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II. Calculation of One-Loop Corrections: Two-Point Functions and Fermion Masses

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Исследование спонтанно нарушенных калибровочных теорий. II. Расчет однопетлевых поправок: двухточечные функции и фермионные массы

В работе определяется однопетлевой порождающий функционал одночастичных неприводимых функций Грина общего СНКТ, который используется для вычисления двухточечных функций теории. Изучаются конечные поправки в фермионной матрице масс.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

Nagy T.

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Investigation of Spontaneously Broken Gauge Theories. II. Calculation of One-Loop Corrections: Two-Point Functions and Fermion Masses

The one-loop generating functional of the 1PI Green functions of a general SBGT is determined and it is used to calculate the two-point functions of the theory. The finite corrections to the fermion mass matrix are studied.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Introduction

In this series of papers we have undertaken a systematic investigation of spontaneously broken gauge theories, having in mind mainly applications to phenomena which can presumably be treated perturbatively. In Part I [1] (hereafter referred to as I) we recapitulated the familiar structure of SBGT's with a gauge group $G = K \times N$, where K is an arbitrary compact semisimple group and N is a possible Abelian factor, and with arbitrary fermion and scalar multiplets. We have also set up a renormalization program, where the counterterms are those of the symmetric theory, and the symmetric parameters and the zeroth order vacuum value of the Higgs field pick up finite corrections. In this and the next Part we calculate the one-loop corrections to proper Green functions which seem to have immediate practical relevance. Calculations are performed in a "renormalizable" gauge, by using a simple quadratic gauge fixing term for the vector fields; this procedure may not be the most convenient one (see, e.g., [2]), but, anyhow, it is interesting to see how unphysical degrees of freedom mix with the physical ones.

The organization of the paper is as follows: In Sec.II. we derive the one-loop generating functional of the proper Green functions. Sec.III gives the general expressions for the 1PI Green functions up to four-point amplitudes (the compact forms for the propagators and vertices can be found in the Appendix) and in Sec. IV we exhibit the detailed form of the two-point functions. Finally, in Sec.V we compute the corrections to the 0^{th} order fermion mass matrix. The three- and four-point case will be treated in a forthcoming paper.

II. The one-loop generating functional of the 1PI Green functions.

For calculating the proper Green functions we perform the loop expansion immediately for the generating functional, using the method described in /3/. Our vacuum functional is (see I. Case of SBGT)

$$Z(J) = e^{-iG(J)} = \int \Pi d\varphi \, \exp i\left[W(\varphi) + J(\varphi+\Lambda)\right], \quad Z(0)=1,$$

where G is the generating functional of the connected Green functions, and

$$W(\varphi) = W_0(\varphi) + W_I(\varphi),$$
$$W_0(\varphi) = \tfrac{1}{2}\varphi_r K_{rs}\varphi_s,$$
$$W_I(\varphi) = \gamma_r\varphi_r + \tfrac{1}{2!}\gamma_{rs}\varphi_r\varphi_s + \tfrac{1}{3!}\gamma_{rst}\varphi_r\varphi_s\varphi_t + \tfrac{1}{4!}\gamma_{rstu}\varphi_r\varphi_s\varphi_t\varphi_u . \qquad (1)$$

The generating functional of the 1PI Green functions is defined by the Legendre transformation

$$\Gamma(\chi) + G(J) + J_r(\chi_r+\Lambda_r) = 0, \quad \chi_r = -\frac{\delta}{\delta J_r}G(J) - \Lambda_r . \qquad (2)$$

The loop expansion can be obtained by writing

$$\exp\left[-\tfrac{i}{\hbar}G\right] = \int \Pi d\varphi \, \exp \tfrac{i}{\hbar}\left[W(\varphi) + J(\varphi+\Lambda)\right]$$

and performing the expansions

$$G = \sum_{L=0}^{\infty}\hbar^L G^{(L)}, \quad \Gamma = \sum_{L=0}^{\infty}\hbar^L\Gamma^{(L)} .$$

Omitting from the action the counterterms, we have

$$W^0(\varphi) = \tfrac{1}{2}\varphi_r K_{rs}\varphi_s + \tfrac{1}{3!}\gamma^0_{rst}\varphi_r\varphi_s\varphi_t + \tfrac{1}{4!}\gamma^0_{rstu}\varphi_r\varphi_s\varphi_t\varphi_u .$$

Let us define $\psi = \psi[J]$ by

$$\left.\frac{\delta W^0}{\delta\varphi_r}\right|_{\varphi=\psi} \equiv K_{rs}\psi_s + \tfrac{1}{2}\gamma^0_{rst}\psi_s\psi_t + \tfrac{1}{6}\gamma^0_{rstu}\psi_s\psi_t\psi_u = -\varepsilon_r J_r$$

($\varepsilon_r = -1$ for fermions and +1 for bosons)

and make the shift $\varphi \to \varphi + \psi$ in the functional integral. It is easy to see that

$$W(\varphi+\psi) = W(\psi) - J_r\varphi_r + \tilde{W}(\varphi),$$

where

$$\tilde{W}(\varphi) = \tilde{W}_0(\varphi) + \tilde{W}_I(\varphi),$$
$$\tilde{W}_0(\varphi) = \tfrac{1}{2}\varphi_r K_{rs}(\psi)\varphi_s, \quad K_{rs}(\psi) = K_{rs} + \gamma^0_{rts}\psi_t + \tfrac{1}{2}\gamma^0_{rtus}\psi_t\psi_u$$
$$\tilde{W}_I(\varphi) = \sum_{\nu=1}^{4}\frac{1}{\nu!}\tilde{\gamma}_{r_1\ldots r_\nu}(\psi)\,\varphi_{r_1}\ldots\varphi_{r_\nu}; \qquad (3)$$

the coefficients $\tilde{\gamma}(\psi)$ can be written down without any difficulty. We get

$$\exp\left\{-\tfrac{i}{\hbar}\left[G + W(\psi) + J_r(\Lambda_r+\psi_r)\right]\right\} = \int \Pi d\varphi \, e^{\frac{i}{\hbar}\tilde{W}(\varphi)} =$$
$$\underset{\varphi\to\hbar^{1/2}\varphi}{=} \int \Pi d\varphi \, \exp i\left\{\tilde{W}_0(\varphi) + \hbar^{-1/2}\tilde{\gamma}_r\varphi_r + \tfrac{1}{2}\tilde{\gamma}_{rs}\varphi_r\varphi_s + \frac{\hbar^{1/2}}{3!}\tilde{\gamma}_{rst}\varphi_r\varphi_s\varphi_t + \frac{\hbar}{4!}\tilde{\gamma}_{rstu}\varphi_r\varphi_s\varphi_t\varphi_u\right\} .$$

Performing the $\hbar$ -expansion and using (cf. /4/)

$$\int \Pi d\varphi \, \exp\left[\tfrac{i}{2}\varphi_r K_{rs}\varphi_s\right] \sim (\det K_B)^{-1/2}\det\vec{K}_F ,$$

where K_B is boson-boson operator ($K_B^{-1} = D_B$) while

$$\vec{K}_F(\psi) = \vec{K}_{FF}(\psi) - \vec{K}_{FB}(\psi)D_B(\psi)\vec{K}_{BF}(\psi) \qquad (4)$$

(the arrow over K indicates that first fermion indices always refer to fermions and second ones to antifermions), one obtains

$$C^{0}(\psi) = -W^{0}(\psi) - J_r(\lambda_r + \psi_r)$$

$$C^{(1)}(\psi) = -W^{1}(\psi) - \frac{i}{2}\,\mathrm{Tr}\ln K_B(\psi)K_B^{-1} + i\,\mathrm{Tr}\ln \vec{K}_F(\psi)\vec{K}_F^{-1} .$$

Here $W^{0(1)}$ is our action with the counterterms taken in zeroth (first) order and $K_{B/F} = K_{B/F}(\psi=0)$.

