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INVESTIGATION OF SPONTANEOUSLY BROKEN GAUGE THEORIES

I. General Structure and Renormalization

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Исследование спонтанно нарушенных калибровочных теорий.
I. Общая структура и перенормировка

В серии работ автором исследованы спонтанно нарушенные калибровочные теории (СНКТ) применительно к эффектам, которые могут быть рассмотрены по теории возмущений. В данной работе представлены общая структура СНКТ и калибровочно инвариантная программа перенормировки для практических расчетов. Доказательство перенормировки Ли и Зинн-Жюстэна распространено на наш общий случай.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Investigation of Spontaneously Broken Gauge Theories.
I. General Structure and Renormalization

In a series of papers we investigate spontaneously broken gauge theories, heaving in mind mainly applications to effects which may be treated perturbatively. In this Part we exhibit the general structure of SBGT's and set up a gauge invariant renormalization program for practical calculations, by extending the proof of renormalizability of Lee and Zinn-Justin to our general case.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Introduction

In the last decade a very promising field theory has emerged which has all the characteristics of being capable to describe elementary particle interactions. We have in mind spontaneously broken gauge theories (SBGT) which are locally gauge invariant Yang-Mills theories /1/, providing well-defined minimal vector interactions, and where the gauge symmetry is spontaneously broken via the Higgs mechanism /2/, so that in spite of the spontaneous symmetry breaking one can get rid off the Goldstone bosons /3/ and in spite of the local gauge invariance one can have massive charged (and neutral) vector particles. It has been shown /4,5,6/ that such theories can be renormalizable. In the framework of SBGT's elegant models were constructed to unify electromagnetic and weak interactions (7) and in the last years these models have been extended and generalized in many directions. It has turned out that gauge theories show also other remarkable features : non-Abelian gauge theories (NAGT) (and under certain conditions only they) can exhibit asymptotic freedom, i.e., vanishing of the invariant coupling constants at high momentum transfers /8/. On the other hand, there is a conjecture /9/, that infrared problems in unbroken NAGT's are so serious that low momentum transfer effects can be treated only non-perturbatively and such effects lead to an unusual particle spectrum in the theory and, in particular, to colour and quark confinement in quantum chromodynamics ("infrared slavery"). The recently discovered instanton solutions /10/ for the unbroken NAGT's indicate that the structure of such theories indeed differs /11/ from the familiar perturbative picture, and there are hopes that confinement effects can be treated by saturating Feynman integrals by instanton-like solutions.

The aim of this work is to set up a general framework for a systematic investigation of gauge theory phenomenology. At the first stage we want to deal mainly with features which can presumably be treated perturbatively, i.e.,electromagnetic and weak phenomena and some strong effects in asymptotically free theories. We will work with a general semisimple compact gauge group, connected with a possible Abelian factor, and consider arbitrary fermion and scalar multiplets. In this part we discuss briefly the structure of the GT Lagrangian and describe a renormalization procedure which is essentially a generalization of /5/. In future publications we shall deal with the Green functions of the theory in one-loop approximation and try to get constraints on the general theory both by comparing it with experience and by making some simple assumptions which may be suggested by the experimental facts.

The plan of this Part is as follows: In Sec. II write down the Lagrangian of the theory and discuss lowest order (tree approximation) features. Sec. III. touches briefly the problem of quantisation and gives the definition of the generating functionals of the Green functions and the Green functions themselves. This section contains also the Ward identities written for the vacuum functional. We discuss the renormalization

of the symmetric theory in Sec.IV and the renormalization of the spontaneously broken theory in Sec.V. Finally, in the Appendix we give the propagators, vertices and gauge invariant counterterms we will use in future calculations.

II. General structure of the gauge theory Lagrangian

In this section we recapitulate the familiar scheme of a SBGT (see,e.g.,/12/) mainly with the purpose of introducing the objects we shall work with. The gauge group is $G = K \times N$; K is an arbitrary compact, semisimple group and N is a possible Abelian factor which is absent in NAGT's. We denote the generators of G by e_α ; they satisfy the commutation relations

$$[e_\alpha, e_\beta] = i g_{\alpha\beta\gamma} e_\gamma \qquad (1)$$

with completely antisymmetric structure constants $g_{\alpha\beta\gamma}$. For the adjoint representation $e_\alpha = T_\alpha$ with $(T_\alpha)_{\beta\gamma} = -i g_{\alpha\beta\gamma}$, for fermions (ψ) we use the notation $e_\alpha = t_\alpha$, and for scalars (φ) $e_\alpha = \vartheta_\alpha$. We note that gauge couplings are included in the normalization of e_α , so that if $\{F_\alpha\}$ are some standard generators, $e_\alpha = g_{\alpha\beta} F_\beta$ with $g_{\alpha\beta} = g_{\beta\alpha}$, $[T_\alpha, g] = 0$ (g can be taken diagonal, with constant elements within a simple factor of K). For $\alpha \in N$ $g_{\alpha\beta\gamma} = 0$ and

$$g_{\alpha\gamma\delta} g_{\beta\gamma\delta} = \mathrm{Tr}\, T_\alpha T_\beta = g^2_{\alpha\beta} C_2(K) \quad \text{for } \alpha, \beta \in K \qquad (2)$$

The Lagrangian, locally invariant under the gauge transformations

$$\delta\psi^a = i\varepsilon_\alpha t_{\alpha b}^{a} \psi^b , \quad \delta\varphi_i = i\varepsilon_\alpha \vartheta_{\alpha ij} \varphi_j \qquad (3)$$
$$\delta A^\mu_\alpha = i\varepsilon_\gamma T_{\gamma\alpha\beta} A^\mu_\beta + \partial^\mu \varepsilon_\alpha$$

(A^{μ}_{α} is the gauge boson field) has the form

$$\mathcal{L} = -\frac{1}{4} F_{\alpha\mu\nu} F_{\alpha}^{\mu\nu} + \frac{1}{2} \nabla_{\mu}\varphi_i \nabla^{\mu}\varphi_i + i\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi - \bar{\psi} m_0 \psi -$$
$$- \bar{\psi}\Gamma_i \varphi_i \psi - P(\varphi) + \bar{\psi}\,\delta m\,\psi + \frac{1}{2}\delta\mu^2_{ij}\varphi_i\varphi_j \,, \tag{4}$$

where

$$F_{\alpha\mu\nu} = \partial_{\mu} A_{\alpha\nu} - \partial_{\nu} A_{\alpha\mu} + g_{\alpha\beta\gamma} A_{\beta\mu} A_{\gamma\nu} \,,$$
$$\nabla_{\mu}\psi^{a} = \partial_{\mu}\psi^{a} - i\, t_{\alpha\,b}^{\,a} A_{\alpha\mu}\psi^{b} \,,\ \nabla_{\mu}\varphi_i = \partial_{\mu}\varphi_i - i\,\vartheta_{\alpha ij} A_{\alpha\mu}\varphi_j \,;$$

$P(\varphi)$, the most general G -invariant quartic polynomial in φ_i (which we take real) can be written as

$$P(\varphi) = \frac{1}{2!}\mu^2_{0\,ij}\varphi_i\varphi_j + \frac{1}{3!}c_{ijk}\varphi_i\varphi_j\varphi_k + \frac{1}{4!}f_{ijkl}\varphi_i\varphi_j\varphi_k\varphi_l \tag{5}$$

with completely symmetric coefficients μ_0^2 , c , and f . The last two terms in (4) are mass counterterms. The fermion matrices t_α , m_0 , δm and Γ_i may contain the Dirac matrices 1 and γ_5 . (We note here that we use the metric $g^{00} = -g^{ii} = 1$; $\bar{\gamma}_\mu \equiv \gamma_0\gamma_\mu^{+}\gamma_0 = \gamma_\mu$, $\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$. Summation is understood over indices occuring twice unless stated otherwise).

Hemiticity of $\mathcal{L}$ gives

$$\bar{m}_0 = m_0 \,,\ \bar{\Gamma}_i = \Gamma_i \,,\ \delta\bar{m} = \delta m \ ; (\bar{a} \equiv \gamma_0 a^{+} \gamma_0). \tag{6}$$

From global G invariance of $\mathcal{L}$ one gets

(T : transposition)

$$t_\alpha = t_\alpha^{+} \,,\ \vartheta_\alpha = \vartheta_\alpha^{+} = -\vartheta_\alpha^{T} \tag{7.a}$$

$$[t_\alpha , \gamma^0 m_0] = 0 \,,\ [t_\alpha , \gamma^0 \delta m] = 0 \,, \tag{7.b}$$

$$[t_\alpha , \gamma^0 \Gamma_i] = -\vartheta_{\alpha,ij}\gamma^0\Gamma_j \ ; \tag{7.c}$$

$$\frac{\partial P}{\partial \varphi_i}\delta\varphi_i = 0 \ : \ [\vartheta_\alpha , \mu_0^2] = 0 \,,\ ([\vartheta_\alpha , \delta\mu^2] = 0) \,, \tag{7.d}$$

$$[\vartheta_\alpha , c_i] = -\vartheta_{\alpha,ij} c_j \,, \tag{7.e}$$

$$[\vartheta_\alpha , f_{ij}] = -\vartheta_{\alpha,ik} f_{kj} - \vartheta_{\alpha,jk} f_{ik} \,. \tag{7.f}$$

