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Связь померонного интерсепта с трехпомеронной константой

Показано, что в партонной модели (ПМ) значения померонного интерсепта $\alpha_P(0)=1+\Delta$ и трехпомеронной константы λ связаны соотношением $\Delta=\lambda$. По этой причине критическое поведение в реджеонной теории поля оказывается невозможным. ПМ позволяет также разделить области применимости подходов Карди и Амати.

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The Relation between the Pomeron Intercept and the Triple-Pomeron Coupling

It is shown in the parton model (PM) that the pomeron intercept $\alpha_p(0)=1+\Delta$ and the triple-pomeron coupling λ are strongly correlated: $\Delta=\lambda$. For this reason the critical behaviour in the reggeon field theory is impossible. In the supercritical case PM allows one to distinguish between the conditions of validity of Amati's and Cardy's approaches.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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1. *INTRODUCTION*

The last years have been marked by the considerable progress in the reggeon field theory (RFT). The approaches studied so far differ in their prescriptions for the two basic parameters: the pomeron intercept $\alpha_P(0)=1+\Delta$ and the triple-pomeron coupling λ. These are: (i) the weak coupling variant [1], which is characterized by the relation $\Delta=\lambda=0$; (ii) the critical pomeron version [2] implemented if $\Delta=\Delta_c\approx\lambda^2\ln\lambda^2$; (iii) the supercritical pomeron with $\Delta>\Delta_c$. Two approaches are known in this case: the first one [3,4] neglects all the multi-pomeron couplings except the triple-pomeron one; the second one [5-9] includes pomeron interaction in all orders.

In the purely phenomenological approach to the RFT the values of Δ and λ could be chosen independently. Here we would like to point out that in the case of $\Delta\geq 0$ the simple parton model (PM) considerations imply the strict relation *:

$$\Delta=\lambda. \tag{1}$$

This means that the critical behaviour in RFT is impossible, if $\Delta\ll 1$. From this point of view the weak and supercritical variants are selfconsistent only.

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* Our normalization for λ corresponds to the triple-pomeron term in the RFT Lagrangian in the form $i\lambda\Psi^+\Psi(\Psi+\Psi^+)$.

The PM allows one also to separate the fields of application of two mentioned above supercritical RFT versions.

2. THE "PARTON" VIEW ON THE RFT [10,11]

The RFT with $\Delta > 0$ seems to be an uneconomical tool for the calculations at least. An input bare pomeron contribution and other graphs of RFT contradict the s -channel unitarity, but after some considerable compensations one obtains a small residue quite different from the starting point. In the parton language this procedure seems to be still stranger. Nevertheless, we try here to carry out the parton model interpretation of the RFT graph contributions to the scattering amplitude.

The basic point of the PM is that fast hadron interacts through its wee-parton component [10]. So, to calculate the scattering amplitude one must consider the parton wave function of the hadron, find the mean number of wee-partons, and then solve the problem of two wee-parton system interaction. But if one wants to find a correspondence with the RFT graphs one must separate certain parts from the parton wave function of the hadron and consider the definite contribution in the scattering amplitude of two wee-parton systems. Namely, the bare pomeron corresponds to such parton comb configurations in the hadron wave function which do not contain the loops (i.e., the fusion of combs). These are tree-type diagrams or diagrams with few noninteracting trees. (See *fig. 1a,b*). Besides, the interaction of two wee-parton systems should be treated in the impulse approximation. All the loop and wee-parton screening corrections correspond to more complicated pomeron graphs. The wee-parton which took part in the interaction is distinguished in fig. 1 by a cross. The parton comb shown in fig. 1 by a heavy line is implemented after the scattering in the form of one multiperipherical chain of particles. Other parton combs in the diagram play the role of vacuum fluctuations [11].