Taking into account Eq.(2), we arrive at the result

$$\Gamma^{(1)}(\varphi) = -C^{(1)}(\varphi) = W^{(1)}(\varphi) + \frac{i}{2}\,\mathrm{Tr}\ln K_B(\varphi)K_B^{-1} - i\,\mathrm{Tr}\ln \vec{K}_F(\varphi)\cdot\vec{K}_F^{-1} \tag{5}$$

with $K_B(\varphi)$ and $\vec{K}_F(\varphi)$ defined by (3) and (4). The 1PI Green functions can be identified with the expansion coefficients in the expansion of $\Gamma^{(1)}(\varphi)$ in powers of φ_r.

III. Calculation of the 1PI Green functions

As we have noted above, to get the Green functions we have to expand $\Gamma^{(1)}(\varphi)$ and take the corresponding expansion coefficients. By introducing the matrices

$$(V_r)_{st} = \gamma^{0}_{srt} \quad , \quad (V_{rs})_{tu} = \gamma^{0}_{trsu} \tag{6}$$

and denoting boson propagators by Δ and fermion propagators by $\vec{\Delta}$, the Green functions in momentum representation can be written in the form (the trace operation includes integration over the loop momentum z):

$$\Gamma_r(p) = \Big[\gamma^{1}_{r} + \frac{i}{2(2\pi)^4}\,\mathrm{Tr}\,\Delta(z)V_r(z,0,-z) - \frac{i}{(2\pi)^4}\,\mathrm{Tr}\,\vec{\Delta}(z)V_r(z,0,-z)\Big]\delta(p) \tag{7}$$

$$\begin{aligned}\Gamma_{rs}(p) = {} & K_{rs}(p) + \gamma^{1}_{rs}(p) + \frac{i}{2(2\pi)^4}\,\mathrm{Tr}\,\big[\Delta(z)V_{rs}(z,p,-p,-z) - \\ & - \Delta(z)V_r(z,p,-z-p)\,\Delta(z+p)\,V_s(z+p,-p,-z)\big] + \\ & + \frac{i}{2(2\pi)^4}\,\mathrm{Tr}\,\big(-2\vec{\Delta}(z)V_s(z,-p,p-z)\,\Delta(z-p)\,V_r(z-p,p,-z) + \\ & + (\vec{\Delta}(z)V_r(z,p,-z-p)\,\vec{\Delta}(z+p)V_s(z+p,-p,-z) + (r\leftrightarrow s\,,\,p\to -p))\big]\end{aligned} \tag{8}$$

$$\begin{aligned}\Gamma_{rst}(pqk) = {} & \gamma^{0}_{rst}(pqk) + \gamma^{1}_{rst}(pqk) + \\ & + \frac{i}{2(2\pi)^4}\,\mathrm{Tr}\sum_{\substack{\text{perm}\\(rp,sq,tk)}}\Big[-\frac{1}{2}\Delta(z)V_r(z,p,-z-p)\,\Delta(p+z)\,V_{st}(p+z,q,k,-z) + \\ & + \frac{1}{3}\Delta(z)V_r(z,p,-z-p)\,\Delta(z+p)\,V_s(p+z,q,k-z)\,\Delta(z-k)\,V_t(z-k,k,-z)\Big] \\ & + \frac{i}{(2\pi)^4}\,\mathrm{Tr}\,\Big[\vec{\Delta}(z)V_s(z,q,-z-q)\,\Delta(z+q)\,V_t(z+q,k,p-z)\,\Delta(z-p)\,\times \\ & \quad \times V_r(z-p,p,-z) + \vec{\Delta}(z)V_t(z,k,-z-k)\,\vec{\Delta}(z+k)\,\times \\ & \quad \times V_s(z+k,q,p-z)\,\Delta(z-p)\,V_r(z-p,p,-z) - \\ & - \frac{1}{3}\sum_{\substack{\text{perm}\\(rp,sq,tk)}}\vec{\Delta}(z)V_r(z,p,-z-p)\,\vec{\Delta}(z+p)\,V_s(z+p,q,k-z)\,\times \\ & \quad \times \vec{\Delta}(z-k)\,V_t(z-k,k,-z)\Big)\end{aligned} \tag{9}$$

The four-point function is given only for bosons:

$$\begin{aligned}\Gamma_{rstu}(pqkl) = {} & \gamma^{0}_{rstu}(pqkl) + \gamma^{1}_{rstu}(pqkl) + \\ & + \frac{i}{2(2\pi)^4}\,\mathrm{Tr}\sum_{\substack{\text{perm}\\(rp,sq,tk,ul)}}\Big[-\frac{1}{8}\Delta(z)V_{rs}(z,p,q,k+l-z)\,\Delta(z-k-l)\,\times \\ & \quad \times V_{tu}(z-k-l,k,l,-z) + \\ & + \frac{1}{2}\Delta(z)V_r(z,p,-p-z)\,\Delta(p+z)\,V_s(p+z,q,k+l-z)\,\times \\ & \quad \times \Delta(z-k-l)\,V_{tu}(z-k-l,k,l,-z) - \\ & - \frac{1}{4}\Delta(z)V_r(z,p,-p-z)\,\Delta(p+z)\,V_s(p+z,q,k+l-z)\,\times \\ & \quad \times \Delta(z-k-l)\,V_t(z-k-l,k,l-z)\,\Delta(z-l)\,V_u(z-l,l,-z)\Big] + \\ & + \frac{i}{4(2\pi)^4}\,\mathrm{Tr}\sum_{\substack{\text{perm}\\(rp,sq,tk,ul)}}\vec{\Delta}(z)V_r(z,p,-p-z)\,\vec{\Delta}(p+z)\,\times \\ & \quad \times V_s(p+z,q,k+l-z)\,\vec{\Delta}(z-k-l)\,V_t(z-k-l,k,l-z)\,\times \\ & \quad \times \vec{\Delta}(z-l)\,V_u(z-l,l,-z) .\end{aligned} \tag{10}$$

It is easy to identify the Feynman diagram structure in these expressions, K , Δ and the vertices $\gamma^{0}\,(V)$ and γ^{1} are given in the Appendix of I, but they can be put in a more compact form which is very convenient, because one can perform the integrations preserving the general form of the Green functions and make use of the 0^{th} order Ward identities. This general form of K, D and V and some useful relations can be found in the Appendix.

The regularization we use is the dimensional regularization of 't Hooft and Veltman /5/. With a little manipulation the one-loop integrals can be written in the following form:

$$\int d^n z \, \frac{f(z)}{(m^2 - z^2 - 2kz)^\alpha} = \frac{i\pi^{n/2}}{\Gamma(\alpha)} \sum_{r=0}^{\infty} \frac{[m^2 + k^2]^{n/2 - \alpha + r}}{4^r \, r!} \times$$
$$\times \Gamma(\alpha - r - \tfrac{n}{2}) \, \Box^r f(-k) \qquad (11)$$

with $\Box = -\frac{\partial}{\partial z^\mu} \frac{\partial}{\partial z_\mu}$. This form is again well suited to preserve generality as far as possible. (Since $f(z)$ is always a polynomial, the sum is in fact finite).

