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Суперток

Аксиальный суперток - простейшее суперполе, которое содержит тензор энергии импульса и спин-векторный ток суперсимметрии. Существование этой фундаментальной величины доказано в общем случае. Найден алгоритм для вывода супертока из произвольного суперполевого лагранжиана.

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Supercurrent

The axial supercurrent is the simplest superfield containing both the energy-momentum tensor and the supersymmetry spin-vector current. The existence of this fundamental object is proved in the general case. An algorithm for its derivation from an arbitrary superfield Lagrangian is given.

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I. Introduction

In 1975 Zumino[1] observed that the energy-momentum tensor $T_{\mu\nu}(x)$ and the Noether supersymmetry current $J_{\mu\alpha}(x)$ of a supersymmetric Lagrangian are components of one supermultiplet. Ferrara and Zumino[2] have suggested that this multiplet can be put into an axial vector superfield $V_\mu(x,\theta)$ which obeys a certain "conservation" law containing the conservation laws for both $T_{\mu\nu}(x)$ and $J_{\mu\alpha}(x)$. They have called this new object $V_\mu(x,\theta)$ "supercurrent" and have found its explicit form for the two simplest examples: those of the chiral and the general scalar superfields. However, they have given neither an algorithm for deriving the supercurrent, nor a proof that it exists in the general case.

The concept of supercurrent has become very actual in connection with the problem of finding a minimal superfield formulation of supergravity. As we proposed in 1976[3], the supercurrent $V_\mu(x,\theta)$ can be viewed as the source of a gravitational axial superfield $h_\mu(x,\theta)$ containing the spin-2-gravitational field and the spin -3/2-"gravitino" field (in direct analogy with Einstein's gravity where the energy-momentum tensor is the source of the graviton).*) It is just the time to answer the question: does the supercurrent really exist and how to find it in the general case?

In the present paper we solve this problem and show how to construct the supercurrent given an arbitrary Lagrangian. The paper is organised as follows. In Section II some general properties of the supercurrent are reminded and discussed. Also, in this section the final formula for the supercurrent is given for convenience of the reader. In Section III some identities following from the basic invariances of the Lagrangian are derived and used in Section IV to prove the existence of the supercurrent. In Section V, the general formula obtained is illustrated by the two simplest examples of Ref.[2]. Appendix A explains the notation. Appendix B is devoted to the γ_5 -current and its connection with the supercurrent.

II. Definition of the Supercurrent

We shall start with formulating the problem. Let $\mathcal{L}(\varphi_i, \mathcal{D}_\alpha \varphi_i, \mathcal{D}_\alpha \mathcal{D}_\beta \varphi_i)$ be the Lagrangian for a supersymmetric system. It depends on the superfields $\varphi_i(x,\theta)$ (i is

*)Several recent papers confirm the adequacy of the supercurrent approach to supergravity. Ferrara and Zumino[4] discuss in a geometrical framework the free Lagrangian for the superfield $h_\mu(x,\theta)$ obtained in ref.[3]. Wess and Zumino[5] reduce the general supergeometrical approach and their results in the linearized limit seem to be consistent with ours[3]. Quite recently Stelle and West and Ferrara and van Nieuwenhuizen[6] succeeded in constructing of a closed local supersymmetry algebra and of an invariant action of supergravity. They used a minimal set of fields which corresponds to the content of the axial superfield $h_\mu(x,\theta)$.

a Lorentz index) and on their first and second spinorial derivatives $\mathcal{D}_\alpha \varphi_i, \mathcal{D}_\alpha \mathcal{D}_\beta \varphi_i$ (for definitions and notation see Appendix A). The action of the system

$$S = \int d^4x\, d^4\theta\, \mathcal{L}(\varphi_i, \mathcal{D}_\alpha \varphi_i, \mathcal{D}_\alpha \mathcal{D}_\beta \varphi_i) \qquad (1)$$

is invariant under translations and supersymmetry transformations. Therefore, following the Noether procedure, two conserved currents can be constructed: the energy-momentum tensor $T_{\mu\nu}(x)$ and the supersymmetry spin-vector current $J_{\mu\alpha}(x)$ obeying the conservation laws

$$\partial^\mu T_{\mu\nu} = 0, \quad \partial^\mu J_{\mu\alpha} = 0 \qquad (2)$$

(Note that $T_{\mu\nu}(x)$ and $J_{\mu\alpha}(x)$ are built out of the ordinary component fields entering into Eq.(1) after the integration over $d^4\theta$ is carried out).

According to the observation of Ref.[1], these two currents are members of a representation of the supersymmetry algebra which is, in general, reducible. Indeed, the symmetrized tensor $T_{\mu\nu}$ describes two spins: 2 and 0 (the latter is represented by the trace $T^\mu{}_\mu$) and the spin-vector $J_{\mu\alpha}$ contains spins 3/2 and 1/2 (the latter being the spinor $(\gamma^\mu J_\mu)_\alpha$). The spins 2, 3/2 and an additional spin 1 can form a supermultiplet with superspin 3/2 [7,8]. The remaining spins 1/2 and 0 can be combined into a chiral multiplet (superspin 0)*).

*) In some particular cases this chiral multiplet may vanish. A necessary condition is $T^\mu{}_\mu = 0$ which means that the action is scale invariant and $T_{\mu\nu}$ is "improved"[9].

The problem now is how to put these representations with superspins 3/2 and 0 into a single superfield connected in some way with the Lagrangian $\mathcal{L}(\varphi_i, \mathcal{D}_\alpha \varphi_i, \mathcal{D}_\alpha \mathcal{D}_\beta \varphi_i)$. The simplest superfield containing superspin 3/2 is the vector one $V_\mu(x,\theta)$ (see Ref. [8]). However, it includes other superspins too (1 and 1/2). So, in order to extract only the two superspins 3/2 and 0 with physical meaning, a certain differential condition has to be imposed on $V_\mu(x,\theta)$. The condition

$$(\gamma^\mu \mathcal{D})_\alpha V_\mu = 0 \tag{3}$$

singles out pure superspin 3/2 [8] but we can modify it slightly to allow for the superspin 0 too:

$$(\gamma^\mu \mathcal{D})_\alpha V_\mu = \mathcal{D}_\alpha \bar{\mathcal{D}} i\gamma_5 \mathcal{D} A + \mathcal{D}_\alpha \bar{\mathcal{D}} \mathcal{D} B. \tag{4}$$

Here A and B are some scalar superfields and $\mathcal{D} i\gamma_5 \mathcal{D} A$ and $\bar{\mathcal{D}}\mathcal{D} B$ describe chiral superfields, i.e. superspin 0 (one can verify the superspin content in Eq.(4) more precisely using the projection operators of Ref. [8]). It is not hard to show that Eq.(4) is equivalent to a higher order differential condition involving V_μ only

$$\mathcal{D}_\alpha \bar{\mathcal{D}}\mathcal{D} V_\mu = 2i\,\partial_\mu (\gamma^\nu \mathcal{D})_\alpha V_\nu . \tag{5}$$