Spontaneous breaking is introduced by postulating that the vaccum value of the scalar field be different from zero and the exact "mass" be positive semidefinite. Then writing $\langle\varphi_i\rangle_0 = \Lambda_i$ and $\varphi_i = \phi_i + \Lambda_i$ (where ϕ_i has zero vacuum expectation value) we obtain the Lagrangian of the SBGT. In tree approximation $\Lambda_i = \lambda_i$ and our postulates take the form

$$\frac{\partial P}{\partial \varphi_i}\Big|_{\varphi_i = \lambda_i} = \mu^2_{0\,ij} + \frac{1}{2}c_{ijk}\lambda_j\lambda_k + \frac{1}{6}f_{ijkl}\lambda_j\lambda_k\lambda_l = 0 \,, \tag{8.a}$$

$$\frac{\partial^2 P}{\partial\varphi_i\partial\varphi_j}\Big|_{\varphi=\lambda} \equiv \mu^2_{ij} = \mu^2_{0\,ij} + c_{ijk}\lambda_k + \frac{1}{2}f_{ijkl}\lambda_k\lambda_l \geq 0 . \tag{8.b}$$

From (7.d-f) and (8.a) we have

$$\mu^2\vartheta_\alpha\lambda = 0 , \tag{8.c}$$

i.e., μ^2 has zero eigenvalues for $\vartheta_\alpha\lambda \neq 0$. (Goldstone's theorem). One can introduce two subspaces in the scalar representation with the following properties

$$F = \{ n_\rho \mid n_\rho\vartheta_\alpha\lambda = 0 \ \text{for all}\ \alpha \} \ ; \ n_\rho n_{\rho'} = \delta_{\rho\rho'}$$
$$F^{\perp} = \{ n_\tau \mid n_\tau \perp F \} = \{\vartheta_\alpha\lambda\} \,,\ n_\tau n_{\tau'} = \delta_{\tau\tau'}$$
$$n_{\tau i}n_{\tau j} + n_{\rho i}n_{\rho j} = \delta_{ij} . \tag{9}$$

Then clearly $\lambda \in F$, $\mu^2 n_\tau = 0$; we choose the vectors in such a way that $\mu^2_{\rho\rho'}$ be diagonal, i.e.,

$$\mu^2_{ij} = \mu_\rho^2 n_{\rho i} n_{\rho j} . \tag{10}$$

One can always use a gauge where all $\phi_T = 0$ /12/, that is there are no Goldstone bosons (unitarity (U-) gauge). In such a gauge the renormalizability of the theory is not manifested; we shall work with gauges where renormalizability can be established but unphysical particles are present (R-gauges).

The scalar vacuum value also gives mass to some of the vector bosons and to the fermions. The vector mass matrix is

$$M^2_{\alpha\beta} = (\lambda, \vartheta_\alpha \vartheta_\beta \lambda). \tag{11}$$

The generators, for which $\vartheta_A \lambda = 0$, form a subgroup S of G which remains unbroken. We can again introduce two subspaces in the adjoint representation space with basis vectors (σ_A, σ_K) having the properties

$$\sigma_{A\alpha}\sigma_{A\beta} + \sigma_{K\alpha}\sigma_{K\beta} = \delta_{\alpha\beta}, \quad \sigma_{A\alpha}\sigma_{A'\alpha} = \delta_{AA'}, \quad \sigma_{K\alpha}\sigma_{K'\alpha} = \delta_{KK'} \tag{12.a}$$
$$\sigma_{A\alpha}\sigma_{K\alpha} = 0,$$

$$e_A = \sigma_{A\alpha} e_\alpha, \quad A^\mu_A = \sigma_{A\alpha} A^\mu_\alpha, \quad \vartheta_A \lambda = 0, \quad M^2_{AA'} = 0 \tag{12.b}$$

$$e_K = \sigma_{K\alpha} e_\alpha, \quad A^\mu_K = \sigma_{K\alpha} A^\mu_\alpha, \quad (\lambda, \vartheta_K \vartheta_{K'} \lambda) = M^2_K \delta_{KK'}. \tag{12.c}$$

The gauge bosons connected with the unbroken subgroup S remain massless. The K- and T- spaces are in one-to-one correspondence thus if we define the matrix

$$\rho_{\alpha i} = \vartheta_{\alpha, ij} \lambda_j, \quad \rho_{Ai} = \rho_{\alpha P} = 0, \tag{13.a}$$

the matrix ρ_{KT} is invertible:

$$\rho_{KT}\,\rho_{K'T} = -M^2_K \delta_{KK'}, \quad \rho^{-1}_{TK} = -\frac{1}{M^2_K}\rho_{KT}. \tag{13.b}$$

Finally, for the fermion mass matrix one gets

$$m = m_0 + \Gamma_i \lambda_i. \tag{14}$$

The following relations can be useful in practical calculations:

$$[\vartheta_\alpha, \mu^2] = \rho_{\alpha i}(c_i + f_{ik}\lambda_k) \rightarrow [\vartheta_A, \mu^2] = 0 \tag{15.a}$$

$$[T_\alpha, M^2]_{\beta\gamma} = -(\lambda, \vartheta_\alpha \{\vartheta_\beta \vartheta_\gamma\} \lambda) \rightarrow [T_A, M^2] = 0 \tag{15.b}$$

$$(\bar{t}_\alpha m - m t_\alpha) = \rho_{\alpha i}\Gamma_i \qquad \rightarrow (\bar{t}_A m - m t_A) = 0 \tag{15.c}$$

$$\vartheta_{A, PT} = T_{AA'K} = (c_T + f_{TP}\lambda_P)_{T_1 T_2} = 0. \tag{15.d}$$

At the end of this section we write down the Lagrangian one obtains by making the substitution $\varphi \rightarrow \phi + \Lambda$ in (4):

$$\begin{aligned}
\mathcal{L} = {} & -\tfrac{1}{2} A^\mu_\alpha [(\Box \delta_{\alpha\beta} - \tilde{M}^2_{\alpha\beta}) g_{\mu\nu} + \partial_\mu \partial_\nu] A^\nu_\beta + \bar{\psi}(i\gamma - \tilde{m})\psi + \\
& + \tfrac{1}{2}\phi_i (\Box \delta_{ij} - \tilde{\mu}^2_{ij})\phi_j - i\partial_\mu \phi_i (\vartheta_\alpha \Lambda)_i A^\mu_\alpha - \\
& - g_{\alpha\beta\gamma}\partial^\nu A_{\alpha\mu} A_{\beta\nu} A^\mu_\gamma - \tfrac{1}{4} g_{\lambda\alpha\beta} g_{\lambda\gamma\delta} A_{\alpha\mu} A_{\beta\nu} A^\mu_\gamma A^\nu_\delta + \\
& + \bar{\psi}\gamma^\mu t_\alpha A_{\alpha\mu}\psi - i\partial_\mu \phi_i \vartheta_{\alpha ij} A^\mu_\alpha \phi_j + \tfrac{1}{4}\phi_i \{\vartheta_\alpha \vartheta_\beta\}_{ij} A_{\alpha\mu} A^\mu_\beta \phi_j - \\
& - \bar{\psi}\Gamma_i \phi_i \psi - \tfrac{1}{3!}(c_{ijk} + f_{ijkl}\Lambda_l)\phi_i \phi_j \phi_k - \\
& - \tfrac{1}{4!} f_{ijkl}\phi_i \phi_j \phi_k \phi_l + \Lambda_i (\vartheta_\alpha \vartheta_\beta)_{ij} A_{\alpha\mu} A^\mu_\beta \phi_j + \\
& + \bar{\psi}\delta m \psi + \tfrac{1}{2}\phi_i \delta\mu^2_{ij}\phi_j + (\Lambda \delta\mu^2 - \tau)_i \phi_i,
\end{aligned} \tag{16}$$

with

$$\tilde{\mu}^2_{ij} = \mu^2_{0ij} + c_{ijk}\Lambda_k + \tfrac{1}{2} f_{ijkl}\Lambda_k \Lambda_l; \quad \tilde{M}^2_{\alpha\beta} = (\Lambda, \vartheta_\alpha \vartheta_\beta \Lambda) \tag{17.a}$$
$$\tilde{m} = m_0 + \Gamma_i \Lambda_i$$

$$\tau_i = \mu^2_{0ij}\Lambda_j + \tfrac{1}{2} c_{ijk}\Lambda_j \Lambda_k + \tfrac{1}{6} f_{ijkl}\Lambda_j \Lambda_k \Lambda_l. \tag{17.b}$$

Since Λ is the exact vacuum expectation value of φ, this Lagrangian contains higher order contributions; in perturbation calculations we shall work with the quantities introduced above and the quantities (17) will be taken into account in the counterterms.