Two points have to be discussed here briefly. One of them is the problem of treating pseudoquantities like γ_5 and $\varepsilon_{\mu\nu\omega\rho}$ for $n \neq 4$; a general discussion of this question can be found in /6/. If there are no ABJ anomalies /7/ in the theory, WT identities will be satisfied if we always commute γ^μ and γ_5 normally. We shall require the absence of anomalies and use normal commutation relations. The second point to mention is the question of the infrared divergencies. Though we don't really want to deal with IR divergent quantities, such divergencies may occur at an intermediate level of the calculations and there are elegant methods to treat them in the framework of the dimensiomal regularization /8/. We simply introduce an infrared cut-off; this step destroys the validity of some of the relations given in the Appendix, but the violation is proportional to the IR cut-off and can be neglected in case of logarithmic divergencies.

IV. Two-point functions

In this section we write down the expressions for the one-loop Green functions $\Gamma_i^{(1)}$, $\Gamma_{\alpha\beta}^{\mu\nu}$, $\Gamma_{\alpha i}^{\mu}$, Γ_{ij}, $\Gamma = (\Gamma_b^a)$ and $\Gamma_{\alpha\beta}$. We use the notations of the Appendix and will explain some other notations here. Let us define the quantity z_{ab} as follows:

$$z_{ab}(p^2, s) = z_{ba}(p^2, 1-s) = m_b^2 + s(m_a^2 - m_b^2 - p^2) + s^2 p^2 . \qquad (12)$$

m_a , m_b may refer to any mass and s is the Feynman parameter in the integrals. Then, denoting the infrared cut-off by Λ , we have

$$A_{ab}^k(p^2, s) = z_{ab}(p^2, s)^{n/2 - k} \, \Gamma(k - \tfrac{n}{2})$$
$$B_{ab}^k(p^2, s) = \frac{1}{\Lambda^2 - m_a^2} [z_{\Lambda b}(p^2, s)^{n/2 - k} - z_{ab}(p^2, s)^{n/2 - k}] \, \Gamma(k - n/2) , \qquad (13)$$
$$\tilde{B}_{ab}^k(p^2, s) = B_{ba}^k(p^2, 1-s) , \qquad (14)$$
$$C_{ab}^k(p^2, s) = \frac{1}{(\Lambda^2 - m_a^2)(\Lambda^2 - m_b^2)} [z_{\Lambda\Lambda}(p^2, s)^{n/2 - k} - z_{\Lambda b}(p^2, s)^{n/2 - k} -$$
$$- z_{a\Lambda}(p^2, s)^{n/2 - k} + z_{ab}(p^2, s)^{n/2 - k}] \, \Gamma(k - \tfrac{n}{2}) . \qquad (15)$$

We shall use the fermion couplings in the representation where the fermion mass is diagonal and free of γ_5 :

$$\hat{m} = \bar{\omega}^+ m \omega^+ , \qquad (16.a)$$
$$\hat{t}_\alpha = \omega t_\alpha \omega^+ , \quad \bar{\hat{t}}_\alpha = \gamma_0 \hat{t}_\alpha^+ \gamma_0 = \bar{\omega}^+ \bar{t}_\alpha \bar{\omega} , \qquad (16.b)$$
$$\hat{\Gamma}_i = \bar{\omega}^+ \Gamma_i \omega^+ . \qquad (16.c)$$

(This notation should not be confused with the familiar expression $\hat{p} = \gamma^\mu p_\mu$!). The relations (6), (7) and (15) of I take the form

$$\hat{t}^+ = \hat{t} , \quad \bar{\hat{\Gamma}} = \hat{\Gamma} , \quad [\hat{t}_\alpha, \gamma^0 \hat{m}] = \varrho_{\alpha i} \gamma_0 \hat{\Gamma}_i$$
$$[\hat{t}_\alpha, \gamma_0 \hat{\Gamma}_i] = -\partial_{\alpha i j} \gamma^0 \hat{\Gamma}_j . \qquad (17)$$

After these preparations we give the list of the Green functions (δm , $\delta\mu^2$, $\delta\lambda$, ξ_s , Z_s , Γ' , c' , f' are the renormalization constants introduced in I). In each case Γ^{CT}, Γ^{B}, Γ^{C}, Γ^{F} refer to the counterterm (and 0^{th} order term), boson, ghost and fermion contribution, respectively and Γ^{ξ} is the term, proportional to the gauge parameter ξ .

A. $$\Gamma_i^{(1)} = (\delta\mu^2\lambda)_i - \{[((\xi-1)\mu^2 + \mu^2(\xi-1))\lambda]_i + \frac{1}{2}(c'_{ijk} - c_{ijk})\lambda_j\lambda_k + \frac{1}{6}(f'_{ijkl} - f_{ijkl})\lambda_j\lambda_k\lambda_l + (\mu^2\delta\lambda)_i\} + \frac{\pi^{n/2}}{2(2\pi)^n}\mathrm{Tr}\{\Gamma(1-\tfrac{n}{2})(m_a^2)^{n/2}[(n-1)P_a\beta_i + \Pi_a\gamma_i + 2\pi_a\hat{m}\hat{\Gamma}_i] - \Gamma(2-\tfrac{n}{2})(\Lambda^2)^{n/2-2}\xi\varrho\gamma_i\bar{\varrho}\}$$

(18)

B. $$\Gamma^{\mu\nu}_{\alpha\beta}(p) = \hat{\Gamma}^{\mu\nu}_{\alpha\beta}(p) - C_{\alpha i}(p^2)\Gamma^{-1}_{ij}(p^2)C_{\beta j}(p^2)p^\mu p^\nu - \xi^{-1}_{\alpha\beta}p^\mu p^\nu + \frac{p^\mu p^\nu}{(p^2)^2}\Gamma_i^{(1)}(\vartheta_\alpha\vartheta_\beta\lambda)_i$$

(cf. Eq. (54) of I and note that we make $\Gamma_i^{(1)} = 0$; for the definition of $C_{\alpha i}(p^2)$ see item C).