The choice of the "conservation law" (5) is finally justified when expanding Eq.(5) in powers of θ_α. It is not difficult to see that the vector superfield $V_\mu(x,\theta)$ obeying Eq.(5) has the decomposition

$$\begin{aligned} V_\mu(x,\theta) &= v_\mu(x) + \bar\theta^\alpha \lambda_{\mu\alpha}(x) + \tfrac{1}{4}\bar\theta\theta\,\partial_\mu f(x) + \tfrac{1}{4}\bar\theta\gamma_5\theta\,\partial_\mu g(x) + \\ &+ \tfrac{1}{4}\bar\theta i\gamma^\nu\gamma_5\theta\left(t_{\nu\mu}(x) + \tfrac{1}{2}\varepsilon_{\nu\mu\lambda\rho}\partial^\lambda v^\rho(x)\right) + \\ &+ \tfrac{1}{4}\bar\theta\theta.\bar\theta^\alpha\left(i\hat\partial\lambda_\mu(x) - 2i\partial_\mu\gamma^\nu\lambda_\nu(x)\right)_\alpha + \tfrac{1}{32}(\bar\theta\theta)^2\left(\partial^2 v_\mu(x) - 2\partial_\mu\partial^\nu v_\nu(x)\right), \end{aligned} \tag{6}$$

where $f(x)$, $g(x)$, $v_\mu(x)$ are arbitrary and $\lambda_{\mu\alpha}(x)$, $t_{\mu\nu}(x)$ satisfy the conditions

$$(\sigma^{\mu\nu}\partial_\mu \lambda_\nu)_\alpha = 0,\ t_{\mu\nu} = t_{\nu\mu},\ \partial^\mu t_{\mu\nu} = \partial_\nu t^\lambda{}_\lambda . \tag{7}$$

Then it is obvious that the combinations

$$T_{\mu\nu} = t_{\mu\nu} - \eta_{\mu\nu} t^\lambda{}_\lambda ,\quad J_{\mu\alpha} = \lambda_{\mu\alpha} - (\gamma_\mu\gamma^\nu\lambda_\nu)_\alpha \tag{8}$$

can serve as conserved energy-momentum tensor and Noether supersymmetry current. So the superfield $V_\mu(x,\theta)$ has to be axial ($t_{\mu\nu}$ stands after $\bar\theta i\gamma_\nu\gamma_5\theta$ in Eq.(6)). One can show that the superspin 3/2 multiplet in V_μ (6) consists of the fields $t_{\mu\nu}$, $\lambda_{\mu\alpha}$ and v_μ (the latter is connected in a way to the γ_5 -current; see Appendix B) and the superspin 0 is formed by $(\gamma^\mu\lambda_\mu)_\alpha, t^\mu{}_\mu, \partial^\mu v_\mu, f$ and g.

The superfield $V_\mu(x,\theta)$ with its conservation law (4) or (5) was defined in Ref. [2] and was called "supercurrent". Now a question arises: does the supercurrent exist in the general case? In Ref. [2] it was constructed by hand for two simple cases only. Here we give the general formula for the supercurrent for an arbitrary Lagrangian $\mathcal{L}(\varphi_i, \mathcal{D}_\alpha\varphi_i, \mathcal{D}_\alpha\mathcal{D}_\beta\varphi_i)$:

$$\begin{aligned} V_\mu &= \bar{\mathcal{D}}\gamma_5\psi_\mu + 2(\bar{\mathcal{D}}\gamma^\nu\gamma_5)^\alpha \bar{\mathcal{D}}\mathcal{D}\,\varphi_{\mu\nu\alpha} + \\ &+ \tfrac{1}{3}\bar{\mathcal{D}} i\gamma_\mu\gamma_5\mathcal{D}\left(a - \bar{\mathcal{D}}\sigma^{\lambda\rho}\varphi_{\lambda\rho}\right) + 2i\,\partial_\mu b . \end{aligned} \tag{9}$$

The following notation is used in Eq.(9):

$$\begin{aligned} \psi_{\mu\alpha} &= \tfrac{i}{2}(\gamma_\mu\mathcal{D})_\alpha \mathcal{L} + \partial_\mu\varphi_i\,\mathcal{L}_\alpha{}^i + \\ &+ (-1)^\varepsilon\varphi_i\,\bar{\mathcal{D}}^\beta\mathcal{L}^i_{\beta\alpha} + \partial_\mu\bar{\mathcal{D}}^\beta\varphi_i\,\mathcal{L}^i_{\alpha\beta} \end{aligned} \tag{10}$$

where $\varepsilon = 0$ if φ_i is a boson superfield and $\varepsilon = 1$

for a fermion superfield; $\mathcal{L}_\alpha{}^i$ and $\mathcal{L}_{\alpha\beta}{}^i$ are variational derivatives of the Lagrangian

$$\mathcal{L}_\alpha{}^i \equiv \frac{\delta\mathcal{L}}{\delta\bar{D}^\alpha\varphi_i}\ , \quad \mathcal{L}_{\alpha\beta}{}^i \equiv \frac{\delta\mathcal{L}}{\delta\bar{D}^\alpha\bar{D}^\beta\varphi_i}\ . \tag{11}$$

Further

$$\varphi_{\mu\nu\alpha} = -\frac{1}{8}(\bar{D}\sigma_{\mu\nu})^\beta\varphi_i\,\mathcal{L}_{\alpha\beta}{}^i + \\ +\frac{i}{4}(\Lambda_{\mu\nu})_{ij}\left(\varphi_j\mathcal{L}_\alpha{}^i + \varphi_j\bar{D}^\beta\mathcal{L}_{\beta\alpha}{}^i + \bar{D}^\beta\varphi_j\,\mathcal{L}_{\alpha\beta}{}^i\right), \tag{12}$$

where $(\Lambda_{\mu\nu})_{ij}$ is the Lorentz generator corresponding to the superfield index (or set of indices) i:

$$a = \mathcal{L} - \frac{1}{4}\left[\bar{D}^\alpha\varphi_i\mathcal{L}_\alpha{}^i + \bar{D}^\alpha\bar{D}^\beta\varphi_i\,\mathcal{L}_{\alpha\beta}{}^i + (-1)^\varepsilon\bar{D}^\alpha\varphi_i\,\bar{D}^\beta\mathcal{L}_{\beta\alpha}{}^i\right] \tag{13}$$

$$b = -\frac{1}{4}\left[(\bar{D}i\gamma_5)^\alpha\varphi_i\mathcal{L}_\alpha{}^i + (\bar{D}i\gamma_5)^\alpha\bar{D}^\beta\varphi_i\,\mathcal{L}_{\alpha\beta}{}^i + (-1)^\varepsilon(\bar{D}i\gamma_5)^\alpha\varphi_i\,\bar{D}^\beta\mathcal{L}_{\beta\alpha}{}^i\right]. \tag{14}$$

Due to the equations of motion for the superfields φ_i the supercurrent (9) obeys the following equation

$$(\gamma^\mu D)_\alpha V_\mu = \frac{1}{3}D_\alpha\,\bar{D}i\gamma_5 D(a - \bar{D}\sigma^{\mu\nu}\varphi_{\mu\nu}) + D_\alpha\bar{D}D\,b \tag{4'}$$

which is in fact the conservation law (4) with

$$A = \frac{1}{3}(a - \bar{D}\sigma^{\mu\nu}\varphi_{\mu\nu})\ , \quad B = b\ . \tag{15}$$

Now we shall prove all these statements using the symmetry properties of the Lagrangian and the equations of motion.

