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NEW DYNAMICAL CHARACTERISTICS: THE ISOSPIN-SPIN POLARIZATIONS OF INTERACTING HADRONS

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NEW DYNAMICAL CHARACTERISTICS: THE ISOSPIN-SPIN POLARIZATIONS OF INTERACTING HADRONS

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Изоспин-спиновые поляризационные характеристики взаимодействующих адронов

Получены изоспин-спиновые поляризационные характеристики взаимодействующих адронов, а также физическая интерпретация насыщения изоспиновых границ. Предполагается приблизительное сохранение изоспин-спиновых поляризаций, как возможного закона сильных взаимодействий адронов.

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New Dynamical Characteristics: the Isospin-Spin Polarizations of Interacting Hadrons

New dynamical characteristics: the isospin-spin polarizations (ISP) of interacting hadrons as well as the saturation of the isospin bounds in terms of the ISP-conservation are obtained using the formalism of the isospin-spin density matrix. The approximate ISP-conservation as a new statistical conservation law of strong interactions (especially at high energies) is suggested.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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It is well known that the isospin symmetry is one of the most important concepts of the particle physics. This symmetry is experimentally satisfied to a surprising accuracy. The breaking of the isospin invariance is usually attributed to the electromagnetic and weak interactions. The conventional electromagnetic corrections are included for the most usual analyses of the experimental data. For hadron-hadron exclusive reactions which differ only by the isospin components of the particle isomultiplets, the isospin invariance implies that transition matrices satisfy linear relations. Hence, it results in that the differential cross sections and spin polarizations of the hadrons, measured in such reactions, must obey a set of the isospin constraints [1-4] more stronger than the usual triangular inequalities. It was pointed out [5-15] that data tend to saturate the usual bounds as well as the stringlet isospin bounds [2,3,13-15]. One can look at this phenomenon in two ways: (i) as to an exact regularity of the strong interactions in the sense that there exists in the physical region a set of lines [3] on which different stringent isospin bounds are exactly saturated; (ii) as to an approximate regularity of the strong interactions in the sense that it is an increasing puzzling of the experimental data that the data tend to saturate the isospin bounds within the experimental error in large regions from the physical domain.

Detailed investigations on the nature of these regularities are necessary since they might provide an important clue towards a better understanding of the hadron interactions. It is natural to assume that the saturations of isospin bounds are closely related to the behaviour of the isospin-spin polarization (ISP) characteristics of the interacting hadrons as we have suggested recently [3]. In this paper, using the formalism [16] of the isospin-spin density matrix, we obtain a new set of (ISP)-characteristics of the interacting hadrons and prove that the saturation of each isospin bound can be expressed in terms of a specific ISP-conservation.

Start with a general reaction

$$a + b \to a + b, \tag{1}$$

where a, b, c, d are isospin multiplets with the isospins I_a, I_b, I_c, I_d and with arbitrary spins. We assume that the reaction (1) is going through two isospin channels. We describe the initial states of the reaction (1) by the isospin-spin density matrix

$$\rho_{in} = \rho_{in}^{(I)} \otimes \rho_{in}^{(J)} \qquad \mathrm{tr}\,\rho_{in} = 1\,, \tag{2a}$$

in the combined isospin-spin spaces of the initial isomultiplets a and b. $\rho_{in}^{(I)}$ is the isospin density matrix in the direct product of the isospin spaces of the initial isomultiplets while $\rho_{in}^{(J)}$ is the spin density matrix describing their polarization states. Since each initial state of reaction (1) is isospin-polarized in the $I^{(3)}$ direction of the isospin space, then $\rho_{in}^{(I)}$ is a diagonal matrix which may be written as:

$$\rho_{in}^{(I)} = \frac{1}{(2I_a+1)(2I_b+1)} \sum \alpha_{I_a^{(3)} I_b^{(3)}} \rho_{in}^{[I_a^{(3)} I_b^{(3)}]}, \tag{2b}$$

$$\sum \alpha_{I_a^{(3)} I_b^{(3)}} = (2I_a+1)(2I_b+1)\,,$$

where the orthogonal primitive idempotents matrices $\rho_{in}^{[I_a^{(3)} I_b^{(3)}]}$ correspond to $(2I_a+1)(2I_b+1)$ -pure isospin polarization states while $\alpha_{I_a^{(3)} I_b^{(3)}}$ are their isospin weights.