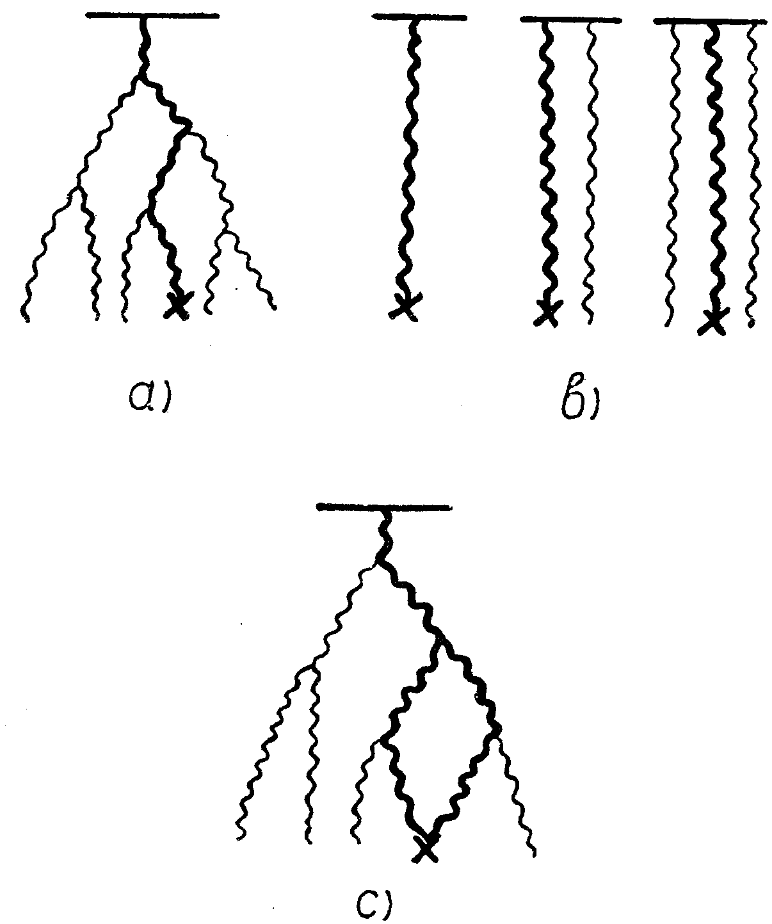

Fig. 1. Different contributions to the parton wave function of a hadron. The wavy lines denote the parton combs. The wee-partons which took part in the interaction are marked by a cross. The heavy lines indicate the combs which give the real particle chains after the interaction. The light lines are the combs which play the role of vacuum fluctuations. a), b) The tree-type and eikonal type diagrams, implemented after the interactions as a bare pomeron exchange in the RFT. c) The tree-type parton diagram, corresponding to the triple-pomeron graph in the RFT.

From comparison with the bare pomeron Green function $\rho(\xi, b)$, where ξ and b are the relative rapidity and impact parameter of two hadrons, one finds that the wee-parton density in a hadron moving with a rapidity y is equal to

$$W(y,b) = \frac{1}{4\pi\alpha' y} e^{y\Delta - \frac{b^2}{4\alpha' y}} . \qquad (2)$$

Indeed, due to the impulse approximation the scattering amplitude is proportional to the product of the wee-parton densities in each hadron

$$\int d^2 b_1 W(y,\vec{b}_1) W(\xi - y, \vec{b} - \vec{b}_1) = \frac{e^{\xi\Delta - \frac{b^2}{4\alpha'\xi}}}{4\pi\alpha'\xi} \equiv \rho(\xi, \vec{b}) . \qquad (3)$$

It is seen from expression (2) that the multiplicity of the wee-partons grows as[11]

$$\langle n(y) \rangle_{wee} = \int d^2 b \, W(y,b) = e^{y\Delta} \qquad (4)$$

when the hadron rapidity y increases.

The problem is: what diagrams are responsible for this behaviour? It is clear that the multi-pomeron graphs of the eikonal type, shown in fig. 1b, cannot explain this effect, because their weights in the hadron wave function do not depend on the y value. So the only graphs which can provide such growth (4) are the tree-diagrams of fig. 1a type.

Note that each vertex of the parton ladder fission contains the triple-pomeron couplings λ. Thus, the dependence of the wee-parton density $W(y,b)$ on the hadron rapidity y is given by the following transport equation (if W is large enough):

$$\frac{\partial W(y,b)}{\partial y} = [\lambda + \alpha'(\vec{\nabla}_b)^2] W(y,b) . \qquad (5)$$

Equations (5) and (2) are compatible only provided the equality (1) takes place.

