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Спектроскопия мезонов, адронных молекул и кварков и вычисление угла Кабиббо

Построена феноменологическая теория спектра масс обычных и очарованных частиц с учетом примеси "молекулярных" состояний. Определены массы кварков, и в модели Вайнберга-Фрича вычислен угол Кабиббо. Предложена теоретическая формула для угла Кабиббо в модели, в которой отношение масс двух последовательных кварков постояно. Теоретическое значение равно 12,80°.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Spectroscopy of Mesons, Hadronic Molecules and Quarks and Calculating the Cabibbo Angle

By carefully analyzing the spectrum of the all known mesons in terms of a universal mass formula the strong interaction symmetry breaking effects are systematized, and the constituent quark masses established. By using the modified Weinberg-Fritzsch formula the Cabibbo angle is expressed in terms of a symmetry breaking parameter r, and by applying the stability condition $d\theta_c/dr = 0$ the formula $tg^2\theta_c = \frac{\sqrt{1/3} - 1/3}{\sqrt{3}+3}$ is derived. The predicted value $\theta_c \approx 12.8°$ and the assumptions on the quark mass spectrum used for the derivation are in nice agreement with experiment.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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In refs./1/ the following mass formula for mesons $(q_i\bar{q}_j)_\alpha$ was suggested (in what follows the masses of mesons and of quarks will be denoted by their respective symbols):

$$M^2_{ij\alpha} = 4m_0^2 + 2(m_i^2+m_j^2) - \frac{(m_i^2-m_j^2)^2}{M^2_{ij\alpha}} + 2(\mu_i^2+\mu_j^2)(L+\beta N) +$$
$$+ 4[\mu_F^2\delta_{L0} + \mu_F'^2(1-\delta_{L0})](\vec{s}_i\vec{s}_j) + \mu_L^2(\vec{L}\vec{S}) - \frac{4}{3}\varepsilon^2_{ij\alpha}(M^2_{ij\alpha}) . \qquad (1)$$

Here $i,j = u,d,s,c$ (and we also use the notation $m_i = q_i$, $m_u = m_d = u$, $m_s = s$, $m_c = c$, etc.), $\alpha = (J,L,S,N)$, or $\alpha = S,V,T,A,P \ldots$, N is the radial quantum number (e.g., $N = 0,1,2,\ldots$). In addition to relativistic kinematics, we here account for a symmetry breaking in the Regge trajectory slope and in the radial excitation energy, which are supposed to be defined by the same effective radius $R_{ij} \sim (\mu_i^2+\mu_j^2)^{-1/2}$. The μ_F^2, μ_L^2 and m_0^2 may have somewhat different values $\bar{\mu}_F^2, \bar{\mu}_L^2$, and $\bar{m}_0^2$ for the charmed mesons. However, the main symmetry breaking effects are supposed to be included in m_i and R_{ij}. For I=0 states we have to take into account the quark mixing effects, and this was successfully done in /1/. Here the effect of the mixing parameter $\varepsilon^2_{ij\alpha}$ is a mere shift in $M^2_{ij\alpha}$ which is significant only for pseudoscalar ($\alpha = P$) mesons, and the dependence of ε^2 on M^2 is appreciable only for $M^2 \leq \eta^2$. For large mixing the mass formulae are neither linear nor quadratic in masses, while for $I = 1, 1/2$ states (and for I=0 states with small mixing, $\varepsilon^2 \to 0$) they are linear. This can easily be proved by defining the effective masses $q_{i\alpha}$ of the quarks. For example, for the vector meson nonet we have

$$s_V^2 = s^2 + m_o^2 + \tfrac{1}{4}\mu_F^2 - \tfrac{4}{3}\varepsilon_V^2\,, \quad u_V^2 = u^2 + m_o^2 + \tfrac{1}{4}\mu_F^2 - \tfrac{4}{3}\varepsilon_V^2\,,$$

and it is easy to verify that

$$K^* \equiv K_V = s_V + u_V\,, \quad \rho = 2u_V \approx \omega\,, \quad \varphi = 2s_V$$

If the parameters m_o^2, μ_F^2, and ε^2 do not depend on the meson masses the simple relation holds: $q_{i\alpha}^2 - q_{j\alpha}^2 = q_i^2 - q_j^2 \equiv \Delta_{ij}^2$

To estimate the parameters let us take as the input masses the following

$$\pi\,,\ K\,,\ \eta\,,\ \rho\,,\ \varphi\,,\ K^* = K_V\,,\ A_2\,,\ K^{**} = K_T\,,\ f\,,\ A_1 \,(\text{or } \delta)\,,\ B.$$

Then $4m_o^2 + 2u^2 + 2s^2 = .7064$, $\Delta_{su}^2 = .1087$,
$\mu_F^2 = .1107$, $\varepsilon_P^2(\eta^2) = \varepsilon_P^2(K^2) = .0605$, $\varepsilon_P^2(\pi^2) = .103$, $\varepsilon_V^2 = .0020$
$\varepsilon_T^2 = .0114$, $\mu_F'^2 = -.0132$, $\mu_L^2 = .2455$, $4\mu_s^2 = 1.163$, $4\mu_u^2 = .9795$.