III. Lagrangian, Invariances, Identities

Here we derive three basic identities which follow from and ensure the invariance of the action (1) under translations, supertransformations and Lorentz transformations.

In general notation the superfields φ_i undergo the following infinitesimal transformations

$$\delta\varphi_i = c_a\left[G_a\varphi_i + (\Gamma_a)_{ij}\varphi_j\right], \tag{16}$$

where $G_a(\Gamma_a)$ is the differential (matrix) part of the generators and c_a are the corresponding parameters (even or odd elements of a Grassmann algebra). Then the variation of the Lagrangian is

$$\delta\mathcal{L} = \delta\varphi_i\frac{\delta\mathcal{L}}{\delta\varphi_i} + \bar{D}^\alpha\delta\varphi_i\,\mathcal{L}_\alpha{}^i + \bar{D}^\alpha\bar{D}^\beta\delta\varphi_i\,\mathcal{L}_{\alpha\beta}{}^i \tag{17}$$

($\mathcal{L}_\alpha{}^i$ and $\mathcal{L}_{\alpha\beta}{}^i$ are defined in Eq.(11)). Using the Euler-Lagrange equations (equations of motion) for the action S (1)

$$\frac{\delta\mathcal{L}}{\delta\varphi_i} - (-1)^\varepsilon\bar{D}^\alpha\mathcal{L}_\alpha{}^i - \bar{D}^\beta\bar{D}^\alpha\mathcal{L}_{\alpha\beta}{}^i = 0 \tag{18}$$

one can rewrite Eq.(17) as follows

$$\delta\mathcal{L} = (-1)^\varepsilon\delta\varphi_i\,\bar{D}^\alpha\mathcal{L}_\alpha{}^i + \bar{D}^\alpha\delta\varphi_i\,\mathcal{L}_\alpha{}^i + \\ +\delta\varphi_i\,\bar{D}^\beta\bar{D}^\alpha\mathcal{L}_{\alpha\beta}{}^i + \bar{D}^\alpha\bar{D}^\beta\delta\varphi_i\,\mathcal{L}_{\alpha\beta}{}^i\ . \tag{19}$$

Now we put Eq.(16) into Eq.(19)

$$\delta\mathcal{L} = c_a\Big\{(-1)^\varepsilon G_a\varphi_i\,\bar{D}^\alpha\mathcal{L}_\alpha{}^i + G_a\bar{D}^\alpha\varphi_i\,\mathcal{L}_\alpha{}^i + \\ + G_a\varphi_i\,\bar{D}^\beta\bar{D}^\alpha\mathcal{L}_{\alpha\beta}{}^i + G_a\bar{D}^\alpha\bar{D}^\beta\varphi_i\,\mathcal{L}_{\alpha\beta}{}^i \pm \\ \pm[\bar{D}^\alpha, G_a]_\mp\varphi_i\mathcal{L}_\alpha{}^i + \bar{D}^\alpha\left([\bar{D}^\beta, G_a]_\mp\varphi_i\right)\mathcal{L}_{\alpha\beta}{}^i \pm [\bar{D}^\alpha, G_a]_\mp\bar{D}^\beta\varphi_i\,\mathcal{L}_{\alpha\beta}{}^i\Big\} + \\ +\bar{D}^\alpha c_a\Gamma_{ij}^a\left[\varphi_j\mathcal{L}_\alpha{}^i + (-1)^\varepsilon\varphi_j\bar{D}^\beta\mathcal{L}_{\beta\alpha}{}^i + \bar{D}^\beta\varphi_j\,\mathcal{L}_{\alpha\beta}{}^i\right]. \tag{20}$$

Here the sign +(−) corresponds to even (odd) c_a.

On the other hand, the invariance of the action integral (1) under the transformations (16) implies that $\delta\mathcal{L}$ has to be a spatial or spinorial divergence. This requirement is often replaced by a stronger one: the Lagrangian $\mathcal{L}$ is supposed to transform as a scalar under the corresponding group, i.e.

$$\delta\mathcal{L} = c_a G_a \mathcal{L} \ . \tag{21}$$

Comparing Eq.(21) with Eq.(20), one can derive identities for the Lagrangian $\mathcal{L}$ which reflect its symmetry properties.

Now we consider the three basic invariances of $\mathcal{L}$ in this aspect.

1) Translation invariance. Here

$$G_a \equiv \partial_\mu \ , \quad \Gamma_a = 0 \ , \quad [\mathcal{D}_\alpha, G_a] = 0 \ .$$

Assuming that Eq.(21) holds (i.e. that $\mathcal{L}$ does not depend on x_μ manifestly) and cancelling the parameters c_μ we find from Eq.(20) and Eq.(21)

$$\partial_\mu \mathcal{L} = (-1)^\varepsilon \partial_\mu \varphi_i \bar{\mathcal{D}}^\alpha \mathcal{L}_\alpha^i + \partial_\mu \bar{\mathcal{D}}^\alpha \varphi_i \mathcal{L}_\alpha^i + \partial_\mu \varphi_i \bar{\mathcal{D}}^\beta \bar{\mathcal{D}}^\alpha \mathcal{L}_{\alpha\beta}^i + \partial_\mu \bar{\mathcal{D}}^\alpha \bar{\mathcal{D}}^\beta \varphi_i \mathcal{L}_{\alpha\beta}^i \ ; \tag{22}$$

2) Superinvariance. Now

$$G_a \equiv S_\alpha = i\left(\frac{\partial}{\partial\bar\theta^\alpha} + \frac{i}{2}(\not{\partial}\theta)_\alpha\right) = i\left(\mathcal{D}_\alpha + i(\gamma^\mu\theta)_\alpha \partial_\mu\right)$$

$$\Gamma_{ij}^\alpha = 0 \ , \quad \{\mathcal{D}_\alpha, S_\beta\} = 0$$

The requirement (21) means that the Lagrangian is a superfield by itself (i.e. it does not depend on θ_α manifestly.) Therefore Eqs. (20), (21) lead to

$$\mathcal{D}_\alpha \mathcal{L} + i(\gamma^\mu\theta)_\alpha \partial_\mu \mathcal{L} = (-1)^\varepsilon \mathcal{D}_\alpha \varphi_i \bar{\mathcal{D}}^\beta \mathcal{L}_\beta^i + \mathcal{D}_\alpha \bar{\mathcal{D}}^\beta \varphi_i \mathcal{L}_\alpha^i + \mathcal{D}_\alpha \varphi_i \bar{\mathcal{D}}^\rho \bar{\mathcal{D}}^\lambda \mathcal{L}_{\lambda\rho}^i + \mathcal{D}_\alpha \bar{\mathcal{D}}^\rho \bar{\mathcal{D}}^\lambda \varphi_i \mathcal{L}_{\rho\lambda}^i + i(\gamma^\mu\theta)_\alpha \Big[(-1)^\varepsilon \partial_\mu \varphi_i \bar{\mathcal{D}}^\beta \mathcal{L}_\beta^i + \partial_\mu \bar{\mathcal{D}}^\beta \varphi_i \mathcal{L}_\beta^i + \partial_\mu \varphi_i \bar{\mathcal{D}}^\rho \bar{\mathcal{D}}^\lambda \mathcal{L}_{\lambda\rho}^i + \partial_\mu \bar{\mathcal{D}}^\rho \bar{\mathcal{D}}^\lambda \varphi_i \mathcal{L}_{\rho\lambda}^i\Big] \ . \tag{23}$$