The final states of the reaction (1) are described by the isospin-spin density matrix defined as

$$\rho_{out} = T\rho_{in}T^{+}\,, \tag{3}$$

where T is the isospin-spin invariant transition operator which may be written as a matrix whose rows and columns are characterized by the isospin and spin quantum numbers of the initial and final hadrons, respectively.

Therefore, the whole information on the reaction (1) is embodied in the isospin-spin density matrices ρ_{out} and ρ_{in}. Any detailed information on the charge channels can be obtained by the evaluation of the expectation values

$$\langle\Lambda\rangle_s = \frac{\mathrm{tr}[\rho_s\Lambda]}{\mathrm{tr}\,\rho_s}, \quad s \equiv \mathrm{in},\ \mathrm{out}, \tag{4a}$$

for the different isospin-spin operators acting on the combined isospin-spin spaces of the final or initial hadrons, for each $\rho_{in}^{[I_a^{(3)} I_b^{(3)}]}$. The expectation values $\langle\Lambda\rangle_s$ will be called the ISP characteristics of the interacting hadrons while by the conservation of an ISP characteristic we mean:

$$\langle\Lambda\rangle_{in} = \langle\Lambda\rangle_{out}\,. \tag{4b}$$

Now, let $\vec{I}_a \times \vec{I}_b$ and $\vec{I}_c \times \vec{I}_d$ be the "vector product" of the isospin operators $(\vec{I}_a, \vec{I}_b)$ and $(\vec{I}_c, \vec{I}_d)$ and let U be an arbitrary spin operator in the combined spin spaces of the final (or initial) hadrons, then using eqs. (2)-(4) we calculate the expectation values $\langle \vec{I}_a \times \vec{I}_b \otimes V_{ab}\rangle_{in}$ and $\langle \vec{I}_c \times \vec{I}_d \otimes V_{cd}\rangle_{out}$, $V \equiv \{1, U\}$, for the most usual reactions $(I_a I_b \to I_c I_d) \equiv (\tfrac{1}{2}\tfrac{1}{2} \to \tfrac{1}{2}\tfrac{1}{2})$, $(1\tfrac{1}{2} \to 1\tfrac{1}{2})$, and different $\rho_{in}^{[I_a^{(3)} I_b^{(3)}]}$. Therefore, using the isospin matrices for the isospin 1/2 and 1, we obtain the results presented in table 1, in terms of the matrices:

$$\sigma_\ell \, \rho^{(\ell)} = T_\ell \, \rho_{in}^{(J)} T_\ell^{+} \; , \; tr \rho^{(\ell)} = 1 \, , \tag{5a}$$

$$\omega_{\ell m} = T_\ell \, \rho_{in} \, T_m^{+} \tag{5b}$$

where σ_ℓ and $\rho^{(\ell)}$, $\ell \neq m = i, j, k$, are the differential cross sections and the final spin density matrices of the charge channels (ℓ) while T_ℓ are their spin transition matrices.

Then, since the isospin invariance implies that the spin transition matrices T_ℓ satisfy the sum rule $\Sigma c_\ell T_\ell = 0$, we obtain the following important equalities:

$$c_i c_j \, Im\{tr[\omega_{ij} V]\} = c_j c_k Im\{tr[\omega_{jk} V]\} = = c_k c_i \, Im\{tr[\omega_{ki} V]\}, \tag{6a}$$

$$2 c_\ell c_m Re\{tr[\omega_{\ell m} V]\} = c_n^2 \sigma_n \, tr[\rho^{(n)} V] - - c_m^2 \sigma_m tr[\rho^{(m)} V] - c_\ell^2 \sigma_\ell \, tr[\rho^{(\ell)} \, V], \tag{6b}$$

where three indices $\ell \neq m \neq n$ represent any permutation of the indices i, j, k while c_ℓ are functions of the Clebsch-Gordan coefficients and are given in table 1.