$$\hat{\Gamma}^{\mu\nu}_{\alpha\beta}(p) = -(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2})A_{\alpha\beta}(p^2),$$

$$A^{CT}_{\alpha\beta}(p^2) = p^2 Z_{3,\alpha\beta} - M^2_{\alpha\beta} - [(Z_1-Z_3)M^2 + M^2(Z_1-Z_3)]_{\alpha\beta} - (\lambda, \vartheta_\alpha\vartheta_\beta(Z-1)\lambda)$$

$$A^{B,C}_{\alpha\beta}(p^2) = \frac{\pi^{n/2}}{2(2\pi)^n}\mathrm{Tr}\{-2\Gamma(1-\tfrac{n}{2})[(n-2)(M^2)^{n/2-1}T_\alpha T_\beta + (\mu^2)^{n/2-1}\vartheta_\alpha\vartheta_\beta] - \int_0^1 ds\, A^2_{ab}(p^2,s)\,2P_a\beta_\alpha\Pi_b\bar{\beta}_\beta + \int_0^1 ds\,[A^1_{ab}(p^2,s)((2n-3)P_aT_\alpha P_bT_\beta + 2\Pi_a\vartheta_\alpha\Pi_b\vartheta_\beta) - B^1_{ab}(p^2,s)(P_a\beta_\alpha\Pi_b\bar{\beta}_\beta - (2(2n-3)p^2 + m_b^2)P_aT_\alpha P_bT_\beta) + C^1_{ab}(p^2,s)\frac{(p^2)^2}{2}P_aT_\alpha P_bT_\beta - A^1_{\Lambda\Lambda}(p^2,s)T_\alpha T_\beta]\}$$

$$A^F_{\alpha\beta}(p^2) = \frac{\pi^{n/2}}{2(2\pi)^n}\mathrm{Tr}\int_0^1 ds\, A^2_{ab}(p^2,s)(4s(1-s)p^2\pi_a f_\alpha\pi_b f_\beta - 2s\pi_a\hat{m}^2\hat{f}_\alpha\pi_b\hat{f}_\beta - 2s\pi_a\hat{m}^2\hat{f}_\beta\pi_b\hat{f}_\alpha + 2\pi_a\hat{m}\hat{f}_\alpha\pi_b\hat{m}\hat{f}_\beta]$$

$$A^\xi_{\alpha\beta}(p^2) = \frac{\pi^{n/2}}{2(2\pi)^n}\mathrm{Tr}\{2\Gamma(2-\tfrac{n}{2})(\Lambda^2)^{n/2-2}[\xi(\varrho\vartheta_\alpha\vartheta_\beta\bar{\varrho} - \varrho\bar{\varrho}T_\alpha T_\beta) - \frac{1}{p^2}M^2_{\alpha\alpha'}\xi T_{\alpha'}T_{\beta'}M^2_{\beta'\beta}] + \int_0^1 ds\, A^2_{\Lambda\Lambda}(p^2,s)[p^2\xi T_\alpha T_\beta - 3\xi(\varrho\vartheta_\alpha\bar{\varrho}T_\beta + \varrho\vartheta_\beta\bar{\varrho}T_\alpha - 2\varrho\bar{\varrho}T_\alpha T_\beta) + \frac{2}{p^2}M^2_{\alpha\alpha'}\xi T_{\alpha'}T_{\beta'}M^2_{\beta'\beta}] + (p^2\delta_{\alpha\alpha'} - M^2_{\alpha\alpha'})\xi T_{\alpha'}\int_0^1 ds\,[\frac{n-4}{p^2}(2+\xi/4)A^2_{\Lambda\Lambda}(p^2,s) + (1-s)B^2_{a\Lambda}(p^2,s)P_a]T_{\beta'}(p^2\delta_{\beta'\beta} - M^2_{\beta'\beta})\}$$

(19)

C. $$\Gamma^\mu_{\alpha i}(p) = -\Gamma^\mu_{i\alpha}(p) = -p^\mu C_{\alpha i}(p^2)$$

$$C^{CT}_{\alpha i}(p^2) = \varrho_{\alpha j}Z_{ij} + (Z_1 - Z_3)_{\alpha\beta}\varrho_{\beta i} + \vartheta_{\alpha ij}\delta\lambda_j$$

$$C^B_{\alpha i}(p^2) = \frac{\pi^{n/2}}{2(2\pi)^n}\mathrm{Tr}\{\int_0^1 ds\,[A^2_{ab}(p^2,s)(2(1+s)P_a\beta_\alpha\Pi_b\vartheta_i + (2+3s+n(1-2s))P_aT_\alpha P_b\beta_i + (1-2s)\Pi_a\vartheta_\alpha\Pi_b\gamma_i) - B^2_{ab}(p^2,s)(2(p^2-m_b^2)(1-s)P_a\beta_\alpha\Pi_b\vartheta_i + (p^2/2 + (p^2-m_b^2)(1-s))P_aT_\alpha P_b\beta_i) - C^2_{ab}(p^2,s)(\frac{p^2}{4})^2(1-2s)P_aT_\alpha P_b\beta_i]\}$$

$$C^F_{\alpha i}(p^2) = \frac{\pi^{n/2}}{2(2\pi)^n}\mathrm{Tr}\{\int_0^1 ds\, 2A^2_{ab}(p^2,s)[(1-s)\pi_a f_\alpha\pi_b\hat{m}\hat{\Gamma}_i - s\pi_a\hat{m}f_\alpha\pi_b\hat{\Gamma}_i^+]$$

$$C^\xi_{\alpha i}(p^2) = \frac{\pi^{n/2}}{2(2\pi)^n}\mathrm{Tr}\{\Gamma(2-\tfrac{n}{2})(\Lambda^2)^{n/2-2}2\xi[\varrho\vartheta_\alpha\vartheta_i - T_\alpha(\varrho\vartheta_i + \vartheta_i\bar{\varrho})] + M^2_{\alpha\alpha'}\xi T_{\alpha'}P_aT_{\beta'}\varrho_{\beta' i}\int_0^1 s\,ds\,(n-1)\tilde{B}^2_{\Lambda a}(p^2,s) + \varrho_{\alpha l}\xi\vartheta_l\pi_a\vartheta_j(p^2\delta_{ji} - \mu^2_{ji})\int_0^1 s\,ds\,[2s^2p^2\tilde{B}^3_{\Lambda a}(p^2,s) - n\tilde{B}^2_{\Lambda a}(p^2,s)] + \int_0^1 s\,ds\,[A^4_{\Lambda\Lambda}(p^2,s)(1-s)(\varrho_{\alpha j}\xi\varrho\vartheta_j + (1-s)M^2_{\alpha\alpha'}\xi T_{\alpha'})\times((1-s)p^2\xi T_\beta\varrho_{\beta i} - \xi\vartheta_l\bar{\varrho}(p^2\delta_{li} - \mu^2_{li})) + A^3_{\Lambda\Lambda}(p^2,s)(2sT_\alpha(\xi\vartheta_l\bar{\varrho}(p^2\delta_{li} - \mu^2_{li}) - (1-s)p^2\xi T_\beta\varrho_{\beta i}) - \frac{1-s}{2}M^2_{\alpha\alpha'}T_{\alpha'}\xi T_\beta\varrho_{\beta i}) - A^2_{\Lambda\Lambda}(p^2,s)\xi T_\alpha T_\beta\varrho_{\beta i}]\}$$

(20)

