^2\sim Q^2\sim S$, taking $\mu=Q_\perp$ *), we have

$$\left.\frac{d\sigma}{dQ^2dQ_\perp^2}\right|_{AB\to\mu^+\mu^-X}=\frac{4\pi\alpha^2}{3Q^6}\,\frac{\tau^2}{N_c}\,\frac{\alpha_s(Q_\perp)}{2\pi}\int_0^1\frac{dx}{x}\int_0^1\frac{dy}{y}$$

$$\frac{\theta(\sqrt{xy}-\sqrt{\tau_\perp}-\sqrt{\tau+\tau_\perp})}{\sqrt{(xy-\tau)^2-4xy\tau_\perp}}\sum_a e_a^2\left\{2C_2(R)\left[\frac{\tau^2+x^2y^2}{xy\tau_\perp}-2\right]\cdot\right.$$

$$f_{a/A}(x,Q_\perp^2) f_{\bar{a}/B}(y,Q_\perp^2)+T^c(R)\big(f_{a/A}(x,Q_\perp^2)+$$

$$+f_{\bar{a}/A}(x,Q_\perp^2)\big) f_{g/B}(y,Q_\perp^2)\left[1+\frac{3\tau}{xy}+\frac{xy-\tau}{\tau_\perp}\left(\left(\frac{\tau}{xy}\right)^2+\right.\right. \tag{37}$$

$$\left.\left.\left.+(1-\tau/xy)^2\right)\right]+\{A\leftrightarrow B\}\right\}\cdot[1+O(\alpha_s(Q_\perp))],$$

where $N_c=3$; $C_2(R)=4/3$, $T^c(R)=1/2$.

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*) In nowaday experiments $Q_\perp<Q$, that is why just the value of $Q_\perp$ characterizes both the effectiveness of α_s- expansion and the justifiability of neglecting the higher twists contribution.

Exclusive Processes

In the preceding sections only inclusive cross-sections have been considered. We assert that the amplitudes of deep elastic scattering processes, in which only colourless bound states of quarks appear in the initial and final states, are also infrared insensitive. No principal modifications are needed to apply the technique we have described earlier for an investigation of these processes. We will illustrate this by investigating the asymptotical behaviour of the pion electromagnetic form factor.

If the pion is treated as a colourless bound state of a quark and an antiquark, the diagrams shown in fig. 9a are summed into (cf. eq. (27))

$$\int d^4\xi\, d^4\eta\, d^4\alpha\, d^4\beta \langle P' | \bar{\psi}(\xi)\gamma_5\gamma_\mu \left(T_c \exp ig\int_\eta^\xi \hat{A}_\mu(z)dz^\mu\right)\cdot \psi(\eta)|0\rangle\Big|_{\mu^2} \cdot E^{\mu\nu}(\xi,\eta,\alpha,\beta,\mu^2)\langle 0|\bar{\psi}(\alpha)\gamma_5\gamma_\nu \cdot \left(T_c \exp ig\int_\beta^\alpha \hat{B}_\mu(z)dz^\mu\right)\psi(\beta)|P\rangle\Big|_{\mu^2} \tag{38}$$

We have written down only the axial projection, because it is just the leading term. The projection onto other structures as well as the configurations fig. 9b are suppressed by a factor (M^2/Q^2). Expanding over local operators

$$A_{\mu_1\ldots\mu_{n+1}} = \bar{\psi}\{\gamma_5\gamma_{\mu_1}\overset{\leftrightarrow}{\hat{D}}_{\mu_2}\ldots\overset{\leftrightarrow}{\hat{D}}_{\mu_{n+1}}\}\psi \tag{39}$$

which may be considered to be the pion interpolating fields, we obtain

$$F_\pi^{(A)}(Q^2) = \frac{1}{Q^2}\sum_{m,n=0}^{\infty} \bar{f}_m(\mu^2)\, E_{mn}\left(\frac{Q^2}{\mu^2}, g(\mu)\right) f_n(\mu^2), \tag{40}$$

where $\bar{f}_m$, f_n are reduced matrix elements of the operators (39):

$$(2i)^n \langle 0| A_{\mu_1\ldots\mu_{n+1}}(0;\mu^2)|P\rangle = \{P_{\mu_1}\ldots P_{\mu_{n+1}}\} f_n(\mu^2)\cdot [1+(-1)^n]/2 \tag{41}$$