Next, let $\lambda[\sigma \, tr[\rho V]]$ be the function

$$\lambda[X] = X_i^2 + X_j^2 + X_k^2 - 2X_i X_j - 2X_j X_k - 2X_k X_i \, , \tag{7}$$

for $X_\ell \equiv c_\ell^2 \sigma_\ell \, tr[\rho^{(\ell)} V]$, ℓ=i,j,k. Then, using eqs. (6a,b) we obtain

$$2|c_\ell c_m \; Im\{tr[\omega_{\ell m} \; V]\}| = \{-4H_V - \lambda[\sigma \, tr[\rho V]]\}^{1/2}, \tag{8}$$

where

Table 1

The expectation values $\langle \vec{I}_c \times \vec{I}_d \otimes V \rangle_{out}$ and $\langle \vec{I}_a \times \vec{I}_b \otimes V \rangle_{in}$ for the isospin $(I_a I_b \to I_c I_d) \equiv (½ ½ \to ½ ½)$, $(1 ½ \to 1 ½)$ reactions with different initial isospin polarized $[I_a^{(3)} I_b^{(3)}]$ states

$I_a I_b \to I_c I_d$	$I_a^{(3)} I_b^{(3)}$	$I_c^{(3)} I_d^{(3)}$	T_ℓ	$c_\ell^{(*)}$	$tr\rho_{out} \langle (\vec{I}_c \times \vec{I}_d)^{(3)} \otimes V \rangle_{out}$	$tr\rho_{out}$	$\langle (\vec{I}_a \times \vec{I}_b)^{(1,2,3)} \otimes V_{ab} \rangle_{in}$ $\langle (\vec{I}_c \times \vec{I}_d)^{(1,2)} \otimes V_{cd} \rangle_{out}$
$(\frac{1}{2} \frac{1}{2} \to \frac{1}{2} \frac{1}{2})$	$\frac{1}{2} \frac{1}{2}$	$\frac{1}{2} \frac{1}{2}$	T_i	+1	0	σ_i	0
	$\frac{1}{2} - \frac{1}{2}$	$\frac{1}{2} - \frac{1}{2}$	T_j	−1	$c_j c_k Im\{tr[\omega_{jk} V]\}$	$\sigma_j + \sigma_k$	0
		$-\frac{1}{2} \frac{1}{2}$	T_k	−1			
	$-\frac{1}{2} \frac{1}{2}$	$\frac{1}{2} - \frac{1}{2}$	T_k	−1	$-c_j c_k Im\{tr[\omega_{jk} V]\}$	$\sigma_j + \sigma_k$	0
		$-\frac{1}{2} \frac{1}{2}$	T_j	−1			
	$-\frac{1}{2} - \frac{1}{2}$	$-\frac{1}{2} - \frac{1}{2}$	T_i	+1	0	σ_i	0
$(1\frac{1}{2} \to 1\frac{1}{2})$	$1 \frac{1}{2}$	$1 \frac{1}{2}$	T_i	+1	0	σ_i	0
	$1 - \frac{1}{2}$	$0 \frac{1}{2}$	T_k	$-\sqrt{2}$	$c_j c_k Im\{tr[\omega_{jk} V]\}$	$\sigma_j + \sigma_k$	0
		$1 - \frac{1}{2}$	T_j	−1			
	$-1 \frac{1}{2}$	$-1 \frac{1}{2}$	T_j	−1	$-c_j c_k Im\{tr[\omega_{jk} V]\}$	$\sigma_j + \sigma_k$	0
		$0 - \frac{1}{2}$	T_k	$-\sqrt{2}$			
	$-1 - \frac{1}{2}$	$-1 - \frac{1}{2}$	T_i	+1	0	σ_i	0

* $\Sigma c_\ell T_\ell = 0$ is implied by the isospin invariance

$$H_V \equiv c_i^2 c_j^2 \sigma_i \sigma_j \{tr[\rho^{(i)} V] tr[\rho^{(j)} V] - tr[(\rho^{(i)} V)(\rho^{(j)} V)]\} =$$

$$= c_j^2 c_k^2 \sigma_j \sigma_k \{tr[\rho^{(j)} V] tr[\rho^{(k)} V] - tr[(\rho^{(j)} V)(\rho^{(k)} V)]\} = \quad (9a)$$

$$= c_k^2 c_i^2 \sigma_k \sigma_i \{tr[\rho^{(k)} V] tr[\rho^{(i)} V] - tr[(\rho^{(k)} V)(\rho^{(i)} V)]\}$$

and

$$4H_V \le -\lambda[\sigma \, tr[\rho V]] . \quad (9b)$$

Hence, for the isospin ($I_a I_b \to I_c I_d$) reactions between the isospin multiplets with arbitrary spins, we obtain the following general results.