D. $$\Gamma^{CT}_{ij}(p^2) = p^2 Z_{ij} - \mu^2_{ij} + \delta\mu^2_{ij} - [(\xi-1)p^2 + \mu^2(\xi-1) + (c^2_k - c_k)\lambda_k + \tfrac{1}{2}(f^2_{kl} - f_{kl})\lambda_k\lambda_l + (c_k + f_{kl}\lambda_l)\delta\lambda_k]_{ij} +$$

$$\Gamma^{B}_{ij}(p^2) = \frac{\pi^{n/2}}{2(2\pi)^4}\,Tr\,\{\Gamma(1-\tfrac{n}{2})(m_a^2)^{n/2-1}[\Pi_a\gamma_{ij} - 2(n-1)P_a\vartheta_i\vartheta_j] + \int_0^1 ds\,[A^2_{ab}(p^2,s)\big((n-1)P_a\beta_i P_b\beta_j + \Pi_a\gamma_i\Pi_b\gamma_j - 4p^2(1+s)P_a\vartheta_i\Pi_b\vartheta_j\big) + B^2_{ab}(p^2,s)\big(\tfrac{2s-3}{2}p^2 P_a\beta_i P_b\beta_j + 4p^2(p^2-m_b^2)(1-s)P_a\vartheta_i\Pi_b\vartheta_j\big) + C^2_{ab}(p^2,s)\frac{(p^2)^2}{4}P_a\beta_i P_b\beta_j]\}$$

$$\Gamma^{F}_{ij}(p^2) = \frac{\pi^{n/2}}{2(2\pi)^4}\,Tr\,\{\Gamma(1-\tfrac{n}{2})(m_a^2)^{n/2-1}\pi_a(\hat\Gamma_i\hat\Gamma_j^+ + \hat\Gamma_j\hat\Gamma_i^+) + \int_0^1 ds\,A^2_{ab}(p^2,s)[p^2\pi_a\hat\Gamma_i\pi_b\hat\Gamma_j^+ - \pi_a\hat m^2\hat\Gamma_i\pi_b\hat\Gamma_j^+ - \pi_a\hat m^2\hat\Gamma_j\pi_b\hat\Gamma_i^+ - 2\pi_a\hat m\hat\Gamma_i\hat m\hat\Gamma_j]\}$$

$$\Gamma^{\xi}_{ij}(p^2) = \frac{\pi^{n/2}}{2(2\pi)^4}\,Tr\,\{\Gamma(2-\tfrac{n}{2})(\Lambda^2)^{n/2-2}\xi[2p^2\vartheta_i\vartheta_j - \mu^2_{ii'}\vartheta_{i'}\vartheta_j - \vartheta_i\vartheta_{j'}\mu^2_{j'j} + \gamma_{ijl}\vartheta_l\bar\xi] - p^2\bar\varrho_{i\alpha}\xi T_\alpha P_a T_\beta\varrho_{\beta j}\int_0^1 s\,ds\,(n-1)\tilde B^2_{\Lambda a}(p^2,s) + (p^2\delta_{ii'} - \mu^2_{ii'})\xi\vartheta_{i'}\Pi_a\vartheta_{j'}(p^2\delta_{j'j} - \mu^2_{j'j}) \times \int_0^1 s\,ds\,[2\tilde B^3_{\Lambda a}(p^2,s)s^2p^2 - n\tilde B^2_{\Lambda a}(p^2,s)] - \int_0^1 s\,ds\,[A^4_{\Lambda\Lambda}(p^2,s)(1-s)\big((p^2\delta_{ii'} - \mu^2_{ii'})\xi\varrho\vartheta_{i'} - (1-s)p^2\bar\varrho_{i\alpha}\xi T_\alpha\big)\big(\xi\vartheta_{j'}\bar\varrho(p^2\delta_{j'j} - \mu^2_{j'j}) - (1-s)p^2\xi T_\beta\varrho_{\beta j}\big) - \tfrac{1}{2}A^3_{\Lambda\Lambda}(p^2,s)(1-s)p^2\bar\varrho_{i\alpha}\xi T_\alpha\xi T_\beta\varrho_{\beta j}]\} \qquad (21)$$

E.

$$\Gamma^{CT}(p) = \hat p - m + \delta m + (\xi_2 - 1)(\hat p - m_0) + (\hat p - m_0)(\bar\xi_2 - 1) - (\Gamma'_i - \Gamma_i)\lambda_i - \Gamma_i\delta\lambda_i$$

$$\Gamma^{BF}(p) = \frac{\pi^{n/2}}{(2\pi)^4}\bar\omega\int_0^1 ds\,[A^2_{ab}(p^2,s)\big((n-1)\bar{\hat t}_a(s\hat p - \hat m)\pi_b\hat t_a + \hat\Gamma_a(s\hat p + \hat m)\pi_b\hat\Gamma_a\big) + B^1_{ab}(p^2,s)(n-1)\bar{\hat t}_a\hat p\,\pi_b\hat t_a]\omega$$

$$\Gamma^{\xi}(p) = \frac{\pi^{n/2}}{(2\pi)^4}\bar\omega\{-\Gamma(2-\tfrac{n}{2})(\Lambda^2)^{n/2-2}\xi_{\alpha\beta}[\bar{\hat t}_\beta(\hat m - \hat p)\hat t_\alpha + \hat\Gamma_i(\vartheta_\beta\vartheta_\alpha\lambda)_i] + \int_0^1 s\,ds\,A^3_{\Lambda a}(p^2,s)\xi_{\alpha\beta}(\hat p - \hat m)\hat t_\beta(\hat m + s\hat p)\times\pi_b\hat t_\alpha(\hat p - \hat m)\}\omega \qquad (22)$$

F. $$\Gamma_{\alpha\beta}(p^2) = p^2 I_{\alpha\beta}(p^2)$$

$$I_{\alpha\beta}(p^2) = Z'_{3,\alpha\beta} + \frac{\pi^{n/2}}{2(2\pi)^4}\,Tr\,\{(n-1)\int_0^1 ds\,B^1_{a\Lambda}(p^2,s)P_a T_\alpha T_\beta + \int_0^1 ds\,A^2_{\Lambda\Lambda}(p^2,s)\,s\,[2(n-1)(1-s)-1]\xi T_\alpha T_\beta\} \qquad (23)$$

For the transition to $n=4$, one can use the rule

$$\pi^{n/2}\Gamma(k-\tfrac{n}{2})F(n) \underset{n\to 4}{\approx} \begin{cases} -\delta_k\pi^2[(\frac{2}{n-4} + \ln\pi + \Gamma_k)F(4) + 2F'(4)], & k \le 2 \\ (k-3)!\,\pi^2 F(4), & k > 2 \end{cases}$$

Here $\delta_k = \dfrac{(-1)^{2-k}}{(2-k)!}$ and Γ_k is defined by

$$\Gamma(z) \underset{z\to k-2\le 0}{\approx} \delta_k/z+2-k - \Gamma_k \qquad (\Gamma_2 = \gamma_E \approx 0{,}57721,\ \Gamma_1 = \gamma_E - 1).$$

A definite choice /9/ of the renormalization constants will be discussed in the next paper of this series.

V. Finite corrections to the fermion mass matrix

As an immediate application of our formulas let us investigate how the one-loop corrections modify the 0^{th} order mass matrix of the fermions. Choosing the counterterms according to 't Hooft's prescription /9/ and performing the integrations in (22) we get

$$\Gamma(p) = \bar\omega[\hat\Gamma(p) - \hat\Gamma_i\delta\lambda_i]\omega, \qquad (24)$$

where

$$\hat\Gamma(p) = \hat p - \hat m - \frac{1}{16\pi^2}\{2\bar{\hat t}_\alpha(\hat p - \hat m)\hat t_\alpha - \ln\Lambda^2\xi_{\alpha\beta}[\bar{\hat t}_\beta(\hat m - \hat p)\hat t_\alpha + \hat\Gamma_i(\vartheta_\beta\vartheta_\alpha\lambda)_i] - 3\bar{\hat t}_a[\hat m L^0_{ab}(p^2) - \hat p L^1_{ab}(p^2)]\pi_b\hat t_a + \hat\Gamma_a[\hat m L^0_{ab}(p^2) + \hat p L^1_{ab}(p^2)]\pi_b\hat\Gamma_a - [L^1_{\Lambda b}(p^2) + 2m_b^2\frac{1}{m_a^2-\Lambda^2}(L^0_{ab}(p^2) - L^0_{\Lambda b}(p^2)) +$$