III. Quantization. Green functions and Ward identities.

We investigate Green functions by using the functional integral method and apply the procedure of Faddeev and Popov /13/ to the quantization of the gauge fields. To simplify things, in what follows we shall systematically use compact notation, i.e., we denote by φ_r any field, the index r refers to any characteristics of the field (including space-time variables) and summation over r contains also space-time integration. In particular, we write

$$A_{\alpha\mu}(x) \to \varphi_\mu \;,\; \psi^a(x) \to \varphi_a \;,\; \bar{\psi}_a(x) \to \varphi_{\dot{a}} \;,\; \varphi_i(x) \to \varphi_i \tag{18.a}$$

$$\varepsilon_\alpha(x) \to \varepsilon_\alpha \text{ (gauge parameter)}, \quad \chi_\alpha(x) \to \chi_\alpha \text{ (ghost field)} \tag{18.b}$$

$$\partial_{\alpha r} : \partial_{\alpha x, \beta\mu y} = \frac{\partial}{\partial y^\mu}\delta(x-y)\delta_{\alpha\beta} \;;\; \bar{\partial}_{r\alpha} = -\partial_{\alpha r} , \tag{18.c}$$

$$\Box_{\alpha\beta} = \partial_{\alpha r}\partial_{\beta r} : \Box_{\alpha x, \beta y} = \delta_{\alpha\beta}\Box\delta(x-y) ; D_{\alpha\beta} \equiv \Box^{-1}_{\alpha\beta} , \tag{18.d}$$

$$e_{\alpha, rs} : e_{\alpha x, Ay, Bz} = e_{\alpha, AB}\,\delta(x-y)\,\delta(x-z) , \tag{18.e}$$

$$e_{\alpha, ij} = \vartheta_{\alpha, ij} , e_{\alpha, \mu\beta, \nu\gamma} = g_{\mu\nu} T_{\alpha\beta\gamma} \;,\; e_{\alpha}{}^a{}_b = t_\alpha{}^a_b \;,\; e_{\alpha a}{}^b = -\bar{t}_\alpha{}^b_a .$$

With these notations the infinitesimal gauge transformations (3) read

$$\delta\varphi_r = f_{\alpha r}(\varphi)\varepsilon_r \quad , \quad f_{\alpha r}(\varphi) = i e_{\alpha rs}\varphi_s + \partial_{\alpha r} . \tag{19}$$

We define $H(\varphi)$ by

$$\begin{aligned} H_{\alpha\beta}(\varphi) &= \partial_{\alpha r} f_{\beta r}(\varphi) \;;\; H = \Box + V = \Box(1 - \Box^{-1}V) \equiv \\ &\equiv \Box\tilde{H} \;;\; V_{\alpha\beta} = i\partial_{\alpha r} e_{\beta rs}\varphi_s . \end{aligned} \tag{20}$$

Now, the generator functional of all Green functions, including the Faddeev-Popov ghosts, can be written in the form

$$Z(J, \eta) = \int \Pi\, d\varphi_r\, d\chi_\alpha\, d\chi^+_\alpha\, e^{i[W(\varphi) + S_\xi(\varphi) + W(\chi, \eta) + J_r\varphi_r]} , \tag{21}$$

where

$$S_\xi(\varphi) = \frac{1}{2}\varphi_r \bar{\partial}_{r\alpha}\,\xi^{-1}_{\alpha\beta}\,\partial_{\beta s}\varphi_s \tag{21.a}$$

is a gauge fixing term with the gauge fixing parameter

$$\xi_{\alpha\beta} : \xi_{\alpha x, \beta y} = \xi_{\alpha\beta}\,\delta(x-y) \;,\; \xi^T = \xi \;,\; [T_\alpha, \xi] = 0 \tag{21.b}$$

and

$$W(\chi, \eta) = \chi^+_\alpha H_{\alpha\beta}\chi_\beta + \eta^+_\alpha\chi_\alpha + \eta_\alpha\chi^+_\alpha \;; \tag{21.c}$$

J_r and $\eta^+_\alpha, \eta_\alpha$ are the corresponding sources. (For details about functional methods see /14/ and /15/. We note here only that $\varphi_a, \varphi_{\dot{a}}, \chi_\alpha$ and their sources are anticommuting quantities and integration over them has to be understood in the sense given in /14/).

The generating functional for the physical Green functions after integrating over the ghost fields, takes the form

$$Z(J) = \int \Pi\, d\varphi_r\, \det\tilde{H}(\varphi)\, \exp i\,[W(\varphi) + S_\xi(\varphi) + J_r\varphi_r] . \tag{22}$$

As Gribov /16/ has remarked, the procedure, leading to this expression, is in fact ambiguous and det $\tilde{H}$ may lose sense for "big" gauge fields. Since we use functional integrals only for the purpose of generating the perturbation series, we ignore this and other mathematical problems related to the existence of the functional integral as a whole.

With the normalization $Z(0) = 1$, the Green functions can be written as

$$Z^{(n)}_{r_1 \dots r_n}(J) = \frac{\delta}{\delta J_{r_1}} \cdots \frac{\delta}{\delta J_{r_n}} Z(J) . \tag{23.a}$$

Let us introduce the generating functional of the connected Green functions as

$$G(J) = i \ln Z(J) \, , \quad G(0) = 0 \, , \tag{23.b}$$

then the definition of the connected Green functions is

$$G^{(n)}_{r_1 \dots r_n}(J) = \frac{\delta}{\delta J_{r_1}} \cdots \frac{\delta}{\delta J_{r_n}} G(J) \, . \tag{23.c}$$

The generating functional of the one-particle irreducible (1PI) or proper Green functions can be defined by the Legendre transformation /17/

$$\begin{aligned} &\Gamma(\chi) + G(J) + J_r \chi_r = 0 \, , \\ &\chi_r = -\frac{\delta}{\delta J_r} G(J) \, , \qquad \varepsilon_r = \begin{cases} +1 & \text{for bosons} \, , \\ -1 & \text{for fermions} \, , \end{cases} \\ &J_r = -\varepsilon_r \frac{\delta}{\delta \chi_r} \Gamma(\chi) \, , \end{aligned} \tag{23.d}$$

and the 1PI Green functions themselves are given by

$$\Gamma^{(n)}_{r_1 \dots r_n}(\chi) = \varepsilon_F^{(n)} \frac{\delta}{\delta \chi_{r_1}} \cdots \frac{\delta}{\delta \chi_{r_n}} \Gamma(\chi) \, , \tag{23.e}$$

$$\varepsilon_F^{(n)} = (-1)^{\Sigma N_r} \, ,$$

$$N_r = \begin{cases} 1 & \text{for fermions} \\ 0 & \text{for antifermions and bosons} \, . \end{cases}$$

The usual Green functions are obtained in the limit $J \to 0$ and $\chi \to 0$, respectively. We note that

$$G_{rs} \Gamma_{st} = \delta_{rt} \, , \quad G_{rst} = \varepsilon_t G_{rr'} G_{ss'} \varepsilon_F^{(3)} \Gamma_{r's't'} G_{t't}$$ etc. (cf. /17/).

For the ghost Green functions we have

$$\begin{aligned} Z_{\alpha\beta r_1 \dots r_n} &= \frac{\delta}{\delta \eta^+_\alpha} \frac{\delta}{\delta \eta_\beta} \frac{\delta}{\delta J_{r_1}} \cdots \frac{\delta}{\delta J_{r_n}} Z(J, \eta) \big|_{J = \eta = 0} = \\ &= H^{-1}_{\alpha\beta} \left(\tfrac{1}{i} \delta / \delta J \right) Z_{r_1 \dots r_n}(J) \big|_{J=0} \, . \end{aligned} \tag{23.f}$$

Finally let us briefly fix the definition of the Fourier transforms.

Introducing the transformations

$$e_{rp} : \; e_{Ax, By} = \frac{\delta_{AB}}{(2\pi)^4} e^{-ixp}$$

we can write

$$\begin{aligned} G_{rs} &= e_{rp} e_{sq} (2\pi)^4 G^{pq}(p) \, \delta(p+q) \, , \\ \Gamma_{r_1 \dots r_n} &= \prod_{i=1}^{n} (e_{r_i p_i}) (2\pi)^4 \delta(\Sigma p_i) \Gamma^{p_1 \dots p_n} \, . \end{aligned}$$

For the formal proof of renormalizability we need the identities of the theory. Let us write Eq.(22) in the form

$$Z(J) = \det \tilde{H} \left(\tfrac{1}{i} \delta / \delta J \right) Z_0(J)$$

and perform in $Z_0(J)$ the transformations (19). With the remark that

$$\delta W(\varphi) = 0 \, , \quad \delta S_\xi = -\xi^{-1}_{\alpha\beta} \partial_{\beta r} \varphi_r H_{\alpha r} \varepsilon_r$$

we get

$$\left[i \xi^{-1}_{\alpha\beta} \partial_{\beta r} H_{\alpha r} \left(\tfrac{1}{i} \delta / \delta J \right) + J_r f_{\gamma r} \left(\tfrac{1}{i} \delta / \delta J \right) \right] Z_0(J) = 0$$

or, using the relation

$$\begin{aligned} \det \tilde{H} \left(\tfrac{1}{i} \delta / \delta J \right) J_r f_{\gamma r} \left(\tfrac{1}{i} \delta / \delta J \right) &= \Big[J_r \det \tilde{H} \left(\tfrac{1}{i} \delta / \delta J \right) + \\ &\quad + H^{-1}_{\alpha\beta} \left(\tfrac{1}{i} \delta / \delta J \right) \partial_{\beta s} e_{\alpha s r} \det \tilde{H} \left(\tfrac{1}{i} \delta / \delta J \right) \Big] f_{\gamma r} \left(\tfrac{1}{i} \delta / \delta J \right) = \\ &= H_{\alpha\gamma} \left(\tfrac{1}{i} \delta / \delta J \right) J_r f_{\beta r} \left(\tfrac{1}{i} \delta / \delta J \right) H^{-1}_{\beta\alpha} \det \tilde{H} \left(\tfrac{1}{i} \delta / \delta J \right) \end{aligned}$$

the derived identity will have the form

$$\left[i \xi^{-1}_{\alpha\beta} \partial_{\beta r} \frac{\delta}{\delta J_r} + J_r f_{\rho r} \left(\tfrac{1}{i} \delta / \delta J_r \right) H^{-1}_{\rho\alpha} \left(\tfrac{1}{i} \delta / \delta J_r \right) \right] Z(J) = 0 \tag{24}$$

(Slavnov /18/, Taylor /19/). In a perturbational sense this identity makes sense only if we introduce an invariant regularization: we shall use the dimensional regularization method of 't Hooft and Veltman /20/, and assume that our theory is free of ABJ anomalies /21/. The Ward identities for the Green functions can be obtained from (24) by differentiating it with resepect to J_r.