1) If the initial state of reaction (1) is an isospin unpolarized state ($\rho_{in}^{(I)} = \frac{1}{(2I_a+1)(2I_b+1)}$) or a state which is going through a single isospin channel, then

$$<\vec{I}_a \times \vec{I}_b \otimes V_{ab}>_{in} = <\vec{I}_c \times \vec{I}_d \otimes V_{cd}>_{out} = 0 . \quad (10)$$

2) If the initial pure isospin polarized state is going through two channels (see table 1), then we have

$$tr\rho_{out} <\vec{I}_c \times \vec{I}_d \otimes V_{cd}>_{out} = \{0, 0, \frac{\epsilon_V}{2}[-4H_V - \lambda[\sigma \, tr[\rho V]]^{1/2}\}, \quad (11a)$$

$$<\vec{I}_a \times \vec{I}_b \otimes V_{ab}>_{in} = \{0, 0, 0\}, \quad (11b)$$

$$\epsilon_V = \pm \, sign\{c_j c_k Im[tr[\omega_{jk} V]]\} \text{ (see table 1)} \quad (12)$$

So, the ISP $<(\vec{I} \times \vec{I}')^{(1,2)} \otimes V>$ characteristics are exactly conserved.

3) The exact saturation of the isospin bound (9b) for a given V implies the exact conservation of the corresponding ISP $<\vec{I} \times \vec{I}' \otimes V>$ characteristic of the interacting hadrons.

The results (8)-(12) and table 1 are sufficient to derive and interpret any isospin bound in terms of the ISP $<\vec{I} \times \vec{I}' \otimes V>$ characteristics of the interacting hadrons. Note that the isospin constraints (9a,b), for $V \equiv 1$, for the many body final state reactions between hadrons with arbitrary spins, have been derived recently in /17/. Detailed results on $<\vec{I}_c \times \vec{I}_d \otimes V_{cd}>_{out}$, for different spin operators $U (U = U^+)$, can be given for each kind of spin reaction, but here we shall discuss only the usual case of the spin ($0^- \frac{1}{2}^+ \to 0^- \frac{1}{2}^+$) reactions.

Therefore, let us assume that the reaction (1) is a spin ($0^- \frac{1}{2}^+ \to 0^- \frac{1}{2}^+$) reaction between the isospin multiplets (e.g., $\pi N \to \pi N$, $KN \to KN$, $\bar{K}N \to \bar{K}N$, $\pi N \to K\Sigma$ (13), etc), and let $\vec{P}_0$ be the initial polarization chosen such that $\vec{P}_0 \perp \vec{n}$ and $|\vec{P}_0| = 1$, where $\vec{n}$ is the normal to the scattering plane. Then, using the spin density matrix $\rho_{in}^{(J)} = \frac{1}{2}(1 + \vec{P}_0 \cdot \vec{\sigma})$, $\vec{\sigma}$ – Pauli's matrices, we obtain the results presented in table 2, for each spin operator $V \equiv 1$, $\vec{\kappa} \cdot \vec{\sigma}$ and $(1 \pm \vec{\kappa} \cdot \vec{\sigma})$, respectively, where $\vec{\kappa}$ is an arbitrary unit vector in the spin space. Let $\vec{P}_\ell = tr[\rho^{(\ell)} \vec{\sigma}]$, be the spin polarization vectors of the final hadrons of the charge channels, then $H_V \equiv H$ (for $V \equiv 1$) can be written as

$$H \equiv c_i^2 c_j^2 H_{ij} = c_j^2 c_k^2 H_{jk} = c_k^2 c_i^2 H_{ki} , \; H_{\ell m} = \frac{1}{2} \sigma_\ell \sigma_m (1 - \vec{P}_\ell \cdot \vec{P}_m) . \quad (14)$$