There are few comments in order:

i) The pomeron which is bare in the RFT is not bare in the PM. As the rapidity y grows, a risk of ruin for each parton comb increases[11]. If γ is a probability for

the comb to be ruined in a unit rapidity interval then the above values of Δ and λ are smaller than PM bare quantities Δ' and λ':

$$\Delta = \Delta' - \gamma ; \quad \lambda = \lambda' - \gamma . \qquad (6)$$

ii) To be convinced of the identity of the constant λ and the bare triple-pomeron coupling in the RFT one can consider the screening corrections to the impulse approximation in the interaction of wee-parton system with a target. The first correction is due to the interaction of a target with two wee-partons. It gives a negative contribution to the total cross section:

$$(\sigma_{tot})_{PPP} = -G e^{2\xi\Delta} \qquad (7)$$

and corresponds to the triple-pomeron graph of the RFT, as shown in fig. 1c. The factor G in expression (7) involves all the particle-pomeron couplings. It is supposed also that $\xi\Delta \gg 1$.

On the other hand, this graph in the RFT gives

$$(\sigma_{tot})_{PPP} = -\lambda G \int d^2 b \, d^2 b_1 \, dy \, \rho(y, \vec{b}_1) \rho^2(\xi - y, \vec{b} - \vec{b}_1) = -\frac{\lambda}{\Delta} G e^{2\xi\Delta} \qquad (8)$$

The comparison of equations (7) and (8) confirms relation (1).

3. *TWO APPROACHES IN THE SUPERCRITICAL POMERON THEORY*

The version of the RFT investigated by Amati et al.[3,4] includes the triple pomeron interaction only. Their results are very natural in the PM. The parton combs fusion adds to the right-hand side of eq. (5) a negative term which is quadratic in W. The new transport equation[4] has a stationary solution which implies a uniform parton comb density inside a disc $b^2 < 4\alpha'\Delta\xi^2$. One can see that the inclusion of another multi-pomeron couplings does not change this pattern if those couplings are small enough. Indeed, the mean free path of the parton comb in the rapidity scale is of the order of $y_2 - y_1 \approx 1/\Delta$. The corresponding pomeron propagator is $\rho(y_2 - y_1) \approx 1$. Thus, an addition of multi-pomeron exchange between these points y_2, y_1 leads to a small correction due to the smallness of the multi-pomeron couplings. Thus, such RFT version seems to be self-consistent under above conditions.

Another approach has been proposed by Cardy [5]. It includes the multi-pomeron interaction couplings g_{mn} from the very beginning and leads to quite different results [6-9]. The pomeron interaction is turned off effectively in asymptotics and does not restrict the wee-parton density

$$W(\xi, b) = \frac{1}{4\pi\alpha'\xi} e^{\xi\Delta_0 - b^2/4\alpha'\xi}. \tag{9}$$

Here Δ_0 is a renormalized value of Δ [6]

$$\Delta_0 = \Delta - g_{11}, \tag{10}$$

g_{11} is a result of the analytical continuation of the g_{mn} series into the point $m = n = 1$.

Thus, we obtain a paradox: two approaches in the RFT seem to be valid in the same conditions but they lead to quite different results.

Note that a positivity of Δ_0 is a one more condition for Cardy's approach validity. This problem is insoluable in the RFT but it can be cleared up in the PM.

The inclusion of multi-pomeron couplings influences, of course, relation (1). In this case one must add the tree-diagrams with parton comb fission $1 \to n$ (but without transitions $m \to n$ with $m > 1$). Then relation (1) is substituted by *

$$\Delta = \sum_{n=2}^{\infty} \frac{n-1}{n!} g_{1n}. \tag{11}$$

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*The couplings g_{mn} are normalized as in ref. [5]. Thus, $g_{21} = 2\lambda$.