IV. Renormalization of the symmetric theory

This and the next section simply generalize the work /5/ of Lee and Zinn-Justin to our general case. In ref./5/ they consider a theory with $G = SU(2)$ and vectors and scalars only; the analogous case for a general semisimple G is studied by Lee in ref./6/ by using a slightly different method.

The task of proving renormalisability is to show that one can eliminate UV divergencies by introducing regulator dependent counterterms in the Lagrangian which are of the same form as the original ones. The counterterms are determined by the divergent parts of the corresponding proper Green functions /22/; one can use subtractions at some suitable momentum or the pole terms of the dimensionally regularized quantities ('t Hooft's method /23/). The derived property of the counterterms is the consequence of the gauge invariance of the theory. In practical calculations we shall work with the pole terms but the investigation of the WT identities was carried out by using the subtraction momentum method. Since the infrared divergences do not allow one to subtract on the mass shell, such an analysis is quite lengthy, and in sketching what is going on we use formal on-mass-shell arguments.

1) Two-point functions

The two-point Green functions are

$$\Gamma^{\mu\nu}_{\alpha\beta}(p) = g^{\mu\nu} A_{\alpha\beta}(p^2) + p^\mu p^\nu B_{\alpha\beta}(p^2) , \qquad (25.a)$$

$$A^T = A , \; B^T = B , \; [T_\alpha , A] = [T_\alpha , B] = 0 ,$$

$$\Gamma^\mu_{\alpha i}(p) = -\Gamma^\mu_{i\alpha}(p) = -p^\mu C_{\alpha i}(p^2) , \qquad (25.b)$$

$$\Gamma(p) = \alpha(p^2) + \beta(p^2)\hat{p} , \; [t_\alpha , \gamma^0 \alpha] = 0 , \; [t_\alpha , \beta] = 0 , \qquad (25.c)$$

$$\Gamma_{ij}(p^2) \; ; \quad \Gamma_{ij} = \Gamma_{ji} , \; [\theta_\alpha , \Gamma] = 0 . \qquad (25.d)$$

The superficial divergence of B, C and β is logarithmic, that of α is linear; A and Γ_{ij} are quadratically divergent. The WT identities read:

$$\xi^{-1}_{\alpha\beta} \partial_{\beta\gamma} G_{\gamma\varepsilon} + \partial_{\beta\varepsilon} G_{\beta\alpha} + e_{\beta\varepsilon t} G_{\beta\alpha t} = 0 . \qquad (26)$$

By defining $\Sigma_{\varepsilon\beta}$ by

$$e_{\beta\varepsilon t} G_{\beta\alpha t} = i \Sigma_{\varepsilon\beta} G_{\beta\alpha} ,$$

one can write for the ghost propagator the equation

$$G_{\alpha\beta} = D_{\alpha\beta} + D_{\alpha\gamma} \Sigma_{\gamma\gamma'} G_{\gamma'\beta}$$

with

$$\Sigma_{\alpha\beta} = -i \partial_{\alpha\gamma} \Sigma_{\gamma\beta} .$$

Since the gauge bosons of the Abelian factor N do not interact with the ghosts, the corresponding ghost quantities are those of the free theory, i.e.,

$$G_{\alpha\beta} = D_{\alpha\beta} , \; \Sigma_{\gamma\alpha} = 0 \quad \text{for } \alpha \text{ or } \beta \in N . \qquad (27)$$

For the proper Green functions (26) gives

$$(-i \partial_{\alpha\varepsilon} + \Sigma_{\varepsilon\alpha}) \Gamma_{\varepsilon\nu} = i \xi^{-1}_{\beta\gamma} \partial_{\gamma\nu} \Gamma_{\beta\alpha}$$

with the inverse ghost propagator

$$\Gamma_{\alpha\beta} = D_{\alpha\beta} + i \partial_{\alpha\nu} \Sigma_{\nu\beta} .$$

Going over to the momentum space we get equations

$$\Gamma_{\alpha\beta}(p^2) = p^2 I_{\alpha\beta}(p^2) , \qquad (28.a)$$

$$[A_{\beta\gamma}(p^2) + p^2 B_{\beta\gamma}(p^2)] I_{\gamma\alpha}(p^2) + \qquad (28.b)$$
$$+ C_{\beta i}(p^2) \Sigma_{i\alpha}(p^2) = -\xi_{\beta\gamma} \Gamma_{\gamma\alpha}(p^2) ,$$

$$\Sigma_{j\alpha}(p^2)\Gamma_{ji}(p^2) - p^2 I_{\beta\alpha}(p^2) C_{\beta i}(p^2) = 0 \tag{28.c}$$

or

$$A_{\alpha\beta} = -p^2 [B_{\alpha\beta} + C_{\alpha i}\Gamma^{-1}_{ij} C_{\beta j} + \xi^{-1}_{\alpha\beta}] \equiv -p^2 \mathcal{F}_{\alpha\beta}(p^2), \tag{28.d}$$

$$\Sigma_{i\alpha} = p^2 \sigma_{i\alpha}, \quad \sigma_{i\alpha}(p^2) = I_{\alpha\alpha'}(p^2) C_{\alpha' j}(p^2) \Gamma^{-1}_{ji}(p^2). \tag{28.e}$$

Taking into account (28.d), the vector Green function can be written in the form

$$\Gamma^{\mu\nu}_{\alpha\beta}(p) = -(g^{\mu\nu}p^2 - p^\mu p^\nu)\mathcal{F}_{\alpha\beta}(p^2) - (\xi^{-1}_{\alpha\beta} + C_{\alpha i}\Gamma^{-1}_{ij} C_{\beta j}) p^\mu p^\nu \tag{29.a}$$

and by inverting (25.a-d) one obtains for the boson propagators

$$G^{\mu\nu}_{\alpha\beta}(p) = -(g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}) \frac{\mathcal{F}^{-1}_{\alpha\beta}(p^2)}{p^2} - \xi_{\alpha\beta}\, p^\mu p^\nu/(p^2)^2, \tag{29.b}$$

$$G^{\mu}_{\alpha i}(p) = -G_{i\,\alpha}^{\,\mu}(p) = -p^\mu/p^2\, \xi_{\alpha\beta} C_{\beta j}(p^2) \Gamma^{-1}_{ji}(p^2), \tag{29.c}$$

$$G_{ij}(p) = \Gamma^{-1}_{ij}(p^2) + \xi_{\alpha\beta} C_{\alpha i'}(p^2) \Gamma^{-1}_{i'i}(p^2) C_{\beta j'}(p^2) \Gamma^{-1}_{j'j}(p^2). \tag{29.d}$$

These equations show that the longitudinal part of the vector propagator is not renormalized and the divergence of $A_{\alpha\beta}$ and $\Gamma_{\alpha\beta}$ is logarithmic. The corresponding function renormalization constants can be defined as

$$Z_{3,\alpha\beta} = \mathcal{F}^{-1}_{\alpha\beta}(0), \quad Z'_{3,\alpha\beta} = I^{-1}_{\alpha\beta}(0); \quad [T_\alpha, Z_3] = [T_\alpha, Z'_3] = 0. \tag{30}$$

We define the scalar renormalization constants by considering the irreducible components of the scalar representation D_S: $D_S = Z \oplus D^{n\sigma}$ (σ is the multiplicity parameter of the IR D^n), and vectors in D_S can be labelled as $\varphi^{n\sigma;m}$.

Then

$$\Gamma_{ij} = Z_i^{n\sigma;m} \Gamma^n_{\sigma\sigma'} Z_j^{n\sigma';m}, \quad \mu^2_{0ij} = Z_i^{n\sigma;m} \mu^2_{n\sigma} Z_j^{n\sigma;m}$$

and with a suitable choice of the mass counterterm $\delta\mu^2$ we

have

$$\Gamma^n_{\sigma\sigma'}(p^2)\Big|_{p^2\approx\mu^2_{n\sigma}} = Z^{-1}_{n\sigma}(p^2 - \mu^2_{n\sigma})\delta_{\sigma\sigma'}.$$

Thus the subtraction for Γ_{ij} will be of the form

$$\Gamma^{sub}_{ij} = p^2\delta_{ij} - \mu^2_{0ij} - [\xi^{-1}(p^2 - \mu_0^2)\xi^{-1}]_{ij},$$
$$\xi^{-1}_{ij} = Z_i^{n\sigma;m} Z_{n\sigma}^{-1/2} Z_j^{n\sigma;m}; \quad \xi^T = \xi, \; [\theta_\alpha, \xi] = 0, \; \xi^2 \equiv Z. \tag{31}$$

(28.c) shows that $C_{\beta,n\sigma m}(p^2)$ must vanish at $p^2 = \mu^2_{n\sigma}$ thus $C_{\beta i}$ does not necessitate any subtraction of its own.