Next, if $V \equiv \vec{\sigma}$ we write $<(\vec{I}_c \times \vec{I}_d)^{(3)} \otimes \vec{\sigma}>_{out}$ as

$$<(\vec{I}_c \times \vec{I}_d)^{(3)} \otimes \vec{\sigma}>_{out} = <(\vec{I}_c \times \vec{I}_d)^{(3)} \otimes \vec{\kappa} \cdot \vec{\sigma}>_{out} \vec{\kappa} + $$
$$+ <(\vec{I}_c \times \vec{I}_d)^{(3)} \otimes \vec{\kappa}' \cdot \vec{\sigma}>_{out} \vec{\kappa}' + <(\vec{I}_c \times \vec{I}_d)^{(3)} \otimes (\vec{\kappa} \times \vec{\kappa}') \cdot \vec{\sigma}>_{out} \vec{\kappa} \times \vec{\kappa}' , \quad (15)$$

where $\vec{\kappa}$, $\vec{\kappa}'$ and $\vec{\kappa} \times \vec{\kappa}'$ are three orthogonal unit vectors (e.g., $\vec{n}$, $\vec{n} \times \vec{P}_0$, $\vec{P}_0$) in the spin space. Then, using the results of table 2 and the identity

$$\lambda(\vec{\kappa} \cdot \vec{P}\sigma] + \lambda[\vec{\kappa}' \cdot \vec{P}\sigma] + \lambda[(\vec{\kappa} \times \vec{\kappa}') \cdot \vec{P}\sigma] = 12H + \lambda[\sigma] , \quad (16)$$

Table 2

The experimental observables $\sigma_\ell \, \mathrm{tr}[\rho^{(\ell)} V]$, H_V, the isospin bounds $4H_V \le -\lambda[\sigma \, \mathrm{tr}[\rho^{(\ell)} V]]$, and the corresponding ISP characteristics $\langle I \times I' \otimes V \rangle$ for the spin $(0^- \tfrac{1}{2}^+ \to 0^- \tfrac{1}{2}^+)$ reactions related by the isospin invariance via two channels.

V	1	$\vec\kappa \cdot \vec\sigma$	$(1 \pm \vec\kappa \cdot \vec\sigma)$
$\sigma_\ell \, \mathrm{tr}[\rho^{(\ell)} V]$	σ_ℓ	$\vec\kappa \cdot \vec P_\ell \sigma_\ell$	$(1 \pm \vec\kappa \cdot \vec P_\ell)\sigma_\ell$
$\mathrm{tr}[\omega_{jk} V]$	$Z^{(0)}_{kj}$ (*)	$Z^{(\kappa)}_{kj}$ (*)	$M^{(\pm\kappa)}_{kj}$ (*)
	$\frac{1}{2}[M^{(+\kappa)}_{kj} + M^{(-\kappa)}_{kj}]$	$\frac{1}{2}[M^{(+\kappa)}_{kj} - M^{(-\kappa)}_{kj}]$	$F^{*(\pm\kappa)}_k F^{(\pm\kappa)}_j$
H_V	H	−H	0
$4H_V \le -\lambda[\sigma \, \mathrm{tr}[\rho V]]$	$4H \le -\lambda[\sigma]$	$\lambda[\vec\kappa \cdot \vec P \sigma] \le 4H$	$0 \le -\lambda^{(\pm)}_\kappa$
	$\lambda[X]$ - eq.(7) $X_\ell = c_\ell^2 \sigma_\ell$	λ - eq.(7) $X_\ell = c_\ell^2 \sigma_\ell \vec\kappa \cdot \vec P_\ell$	λ - eq.(7) $X_\ell = c_\ell^2 \sigma_\ell (1 \pm \vec\kappa \cdot \vec P_\ell)$
$\mathrm{tr}\rho_{out}$	$\pm\frac{\epsilon}{2}[-4H - \lambda[\sigma]]^{1/2}$	$\pm\frac{\epsilon_\kappa}{2}[4H - \lambda[\vec\kappa \cdot \vec P \sigma]]^{1/2}$	$\pm\frac{\epsilon^{(\pm)}_\kappa}{2}[-\lambda^{(\pm)}_\kappa]^{1/2}$
$\langle (\vec I_c \times \vec I_d)^{(3)} \otimes V_{cd} \rangle_{out}$	(**) $\epsilon \equiv \mathrm{sign}\{c_j c_k \mathrm{Im} Z^{(0)}_{kj}\}$	(**) $\epsilon_\kappa \equiv \mathrm{sign}\{c_j c_k \mathrm{Im} Z^{(\kappa)}_{jk}\}$	(**) $\epsilon^{(\pm)}_\kappa \equiv \mathrm{sign}\{c_j c_k \mathrm{Im} M^{(\pm\kappa)}_{jk}\}$
$\langle (\vec I_a \times \vec I_b)^{(3)} \otimes V_{ab} \rangle_{in}$	0	0	0

* For the definitions of the bilinear forms $Z^{(0)}_{kj}, Z^{(\kappa)}_{kj}, M^{(\pm\kappa)}_{kj}$ and of the amplitudes $F^{(\pm\kappa)}_\ell$, see eqs.(9a)–(9c) and (12b) in /3/.