$$+(m_a^2-m_b^2-p^2)\frac{1}{m_a^2-\Lambda^2}\left(L^1_{ab}(p^2)-L^1_{\Lambda b}(p^2)\right)+$$
$$+\frac{1}{m_a^2-\Lambda^2}\left(m_a^2\ln m_a^2-\Lambda^2\ln\Lambda^2\right)\Big]\hat{\bar{f}}_a\hat{p}\pi_b f_a-$$
$$-\xi_{\alpha\beta}(\hat{p}-\hat{m})f_\beta[\hat{m}I^1_{\Lambda a}(p^2)+\hat{p}I^2_{\Lambda a}(p^2)]\pi_a\bar{f}_\alpha(\hat{p}-\hat{m}) \quad (25)$$

with

$$I^k_{ab}(p^2)=\int_0^1\frac{s^k ds}{z_{ab}(p^2,s)}\,,\quad L^k_{ab}(p^2)=\int_0^1 ds\, s^k\ln|z_{ab}(p^2,s)| \quad (26)$$

and $\delta\lambda_i$ is the modification of the scalar vacuum expectation value, the form of which we anticipate from our next paper:

$$\delta\lambda_i=\frac{1}{32\pi^2}\mu^{-2}_{ij}\,\mathrm{Tr}\{m_a^2\ln m_a^2[\Pi_a\gamma_j-3P_a\{\partial_j,\partial_k\}\lambda_k+$$
$$+2\pi_a\hat{m}\hat{\Gamma}_j]+\ln\Lambda^2\,\xi\,\xi\,\gamma_j\,\bar{\xi}\,\}. \quad (27)$$

The one-loop corrections to the fermion masses in a general gauge theory have been determined by Weinberg [10]. Our calculation differs from his in that we work with diagonal fermion masses and obtain somewhat more explicit results. In the renormalization scheme we work with there is no question of divergent corrections changing the general structure of the fermion mass matrix.

To determine the mass correction we write $\Gamma(p)$ in the form

$$\Gamma(p)=A(p^2)\hat{p}-B(p^2)=\bar{C}(p^2)(\hat{p}-m-\delta M(p^2))C(p^2)$$

and by using standard perturbation theory we get that in a subspace with a given m_a

$$\underset{(m_a=m_b)}{\delta m^a_b}=\frac{1}{2}[\bar{\omega}^+\delta M(m_a^2)\omega^++\gamma_0\bar{\omega}^+\delta M(m_a^2)\omega^+\gamma_0]^a_b. \quad (28)$$

This expression gives the splitting of the degenerate fermion masses, i.e.,gives the symmetry breaking corrections to the

so-called natural symmetries [11,12] of the mass matrix. Since it is hermitian and free of γ_5, it can be diagonalized in the corresponding subspace by an ordinary unitary transformation.

Putting together (24) and (28) one obtains

$$\underset{(m_a=m_b)}{\delta m^a_b}=\delta_D m^a_b+\delta_{(A)}m^a_b+\delta_{(K)}m^a_b+\delta_{(P)}m^a_b+$$
$$+\delta_{(T)}m^a_b+\delta_{\delta\lambda}m^a_b. \quad (29)$$

$\delta_D m$ is a correction which can be incorporated into the original mass without changing its degeneracy structure:

$$32\pi^2\delta_D m^a_b=[4m_a(\hat{f}_\alpha\hat{f}_\alpha+\hat{\bar{f}}_\alpha\hat{\bar{f}}_\alpha)+(\partial_\alpha\partial_\alpha\lambda)_i(\hat{\Gamma}_i+\hat{\Gamma}_i^+)]^a_b \quad (29.a)$$

$\delta_{(A)}m$ and $\delta_{(K)}m$ are the contributions of the zero mass and massive vector bosons, respectively:

$$32\pi^2\delta_{(A)}m^a_b=-3m_a\ln m_a^2(\hat{f}_A f_A+\hat{\bar{f}}_A\hat{\bar{f}}_A)^a_b$$
$$32\pi^2\delta_{(K)}m^a_b=m_a\{L^0_{Kc}(m_a^2)\Big(-\frac{M_K^2}{m_a^2}+\frac{(m_a^2-m_c^2)^2}{2m_a^2M_K^2}+\frac{m_a^2+m_c^2}{2m_a^2}\Big)+$$
$$+\frac{M_K^2}{m_a^2}(\ln M_K^2-1)-4-\frac{m_c^2}{m_a^2}\ln m_c^2-\frac{m_a^2-m_c^2}{2m_a^2}(\ln M_K^2+1)-$$
$$-\frac{m_c^2(m_a^2+m_c^2)}{2m_a^2M_K^2}(\ln m_c^2-1)\}(\hat{f}_K\pi_c\hat{f}_K+\hat{\bar{f}}_K\pi_c\hat{\bar{f}}_K)^a_b \quad (29.b)$$
$$-3m_c L^0_{Kc}(m_a^2)(\hat{\bar{f}}_K\pi_c\hat{f}_K+\hat{f}_K\pi_c\hat{\bar{f}}_K)^a_b \quad (29.c)$$

$\delta_{(P)}m$ gives the correction due to the physical Higgs scalars:

$$32\pi^2\delta_{(P)}m^a_b=m_c L^0_{Pc}(m_a^2)(\hat{\Gamma}_P\pi_c\hat{\Gamma}_P+\hat{\Gamma}_P^+\pi_c\hat{\Gamma}_P^+)^a_b+$$
$$+m_a L^1_{Pc}(m_a^2)(\hat{\Gamma}_P\pi_c\hat{\Gamma}_P^++\hat{\Gamma}_P^+\pi_c\hat{\Gamma}_P)^a_b. \quad (29.d)$$

$\delta_{(T)} m$, the Goldstone boson contribution can be transformed into an additional vector boson term by using third equation of (17)

$$32\pi^2 \delta_{(T)} m^a_b = -m_c \frac{m_c^2}{M_K^2} (1-\ln m_c^2)(\hat{f}_K \pi_c \bar{\hat{f}}_K + \bar{\hat{f}}_K \pi_c \hat{f}_K)^a_b \tag{29.e}$$

and finally, $\delta_{\delta\lambda} m$ (the "tadpole term" in /10/) is

$$32\pi^2 \delta_{\delta\lambda} m^a_b = \frac{1}{\mu_\rho^2} \operatorname{Tr}\{\mu^2 \ln \mu^2 \gamma_\rho - 3 M^2 \ln M^2 \vartheta_\rho \vartheta_{\rho'} \lambda_{\rho'} + 2\hat{m}^3 \ln \hat{m}^2 \hat{\Gamma}_\rho\} \frac{1}{2} (\hat{\Gamma}_\rho + \hat{\Gamma}_\rho^+)^a_b . \tag{29.f}$$

As it has been shown in /10/ all logarithms can be calculated by using an arbitrary mass unit: a change in the mass unit influences only $\delta_D m$. δm does not depend on the gauge parameter ξ and is free of infrared divergencies.