The fermion Green function can be treated on the same line as the scalar case and we may write

$$\Gamma^{sub} = \hat{p} - m_0 - \xi_2^{-1}(\hat{p} - m_0)\bar{\xi}_2^{-1},$$
$$\xi_2^+ = \xi_2, \quad [t_\alpha, \xi_2] = 0, \quad \xi_2^2 \equiv Z_2 \tag{32}$$

2. <u>Three-point functions</u>

According to the final analysis the only functions which show overall divergence are Γ^a_{bi} , Γ_{ijk} and the following pieces of $\Gamma^{\mu\nu\rho}_{\alpha\beta\gamma}$, $\Gamma_{\alpha\beta}{}^{\mu}_{\gamma}$, $\Gamma_{ij}{}^{\mu}_{\alpha}$ and $\Gamma^a_b{}^\mu_\alpha$:

$$i\Gamma^{\mu\nu\rho}_{\alpha\beta\gamma}(pqk) = g^{\rho\nu}a_{1\alpha\beta\gamma}(k^\mu - q^\mu) + g^{\mu\rho}a_{2\alpha\beta\gamma}(p^\nu - k^\nu) + g^{\mu\nu}a_{3\alpha\beta\gamma}(q^\rho - p^\rho),$$

$$i\Gamma_{\alpha\beta}{}^{\mu}_{\gamma}(pqk) = p^\mu b_{\alpha\beta\gamma},$$

$$\Gamma_{ij}{}^{\mu}_{\alpha}(pqk) = c_{\alpha ij}(p^\mu - q^\mu),$$

$$\Gamma^a_b{}^\mu_\alpha(pqk) = \gamma^\mu d_\alpha{}^a_b,$$

where a,b,c and d are functions of the invariants p^2, q^2, k^2.

The WT identities for the 3-point functions are

$$\xi^{-1}_{\alpha\beta}\partial_{\beta\gamma}G_{\gamma st} + \partial_{\beta s}G_{\beta\alpha t} + \partial_{\beta t}G_{\beta\alpha s} + e_{\beta sw}\, iZ_{\beta\alpha wt} + e_{\beta tw}\, iZ_{\beta\alpha sw} = 0. \tag{33}$$

Let us define the function $\Sigma_{s\alpha\gamma}$ by

$$e_{\beta s w} i Z_{\beta\alpha w\gamma} = i\Sigma_{s\alpha'\gamma'} G_{\alpha'\alpha} G_{\gamma'\gamma} + i\Sigma_{s\gamma} G_{\gamma\gamma'} \Gamma_{\gamma'\alpha'\gamma'} G_{\alpha'\alpha} G_{\gamma'\gamma}. \tag{34}$$

We have

$$\Gamma_{\alpha\beta\gamma} = -i\partial_{\alpha s}\Sigma_{s\beta\gamma} \quad ; \quad \Sigma_{s\alpha\gamma} = -e_{\alpha s\gamma} \quad \text{for } \alpha \in N.$$

Then in case of the bosons we get for the proper Green functions:

$$(-i\partial_{\alpha\gamma} + \Sigma_{\gamma\alpha})\Gamma_{\gamma s t} = \Gamma_{ss'}\Sigma_{s'\alpha t} + i\xi^{-1}_{\beta\gamma}\partial_{\gamma s}\Gamma_{\beta\alpha t} + (s \leftrightarrow t), \tag{35}$$

The on-mass-shell analysis of these equations yields the following results:

$$a_1 = a_2 = a_3 \equiv a \quad , \quad a_{\alpha\beta\gamma} \text{ is completely antisymmetric}$$

$$b_{\alpha\beta\gamma} = I_{\beta\beta'} a_{\alpha'\beta'\gamma} \mathfrak{Z}^{-1}_{\alpha'\alpha} \tag{36.a}$$

$$a_{\alpha\beta\gamma} = b_{\alpha\beta\gamma} = 0 \quad \text{for } \alpha \text{ or } \beta \text{ or } \gamma \in N, \tag{36.b}$$

furthermore, we get a relation between $c_{\alpha ij}$ and $\Sigma_{i\alpha j}$.

The WT identities for the fermion proper Green functions are

$$(-i\partial_{\alpha s} + \Sigma_{s\alpha})\Gamma_{sai} = \Gamma_{aa'}\Sigma_{a'\alpha i} + \Sigma_{i'\alpha a}\Gamma_{i'i} \tag{37.a}$$

or in momentum space

$$I_{\alpha\alpha'}(k^2)\, k_\mu \Gamma^{a\ \mu}_{i\ \alpha'}(pqk) + k^2\sigma_{i\alpha}(k^2)\Gamma^a_{i\ i}(pqk) =$$

$$= \Gamma^a_c(p)\Sigma^c{}_{\alpha i}(pkq) + \Sigma_{c\alpha}{}^a(qkp)\Gamma^c_i(-q). \tag{37.b}$$

These identities lead to a relation between $d_{\alpha}{}^a_i$ and $\Sigma^a{}_{\alpha i}$.

Γ^a_{ii} and Γ_{ijk} are logarithmically divergent and can be subtracted in the usual way. The corresponding renormalization constants will be given below.

3. Four-point functions

The functions which are actually overall divergent are Γ_{ijkl} (it can be treated as in the usual φ^4 theory) and

the following pieces of $\Gamma^{\mu\nu\omega\varrho}_{\alpha\beta\gamma\delta}$ and $\Gamma^{\mu\nu}_{\alpha\beta ij}$:

$$\Gamma^{\mu\nu\omega\varrho}_{\alpha\beta\gamma\delta}(pqkl) = g^{\mu\nu}g^{\omega\varrho}X_{1\alpha\beta\gamma\delta} + g^{\mu\omega}g^{\nu\varrho}X_{2\alpha\beta\gamma\delta} + g^{\mu\varrho}g^{\nu\omega}X_{3\alpha\beta\gamma\delta},$$

$$\Gamma^{\mu\nu}_{\alpha\beta ij}(pqkl) = g^{\mu\nu}Y_{\alpha\beta ij};$$

the X's and Y are functions of six invariants.

The WT identities, which we do not reproduce here, give:

$$X_{1\alpha\beta\gamma\delta} = X_{2\alpha\gamma\beta\delta} = -X_{3\alpha\gamma\delta\beta} = -q_{\alpha\gamma\beta\delta} - q_{\alpha\delta\beta\gamma}$$

with

$$q_{\alpha\beta\gamma\delta} = a_{\alpha\beta\varepsilon}\mathfrak{Z}^{-1}_{\varepsilon\varepsilon'}a_{\varepsilon'\gamma\delta}\,;\; q_{\alpha\beta\gamma\delta} + q_{\alpha\delta\beta\gamma} + q_{\alpha\gamma\delta\beta} = 0. \tag{38}$$

Writing

$$Z_{\alpha\beta\gamma} = \mathfrak{Z}^{-1}_{\alpha\alpha'}a_{\alpha'\beta\gamma}$$

we have in tree approximation $Z^0_{\alpha\beta\gamma} = g_{\alpha\beta\gamma}$. Then assuming that up to $n-1$ loop approximation

$$Z^k_{\alpha\beta\gamma} = Z^k_{\alpha\alpha'}g_{\alpha'\beta\gamma} \;(k = 1,\dots,n-1),\; (Z^k)^T = Z^k,\; [T_\alpha, Z^k] = 0, \tag{39}$$

and writing down the second equation of (38) in n-loop approximation we get

$$[T_\alpha, Z^n_\beta] - [T_\beta, Z^n_\alpha] = i q_{\alpha\beta\varepsilon}Z^n_\varepsilon - Z^n_{\alpha\beta\varepsilon}T_\alpha \qquad ((Z^k_\alpha)_{\beta\gamma} \equiv Z^k_{\alpha\beta\gamma}).$$

This and the global gauge symmetry relation

$$[T_\alpha, Z^n_\beta] = i q_{\alpha\beta\gamma}Z^n_\gamma$$

yield

$$q_{\alpha\gamma\varepsilon}Z^n_{\varepsilon\beta\delta} = Z^n_{\alpha\gamma\varepsilon}q_{\varepsilon\beta\delta},$$

whence it follows that $Z^n_{\alpha\beta\gamma}$ is also of the form given in (39), i.e., by induction

$$a_{\alpha\beta\gamma} = Z^{-1}_{1,\alpha\alpha'}q_{\alpha'\beta\gamma}, \quad Z_1^T = Z_1, \quad [T_\alpha, Z_1] = 0. \tag{40}$$

Now (36.b) can be written in the form

$$b_{\alpha\beta\gamma} = Z^{)-1}_{1,\alpha\alpha'}q_{\alpha'\beta\gamma} = (Z_3^{)-1}Z_3Z_1^{-1})_{\alpha\alpha'}q_{\alpha'\beta\gamma}. \tag{41}$$

For $c_{\alpha ij}$, taking into account the relation mentioned in the previous section, one obtains

$$\tilde{c}_{\alpha i l}\tilde{c}_{\beta l j}-\tilde{c}_{\beta i l}\tilde{c}_{\alpha l j}=i\,z_{3\,\alpha\alpha'}^{-1}z_{1\,\alpha'\alpha''}^{-1}\,g_{\alpha''\beta\gamma}\,\tilde{c}_{\gamma i j}$$

$$(\tilde{c}_{\alpha i j}=\xi_{i l}\,c_{\alpha l l'}\,\xi_{l' j}).$$

Using the global symmetry relation

$$\vartheta_{\alpha, i l}\,\tilde{c}_{\alpha l j}-\tilde{c}_{\beta i l}\,\vartheta_{\alpha l j}=i\,g_{\alpha\beta\gamma}\,\tilde{c}_{\gamma i j}$$

we get from here again by induction

$$c_{\alpha i j}=[z_3 z_1^{-1}]_{\alpha\beta}\,(\xi^{-1}\vartheta_\beta\,\xi^{-1})_{ij} \tag{42}$$

where, by definition $(z_3 z_1^{-1})_{\alpha\beta}=\delta_{\alpha\beta}$ for $\alpha,\beta\in N$).