Eq.(23) splits into two identities. The first one is obtained from the terms with $i(\gamma^\mu\theta)_\alpha$ and it coincides with Eq.(22). This is not a surprise because the translation invariance is in fact a consequence of the superinvariance. The second identity is

$$\mathcal{D}_\alpha \mathcal{L} = (-1)^\varepsilon \mathcal{D}_\alpha \varphi_i \bar{\mathcal{D}}^\beta \mathcal{L}_\beta^i + \mathcal{D}_\alpha \bar{\mathcal{D}}^\beta \varphi_i \mathcal{L}_\beta^i + \mathcal{D}_\alpha \varphi_i \bar{\mathcal{D}}^\rho \bar{\mathcal{D}}^\lambda \mathcal{L}_{\lambda\rho}^i + \mathcal{D}_\alpha \bar{\mathcal{D}}^\rho \bar{\mathcal{D}}^\lambda \varphi_i \mathcal{L}_{\rho\lambda}^i \ . \tag{24}$$

With the help of identities (22), (24) formula (20) for the variation of the Lagrangian can be simplified. If the generators G_a of some other invariance group have the form

$$G_a = A_a^\alpha \frac{\partial}{\partial\bar\theta^\alpha} + B_a^\mu \partial_\mu = A_a^\alpha \mathcal{D}_\alpha + B_a'^\mu \partial_\mu \ , \tag{25}$$

we can rewrite Eq. (20) as follows

$$\delta\mathcal{L} = c_a G_a \mathcal{L} + c_a \Big\{ \pm [\bar{\mathcal{D}}^\alpha, G_a]_\mp \varphi_i \mathcal{L}_\alpha^i \pm [\bar{\mathcal{D}}^\alpha, G_a]_\mp \bar{\mathcal{D}}^\beta \varphi_i \mathcal{L}_{\alpha\beta}^i + \bar{\mathcal{D}}^\alpha \left([\bar{\mathcal{D}}^\beta, G_a]_\mp \varphi_i\right) \mathcal{L}_{\alpha\beta}^i \Big\} + \bar{\mathcal{D}}^\alpha c_a \Gamma_{ij}^a \Big[\varphi_j \mathcal{L}_\alpha^i + (-1)^\varepsilon \varphi_j \bar{\mathcal{D}}^\beta \mathcal{L}_{\beta\alpha}^i + \bar{\mathcal{D}}^\beta \varphi_j \mathcal{L}_{\alpha\beta}^i\Big] \ . \tag{26}$$

3) Lorentz invariance. The differential part of the generators

$$G_\alpha \equiv J_{\mu\nu} = x_\mu \partial_\nu - x_\nu \partial_\mu - \frac{i}{2}\bar{\theta}\sigma_{\mu\nu}\frac{\partial}{\partial\bar{\theta}}$$

is of the type (25). Further

$$[\mathcal{D}_\alpha, J_{\mu\nu}] = -\frac{i}{2}(\sigma_{\mu\nu}\mathcal{D})_\alpha, \quad \Gamma^{\,a}_{i\,j} \equiv (\Lambda_{\mu\nu})_{ij},$$

where $(\Lambda_{\mu\nu})_{ij}$ is the matrix generator of the Lorentz group representation corresponding to the index (or set of indices) i of the superfield φ_i. The invariance condition (21) now means that the Lagrangian is a Lorentz scalar. Putting all this into Eq. (26) we obtain the identity

$$0 = (\bar{\mathcal{D}}\sigma_{\mu\nu})^\alpha \varphi_i \mathcal{L}^i_\alpha + (\bar{\mathcal{D}}\sigma_{\mu\nu})^\alpha \bar{\mathcal{D}}^\beta \varphi_i \mathcal{L}^i_{\alpha\beta} + \bar{\mathcal{D}}^\alpha(\bar{\mathcal{D}}\sigma_{\mu\nu})^\beta \varphi_i \mathcal{L}^i_{\alpha\beta} - 2i\bar{\mathcal{D}}^\alpha(\Lambda_{\mu\nu})_{ij}\left[\varphi_j \mathcal{L}^i_\alpha + \varphi_j \bar{\mathcal{D}}^\beta \mathcal{L}^i_{\beta\alpha} + \bar{\mathcal{D}}^\beta\varphi_j \mathcal{L}^i_{\alpha\beta}\right]. \quad (27)$$

Concluding this section we wish to stress once more that the identities derived (Eqs.(22), (24), (27)) are based on the symmetry properties of the Lagrangian as well as on the equations of motion (18).

IV. The Derivation of the Supercurrent

Our goal can be formulated as follows. We wish to find an axial superfield $V_\mu(x,\theta)$ constructed out of the Lagrangian $\mathcal{L}$, its variational derivatives $\mathcal{L}^i_\alpha$, $\mathcal{L}^i_{\alpha\beta}$ and of the superfields φ_i and their derivatives $\mathcal{D}_\alpha\varphi_i$, $\mathcal{D}_\alpha\mathcal{D}_\beta\varphi_i$. The dimensionality of V_μ has to be cm^{-3} ($[V_\mu] = [\bar{\theta}i\gamma^\nu\gamma_5\theta]\cdot[T_{\nu\mu}] = cm^{1}\cdot cm^{-4} = cm^{-3}$; see Eqs.(6),(8)).

This supercurrent $V_\mu(x,\theta)$ must obey the conservation law (4) due to the symmetry identities (22),(24) and (27).

The starting point in our construction is the observation that the translation identity (22) can be rewritten in the form of a "conservation law" for a spin-vector "supercurrent"

$$\bar{\mathcal{D}}^\alpha \Psi_{\mu\alpha}(x,\theta) = 0, \quad (28)$$

where

$$\Psi_{\mu\alpha} = \frac{i}{2}(\gamma_\mu\mathcal{D})_\alpha \mathcal{L} + \partial_\mu\varphi_i \mathcal{L}^i_\alpha + (-1)^\varepsilon \partial_\mu\varphi_i \bar{\mathcal{D}}^\beta \mathcal{L}^i_{\beta\alpha} + \partial_\mu\bar{\mathcal{D}}^\beta\varphi_i \mathcal{L}^i_{\alpha\beta}. \quad (29)$$

This new object contains the energy-momentum tensor. Indeed, let us integrate Eq. (28) over $d^4\theta$:

$$0 = \int d^4\theta\, \bar{\mathcal{D}}^\alpha \Psi_{\mu\alpha} = \int d^4\theta\left[-\frac{\partial}{\partial\theta_\alpha} + \frac{i}{2}\partial^\nu(\bar{\theta}\gamma_\nu)^\alpha\right]\Psi_{\mu\alpha} = \frac{i}{2}\partial^\nu\int d^4\theta.\, \bar{\theta}\gamma_\nu\Psi_\mu \equiv \partial^\nu\theta_{\nu\mu}(x). \quad (30)$$

The conserved tensor quantity $\theta_{\nu\mu}(x)$ obtained has in fact the right dimensionality cm^{-4} although it is not symmetric (the latter can be achieved only with the help of Lorentz invariance).