** For determination of these signs see also ref. /3/; the signs ± are determined as in table I.

we get

$$[\mathrm{tr}\rho_{out}]^2 \, |\langle(\vec I_c \times \vec I_d)^{(3)} \otimes \vec\sigma\rangle|^2 = -\frac{1}{4}\lambda[\sigma] \ge 0 , \qquad (17a)$$

$$|\langle(\vec I_a \times \vec I_b)^{(3)} \otimes \vec\sigma\rangle|^2 = 0 , \qquad (17b)$$

where λ -functions are defined by eq. (7) for

$$X_\ell \equiv c_\ell^2 \sigma_\ell \vec\kappa \cdot \vec P_\ell , \; c_\ell^2 \sigma_\ell \vec\kappa' \cdot \vec P_\ell , \; c_\ell^2 \sigma_\ell (\vec\kappa \times \vec\kappa') \cdot \vec P_\ell , \; c_\ell^2 \sigma_\ell ,$$

respectively.

Therefore, the results (17a,b) allow us to discuss the observed saturation /5-13/ of the usual triangle inequalities $(-\lambda[\sigma] \ge 0)$ in terms of the approximate conservation of the isospin-spin polarization vector $\langle \vec I \times \vec I' \otimes \vec\sigma \rangle$ in any direction of the isospin and spin spaces. Moreover, the results presented in tables 1,2 are important for the interpretation of the observed lines /2,3,13-15/ from the physical domain where the stringent isospin bounds $[4H \le -\lambda[\sigma], \quad \lambda[\vec\kappa \cdot \vec P \sigma] \le 4H, \; 0 < -\lambda^{(\pm)}_\kappa]$ are exactly saturated as the lines of exact conservation of the isospin-spin polarization (LECISP) characteristics $[\langle \vec I \times \vec I' \otimes 1 \rangle, \; \langle \vec I \times \vec I' \otimes \vec\kappa \cdot \vec\sigma \rangle, \; \langle \vec I \times \vec I' \otimes (1 \pm \vec\kappa \cdot \vec\sigma) \rangle]$ respectively. Similar interpretation is also obtained for the isospin bound /3/: $\lambda(\vec\kappa \cdot \vec P \sigma] \le -\lambda[\sigma]$, since according to table 2 we have:

$$4[\mathrm{tr}\rho_{out}]^2 \{ [\langle(\vec I_c \times \vec I_d)^{(3)} \otimes 1\rangle_{out}]^2 + \qquad (18a)$$

$$+ [\langle(\vec I_c \times \vec I_d)^{(3)} \otimes \vec\kappa \cdot \vec\sigma\rangle_{out}]^2 \} =$$

$$= -\lambda[\sigma] - \lambda[\vec\kappa \cdot \vec P \sigma] \ge 0 ,$$

$$[\langle \vec I_a \times \vec I_b \otimes 1 \rangle_{in}]^2 + [\langle \vec I_a \times \vec I_b \otimes \vec\kappa \cdot \vec\sigma \rangle_{in}]^2 = 0 . \qquad (18b)$$

Hence, for the spin $(0^- \tfrac{1}{2}^+ \to 0^- \tfrac{1}{2}^+)$ reactions, the results presented in table 2 are sufficient for studying any experimental situation in terms of the ISP-characteristics $\langle \vec I \times \vec I' \otimes V \rangle$. It is important to note that the formalism presented here can be extended to the many body final state reactions with arbitrary spins, and also to a straightforward development of the [U-spin, V-spin, unitary spin, etc.] -polarization characteristics of interacting hadrons.

Finally, we suggest that the experimental investigation of the (LECISP)-topology (see properties (i)-(vii), ref. [3]) is important for the development of a new phenomenology based on the interpretation of the experimental data in terms of the ISP-conservation. On the other hand, a detailed experimental study of the phenomenon of near saturation of isospin bound is necessary since the ISP-conservation may be a new approximate statistical conservation law of the strong interactions, especially at high energies.

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