Then the WT identity for Y gives

$$Y_{\alpha\beta i j}=(z_3 z_1^{-1})_{\alpha\alpha'}(z_3 z_1^{-1})_{\beta\beta'}\,[\xi^{-1}\{\vartheta_{\alpha'},\vartheta_{\beta'}\}\xi^{-1}]_{ij} \tag{43}$$

The treatment of the fermion case is similar and the final result is

$$\alpha_\alpha=(z_3 z_1^{-1})_{\alpha\alpha'}\,\bar{\xi}_2^{-1}\,t_{\alpha'}\,\bar{\xi}_2^{-1}\,, \tag{44}$$

with $(z_3 z_1^{-1})_{\alpha\beta}=\delta_{\alpha\beta}$ for $\alpha,\beta\in N$ by definition.

On the basis of (30), (31), (32), (40), (41), (42) and (44) the wave function and coupling constant renormalizations can be written as follows (renormalized quantities are denoted by tilde):

$$A^\mu_\alpha=z_{3\,\alpha\beta}^{1/2}\tilde{A}^\mu_\beta\,,\quad \varphi_i=\xi_{ik}\tilde{\varphi}_k\,,\quad \psi^a=\bar{\xi}_{2\,b}^{\,a}\tilde{\psi}^b\,,\quad \chi_\alpha=z_{3\,\alpha\beta}^{1/2}\tilde{\chi}_\beta \tag{45.a}$$

$$\xi_{\alpha\beta}=z_{3\,\alpha\alpha'}^{1/2}\tilde{\xi}_{\alpha'\beta'}z_{3\,\beta'\alpha}^{1/2}\,,\quad g_{\alpha\beta\gamma}=(z_1 z_3^{-3/2})_{\alpha\alpha'}\tilde{g}_{\alpha'\beta\gamma} \tag{45.b}$$

$$t_\alpha=(z_1 z_3^{-3/2})_{\alpha\alpha'}\tilde{t}_{\alpha'}\,,\quad \vartheta_\alpha=(z_1 z_3^{-3/2})_{\alpha\alpha'}\tilde{\vartheta}_{\alpha'} \tag{45.c}$$

$$\Gamma_{i\,b}^{\,a}=\Gamma_s P^{s\,a}_{i\,b}=(\xi_2^{-1})^a_{a'}\tilde{\Gamma}_s z_{2s}P^{s\,a'}_{i'\,b'}(\bar{\xi}_2^{-1})^{b'}_b\,\xi^{-1}_{i'i}\equiv$$
$$\equiv(\xi_2^{-1})^a_{a'}\tilde{\Gamma}^{\,a'}_{i'\,b'}(\bar{\xi}_2^{-1})^{b'}_b \tag{45.d}$$

$$c_{ijk}=c_s P^s_{ijk}=\tilde{c}_s z_s P^s_{i'j'k'}\xi^{-1}_{i'i}\xi^{-1}_{j'j}\xi^{-1}_{k'k}\equiv$$
$$\equiv\tilde{c}_{i'j'k'}\xi^{-1}_{i'i}\xi^{-1}_{j'j}\xi^{-1}_{k'k} \tag{45.e}$$

$$f_{ijkl}=f_s P^s_{ijkl}=\tilde{f}_s z_{4s}P^s_{i'j'k'l'}\xi^{-1}_{i'i}\dots\xi^{-1}_{l'l}\equiv$$
$$\equiv\tilde{f}_{i'j'k'l'}\xi^{-1}_{i'i}\dots\xi^{-1}_{l'l}$$

$$\xi^2=z\,,\quad \xi_2^2=z_2\,;\quad z_3' z_1'^{-1}=z_3 z_1^{-1}\ (=1 \text{ for } N) \tag{45.f}$$

$P^{s\,a}_{i\,b}$, P^s_{ijk} and P^s_{ijkl} are all scalar projections in the corresponding representation spaces.

Then writing

$$L(\varphi)=L(\tilde{\varphi})-L_{CT}(\tilde{\varphi})$$

we have

$$\begin{aligned}-L_{CT}(\tilde{\varphi})=&-\tfrac{1}{2}\tilde{A}^\mu_\alpha(z_3-1)_{\alpha\beta}(\Box g_{\mu\nu}+\partial_\mu\partial_\nu)\tilde{A}^\nu_\beta+\bar{\tilde{\psi}}(z_2-1)\,i\hat{\partial}\,\tilde{\psi}-\\&-\bar{\tilde{\psi}}(\xi_2 m_0\bar{\xi}_2-m_0)\tilde{\psi}+\tfrac{1}{2}\tilde{\varphi}(z-1)(\Box-\mu_0^2)\tilde{\varphi}+\\&+\bar{\tilde{\psi}}\xi_2\,\delta m\,\bar{\xi}_2\tilde{\psi}+\tfrac{1}{2}\tilde{\varphi}\,\xi\,\delta\mu^2\,\xi\,\tilde{\varphi}-(z_1-1)_{\alpha\alpha'}\tilde{g}_{\alpha'\beta\gamma}\partial^\nu\tilde{A}_{\alpha\mu}\tilde{A}_{\beta\nu}\tilde{A}^\mu_\gamma-\\&-\tfrac{1}{4}(z_1^2 z_3^{-1}-1)_{\alpha\alpha'}\tilde{g}_{\lambda\alpha'\beta}\tilde{g}_{\lambda\gamma\delta}\tilde{A}_{\alpha\mu}\tilde{A}_{\beta\nu}\tilde{A}^\mu_\gamma\tilde{A}^\nu_\delta+\\&+\bar{\tilde{\psi}}\gamma^\mu[(z_1 z_3^{-1})_{\alpha\alpha'}\bar{z}_2-\delta_{\alpha\alpha'}]\tilde{t}_{\alpha'}\tilde{A}_{\alpha\mu}\tilde{\psi}-\\&-i\partial_\mu\tilde{\varphi}\,[(z_1 z_3^{-1})_{\alpha\alpha'}(z_1 z_3^{-1})_{\beta\beta'}z-\delta_{\alpha\alpha'}\delta_{\beta\beta'}]\times\\&\times\{\tilde{\vartheta}_{\alpha'},\tilde{\vartheta}_{\beta'}\}\tilde{A}_{\alpha\mu}\tilde{A}^\mu_\beta\tilde{\varphi}-\\&-\tfrac{1}{3!}(\tilde{c}'_{ijk}-\tilde{c}_{ijk})\tilde{\varphi}_i\tilde{\varphi}_j\tilde{\varphi}_k-\tfrac{1}{4!}(\tilde{f}'_{ijkl}-\tilde{f}_{ijkl})\tilde{\varphi}_i\tilde{\varphi}_j\tilde{\varphi}_k\tilde{\varphi}_l\\&-\bar{\tilde{\psi}}(\tilde{\Gamma}'_i-\tilde{\Gamma}_i)\tilde{\varphi}_i\tilde{\psi}\,,\end{aligned} \tag{46}$$

where $\tilde{\Gamma}^{\,a}_{i\,b}=\tilde{\Gamma}_s P^{s\,a}_{i\,b}$, $\tilde{c}_{ijk}=\tilde{c}_s P^s_{ijk}$, $\tilde{f}_{ijkl}=\tilde{f}_s P^s_{ijkl}$.

We note at the end of this section that by introducing the renormalized sources by

$$J_\nu=\xi^{-1}_{\nu\nu'}\tilde{J}_{\nu'}\,,$$

the renormalized Green functions can be obtained as derivatives with respect to $\tilde{J}_\nu$, and the WT identities for the renormalized Green functions read:

$$[\,i\,\xi^{-1}_{\alpha\beta}\,\partial_{\beta\nu}\tfrac{\delta}{\delta\tilde{J}_\nu}+\tilde{J}_\nu\,(z_{1\beta\beta'}\tilde{c}_{\beta'\nu s}\,\delta/\delta\tilde{J}_s+z'_{3\beta\beta'}\,\partial_{\beta'\nu})\,H^{-1}_{R,\beta\alpha}(\tfrac{1}{i}\delta/\delta\tilde{J})\,]\,Z(\tilde{J})=0, \tag{47}$$

where (cf.(20))

$$H_R=z_3'\,D+z_1'\,V\,.$$

V. Renormalization of the spontaneously broken theory

It is important, that the spontaneously broken theory can be made finite with the counterterms of the symmetric one. One can demonstrate this by using the spurion method of /5/.