Experimental facts indicate that vector boson and possibly also Higgs scalar masses must be higher than the known fermion masses. Therefore, it is reasonable to consider (29) in an approximation with m/M, m/μ small. Below we give δm up to order m^2/M^2 and m^2/μ^2 :

$$\delta m^a_b = \delta_D m^a_b + \delta_S m^a_b \qquad (m_a = m_b) \tag{30}$$

where $\delta_D m$ is again a term which does not split natural symmetries and

$$\begin{aligned} 32\pi^2 \delta_S m^a_b = {} & m_a \left(\tfrac{3}{2} - 3\ln m_a^2\right)(\hat{f}_A \hat{f}_A + \bar{\hat{f}}_A \bar{\hat{f}}_A)^a_b - \\ & - 3 m_c \left\{\ln M_K^2 - \frac{m_a^2}{M_K^2} + \frac{m_c^2}{M_K^2}\left[\ln \frac{M_K^2}{m_c^2} + \tfrac{1}{3}(1-\ln m_c^2)\right]\right\} \times \\ & \times (\bar{\hat{f}}_K \pi_c \hat{f}_K + \hat{f}_K \pi_c \bar{\hat{f}}_K)^a_b \\ & + m_a \frac{m_a^2 - 3m_c^2}{2M_K^2} (\ln M_K^2 - 5/3)(\hat{f}_K \pi_c \hat{f}_K + \bar{\hat{f}}_K \pi_c \bar{\hat{f}}_K)^a_b + \\ & + m_a \left[-\tfrac{1}{4} + \tfrac{1}{2} \ln \mu_\rho^2 - \frac{1}{2\mu_\rho^2}\left(\tfrac{1}{3} m_a^2 - m_c^2\right)\right](\hat{\Gamma}_\rho \pi_c \hat{\Gamma}_\rho^+ + \hat{\Gamma}_\rho^+ \pi_c \hat{\Gamma}_\rho)^a_b \\ & + m_c \left[-1 + \ln \mu_\rho^2 + \frac{m_c^2}{\mu_\rho^2} \ln \frac{\mu_\rho^2}{m_c^2} - \frac{m_a^2}{2\mu_\rho^2}\right](\hat{\Gamma}_\rho \pi_c \hat{\Gamma}_\rho + \hat{\Gamma}_\rho^+ \pi_c \hat{\Gamma}_\rho^+)^a_b \\ & + \frac{1}{\mu_\rho^2} \operatorname{Tr}\{\mu^2 \ln \mu^2 \gamma_\rho - 3M^2 \ln M^2 \vartheta_\rho \vartheta_{\rho'} \lambda_{\rho'} + \\ & + 2\hat{m}^3 \ln \hat{m}^2 \hat{\Gamma}_\rho\} \frac{1}{2} (\hat{\Gamma}_\rho + \hat{\Gamma}_\rho^+)^a_b . \end{aligned} \tag{30.a}$$

We did not check the detailed coincidence of the expression (29) with that of /10/: the two expressions are of the same structure and in the special cases, analyzed in /10/, they give the same result.

As an interesting case for natural symmetries it was suggested by Weinberg /13/ that in some theories the electron mass may naturally be zero in lowest order, and, if appropriate vector particles are present, higher order corrections can give $m_e = \alpha m_\mu$ (m_μ : muon mass). Such corrections are given by the second term of (30.a). Now, if the masslessness of the electron is the consequence of the fact that the electron has no Yukawa couplings to the scalar mesons, the corrections, as (30.a) shows, are proportional to m_c^2 / M_K^2 . Indeed,

$$(\bar{\hat{f}}_K \hat{m} \hat{f}_K)^a_b = \frac{1}{2} m_a (\hat{f}_K \hat{f}_K + \bar{\hat{f}}_K \bar{\hat{f}}_K)^a_b - \frac{1}{2} (\vartheta_K^2 \lambda)_i (\Gamma_i)^a_b$$

thus the $\ln M_K^2$ term does not contribute. Georgi and Glashow /14/ give an example where the electron picks up a mass of order αm_μ by such a mechanism; the trick is that a heavy lepton c has a coupling $\sim \alpha m_\mu / m_c$ as a result of convenient mixings. Cases, where the masslessness of the electron comes from a special choice of the scalar vacuum expectation value λ seem to be much more complicated /14/.

Appendix

We give here the compact form of the propagators and vertices we use in calculating Green functions (the Faddeev-Popov ghost contributions are treated separately). For this purpose let us introduce the following matrices acting on the indices of the fields (we write down only the non-zero elements in the definitions):

$$p:\ p_{\alpha,\beta\mu} = p_\mu \delta_{\alpha\beta} \quad ; \quad \bar p:\ \bar p_{\beta\mu,\alpha} = -p_\mu \delta_{\alpha\beta} \ ; \quad e:\ e_{\alpha\beta} = \delta_{\alpha\beta}$$

$$\frac{\partial}{\partial p^\mu} p = \varepsilon_\mu \ , \quad \frac{\partial}{\partial p^\mu} \bar p = \bar\varepsilon_\mu \quad ; \quad \varepsilon^\mu \bar\varepsilon^\nu = -g^{\mu\nu} e \tag{A.1}$$

$$\varrho:\ \varrho_{\alpha\gamma} = e_{\alpha\gamma\delta}\lambda_\delta \quad ; \quad \bar\varrho:\ \bar\varrho_{\gamma\alpha} = \varrho_{\alpha\gamma} \tag{A.2}$$

$$\varepsilon:\ \varepsilon_{\gamma\delta} = \delta_{\gamma\delta}\,\varepsilon_\gamma \quad ; \quad \varepsilon_{(\mu\alpha)} = +1 \ , \ \varepsilon_i = -1 \ , \ \varepsilon_a = 0 \tag{A.3}$$

$$e_\gamma:\ (e_\gamma)_{\alpha\delta} = (e_\delta)_{\gamma\alpha} = e_{\alpha\gamma\delta}$$

$$\bar e_\gamma:\ (\bar e_\gamma)_{\alpha\delta} = (\bar e_\delta)_{\gamma\alpha} = \bar e_{\alpha\gamma\delta} = -e_{\alpha\delta\gamma} \tag{A.4}$$

($\bar e_\gamma = e_\gamma$ for bosons)

$$P_s:\ (P_s)_{\alpha\beta} = \delta_{s\alpha}\delta_{s\beta} \tag{A.5}$$

$\Pi_s:\ (\Pi_s)_{ij} = u_{si}u_{sj}$ (no summation over s)

$$\pi_s:\ (\pi_s)^a_b = \delta^a_s \delta^s_b$$

$\sum_s P_s = 1$ (vector), $\sum_s \Pi_s = 1$ (scalar), $\sum_s \pi_s = 1$ (fermion)

The definition of $e_{\alpha\gamma\delta}, \lambda_s, \delta_{s\alpha}, u_{si}$ can be found in I. We note here some useful relations:

$p\bar q = -(pq)e$, $\varrho\bar\varrho = -M^2$ (M : vector mass), $\varrho\mu^2 = \mu^2\bar\varrho = 0$ (μ : scalar mass)

$$p\varepsilon = p \ , \ \varepsilon\bar p = \bar p \ , \ \varrho\varepsilon = -\varrho \ , \ \varepsilon\bar\varrho = -\bar\varrho \ ; \ p\bar\varrho = \varrho\bar p = 0$$

$$p e_\gamma = -e_\gamma \bar p = -p_{\alpha\gamma} T_\alpha \ , \quad \varrho e_\gamma + e_\gamma \bar\varrho = -\varrho_{\alpha\gamma} T_\alpha \ ,$$

$$p^T = -\bar p \ , \ \varrho^T = \bar\varrho \ , \ e_\gamma^T = -\bar e_\gamma \ . \tag{A.6}$$