Moreover, the spin-vector supercurrent contains the spin-vector supersymmetry current $J_{\mu\alpha}(x)$ too. To see this we have to exploit the fact that the action is supersymmetric. Let us integrate Eq.(28) over $d^4\theta.(\gamma^\mu\theta)_\alpha$:

$$0 = \int d^4\theta\,(\gamma^\mu\theta)_\alpha\bar{\mathcal{D}}^\beta\Psi_{\mu\beta} = \int d^4\theta.(\gamma^\mu\theta)_\alpha\left[-\frac{\partial}{\partial\theta_\beta} + \frac{i}{2}\partial^\nu(\bar{\theta}\gamma_\nu)^\beta\right]\Psi_{\mu\beta} = \int d^4\theta\left[-(\gamma^\mu\Psi_\mu)_\alpha\right] + \frac{i}{2}\partial^\nu\int d^4\theta\,(\gamma^\mu\theta)_\alpha\,\bar{\theta}\gamma_\nu\Psi_\mu. \quad (31)$$

The second term in Eq. (31) has already the form of a conservation law, so we have to examine the first one. Multiplying Eq. (29) by γ_μ and using the supersymmetry identity (24) one finds

$$(\gamma^\mu \Psi_\mu)_\alpha = \bar{\mathcal{D}}^\beta F_{\beta\alpha} \equiv$$

$$\equiv -i \bar{\mathcal{D}}^\beta \left(1_{\alpha\beta} \mathcal{L} + \mathcal{D}_\alpha \varphi_i \mathcal{L}_\beta^i + \mathcal{D}_\alpha \bar{\mathcal{D}}^\rho \varphi_i \mathcal{L}_{\beta\rho}^i + (-1)^\varepsilon \mathcal{D}_\alpha \varphi_i \bar{\mathcal{D}}^\rho \mathcal{L}_{\rho\beta}^i \right) =$$

$$= i \mathcal{D}_\alpha a - (\gamma_5 \mathcal{D})_\alpha b + i (\gamma^\mu \mathcal{D})_\alpha c_\mu - (\gamma^\mu \gamma_5 \mathcal{D})_\alpha d_\mu + i (\sigma^{\mu\nu} \mathcal{D})_\alpha e_{\mu\nu}, \tag{32}$$

where

$$a = \mathcal{L} - \frac{1}{4} \left[\bar{\mathcal{D}}^\alpha \varphi_i \mathcal{L}_\alpha^i + \bar{\mathcal{D}}^\alpha \bar{\mathcal{D}}^\beta \varphi_i \mathcal{L}_{\alpha\beta}^i + (-1)^\varepsilon \bar{\mathcal{D}}^\alpha \varphi_i \bar{\mathcal{D}}^\beta \mathcal{L}_{\beta\alpha}^i \right] \tag{33.a}$$

$$b = -\frac{1}{4} \left[(\bar{\mathcal{D}} i \gamma_5)^\alpha \varphi_i \mathcal{L}_\alpha^i + (\bar{\mathcal{D}} i \gamma_5)^\alpha \bar{\mathcal{D}}^\beta \varphi_i \mathcal{L}_{\alpha\beta}^i + (-1)^\varepsilon (\bar{\mathcal{D}} i \gamma_5)^\alpha \varphi_i \bar{\mathcal{D}}^\beta \mathcal{L}_{\beta\alpha}^i \right] \tag{33.b}$$

$$c_\mu = \frac{1}{4} \left[(\bar{\mathcal{D}} \gamma_\mu)^\alpha \varphi_i \mathcal{L}_\alpha^i + (\bar{\mathcal{D}} \gamma_\mu)^\alpha \bar{\mathcal{D}}^\beta \varphi_i \mathcal{L}_{\alpha\beta}^i + (-1)^\varepsilon (\bar{\mathcal{D}} \gamma_\mu)^\alpha \varphi_i \bar{\mathcal{D}}^\beta \mathcal{L}_{\beta\alpha}^i \right] \tag{33.c}$$

$$d_\mu = \frac{1}{4} \left[(\bar{\mathcal{D}} i \gamma_\mu \gamma_5)^\alpha \varphi_i \mathcal{L}_\alpha^i + (\bar{\mathcal{D}} i \gamma_\mu \gamma_5)^\alpha \bar{\mathcal{D}}^\beta \varphi_i \mathcal{L}_{\alpha\beta}^i + (-1)^\varepsilon (\bar{\mathcal{D}} i \gamma_\mu \gamma_5)^\alpha \varphi_i \bar{\mathcal{D}}^\beta \mathcal{L}_{\beta\alpha}^i \right] \tag{33.d}$$

$$e_{\mu\nu} = \frac{1}{8} \left[(\bar{\mathcal{D}} \sigma_{\mu\nu})^\alpha \varphi_i \mathcal{L}_\alpha^i + (\bar{\mathcal{D}} \sigma_{\mu\nu})^\alpha \bar{\mathcal{D}}^\beta \varphi_i \mathcal{L}_{\alpha\beta}^i + (-1)^\varepsilon (\bar{\mathcal{D}} \sigma_{\mu\nu})^\alpha \varphi_i \bar{\mathcal{D}}^\beta \mathcal{L}_{\beta\alpha}^i \right] \tag{33.e}$$

Now we insert Eq. (32) into Eq.(31) and get

$$0 = \frac{i}{2} \partial^\nu \int d^4\theta \left[-(\bar{\theta} \gamma_\nu F)_\alpha + (\gamma^\mu \theta)_\alpha \bar{\theta} \gamma_\nu \Psi_\mu \right] \equiv \partial^\nu j_{\nu\alpha}(x), \tag{34}$$

i.e. this is the conservation law for a spin-vector current $j_{\nu\alpha}(x)$, having the same dimensionality as the Noether supersymmetry current $J_{\mu\alpha}(x)$.

Thus we see that the spin-vector supercurrent $\Psi_{\mu\alpha}(x, \theta)$ (29) contains, in principle, the two currents which we are

trying to combine into a superfield. However, in comparison with the axial supercurrent $V_\mu(x, \theta)$ the spin-vector one has too many superfluous components*). So $\Psi_{\mu\alpha}(x, \theta)$ can serve as a ground for deriving the axial supercurrent.