In the SBT one generally has $\mu_0^2 < 0$ (see(5)): this condition may not be necessary, if cubic scalar couplings are present, i.e., $c_{ijk} \neq 0$. In any case, let us consider a symmetric theory where μ_0^2 is substituted by a positive definite μ_s^2 and let us consider the Green functions of the theory at $J_i = \gamma_i$, $J_\mu = J_a = 0$. Then

$$\left.\frac{\delta G}{\delta J_i}\right|_{J_i=\gamma_i,\, J_\mu = J_a = 0} = -\Lambda_i(\gamma), \quad \left.\frac{\delta \Gamma}{\delta \chi_i}\right|_{\chi_i = \Lambda_i(\gamma),\, \chi_\mu = \chi_a = 0} = -\gamma_i . \tag{48}$$

The formal action, which gives these Green functions in the usual way (for all $J_r = 0$), is

$$W(\varphi, \gamma) = W(\varphi) + \gamma_i \varphi_i .$$

Writing

$$\varphi_i = \phi_i + \Lambda_i(\gamma)$$

we get for $W(\varphi,\gamma)$ the form (16), with the substitutions

$$\tilde{\mu}^2 \to \tilde{\mu}_1^2(\Lambda(\gamma)) = \tilde{\mu}^2(\Lambda(\gamma)) + \kappa^2 , \quad \kappa^2 = \mu_s^2 - \mu_0^2 > 0$$

$$\Lambda\delta\mu^2 - \tau \to \Lambda(\gamma)\,\delta\mu^2(\Lambda(\gamma), \kappa^2, \mu_0^2) - \tau_1(\Lambda(\gamma)) + \gamma$$

$$\tau_1(\Lambda(\gamma)) = \tau(\Lambda(\gamma)) + \kappa^2 \Lambda(\gamma) . \tag{49}$$

Now the proper Green functions with $\Lambda(\gamma) \neq 0$ can be expressed by the Green functions of the $\Lambda(\gamma) = 0$ case as follows:

$$\Gamma_r(p \,|\, \Lambda(\gamma), q, \Gamma, c, f) = \sum_{\substack{s=0 \\ i_1,\dots,i_s}}^{\infty} \frac{\Lambda_{i_1} \dots \Lambda_{i_s}}{s!} \Gamma_{r;i_1 \dots i_s}(p; 0, \dots, 0 \,|\, 0, q, \Gamma, c, f) .$$

This gives, that if $\tilde{\Gamma}_{r;i_1 \dots i_s}(p; 0 \dots 0 \,|\, 0, \tilde{q}, \tilde{\Gamma}, \tilde{c}, \tilde{f})$ are Green

functions renormalized according to the prescriptions of the previous section, the functions

$$\tilde{\Gamma}_r(p \,|\, \tilde{\Lambda}(\gamma), \tilde{q}, \tilde{\Gamma}, \tilde{c}, \tilde{f}) = \sum_{\substack{s=0 \\ i_1,\dots,i_s}}^{\infty} \frac{\tilde{\Lambda}_{i_1} \dots \tilde{\Lambda}_{i_s}}{s!} \tilde{\Gamma}_{r;i_1 \dots i_s}(p; 0, \dots, 0 \,|\, 0, \tilde{q}, \tilde{\Gamma}, \tilde{c}, \tilde{f})$$

will be finite for finite $\tilde{\Lambda}$, where

$$\Lambda_i = \xi_{ij} \tilde{\Lambda}_j . \tag{50}$$

With $\tilde{\gamma} = \xi \gamma$ one has in tree approximation (see (49))

$$\tau_1(\tilde{\Lambda}_0(\tilde{\gamma})) = \tilde{\gamma} ; \tag{51}$$

$\delta\mu^2$ and $\delta\lambda = \tilde{\Lambda} - \tilde{\Lambda}_0$ have to be chosen so as to kill the tadpole contributions to (48). This last condition can be reformulated by considering the global symmetry relation

$$\int d^4x_\alpha\, t_{\alpha rs} J_r \frac{\delta}{\delta J_s} Z(J) = 0 ,$$

which gives, by differentiating with respect to J_i and putting $J_i = \gamma_i$: $\Gamma_{ij}(p=0 \,|\, \Lambda(\gamma))\, T_{\alpha jk} \Lambda_k(\gamma) = -T_{\alpha ij} \gamma_j$

or

$$\tilde{\Gamma}(p=0 \,|\, \tilde{\Lambda}(\tilde{\gamma}))\, \tilde{T}_\alpha \tilde{\Lambda}(\tilde{\gamma}) = -\tilde{T}_\alpha \tilde{\gamma} . \tag{52}$$

Thus the role of $\delta\mu^2$ is to eliminate the divergences of the transversal part of $\tilde{\Gamma}(p=0)$ left over after the wave function renormalization and $\delta\lambda$ has to be adjusted according to this last equation.

Now, if we fix $\tilde{\Lambda}_0(\tilde{\gamma}) = \lambda$ to satisfy $\tau(\lambda) = 0$, (49) and (51) yield $\tilde{\gamma} = \kappa^2 \lambda$. Then in the limit $\kappa^2 \to 0$, $\tilde{\gamma} \to 0$

$$\tilde{\Lambda}(\tilde{\gamma}) \to \tilde{\Lambda} , \quad \delta\mu^2(\tilde{\Lambda}(\tilde{\gamma}), \kappa^2, \mu_0^2) \to \delta\mu^2(\Lambda^\sim, 0, \mu_0^2)$$

and $W(\varphi,\gamma)$ goes over into the spontaneously broken Lagrangian (16). Furthermore, (52) will take the form

$$\tilde{\Gamma}(p=0\,|\tilde{\Lambda})\,\tilde{J}_\alpha\tilde{\Lambda} = 0 \tag{53}$$

which is just the Goldstone theorem.

Thus, to renormalise the SBGT we have to use the counter-terms of the symmetric theory, renormalize Λ according to (50) and choose $\delta\mu^2$ and $\delta\lambda = \tilde{\Lambda} - \lambda$ so as to make $\tilde{\Gamma}_i$ to vanish (or equivalently, to satisfy (53), i.e., make the transversal part of the inverse scalar propagator to vanish; we remark that, as it was shown in /5/ $\tilde{\Gamma}^T(p=0)$ is free of infrared divergencies in the Landau gauge).

In R-gauges the theory is not manifestly unitary; one has to show that unphysical poles cancel in the S-matrix elements. Here we don't want to study such problems: we shall assume that they can be treated on the lines given in /5/.

As a last remark we notice that the WT identities (24) and (47) are valid also for the SBGT. In particular, we get that the longitudinal part of the vector propagator is not renormalized in the SBT either and by using $\Gamma_{rs}C_{st} = \delta_{rt}$ we obtain (see 29.a)

$$\Gamma^{\mu\nu}_{\alpha\beta}(p) = -\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\lambda_{\alpha\beta}(p^2) - \left(\xi^{-1}_{\alpha\beta} + C_{\alpha i}\Gamma^{-1}_{ij}C_{\beta j}\right)p^\mu p^\nu \tag{54}$$

and Eq's (29,b,c,d) with $7^{-1}/p^2 \to A^{-1}$. The fermion identities (37.b) are valid too and we will check them for the functions calculated in one loop approximation.

In the Appendix we write down the Lagrangian we use in the actual calculations. We give the general form of the action and the propagators and vertices in momentum representation with the corresponding counterterms. All quantities are renormalized

ones but we don't denote them by tilde any more. The counter-terms have to be determined by appropriate prescriptions for the renormalised Green functions. The vertices for the usual Feynman rules can be obtained by putting all Z's and ξ's equal to one and taking $\delta m = 0$, $\delta\mu^2 = 0$, $\Lambda = \lambda$, $c' = c$, $f' = f$.

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Appendix

In the compact notation, introduced in Sec. III, we have

$$W(\varphi) = W_0(\varphi) + W_I(\varphi) \tag{A.1}$$

where

$$\begin{aligned} W_0(\varphi) &= \tfrac{1}{2}\varphi_r K_{rs}\varphi_s\,, \\ W_I(\varphi) &= Y_r\varphi_r + \tfrac{1}{2!}Y_{rs}\varphi_r\varphi_s + \tfrac{1}{3!}Y_{rst}\varphi_r\varphi_s\varphi_t + \\ &\quad + \tfrac{1}{4!}Y_{rstu}\varphi_r\varphi_s\varphi_t\varphi_u\,, \\ K_{rs} &= \delta_F K_{sr}\,, \quad Y_{r_{i_1}\dots r_{i_m}} = \delta_F Y_{r_1\dots r_m} \end{aligned} \tag{A.2}$$

(δ_F : parity of the fermion permutations).