With these matrices the inverse propagator K , the propagator Δ and the vertices V_γ , $V_{\gamma\delta}$ can be expressed as follows:

$K(p) = K^1(p) + \bar p\, \xi^{-1} p$ (ξ : gauge parameter matrix)

$$K^1(p) = K^0(p) - \bar p p + \bar p \varrho + \bar\varrho p$$

$K^0(p) = -\varepsilon(p^2 - m^2)$ for bosons

$= \hat p - m$ for fermions

$$(p+\varrho)K(p) = -p^2 \xi^{-1} p \ , \quad (p+\varrho)K^1(p) = K^1(p)(\bar p + \bar\varrho) = 0$$

$$K(p)(\bar p + \bar\varrho) = -p^2 \bar p\, \xi^{-1} \tag{A.7}$$

$$\Delta(p) = \Delta^1(p) + \frac{1}{(p^2)^2}(\bar\varrho + \bar p)\xi(\varrho + p)$$

$$\Delta^1(p) = \Delta^0(p) - \frac{1}{p^2}\bar p\, \tilde\delta(p)\, p$$

$$\Delta^0(p) = \frac{\Delta^0_s(p)}{p^2 - m_s^2} \ ; \quad (\Delta^0_s)^{\mu\nu}_{\alpha\beta} = -g^{\mu\nu} P_{s,\alpha\beta}$$

$$(\Delta^0_s)_{ij} = \Pi_{s,ij}$$

$$(\Delta^0_s)^a_b = [\omega^+ \pi_s (\hat p + \hat m)\bar\omega^+]^a_b$$

$$\tilde\delta(p) = \frac{\delta_s}{p^2 - m_s^2} \ ; \quad \delta_s = P_s$$

$$\bar p P_s p = -\bar p p \Delta^0_s = -\Delta^0_s \bar p p$$

$$p\Delta(p) = -\frac{1}{p^2}\xi(\varrho + p) \ , \quad p\Delta^1(p) = \Delta^1(p)\bar p = 0$$

$$\Delta(p)\bar p = -\frac{1}{p^2}(\bar\varrho + \bar p)\xi \ , \quad \varrho\Delta^0(p) = \frac{1}{p^2}\varrho \ , \ \Delta^0(p)\bar\varrho = \frac{1}{p^2}\bar\varrho \tag{A.8}$$

$$V_\gamma(pqk) = V^1_\gamma(pqk) + \bar p e_\gamma \varepsilon + \varepsilon e_\gamma k$$

$$V^1_\gamma(q) = V^0_\gamma + 2\bar q e_\gamma \varepsilon + 2\varepsilon e_\gamma q$$

$$V^0_{\alpha\mu} = -\bar\varepsilon_\mu \beta_\alpha - \bar\beta_\alpha \varepsilon_\mu + \gamma^\mu t_\alpha \ ; \quad \beta_\alpha = T_\alpha \varrho - 2\varrho\vartheta_\alpha$$

$$\bar\beta_\alpha = -\beta_\alpha^T = \bar\varrho T_\alpha - 2\vartheta_\alpha \bar\varrho$$

$$V^0_i = \bar\varepsilon_\mu \beta_i \varepsilon^\mu + \gamma_i - \Gamma_i \ ; \quad \beta_i = \beta_i^T = \varrho\vartheta_i - \vartheta_i\bar\varrho$$

$$\gamma_i = \gamma_i^T = -c_i - f_{ij}\lambda_j$$

$$(V_0{}^a)^\mu_{\alpha b} = (V^0{}_b)^{a\mu}_\alpha = (\gamma^\mu t_\alpha)^a_b$$

$$(V^{0a})_{ib} = (V^0{}_b)^a{}_i = -\Gamma_i{}^a_b \tag{A.9}$$

$$(\not{q}+\not{p})V_r(pqk) = \bar{e}_r K'(-k) - K'_{r r'}(q) e_{r'}$$
$$V_r(kqp)(\bar{\not{q}}-\bar{\not{p}}) = -K'(k)e_r + \bar{e}_{r'} K'_{r' r}(-q)$$

(0^{th} order WT identities)

$$p V^0_{\alpha\mu} = p_\mu \beta_\alpha \;, \quad V^0_{\alpha\mu}\bar{p} = p_\mu \bar{\beta}_\alpha$$
$$p V_i = -\beta_i p \;, \quad V^0_i \bar{p} = -\bar{p}\beta_i \,.$$

$$V_{rstu}(pqkl) = V^0_{rstu} \qquad \text{(A.10)}$$

$$V^0_{\mu\alpha,\nu\beta} = -g_{\mu\nu}(e_\alpha \varepsilon e_\beta + e_\beta \varepsilon e_\alpha) + \bar{\varepsilon}_\mu \left(\tfrac{3}{2}[T_\alpha, T_\beta] - \tfrac{1}{2}\{T_\alpha, T_\beta\}\right) e_\nu$$
$$- \bar{\varepsilon}_\nu \left(\tfrac{3}{2}[T_\alpha, T_\beta] + \tfrac{1}{2}\{T_\alpha, T_\beta\}\right) \varepsilon_\mu \,,$$

$$V^0_{ij} = -\bar{\varepsilon}_\mu \{e_i e_j\} \varepsilon^\mu + \gamma_{ij} \;; \quad \gamma_{ij} = -f_{ij}$$

$$\text{(A.11)}$$

For the ghost one has (see I., Appendix)

$$\Delta_{\alpha\beta}(p^2) = \frac{\delta_{\alpha\beta}}{p^2} \,,$$
$$\gamma_{\alpha\beta\zeta}(pqk) = -i g_{\alpha\beta\gamma} p^\mu \,. \qquad \text{(A.12)}$$

References:

/1/ T.Nagy, to be published.

/2/ K.Fujikawa, B.W.Lee and A.I.Sanda, Phys.Rev., D6, 2923 (1972).

/3/ J.Iliopoulos, C.Itzykson and A.Martin, Rev.Mod.Phys., 47, 165 (1975).

/4/ F.A.Berezin, The method of second quantization, Academic Press, N.Y. (1966).

/5/ G.'t Hooft and M.Veltman, Nucl.Phys., B44, 189 (1972).

/6/ G.Costa, T.Marinucci, M.Tonin and J.Julve, Nuovo Cim., 38A, 373 (1977).

/7/ S.L.Adler, Phys.Rev., 177, 2426 (1969).
J.S.Bell and R.Jackiw , Nuovo Cim., 60, 47 (1969).
D.J.Gross and R.Jackiw, Phys.Rev., D6, 477 (1972).

/8/ R.Gastmans and R.Meuldemaus, Nucl.Phys., B63, 277 (1973).

/9/ G.t(Hooft, Nucl.Phys., B61, 455 (1973).

/10/ S.Weinberg, Phys.Rev., D7, 2887 (1973).

/11/ S.Weinberg, Phys.Rev.Letters, 29, 388 (1972).

/12/ H.Georgi and S.L.Glashow, Phys.Rev., D6, 2977 (1972).

/13/ S.Weinberg, Phys.Rev., D5, 1962 (1972).

/14/ H.Georgi and S.L.Glashow, Phys.Rev., D7, 2457 (1973).

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