The functions $\bar{f}_m$, f_n may be considered to be moments of parton wave functions /1./ $\varphi(\xi,\mu^2)$:

$$\int_0^1 \xi^n \varphi(\xi,\mu^2)\, d\xi = f_n(\mu^2). \tag{42}$$

The wave function $\varphi(\xi,\mu^2)$ describes the decomposition of the pion into two quarks having momenta $(1+\xi)P/2$ and $(1-\xi)P/2$, respectively (fig. 10a). Then

$$F_\pi^{(A)}(Q^2) = \int_0^1 d\xi \int_0^1 d\eta\, \varphi^*(\eta,\mu^2)\frac{1}{Q^2}E\left(\xi,\eta,\frac{Q^2}{\mu^2}, g\right)\varphi(\xi,\mu^2) \tag{43}$$

The function $\frac{1}{Q^2}E$ is the amplitude of the parton subprocess. Our wave function obeys a peculiar normalization condition

$$P_\nu\int_0^1 \varphi(\xi,\mu^2)\, d\xi = \langle 0|\bar{\psi}\gamma_5\gamma_\nu\psi|P\rangle = f_\pi P_\nu \tag{44}$$

because this matrix element is known from the data on decay $\pi\to\mu\nu$: $f_\pi \simeq 0.95\, m_\pi$ *). Formulas (39), (40) represent the sum of leading asymptotical forms (i.e., of the terms $(\ln Q^2)^N/Q^2$) of the diagrams 9a. This sum, of course, does not depend on the choice of parameter μ. Differentiating eq. (40) with respect to μ, we obtain the renormalization group equation

$$\sum_{m'=m}^{\infty}\sum_{n'=n}^{\infty}\left[\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right)\delta_{nn'}\delta_{m'm} + Z_{n'n}(g)\delta_{mm'} + \bar{Z}_{m'm}(g)\delta_{nn'}\right] E_{m'n'}\left(\frac{Q^2}{\mu^2}, g\right) = 0. \tag{45}$$

---

*) We use here the definition of f_π which differs from that used in ref. /12/ by a factor $i\sqrt{2}$.

Anomalous dimension matrix $Z_{nn'}$ is triangular in the basis chosen (39) rather than diagonal one. Furthermore $Z_{00}=0$, whereas other diagonal terms are negative: $Z_{nn}<0$ for $n\geq 1$. Hence, as $\mu^2\to\infty$ we have $f_n(\mu^2)\to f_n^{(0)}$, where $f_n^{(0)}$ is a vector satisfying

$$\sum_{n'=0}^{\infty} Z_{nn'} f_{n'}^{(0)} = 0 . \qquad (46)$$

This equation is easily solved, and as a result

$$\lim_{\mu^2\to\infty} \varphi(\xi,\mu^2) = \frac{3}{2} f_\pi (1-\xi^2). \qquad (47)$$

The factor 3/2 is due to the normalization condition (44). Taking $\mu=Q$ and using the Born approximation for E (fig. 10b), we obtain

$$F_\pi^{(A)}(Q^2) = \frac{8\pi\alpha_s(Q^2) f_\pi^2}{Q^2}\cdot\frac{C_2(R)}{N_c}\left|\int_0^1 \frac{a(\xi,Q^2)}{1-\xi^2}\,d\xi\right|^2, \qquad (48)$$

where $a(\xi)=\varphi(\xi)/f_\pi$.

The limiting curve (47) for the wave function has a very natural shape: the function is maximal at $\xi=0$ (when the quarks have equal momenta) and is zero at $\xi=1$ (when one of the quarks takes the whole momentum of the pion). Due to the normalization condition (44), the magnitude of the integral entering into eq. (48) is very close to its limiting value equal to 3/2 for all functions of this type. Substituting this value into eq. (48) and taking for the strong coupling constant $\alpha_s(Q)$ the asymptotic freedom result $\alpha_s(Q)=4\pi/9\ln Q^2/\Lambda^2$, where $\Lambda=$ $=0.5$ GeV/22/, we obtain for $Q^2=2$ GeV^2 the value $F_\pi^{(A)}(Q^2=2)=$ $=0.18$ which is very close to that dictated by the ρ -pole fit

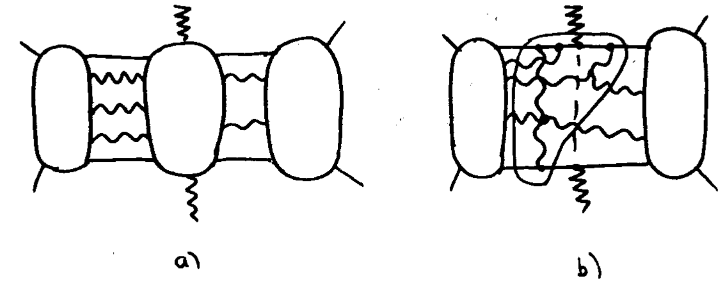

Fig. 8. The structure of contributions in a gauge theory.