The predicted masses of all other states with N=0 prove to be in good agreement with experiment (up to some unknown mixing effects, which seem to be appreciable only for scalar mesons)

Assuming that $\bar{m}_o^2 = m_o^2$, $\varepsilon_{ci}^2 = 0$ and using $\mathcal{D}$, $\mathcal{D}^* = \mathcal{D}_V$ as the input masses we find

$$\Delta^2_{cu} = 2.521\,,\ \bar{\mu}_F^2 = .075\,,\ \bar{u}_P = .258\,,\ \bar{s}_P = .418\,,\ c_P = 1.609,$$
$$\bar{u}_v = .376\,,\ \bar{s}_v = .500\,,\ c_v = 1.632\,.$$

The corresponding predictions are

$$F_V = 2.131\,,\ F = 2.007\,,\ \psi = 3.263\,,\ \eta_c = 3.217.$$

The predictions for ψ and η_c can be significantly changed by taking into account $\psi - \psi'$, $\eta_c - \eta_c'$ mixings through $\mathcal{D}\bar{\mathcal{D}}, \mathcal{D}\bar{\mathcal{D}}_V$ and $\mathcal{D}_V\bar{\mathcal{D}}_V$ channels /1/ .

If $\bar{m}_o^2 \neq m_o^2$ we need one more input parameter (eq. ψ). Then we can obtain the following prediction

$$F_V = 2.114\,,\ F = 1.997\,,\ \eta_c = 3.040.$$

Without incorporating the $\mathcal{D}\bar{\mathcal{D}}$ channels effects the smaller value of η_c is extremely difficult (if not impossible) to obtain, and so it is quite probable that $\eta_c \neq X(2.82)$ (a possibility of saving the hypothesis $\eta_c = X(2.82)$ was discussed in /1/ , in this case the decays of X (2.82) are somewhat difficult to explain).

The parameters $4\mu_c^2$, $\bar{\mu}_L^2$ are to be obtained by using the masses $\chi_2(3.55), \chi_1(3.51), \chi_0(3.41)$ which however do not satisfy the simple LS-splitting rule: $\chi_2^2 + 2\chi_0^2 = 3\chi_1^2$.
Assuming that $\bar{\mu}_L^2/\mu_L^2 \sim \bar{\mu}_F^2/\mu_F^2$ we can predict the value of $\bar{\mu}_L^2$ ($\bar{\mu}_L^2 \sim .16$) which is rather close to $\frac{1}{2}(\chi_2^2 - \chi_1^2)$.
This probably tells us that χ_0 is rather strongly mixed with $\mathcal{D}\bar{\mathcal{D}}$ -states, e.g., with a 0^{++} $\mathcal{D}\bar{\mathcal{D}}$ bound state which must exist if the molecular charmonium model is true (see 1,2) . An uncertainty of the mixing effects prevents us from finding the exact value of $4\mu_c^2$. In the two models mentioned above we only obtain $4\mu_c^2 \sim 2 \div 3$.
A more reasonable estimate can be found by considering the radially excited states $\rho'(1.6)$, $\psi'(3.7)$:

$$\beta \approx 2.00\,, \quad 4\mu_c^2 \approx 2.04$$

The corresponding predictions are
$\varphi' = 1.83$, $\overset{*}{K}{}' = 1.72$ ($N=1$), $\psi'' = 4.19$ ($N=2$), $\psi''' = 4.6$ ($N=3$),

and the states ψ'', ψ''' may be reasonably identified with 1^{--} $\psi(4.18), \psi(4.14)$. Using this value of $4\mu_c^2$ it is easy to predict also the radially and orbitally excited states of the D-particles. The experimental observation of such states would allow one to solve the problem of finding the exact value of $4\mu_c^2$ as well as to prove (or disprove) our ideas on the phenomenological description of the symmetry breaking effects.

To calculate $\Gamma(V, V' \to e^+e^-)$ we have to use some equation for the wave functions of quarks (u_{ij}) and to choose a potential. The simplest model which is consistent with our mass formula is provided by the equation

$$\frac{d^2 u_{ij}}{d\tau^2} - \left[\varkappa^2_{ij} + \frac{(L+1)^2 - 1/4}{\tau^2} + V_{ij}(\tau) \right] u_{ij}(\tau) = 0, \qquad (2)$$

where $\vec{k}^2 = -\varkappa^2_{ij}$ is the relative momentum squared of the quarks in the c.m.s. $\tau^2 = (x_{i0} - x_{j0})^2 + (\vec{x}_i - \vec{x}_j)^2$ and the metrics is Euclidean. For the pion this equation essentially coincides with the Euclidean Bethe-Salpeter equation. Neglecting the spin- spin interaction we may choose the potnetial in the form

$$V(\tau) = \frac{1}{R^2}\left[g_2 \rho^2 + g_1 \rho + g_0 \ln\rho + g_{-1}\rho^{-1} \right], \quad \rho = \frac{\tau}{R}.$$