The only vector superfield with proper dimensionality obtainable from $\Psi_{\mu\alpha}$ is

$$V_\mu' = \bar{\mathcal{D}} \gamma_5 \Psi_\mu \tag{35}$$

(in principle one can add a term $\bar{\mathcal{D}} \gamma_5 \gamma_\mu \gamma^\nu \Psi_\nu$ but it can be shown that it does not change the final result). So let us calculate $(\gamma^\mu \mathcal{D})_\alpha V_\mu'$ using Eq. (28):

$$(\gamma^\mu \mathcal{D})_\alpha V_\mu' = \frac{1}{2} \left[(\gamma_5 \bar{\mathcal{D}} \mathcal{D} - \bar{\mathcal{D}} \gamma_5 \mathcal{D}) \gamma^\mu \Psi_\mu \right]_\alpha . \tag{36}$$

Putting the expansion (32), (33) into Eq. (36) we find

$$(\gamma^\mu \mathcal{D})_\alpha V_\mu' = \bar{\mathcal{D}} \mathcal{D} (i \gamma_5 \mathcal{D})_\alpha a + \bar{\mathcal{D}} \mathcal{D} . \mathcal{D}_\alpha b + \bar{\mathcal{D}} \mathcal{D} (i \gamma_5 \sigma^{\mu\nu} \mathcal{D})_\alpha e_{\mu\nu} . \tag{37}$$

The terms in the r.h.s. of Eq. (37) can be rewritten as follows:

$$\bar{\mathcal{D}} \mathcal{D} (i \gamma_5 \mathcal{D})_\alpha a = -\frac{1}{3} (i \gamma_5 \mathcal{D})_\alpha \bar{\mathcal{D}} \mathcal{D} a - (\gamma^\mu \mathcal{D})_\alpha \frac{1}{3} \bar{\mathcal{D}} i \gamma_\mu \gamma_5 \mathcal{D} a \tag{38}$$

$$\bar{\mathcal{D}} \mathcal{D} . \mathcal{D}_\alpha b = \mathcal{D}_\alpha \bar{\mathcal{D}} \mathcal{D} b - (\gamma^\mu \mathcal{D})_\alpha 2 i \partial_\mu b . \tag{39}$$

To proceed with the third term in Eq. (37) we have to use the remaining symmetry property of the Lagrangian**).

*) It is not the case in 2 or 3-dimensional space. There $\Psi_{\mu\alpha}(x, \theta)$ is the simplest superfield containing $T_{\mu\nu}$ and $J_{\mu\alpha}$.

**) This latter step in the derivation of the supercurrent is in fact analogous to the well known Belinfante procedure for symmetrization of the energy - momentum tensor /10/.

$$e_{\mu\nu} = \bar{D}^{\alpha} \varphi_{\mu\nu\alpha} , \tag{40}$$

where

$$\varphi_{\mu\nu\alpha} = -\frac{1}{8} (\bar{D}\sigma_{\mu\nu})^{\beta} \varphi_i Z^i_{\alpha\beta} + \frac{i}{4} (\Lambda_{\mu\nu})_{ij} (\varphi_j Z_\alpha{}^i + \varphi_j \bar{D}^{\beta} Z_{\beta\alpha}{}^i + \bar{D}^{\beta} \varphi_j Z_{\alpha\beta}{}^i) , \tag{41}$$

therefore

$$\bar{D}D(i\gamma_5 \sigma^{\mu\nu} D)_\alpha e_{\mu\nu} = \frac{1}{3} (i\gamma_5 D)_\alpha \bar{D}D . \bar{D}\sigma^{\mu\nu} \varphi_{\mu\nu} + (\gamma^\mu D)_\alpha \left[-2 (\bar{D}\gamma_\nu\gamma_5)^{\beta} \bar{D}D \varphi_{\mu\nu\beta} + \frac{1}{3} \bar{D} i\gamma_\mu\gamma_5 D . \bar{D}\sigma^{\lambda\rho} \varphi_{\lambda\rho} \right] . \tag{42}$$

So, the r.h.s. of Eq. (37) according to Eqs. (38), (39),(42), contains two kinds of terms: some of them correspond to the r.h.s. of the conservation law (4) and the others have the form $(\gamma^\mu D)_\alpha \Delta V_\mu$ and can be used for redefining the initial supercurrent V_μ' (35). After making all these rearrangements we arrive at the final formula for the supercurrent V_μ given in Section II (see Eqs.(9) and (4')).

Now we are going to discuss the uniqueness of the result obtained. The supercurrent (9) is conserved nontrivially, i.e. it obeys the conservation law (4') due to the equations of motion. Therefore it contains nontrivial conserved currents (the energy-momentum thensor and the supersymmetry current). However, it is well known that these currents are determined as charge densities up to terms which are automatically conserved (i.e. without using the equations of motion). For instance, the following terms can be added to the symmetric energy-momentum tensor $T_{\mu\nu}$:

$$\Delta T_{\mu\nu} = \partial^2 \sigma_{\mu\nu} - \partial_\mu \partial^\lambda \sigma_{\lambda\nu} - \partial_\nu \partial^\lambda \sigma_{\lambda\mu} + \eta_{\mu\nu} \partial^\lambda \partial^\rho \sigma_{\lambda\rho} - \frac{1}{3} (\eta_{\mu\nu} \partial^2 \sigma^\lambda{}_\lambda - \partial_\mu \partial_\nu \sigma^\lambda{}_\lambda) , \tag{43}$$

where $\sigma_{\mu\nu}$ is an arbitrary symmetric tensor. Indeed, $\partial^\mu \Delta T_{\mu\nu} = 0$ identically and $\Delta T_{\mu\nu}$ does not change the four-momentum $P_\mu = \int d^3x \, T_{0\mu}$ *). Such additions are, in fact, used to "improve" the energy-momentum tensor /9/. In particular, the term ($\sigma_{\mu\nu} = \frac{3}{2} \eta_{\mu\nu} \sigma$ in Eq.(43))

$$\Delta T_{\mu\nu} = \eta_{\mu\nu} \partial^2 \sigma - \partial_\mu \partial_\nu \sigma \tag{44}$$

improves the energy-momentum tensor for the scalar field.

Just the same situation is observed in the supercurrent case. There are terms which obey Eq. (4) automatically and do not change substantially the currents in the supercurrent decomposition. One can prove that such terms cannot be constructed with the help of one, two or three spinorial derivatives. Four derivatives are already sufficient for the following automatically conserved combination

$$\Delta V_\mu = \bar{D} i\gamma_\mu\gamma_5 D . \bar{D} i\gamma^\nu\gamma_5 D W_\nu + 3 \bar{D} i\gamma^\nu\gamma_5 D . \bar{D} i\gamma_\mu\gamma_5 D W_\nu , \tag{45}$$

where W_ν is an arbitrary vector superfield. The operator in Eq.(45) is in fact the operator of the free equation of motion for the vector superfield /3/. This term (45) includes the addition (43) to the energy-momentum tensor and the following automatically

*) Note that the operator in Eq. (43) is identical with the operator of the free equation of motion for a massless spin 2 field.

conserved addition to the supersymmetry current

$$\Delta J_{\mu\alpha} = (\not\partial \varphi_\mu - \gamma_\mu \partial^\nu \varphi_\nu - \tfrac{1}{3}\partial_\mu \gamma^\nu \varphi_\nu + \tfrac{1}{3}\gamma_\mu \not\partial \gamma^\nu \varphi_\nu)_\alpha \quad (46)$$

($\varphi_{\mu\alpha}(x)$ is an arbitrary spin-vector field). The particular case (44) corresponds to the particular case

$$\Delta V_\mu = \partial_\mu \bar{\mathcal{D}}\mathcal{D} P + \partial_\mu \bar{\mathcal{D}}\gamma_5 \mathcal{D} S \quad (47)$$

which is obtained from Eq. (45) putting $W_\mu = \frac{1}{16\,?^2}\partial_\mu(\bar{\mathcal{D}}\mathcal{D}P + \bar{\mathcal{D}}\gamma_5\mathcal{D}S)$ (here P and S are arbitrary pseudoscalar and scalar superfields). In this case we have the identity

$$(\gamma^\mu \mathcal{D})_\alpha \Delta V_\mu = -\tfrac{i}{2}\mathcal{D}_\alpha(\bar{\mathcal{D}}\mathcal{D})^2 P - \tfrac{i}{2}(\gamma_5 \mathcal{D})_\alpha (\bar{\mathcal{D}}\mathcal{D})^2 S \,. \quad (48)$$

This means that the quantities A and B in the r.h.s. of the conservation law (4) are changed by purely chiral terms, for instance

$$\Delta A = \tfrac{1}{2}\bar{\mathcal{D}}\mathcal{D} S \;, \quad \Delta B = -\tfrac{i}{2}\bar{\mathcal{D}}\mathcal{D} P, \quad (49)$$

or $\Delta A = \frac{1}{2}\bar{\mathcal{D}}\gamma_5\mathcal{D} P, \quad \Delta B = -\frac{i}{2}\bar{\mathcal{D}}\gamma_5 \mathcal{D} S$, etc.