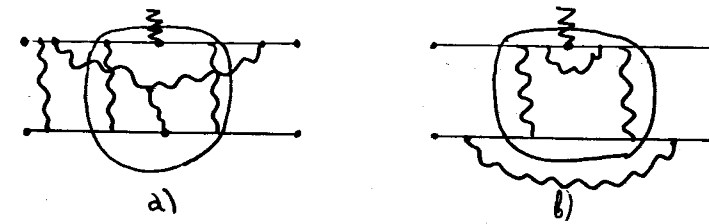

Fig. 9. The diagrams contributing to the form factor of a composite particle.

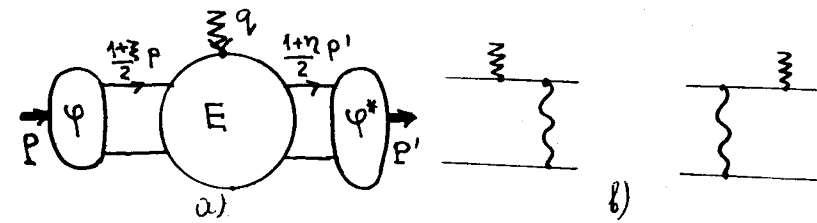

Fig. 10. a) Parton interpretation of eq. (38).
b) Born approximation for the amplitude of the parton subprocess.

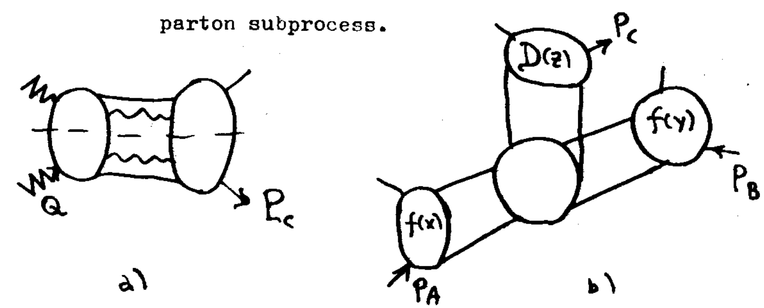

Fig. 11. The processes involving a hadron detected in the final state.

$F_\pi^{(\rho)}=(1+Q^2/m_\rho^2)^{-1}$, namely $F_\pi^{(\rho)}(Q^2=2)$ =0.19. But for $Q^2>2$ GeV^2 the curve $F_\pi^{(A)}$ decreases more rapidly than the curve $F_\pi^{(\rho)}$ because of the presence of $\alpha_s(Q)$ term.

QCD on the Light Cone

In this paper we have summarized the physical ideas underlying our approach and have omitted some details of mathematical nature.

For example: we have discussed the situation when all the lines inside a subgraph $\mathcal{V}$ carry large virtual momenta $k_i^2=O(Q^2)$. On the other hand, we believe that final state quarks are near their "would be" mass shell, i.e.,for these lines (also belonging to the subgraph $\mathcal{V}$) $k^2\sim 0$. Our treatment is valid indeed for a zero-angle scattering amplitude $T(\omega,Q^2)$ in the Euclidean region (e.g.,at $|\omega|<1$ for deep inelastic scattering). A rigorous analysis leads, as is well-known, to a moment statement

$$\int_0^1 F_2(x,Q^2)x^n\frac{dx}{x^2}=\sum_a E_n^a(1,\bar g(Q))\tilde f_a(n,Q^2)+ + O(M^2/Q^2). \tag{49}$$

In the lowest approximation $E_n^a=e_a^2$. Treating $\tilde f_a(n,Q^2)$ to be the moment of a parton distribution function $f(x,Q^2)$, we obtain

$$\int_0^1 F_2(x,Q^2)x^n\frac{dx}{x^2}=\sum_a e_a^2\int_0^1\frac{dx}{x}x^n f_a(x,Q^2)+ + O(M^2/Q^2)+O(\alpha_s(Q)). \tag{50}$$

In general, however, one cannot derive from eq. (50) that

$$F_2(x,Q^2)=\sum_a e_a^2\, x f_a(x,Q^2)\{1+O(M^2/Q^2)+O(\alpha_s(Q))\} \tag{51}$$

because the moment inversion is a rather delicate procedure. But a more detailed analysis/23/ shows that outside the resonance region (i.e.,at x not too close to 1) the relation (51) is justifiable. This provides a QCD-basis for parton model ideas.

