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Кварковая спектроскопия и угол Кабиббо

На основе уточнения и обобщения идеи Вайнберга и Фритча получена формула, выражающая угол Кабиббо θ_c через параметр нарушения симметрии сильного взаимодействия r. Из условия стационарности $\delta\theta_c/\delta r = 0$ найдено значение θ_c при достаточно простых предположениях о характере нарушения симметрии сильного взаимодействия. Простейшая полученная таким способом формула $tg^2\theta_c = (\sqrt{\frac{1}{3}} - \frac{1}{3})(\sqrt{3}+3)^{-1}$ дает значение $\theta_c = 12{,}794°$, блестящее согласующееся с экспериментом. Показано, что феноменологический анализ нарушения SU(4) -симметрии в спектре мезонов хорошо согласуется с предположениями, сделанными при выводе формулы для угла Кабиббо.

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Quark Spectroscopy and Cabibbo Angle

The expression of the Cabibbo angle θ_c in terms of the u, d, s, c quark masses is obtained by using a generalization of Weinberg-Fritzsch arguments. Introducing as the symmetry breaking parameter $r = d/s$ and requiring that $\frac{d\theta_c}{dr} = 0$ we find for the simplest quark mass spectrum the formula: $tg^2\theta_c = (\sqrt{\frac{1}{3}} - \frac{1}{3})(\sqrt{3}+3)^{-1}$. The corresponding values $\theta_c \approx 12.8°$ and $r = 0.454$ are in very good agreement with experiment.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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Since the introduction of the generalized universality of the weak interaction, based on a possibility of rotating the hadronic weak current by the Cabibbo angle θ_c, there were many attempts to calculate this angle or, at least, to explain its small magnitude ($\theta_c \simeq 13°$). The idea of estimating θ_c is suggested by an empirical relation $tg\,\theta_c \sim \frac{m_\pi}{m_K}$, which possibly tells us that θ_c is somehow related to strong interaction symmetry breaking. In refs. /1/, /2/ it was shown how the condition of cancellation of divergences, induced in the quark matrix by weak interactions, might give a relation like $tg^2\theta_c \sim d/s$ (d, s denote the masses of the quarks d and s). In the approach /1/,/2/ the Cabibbo angle proved to be intrinsically related to the strong chiral symmetry $SU_{2L} \times SU_{2R}$, and the small magnitude of the θ_c was explained in terms of the small symmetry breaking parameter.

The invention of the c -quark made the theory of the weak interaction more symmetric /3/ and gave rise to new approaches for calculating θ_c. The pairs $(uc)(ds)$ can independently be rotated by angles θ_1 and θ_2, resp., while the hadronic current $J = \bar{c}_{\theta_1} s_{\theta_2} + \bar{u}_{\theta_1} d_{\theta_2}$ can be represented in the standard form $J = c s_{\theta_c} + \bar{u} d_{\theta_c}$, where $\theta_c = \theta_2 - \theta_1$, $d_{\theta_c} = d\cos\theta_c + s\sin\theta_c$, etc. Using such rotations and some plausible assumptions on the form of the quark mass matrix in one of the bases, Weinberg /4/ and Fritzsch /5/ discovered new formulae which express θ_c in terms of u, d, s, c. These formulae coinside with the Gatto et al. old formula in the limit $\frac{u}{c} \ll \frac{d}{s} \ll 1$. As assumed in these papers, the mass matrices of the pairs (uc) and (ds) satisfy, in the original "weak" basis, the relations $Q^{(1)}_{uc} = \overset{*}{Q}{}^{(1)}_{cu}$, $Q^{(2)}_{ds} = Q^{(2)}_{sd}$, $Q^{(1)}_{uu} = Q^{(2)}_{dd} = 0$. It follows that $\det Q^{(1)} = -|Q^{(1)}_{uc}|^2$, $\det Q^{(2)} = -|Q^{(2)}_{ds}|^2$, i.e., eigenvalues of the matrices, which are to be identified with the quark masses, must be of opposite signs.