If one supposes the eikonal-type dependence: $g_{mn} = g_{00} g^{m+n}$, then one finds from (11)

$$\Delta_0 = g_{00} g (g-1)(e^g - 1). \tag{12}$$

Thus, the positivity condition for Δ_0 in eq. (10) implies that $g > 0$. This is the condition for validity of Cardy's approach.

On the other hand, Amati's approach needs for $g \ll 1$, as has been mentioned above. Those conclusions explain the paradox

Note that the estimates [6] in the one pion exchange model give for g the value of about unity.

4. CONCLUSION

We have proved relation (1) and shown that the weak coupling and supercritical versions of the RFT are self-consistent from the point of view of the PM. The critical behaviour in the RFT is impossible.

It has been shown that in the supercritical case two known variants of the RFT are additional to each other and its implementation depends on RFT parameter values.

The value of Δ is defined [6-8]* from the total cross section data and is equal to $\Delta \approx 0.07$. It is supposed in that analysis that the enhanced graph contribution is negligible due to the smallness of the triple-pomeron coupling λ. Such procedure is self-consistent due to the relation (1) and because Δ turns out to be small.

The value of λ is related to the effective coupling $G_{PPP}(0)$ extracted from the experimental data on the inclusive reactions in paper [12]* as

$$\lambda = \left[\frac{8\pi}{\alpha'(\sigma_{tot}^{pp})^3}\right]^{\frac{1}{2}} G_{PPP}(0). \tag{13}$$

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* See also references therein.

Substituting values of $\alpha' = 0.3\ (GeV/c)^{-2}$; proton-proton total cross section $\sigma_{tot}^{pp} = 40\ mb$ and $G_{PPP}(0) = 3.2\ mb/GeV^2$ one obtains $\lambda = 0.07$. This value of λ can differ significantly from the bare one due to the large cut corrections /13/. The comparison with the relation (1) shows that this is not the case. Moreover the wonderful precision of the relation (1) confirmation by the experimental data puts a question: is a reduction of cut contributions in the inclusive cross section occasional or not?

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REFERENCES

1. *Gribov V.N. Yad.Fiz., 1973, 17, p.603 (Sov. J.Nucl. Phys., 1973, p.410); Proc. XVI Int. Conf. on High Energy Phys., Chicago, 1973, 3, ed. NAL, Batavia, 1973, p.491.*
2. *Migdal A.A., Polyakov A.M., Ter-Martirosyan K.A. JETP (Sov.Phys.),1974, 67, p.84, JETP (Sov.Phys.), 1975, 40, p.420.*
   Abarbanel H.D.I., Bronzan J.B. Phys. Rev., 1974, D9, p.2397.
3. *Amati D., Caneschi L., Jengo R. Nucl.Phys., 1975, B101, p.397.*
   Alessandrini V., Amati D., Jengo R. Nucl.Phys., 1976, B108, p.425.
   Amati D. et al. Nucl.Phys., 1976, B112, p.107; Nucl. Phys., 1976, B114, p.483.
4. *Ciafaloni M., Marchesini G. Nucl.Phys., 1976, B109, p.261.*
5. *Cardy J.L. Nucl.Phys., 1974, B75, p.413.*
6. *Kopeliovich B.Z., Lapidus L.I. JETP (Sov.Phys.), 1976, 71, p.61; JINR, E2-9537, Dubna, 1976.*
   Kopeliovich B.Z. Preprint LNPI-269, Leningrad, 1976.
7. *Dubovikov M.S., Ter-Martirosyan K.A. Preprint ITEP-37, Moscow, 1976.*
8. *Dubovikov M.S. et al. Nucl.Phys., 1977, B123, p.147.*

9. *Dubovikov M.S., Ter-Martirosyan K.A. Nucl.Phys., 1977, B124, p.163; JETP (Sov.Phys), 1977, 73, p.2008.*
10. *Gribov V.N. Proc. VIII Winter LNPI School of Physics, Leningrad, 1973, v.II, p.5.*
11. *Glassberger P. Nucl.Phys., 1977, B125, p.84.*
12. *Kazarinov Yu.M. et al. JETP (Sov.Phys.), 1976, 70, p.1152.*
13. *Abramovsky V.A. JETP Lett. (Sov.Phys.), 1976, 23, p.228.*

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