Treating the process $AB\to\mu^+\mu^-X$ we must also consider the amplitude $T(\tilde\omega,Q^2)$ at $|\tilde\omega|<1$ (where $\tilde\omega=1/\tau=S/Q^2$). The function $T(\tilde\omega,Q^2)$ has two cuts: at $\tilde\omega>1$ and at $\tilde\omega<-1$. The right cut discontinuity is proportional to a cross-section $d\sigma/dQ^2$ for the process $AB\to\mu^+\mu^-X$, whereas the left cut discontinuity is related to the process $A\bar B\to\mu^+\mu^-X$ (or $\bar AB\to\mu^+\mu^-X$). Hence it is a moment relation

$$\int_0^1 W(Q^2,\tau)\tau^{n-1}d\tau=\sum_{a,b} w_n^{ab}(Q^2/\mu^2,g(\mu))\tilde f_{a/A}(n,\mu^2)\cdot \cdot\tilde f_{b/B}(n,\mu^2)+O(M^2/Q^2) \tag{52}$$

which is the result of our analysis. But by analogy with deep inelastic scattering one should expect that eq. (35), which is an inverted moment version of eq. (52), is valid for τ not too close to 1.

The treatment of processes having a detected hadron in the final state ($e^+e^-\to CX$, $AB\to CX$, etc.) is much more complicated. Using the methods sketched above it is easy to see that the gluon insertions into the corresponding subprocesses (fig. 11) result in the manifest recovery of gauge invariance. Hence, one must expect that all the double logarithmic factors $(g^2\ln^2 Q^2/\mu^2)^N$ will cancel with each other, and that the remaining logarithmic factors $(g^2\ln Q^2/\mu^2)^N$ together with the μ^2 -dependent parton distribution functions $f(x,\mu^2)$ and parton decay functions $D(z,\mu^2)$ will combine into a μ^2 - independent combination*). But there is a difficulty connected with the parton decay functions: it is well-known that they cannot be related to matrix elements of local operators/24/.

In conclusion we want to emphasize the decisive role of the light cone in our treatment. We were able to obtain a parton picture mainly due to the fact that the contribution of higher twist operators has been suppressed by powers of (M^2/Q^2). On the other hand, for the processes which have more complicated kinematics than that of deep inelastic scattering, the connection between the light-cone singularities and asymptotical behaviour of the process is not straightforward due to $(x^2)^k s^k$ terms. But the effect of $(x^2)^k$ factor is compensated by s^k term, and as a result we obtain a relation analogous to eq. (5):

$$M \sim Q^{4-\Sigma t_{O_i}}, \tag{53}$$

where t_{O_i} is the twist of a composite operator $O^i_{\mu_1 \ldots \mu_n}$. These operators appear after a contraction of the subgraph V describing a parton subprocess, into point. It is also necessary that after such a contraction the resulting diagrams do not depend on large variables. The statement that the asymptotical behaviour of some process in which the hadrons are involved is determined by that of a parton subprocess, is completely equivalent to the statement that the asymptotical behaviour of the hadronic process is dominated by the light-like distances inside the subgraphs, the contraction of which into point eliminates the dependence of the whole diagram on large variables. The parton picture and the light-cone dominance are the same phenomenon described in terms of two different languages. It is worth noting that our approach differs from an old LC-analysis in that there is no need to have a product of two currents to begin with. Treating the pion EM form factors one starts with an expression $\langle P'|J^\mu(0)|P\rangle$ possessing the only current. It is also possible (as for $AB \to CX$ process) that an asymptotical behaviour is dominated by lightlike distances between the internal points of a diagram.

*) Taking $\mu = P_T$ one gets rid of logarithmical contributions to the subprocess cross section. One can take also $\mu = P_T/10$, or $\mu^2 = 2(P_A P_c)$; but it is inconsistent to take $\mu^2 = 2(P_A P_c) x/z$, where $x P_A$ and P_c/z are the momenta of partons taking part in the subprocess.

A more detailed treatment of all the questions touched upon in this paper will be published elsewhere.

It is a pleasure to express here our gratitude to D.I.Blokhintsev, V.L.Chernyak, A.De Rújula, Y.L.Dokshitzer, D.I.Dyakonov, R.N.Faustov, I.F.Ginzburg, B.L.Ioffe, E.M.Levin, L.N.Lipatov, V.A.Meshcheryakov, V.G.Serbo, D.V.Shirkov, S.I.Troyan, A.A.Vladimirov and V.I.Zakharov for helpful discussions and stimulating remarks.

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Received by Publishing Department
on April 27 1978.