The value of β obtained above agrees well with the oscillator potential. However, the linear one, gives rather similar results, and even an admixture of the Coulomb and logarithmic potentials is impossible to exclude provided that g_{-1} and g_0 are not large . Such an admixture changes the dependence of $\Gamma(V_N \to e^+e^-)$ on N, as

$$\Gamma(V_N \to e^+e^-) \sim |v_N(0)|^2 M_{V_N}^{-3}; \quad v_N(\tau) \equiv \tau^{-3/2} u_N(\tau).$$

Using this fact one can obtain a good fit for $\Gamma(\psi') : \Gamma(\psi'') : \Gamma(\psi''')$ with reasonably small admixtures of the logarithmic and Coulomb potentials, not spoiling the nice result for the β .

In addition to the notorious $\eta_c - X$ problem we have a problem of explaining the states $\psi(3.77)$, $\psi(3.95)$, $\psi(4.03)$. As their masses are almost equal to $2D$, $D + D_V$, and $2D_V$ resp., and there are some arguments in favour of a rather strong interaction between the pairs $D\bar{D}$, $D\bar{D}_V$ and $D_V\bar{D}_V$ in the $J = 1$ state /2/ , we may try to interprete these resonances as threshold resonances in corresponding channels. If $\psi(3.77)$ is the $L = 1$ $D\bar{D}$ resonance, and $\psi(4.03)$ – $L=1$ $D_V\bar{D}_V$ resonance, the 0^{++} $D\bar{D}$ and $D_V\bar{D}_V$ bound states must exist due to exchange degeneracy of the forces. Following this reasoning we speculated in /1/ that the $X(2.82)$ could be such a bound state [x)] . As the effective radii of the charmed particles are smaller than those ofthe old particles, and as the forces between D and $\bar{D}$ have practically the same radius $\sim (2m_\pi)^{-1}$, it is legitimate to use for describing the $D\bar{D}$ bound states the relativistic quasipotential equation /3/, neglecting the structure of D-particles. For scalar particles interacting via the Yukawa exchange $\sim g\,\frac{e^{-2m_\pi \tau}}{\tau}$ the Todorov's version of the Logunov-Tavkhelidze equation seems to be most appropriate. (One can formally derive the equation by considering τ in eq. (2) as a three dimensional vector and by introducing in the potential the dependence on the mass of

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x) This possibility was later considered in more detail by a student of mine O.Rassi-Zadeh; to be published.

the bound state: $V \to V/M$). Then, by fitting the mass of the $L=0$ state χ (2.82) we predict the mass of the $L=1$ state $\approx$ 3.74 which reasonably agrees with the ψ (3.77) mass. Describing 0^{++} and 1^{--} $\mathcal{D}_V \bar{\mathcal{D}}_V$ states by the same equation with the same potential we predict their masses to be 3.53 and 4.02 resp., and this allows us to identify them with χ' (3.454), ψ (4.03) .

Having understood the pattern of the meson spectrum and of the symmetry breaking, we are now in a position to make a definite statement about the quark mass spectrum. Modifying the arguments of Weinberg and Fritzsch /4/ , we can also give a very reasonable prediction for the Cabibbo angle θ_c . Assume that $m_\pi \to 0$ when $u, d \to 0$ while the parameters m_o^2, μ_F^2, and $\varepsilon_P^2(\pi^2)$ remain essentially constant. Then $u \approx d \approx \frac{m_\pi}{2}$, and we can obtain the "true" masses of the constituent quarks:

$$u \approx .070\,, \quad s \approx .33\,, \quad c \approx 1.59\,.$$

These masses satisfy a remarkably simple relation

$$\tau^2 = \frac{u}{s} = \frac{d}{s} \approx \frac{s}{c} \approx .21$$

so as the Weinberg-Fritzsch formula /4/ may be written in the form:

$$\theta_c = \mathrm{arctg}\, \tau - \mathrm{arctg}\, \tau^2 \qquad (3)$$

In the original derivation this formula related θ_c to the current quark masses and here we used instead the "constituent" ones . In fact, we have presently derived the same

formula starting from other assumptions which are, to our mind, more consistent than the original ones (to be published). In our derivation the quark masses in the Weinberg-Fritzsch formula are to be unambiguously interpreted as the "constituent" masses. If, in addition, we subject $\theta_c(\tau)$ to a natural stability condition

$$d\,\theta_c(\tau)/d\tau = 0\,, \quad \tau = \tau_o$$

we can derive from (3) the τ_o . After simple calculations we finally obtain an amusing formula

$$\tau_o = \frac{1+\sqrt{3}}{2} - \left(\frac{3}{4}\right)^{1/4}, \quad tg^2\theta_c = \frac{\sqrt{\frac{1}{3}} - \frac{1}{3}}{\sqrt{3}+3}, \quad \theta_c \approx 12{,}79^\circ . \qquad (4)$$

This result is dependent on neglecting the heavier quarks, on simplest possible assumption about the quark mass ratios, etc. However, the agreement of (3) and (4) with experiment is impressive!

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