It is clear that if A and B (Eq.(15) were chiral or, at least, if they could be split locally (i.e. without using projection operators) into parts with superspins 1/2 and 0, then one could redefine V_μ to eliminate the r.h.s. in Eq.(4). We wish to point out here that neither γ_5 -invariance, nor scale invariance of the action are conditions sufficient for vanishing the r.h.s. of Eq. (4).

V. Examples of Supercurrents

To illustrate our procedure,we shall obtain in this section the supercurrent for two particular cases, just for the general and chiral scalar superfields. The resulting expressions are the same as those given in Ref. /2/ without derivation.

1) General scalar superfield. The Lagrangian for the free massive general scalar superfield $\varphi(x,\theta)$ is

$$\mathcal{L} = \tfrac{1}{2}\bar{\mathcal{D}}^\alpha \bar{\mathcal{D}}^\beta \varphi \mathcal{D}_\alpha \mathcal{D}_\beta \varphi - 2m^2\varphi^2 \;; \quad \mathcal{L}_{\alpha\beta} = \mathcal{D}_\alpha \mathcal{D}_\beta \varphi \,. \quad (50)$$

Then one has to calculate the quantities (10)-(14) and insert them into Eq.(9). We are not going to do all this here. Instead, we should examine only the terms a,b, and $\bar{\mathcal{D}}\sigma^{\mu\nu}\varphi_{\mu\nu}$ in order to clarify the situation discussed at the end of Section IV. The corresponding expressions are

$$a = -\tfrac{1}{8}\bar{\mathcal{D}}\mathcal{D}(\bar{\mathcal{D}}^\alpha\varphi . \mathcal{D}_\alpha\varphi) - 2m^2\varphi^2 \quad (51)$$

$$b = \tfrac{i}{8}\bar{\mathcal{D}}\gamma_5\mathcal{D}(\bar{\mathcal{D}}^\alpha\varphi . \mathcal{D}_\alpha\varphi) \quad (52)$$

$$\bar{\mathcal{D}}\sigma^{\mu\nu}\varphi_{\mu\nu} = \tfrac{1}{8}\bar{\mathcal{D}}\mathcal{D}(\varphi\bar{\mathcal{D}}\mathcal{D}\varphi) - \tfrac{1}{8}\bar{\mathcal{D}}\gamma_5\mathcal{D}(\varphi\bar{\mathcal{D}}\gamma_5\mathcal{D}\varphi) \,. \quad (53)$$

When $m = 0$ the superfields (51),(52),(53) are chiral and so are A and B (Eq.(15)). Therefore, according to Eq.(47) we can redefine the supercurrent V_μ in order to eliminate the corresponding terms in Eq. (9):

$$\tilde{V}_\mu = \bar{\mathcal{D}}\gamma_5 \Psi_\mu + 2(\bar{\mathcal{D}}\gamma^\nu\gamma_5)^\alpha \bar{\mathcal{D}}\mathcal{D}\varphi_{\mu\nu\alpha} \,. \quad (54)$$

Eq.(49) now implies that $\tilde{A} = \tilde{B} = 0$ so the r.h.s. in Eq.(4) can be made zero in the massless case. This means, in particular, that the redefined supercurrent (54) contains just the improved energy-momentum tensor (see Ref. /2/).

The final result for the supercurrent (including the massive case) is as follows

$$\tilde{V}_\mu = -\frac{i}{2}\bar{\mathcal{D}}\mathcal{D}.\bar{\mathcal{D}}^\alpha\Phi.\bar{\mathcal{D}}\mathcal{D}(\gamma_\mu\gamma_5\mathcal{D})_\alpha\Phi +$$

$$+2im^2\bar{\mathcal{D}}^\alpha\Phi(\gamma_\mu\gamma_5\mathcal{D})_\alpha\Phi - \frac{2}{3}im^2\bar{\mathcal{D}}\gamma_\mu\gamma_5\mathcal{D}(\Phi^2) +$$

$$+2i(\partial^2+\frac{1}{4}(\bar{\mathcal{D}}\mathcal{D})^2+m^2)\Phi.\bar{\mathcal{D}}\gamma_\mu\gamma_5\mathcal{D}\Phi \tag{55a}$$

$$(\gamma^\mu\mathcal{D})_\alpha V_\mu = -\frac{2i}{3}m^2\mathcal{D}_\alpha\bar{\mathcal{D}}\gamma_5\mathcal{D}(\Phi^2). \tag{55b}$$

The last term in Eq. (55a) vanishes due to the equations of motion so it does not effect the conservation of the supercurrent. However, it could become crucial when discussing the transformation properties of the superfield $\Phi(x,\theta)$ (see Ref. /3/).

2) Chiral scalar superfields. Here we prefer the two component Van der Vaerden notation (see Appendix A). The action for the free massless chiral superfield Φ and its conjugate Φ^+ is traditionally written in the form /11/

$$S = \int d^4x\, d^4\theta\,[\Phi\Phi^+ +$$

$$+\frac{m}{4}(\delta(\theta)\Phi^2 - \delta(\bar{\theta})\Phi^{+2})], \tag{56}$$

where $\delta(\theta)$ and $\delta(\bar{\theta})$ are δ -functions on Grassmann algebra. In order to derive equations of motion from this action one has to vary the superfields Φ, Φ^+ in Eq. (56) under supplementary conditions

$$\mathcal{D}_\alpha\Phi = 0, \quad \bar{\mathcal{D}}^{\dot{\alpha}}\Phi^+ = 0. \tag{57}$$

This approach is not convenient for our purposes so we shall

modify Eq. (56). Let us introduce a new general superfield $\varphi(x,\theta,\bar{\theta})$ and its conjugate $\varphi^+(x,\theta,\bar{\theta})$ and define

$$\Phi = \mathcal{D}\mathcal{D}\varphi, \quad \Phi^+ = \bar{\mathcal{D}}\bar{\mathcal{D}}\varphi^+. \tag{58}$$