For this reason we first propose a new derivation of similar formulae starting from somewhat different assumptions. When the weak interaction is switched off, the matrices $Q^{(1)}_{ij}$ and $Q^{(2)}_{ij}$ are supposed to be diagonal, their eigenvalues being equal to the masses of the quarks (uc) and (ds), resp. These are the "constituent" quark masses, in terms of which the masses of the hadrons are defined. Switching on the weak interaction makes $Q^{(1)}$ and $Q^{(2)}$ non-diagonal: $Q^{(1)}_{uc} = Q^{(1)}_{cu} = a_1$, $Q^{(2)}_{ds} = Q^{(2)}_{sd} = a_2$, where $a_1, a_2 \in Re$. Let us assume that the weak interaction "chooses" such a basis $(u_{\theta_1}, c_{\theta_1}), (d_{\theta_2}, s_{\theta_2})$ that $Q^{(i)}$ are diagonal and, moreover, $Q^{(1)}_{uu} = Q^{(2)}_{dd} = 0$. This condition is essentially equivalent to the vanishing of u and d "current" quark masses, and in the new basis the chiral $SU_{2L} \times SU_{2R}$ symmetry will be exact. An elementary calculation, with the condi-

tions $Q^{(1)}_{uu} = Q^{(2)}_{dd} = 0$ properly taken into account, gives the relations

$$tg\, 2\theta_1 = 2\sqrt{cu}\,(c-u)^{-1}, \quad tg\, 2\theta_2 = 2\sqrt{sd}\,(s-d)^{-1}.$$

This results in the Weinberg - Fritzsch /5/ formula which we rewrite in the form

$$tg\,\theta_c = \frac{\tau - \tau_1}{1 + \tau\tau_1}, \quad \tau^2 = \frac{d}{s}, \quad \tau_1^2 = \frac{u}{c}. \qquad (2)$$

If we had added a_i also to the diagonal elements of $Q^{(i)}$, we would obtain the same eq. (2) for θ_c, except for the relation between quark masses and τ, τ_1: $\tau \to \bar{\tau} = \frac{d}{s}$, $\tau_1 \to \bar{\tau}_1 = \frac{u}{c}$. If the weak interaction terms in the $Q^{(i)}$-matrices had a more complicated structure (e.g., a_i are added to all the elements except for $Q^{(1)}_{uu}, Q^{(2)}_{dd}$), the formula for θ_c would be the same, but the relation of τ, τ_1 to the quark mass ratios would be more complicated. The structure of the "weak" mass matrix is essentially dependent on a weak interaction model used. As expressed in refs. /4/,/5/ and in references therein, the matrices in the $SU^W_{2L} \times SU^W_{2R} \times U^W_1$ type theories probably meet our requirements.

The hypothesis of vanishing the "current" quark masses u, d gave us the non-trivial relations $a_1 = \sqrt{cu}$, $a_2 = \sqrt{ds}$, which in principle could be satisfied in a unified theory of strong, electromagnetic and weak interactions (cf.,e.g.,/4/). As we are not in a position to really calculate a_i and u, d, s, c we instead will use a necessary condition for the existence of such a relation - a principle of extremality of θ_c with respect to small variations of a symmetry breaking parameter near its true value. As such a parameter we choose τ, implying that $\tau_1 = \tau_1(\tau)$, $\tau_1 < \tau$, $0 < \tau_1'(\tau) < 1$. This assumption can be approximately satisfied in a theory of spontaneously broken $U_{nL} \times U_{nR}$ symmetry /6/. In this theory the spontaneous breaking of the U_n -symmetry occurs through an interference of "diagonal" $\mathcal{M}(q_i\bar{q}_j \to q_i\bar{q}_j) \sim g_D$ and of "non-diagonal" $\mathcal{M}(q_i\bar{q}_i \to q_j\bar{q}_j) \sim g_E$ quark-transitions. For $g_E/g_D \ll 1$ the nonsymmetric solutions of the self-consistent equations for the quark propagators are energetically more favourable than the symmetric ones, and the sequential symmetry breaking $U_n \supset U_{n-1} \supset \ldots \supset U_2$ can be realised under conditions $\frac{u}{s} \sim \frac{d}{s} \sim \frac{s}{c} \sim (g_E/g_D)$ /6/. The same mechanism of the quark-mixing explains the large $\eta-\eta'$ - mixing, and $\varepsilon_\eta^2/m_K^2 \sim g_E/g_D$ (from the mass formulae we obtained $\varepsilon_\eta^2/m_K^2 \sim 1/5$) /7/.

As the a_i implicitly depend on θ (e.g., in the simplest model of ref. /2/ $a_1 \sim \sqrt{uc}\,\sin 2\theta$), the requirement $\frac{d\theta_c}{d\tau} = 0$ seems to be necessary to guarantee a self - consistent solution of all the constraints. A more formal argument in favour of this requirement can be deduced from the quark confinement hypothesis. As soon as there are no free quarks, their masses q_i and the symmetry breaking parameter are defined by some averaging process /8/. Any physically acceptable solution has to be stable with respect to corresponding fluctuations of τ hence $\theta'(\tau)=0$.

From the equation $\theta'(\tau_0)=0$ we now obtain τ_0 and $\theta_c(\tau_0)$

$$\tau_1'(\tau_0)=[1+\tau_1^2(\tau_0)](1+\tau_0^2)^{-1}, \quad \mathrm{tg}\,\theta_c(\tau_0) = \frac{\tau_0 - \tau_1(\tau_0)}{1 + \tau_0\tau_1(\tau_0)} \quad . \quad (3)$$

With the simplest assumption $\tau_1 = \tau^2$ one can easily deduce from eq. (3) the amusing formula

$$\tau_0 = \frac{1+\sqrt{3}}{2} - \left(\frac{3}{4}\right)^{1/4} \approx .4354\,, \quad \mathrm{tg}^2\theta_c = \frac{\sqrt{\frac{1}{3}} - \frac{1}{3}}{\sqrt{3}+3}\,, \quad \theta_c \approx 12.794^\circ \quad (4)$$

The relation $\tau_1 = \tau^2$ and the "extremal" value of the symmetry breaking parameter τ_0 are to be compared with the empirical relation between $\frac{u}{c}$ and $\frac{d}{s}$ obtained in /7/. One can roughly reproduce the results of this paper by introducing the effective mass $q_{i\alpha}$ of the i-th quark in a vector ($\alpha = V$) or pseudoscalar ($\alpha = P$) meson: $q_{i\alpha}^2 = q_i^2 + m_\alpha^2$,where q_i is the "true" mass of the quark. Then, for states with small mixing of quarks ($\varepsilon_\alpha^2 \to 0$) the $(q_i\bar{q}_j)_\alpha$ meson mass is $M_{ij\alpha} = q_{i\alpha} + q_{j\alpha}$ (see /6/,/7/). Using the masses of $\rho, \varphi, \mathcal{D}, \mathcal{D}^*$ as the input parameters and exploiting the obvious relation $q_{i\alpha}^2 - q_{j\alpha}^2 = q_i^2 - q_j^2$,one can easily calculate $u_\alpha \approx d_\alpha$, s_α and c_α . To find the true masses u, d, s, c we observe that in the limit $u, d \to 0$ the pion mass has to vanish, therefore $m_\pi^2 = 2(u^2+d^2)$. Performing a more rigorous calculation, which accounts for the $u-d$ mass difference and the quark mixing (see /7/) we arrive at the final result

$$u \approx .063\,, \quad d \approx .073\,, \quad s \approx .337\,, \quad c \approx 1.59 \quad (5)$$

For these masses of quarks the relation $\tau_1 \approx \tau^{2.11}$ is satisfied and the corresponding θ_c (as obtained from eq. (2)) is slightly larger than that given by eq. (4). Neglecting the $u-d$ splitting we have a remarkably simple pattern of symmetry breaking in the quark masses: $\frac{u+d}{2} : s \approx \frac{s}{c} \approx \tau^2$, where $\tau = .454$ is rather close to the "extremal" value τ_0 in eq. (4). Remark in passing that the relation $\tau^2 \sim (\varepsilon_\eta^2/m_K^2) \sim (g_E/g_D)$ is also satisfied.

Other assumptions on "weak" corrections and on u, d, s, c

mass spectrum (e.g., $\tau_1 = \tau^{2+\varepsilon}$) would give somewhat different formulae for θ_c . However, the simplest one, given by eq. (4), is in the best agreement with the experimental value $\theta_c \approx 13^o$ and with the empirical mass spectrum (5).

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