Thus the chirality conditions (57) are automatically fulfilled. Then Eq. (56) becomes

$$S = \int d^4x\, d^4\theta\,[\mathcal{D}\mathcal{D}\varphi\bar{\mathcal{D}}\bar{\mathcal{D}}\varphi^+ - \frac{1}{2}m(\varphi\mathcal{D}\mathcal{D}\varphi + \varphi^+\bar{\mathcal{D}}\bar{\mathcal{D}}\varphi^+)] \tag{59}$$

and the equations of motion are obtained by the standard variational procedure. Now we can go on to the supercurrent. First we examine the terms A and B of Eq. (15):

$$A = \frac{1}{12}\{-5\Phi\Phi^+ + \frac{3}{2}m(\varphi\Phi+\varphi^+\Phi^+) -$$

$$-\frac{5}{2}[\mathcal{D}\mathcal{D}(\varphi\Phi^+)+\bar{\mathcal{D}}\bar{\mathcal{D}}(\varphi^+\Phi)] + \frac{5}{4}m[\mathcal{D}\mathcal{D}(\varphi^2)+\bar{\mathcal{D}}\bar{\mathcal{D}}(\varphi^+)^2]\} \tag{60}$$

$$B = -\frac{1}{8}[m(\varphi\Phi-\varphi^+\Phi^+) + \mathcal{D}\mathcal{D}(\varphi\Phi^+) - \bar{\mathcal{D}}\bar{\mathcal{D}}(\varphi^+\Phi) -$$

$$-\frac{1}{2}m(\mathcal{D}\mathcal{D}(\varphi^2) - \bar{\mathcal{D}}\bar{\mathcal{D}}(\varphi^+)^2]. \tag{61}$$

Once more one can omit the manifestly chiral terms in Eqs. (60), (61) for the reasons discussed above. Then after some algebra we obtain the final result

$$V_\mu = \frac{2}{3}[\Phi^+ i\partial_\mu\Phi - \Phi i\partial_\mu\Phi^+ - \frac{1}{4}\mathcal{D}^\alpha\Phi^+(\sigma_\mu\bar{\mathcal{D}})_\alpha\Phi]$$

$$(\sigma^\mu\bar{\mathcal{D}})_\alpha V_\mu = -\frac{m}{12}\mathcal{D}_\alpha(\Phi^+)^2. \tag{62}$$

Both expressions (55) and (62) coincide exactly with those in Ref./2/.

Some applications of the general supercurrent formula in the supergravity theory will be given elsewhere.

It is a pleasure for us to thank Dr.E.A.Ivanov for numerous discussions.

Appendix A: Notations

In the present paper the four-component Majorana notations are mainly used:

$$\frac{1}{2}\{\gamma_\mu,\gamma_\nu\}=\eta_{\mu\nu}=diag(+---)\,;\quad \gamma_5=\gamma_0\gamma_1\gamma_2\gamma_3\,;$$

$$\sigma_{\mu\nu}=\frac{i}{2}[\gamma_\mu,\gamma_\nu]\,;\quad \varepsilon^{0123}=1\,.$$

The spinorial variables θ_α are Majorana spinors

$$\bar\theta^\beta=(C^{-1})^{\beta\alpha}\theta_\alpha\,,$$

where $C=i\gamma^0\gamma^2$ is the charge conjugation matrix.

The notation $\Gamma_{\alpha\beta}$ for the matrix Γ means $C_{\beta\rho}\Gamma_\alpha{}^\rho$.

The spinor derivatives D_α are defined as follows

$$D_\alpha=\frac{\partial}{\partial\bar\theta^\alpha}-\frac{i}{2}(\not\partial\theta)_\alpha\;;\quad \{D_\alpha,D_\beta\}=(\gamma^\mu)_{\alpha\beta}\,i\partial_\mu\,.$$

In the chiral superfield case the two component Van der Vaerden formalism is preferable. It is the same as in Refs. /2/,/11/ except for the sign of the anticommutator

$$\{D_\alpha,\bar D_{\dot\alpha}\}=2i\,\not\partial_{\alpha\dot\alpha}\,.$$

Appendix B: The γ_5-Invariance

In Ref./2/ a guess was made that the axial vector field $v_\mu(x)$ in the decomposition of the supercurrent (6) is the γ_5-current of the Lagrangian and its conservation was connected with the vanishing of the r.h.s. of Eq.(4). Here we show that this is not true in general.

Let us examine the consequences of γ_5-invariance of the action (1). For this case (see Eq.(16))

$$G_a\equiv G_5=\bar\theta\, i\gamma_5\frac{\partial}{\partial\bar\theta}\;;\quad [D_\alpha,G_5]=(i\gamma_5 D)_\alpha \tag{63}$$

$(\Gamma_a)_{ij}\equiv(\gamma_5)_\alpha{}^\beta$ for the fermion superfields. Now there is no ground to write the invariance condition in the strong form (21) and we shall weaken it:

$$\delta\mathcal{L}=c\,G_5\mathcal{L}+c\,\bar D^\alpha\Delta_\alpha\,, \tag{64}$$

where $\bar D^\alpha\Delta_\alpha$ is some divergence and c is the parameter. The transformation (63) has the form (25) so we can use Eq. (26) and find

$$\bar D^\alpha\Sigma_\alpha=$$

$$=(\bar D\gamma_5)^\alpha\varphi_i\mathcal{L}^i_\alpha+(\bar D\gamma_5)^\alpha\bar D^\beta\varphi_i\mathcal{L}^i_{\alpha\beta}+\bar D^\alpha(\bar D\gamma_5)^\beta\varphi_i\mathcal{L}^i_{\alpha\beta}\,. \tag{65}$$

The divergence $\bar D^\alpha\Delta_\alpha$ from Eq.(64) and the Γ^a_{ij} term from Eq. (26) are combined together into a symbolic divergence $\bar D^\alpha\Sigma_\alpha$ in the l.h.s. of Eq. (65).

Now we multiply the conservation law (4') by $-\frac{i}{2}\bar D^\alpha$ and find

$$\partial^\mu V_\mu = \frac{1}{3}\partial^\mu \bar{\mathcal{D}} i\gamma_\mu\gamma_5 \mathcal{D}(a - \bar{\mathcal{D}}\sigma^{\lambda\rho}\varphi_{\lambda\rho}) - \frac{i}{2}(\bar{\mathcal{D}}\mathcal{D})^2 b \; . \qquad (66)$$

If the theory is γ_5 -invariant, then the identity (65) takes place and one can show that b (14) is a divergence: $b = \bar{\mathcal{D}}^\alpha \Omega_\alpha$. Therefore

$$-\frac{i}{2}(\bar{\mathcal{D}}\mathcal{D})^2 b = \partial^\mu \bar{\mathcal{D}}\mathcal{D}.\bar{\mathcal{D}}\gamma_\mu\Omega$$

and Eq. (66) becomes

$$\partial^\mu\left[V_\mu - \frac{1}{3}\bar{\mathcal{D}} i\gamma_\mu\gamma_5\mathcal{D}(a - \bar{\mathcal{D}}\sigma^{\lambda\rho}\varphi_{\lambda\rho}) - \bar{\mathcal{D}}\mathcal{D}.\bar{\mathcal{D}}\gamma_\mu\Omega\right] = 0. \qquad (67)$$

Integrating Eq.(67) over $d^4\theta$ we obtain just the γ_5 -current conservation law. However, it is clear that this current does not coincide with the $v_\mu(x)$ -component in Eq.(6). It would coincide if the r.h.s. of Eq. (4) were 0, but γ_5 -invariance is not a condition sufficient for this. A convincing example is the general scalar superfield of Section V. It is not difficult to show that that theory possesses γ_5 -invariance even in the massive case (when the r.h.s. in Eq.(55) does not vanish).

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