In W we include ghost fields too. Below we give K_{rs}, $D_{rs} = K^{-1}_{rs}$ and $Y_{r_1\dots r_m}$ in momentum representation. The diagonal fermion mass $\hat{m}$ is supposed to be free of γ_5 and positive semi-definite /24/.

$$\begin{aligned} K^{\mu\nu}_{\alpha\beta}(p) &= \left[-(p^2-M^2)_{\alpha\beta}g^{\mu\nu} + p_\mu p_\nu\left(\delta_{\alpha\beta} - \xi^{-1}_{\alpha\beta}\right)\right] \\ &= \sigma_{s\alpha}\left[-(p^2-M_s^2)g^{\mu\nu}\delta_{ss'} + p_\mu p_\nu\left(\delta_{ss'} - \xi^{-1}_{ss'}\right)\right]\sigma_{s\beta} \\ M_A^2 &= 0\,, \quad M_k^2 = (\lambda, \theta_k\theta_k\lambda) \end{aligned} \tag{A.3}$$

$$K_{ij}(p) = (p^2-\mu^2)_{ij} = u_{si}(p^2-\mu_i^2)u_{sj}$$

$$\mu^2_{ij} = \mu^2_{0\,ij} + c_{ijk}\lambda_k + \tfrac{1}{2} f_{ijkl}\lambda_k\lambda_l \; ; \; \mu_T^2 = 0 \, , \; \mu_P^2 = \mu^2_{PP}$$

$$K^a_b(p) = (\hat{p}-m)^a_b = [\bar{\omega}(\hat{p}-\hat{m})\omega]^a_b$$

$$m = m_0 + \lambda_i \Gamma_i = \bar{\omega}\hat{m}\omega \quad (\hat{m}^a_b = m_0\delta^a_b) \, , \; \omega^+\omega = 1$$

$$K_i{}^\mu_\alpha(p) = -K^\mu_{\alpha i}(p) = \rho_{\alpha i}\, p^\mu$$

$$K_{\alpha\beta}(p) = p^2\delta_{\alpha\beta}$$

$$\Delta^{\mu\nu}_{\alpha\beta}(p) = -\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(p^2-M^2)^{-1}_{\alpha\beta} - \xi_{\alpha\beta}\frac{p^\mu p^\nu}{(p^2)^2} \qquad \text{(A.4)}$$

$$= \sigma_{s\alpha}\left[-\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\frac{\delta_{ss'}}{p^2-M_s^2} - \xi_{ss'}\frac{p^\mu p^\nu}{(p^2)^2}\right]\sigma_{s'\beta}$$

$$D_{ij}(p) = (p^2-\mu^2)^{-1}_{ij} + \frac{\rho_{i\alpha}\xi_{\alpha\beta}\rho_{j\beta}}{(p^2)^2} = u_{si}\left(\frac{\delta_{ss'}}{p^2-\mu_s^2} + \frac{\rho_{sk}\xi_{kk'}\rho_{s'k'}}{(p^2)^2}\right)u_{s'j}$$

$$D^a_b(p) = [(\hat{p}-m)^{-1}]^a_b = (\omega^+)^a_c\,\frac{(\hat{p}+\hat{m})^c_d}{p^2-m_c^2}\,(\bar{\omega}^+)^d_b$$

$$D_i{}^\mu_\alpha(p) = -D^\mu_{\alpha,i}(p) = \xi_{\alpha\beta}\rho_{\beta i}\,p^\mu/(p^2)^2$$

$$D_{\alpha\beta}(p) = \delta_{\alpha\beta}/p^2$$

$$\gamma_i(p) = (\Lambda\xi\delta\mu^2\xi)_i - \tau_i' \qquad \text{(A.5)}$$

$$\tau_i' = (\xi\mu_0^2\xi\Lambda)_i + \tfrac{1}{2}c'_{ijk}\Lambda_j\Lambda_k + \tfrac{1}{6}f'_{ijkl}\Lambda_j\Lambda_k\Lambda_l$$

$$\gamma_{ij}(p) = [p^2(Z-1) - \mu'^2 + \mu^2 + \xi\,\delta\mu^2\,\xi]_{ij} \qquad \text{(A.6)}$$

$$\mu'^2 = \xi\mu_0^2\xi + c'_i\Lambda_i + \tfrac{1}{2}f'_{ij}\Lambda_i\Lambda_j$$

$$\gamma_i{}^\mu_\alpha(p) = -\gamma^\mu_{\alpha i}(p) = p^\mu[((Z_1Z_3^{-1})_{\alpha\alpha'}Z - \delta_{\alpha\alpha'})\vartheta_{\alpha'}]_{ij}\Lambda_j$$

$$\gamma^{\mu\nu}_{\alpha\beta}(p) = -[(p^2g^{\mu\nu} - p^\mu p^\nu)(Z_3-1)_{\alpha\beta} - (M'^2-M^2)_{\alpha\beta}\,g^{\mu\nu}]$$

$$M'^2_{\alpha\beta} = (\Lambda,\xi\vartheta_{\alpha'}\vartheta_{\beta'}\xi\Lambda)(Z_1Z_3^{-1})_{\alpha\alpha'}(Z_1Z_3^{-1})_{\beta'\beta}$$

$$\gamma^a_b(p) = [(Z_2-1)\hat{p} - m' + m + \xi_2\,\delta m\,\bar{\xi}_2]^a_b$$

$$m' = \xi_2 m_0\bar{\xi}_2 + \Gamma'_i\Lambda_i$$

$$\gamma_{\alpha\beta}(p) = p^2(Z_3'-1)_{\alpha\beta}$$

$$\text{(A.7)}$$

$$\gamma_{ijl}(pqk) = -c'_{ijl} - f'_{iljm}\Lambda_m$$

$$\gamma_{ij}{}^\mu_\alpha(pqk) = (p^\mu - q^\mu)(Z_1Z_3^{-1})_{\alpha\alpha'}(Z\vartheta_{\alpha'})_{ij}$$

$$\gamma_i{}^{\mu\nu}_{\alpha\beta}(pqk) = \Lambda_j(Z_1Z_3^{-1})_{\alpha\alpha'}(Z_1Z_3^{-1})_{\beta\beta'}(Z\{\vartheta_{\alpha'}\vartheta_{\beta'}\})_{ji}\,g^{\mu\nu}$$

$$\gamma^a_{bi}(pqk) = -\Gamma'^a_{i\,b}$$

$$\gamma^{\mu\nu\rho}_{\alpha\beta\gamma}(pqk) = -iZ_{1\alpha\alpha'}g_{\alpha'\beta\gamma}[g^{\nu\rho}(k-q)^\mu + g^{\mu\rho}(p-k)^\nu + g^{\mu\nu}(q-p)^\rho]$$

$$\gamma^a_b{}^\mu_\alpha(pqk) = [\gamma^\mu(Z_1Z_3^{-1})_{\alpha\alpha'}\bar{Z}_2\,t_{\alpha'}]^a_b$$

$$\gamma_{\alpha\beta}{}^\mu_\gamma(pqk) = -iZ'_{1\gamma\gamma'}\,g_{\alpha\beta\gamma'}\,p^\mu$$

$$p + q + k = 0$$

$$\gamma^{\mu\nu\omega\rho}_{\alpha\beta\gamma\delta}(pqkl) = -(Z_1^2Z_3^{-1})_{\alpha\alpha'}\{g_{\lambda\alpha'\beta}\,g_{\lambda\gamma\delta}(g^{\mu\omega}g^{\nu\rho} - g^{\mu\rho}g^{\nu\omega}) + g_{\lambda\alpha'\gamma}\,g_{\lambda\beta\delta}(g^{\mu\nu}g^{\omega\rho} - g^{\mu\rho}g^{\nu\omega}) + g_{\lambda\alpha'\delta}\,g_{\lambda\beta\gamma}(g^{\mu\nu}g^{\omega\rho} - g^{\mu\omega}g^{\nu\rho})\}$$

$$\gamma_{ij}{}^{\mu\nu}_{\alpha\beta}(pqkl) = (Z_1Z_3^{-1})_{\alpha\alpha'}(Z_1Z_3^{-1})_{\beta\beta'}[Z\{\vartheta_{\alpha'},\vartheta_{\beta'}\}]_{ij}\,g^{\mu\nu}$$

$$\gamma_{ijmn}(pqkl) = -f'_{ijmn}\,.$$

References:

/1/ C.N.Yang and R.L.Mills, Phys.Rev., 96, 191 (1954).

/2/ P.W.Higgs, Phys.Letters, 12, 132 (1964); Phys.Rev., 145, 1156 (1966).
T.W.Kibble, Phys.Rev., 155, 1554 (1967).

/3/ J.Goldstone, Nuovo Cim., 19, 154 (1961).

/4/ G. 't Hooft, Nucl.Phys., B35, 167 (1972).
B.W.Lee, Phys.Rev., D5, 823 (1972).

/5/ B.W.Lee and J.Zinn-Justin, Phys.Rev., D5, 3121, 3137, 3155 (1972).

/6/ B.W.Lee, Phys.Rev., D9, 933 (1974).

/7/ S.Weinberg, Phys.Rev.Letters, 19, 1264 (1967).
A.Salam, in Elementary Particle Physics, ed. N.Svartholm (Almquist and Wiksells, Stockholm, 1968).

/8/ S.Coleman and D.J.Gross, Phys.Rev.Letters, 31, 851 (1973).
D.J.Gross and F.Wilczek, Phys.Rev.Letters, 30, 1343 (1973).
H.D.Politzer, Phys.Rev.Letters, 30, 1346 (1973).

/9/ S.Weinberg, Phys.Rev.Letters, 31, 494 (1973); Phys.Rev., D8, 4482 (1973).

/10/ A.A.Belavin, A.M.Polyakov. A.S.Schwartz and Yu.S.Tyupkin, Phys.Letters, 59B, 85 (1975).

/11/ C.G.Callan, R.F.Dashen and D.J.Gross, Phys.Letters, 63B, 334 (1976).

/12/ S.Weinberg, Phys.Rev., D7, 1068 (1973).

/13/ L.D.Faddeev and V.N.Popov, Phys.Letters, 25B, 29 (1967).

/14/ F.A.Berezin, The method of second quantization, Academic Press, N.Y. (1966).

/15/ I.Iliopoulos, C.Itzykson and A.Martin, Rev.Mod.Phys., 47 165 (1975).

/16/ V.N.Gribov, in Proceedings of the XII Winter School of LIYaF, Leningrad, 1977.

/17/ G.Jona-Lasinio, Nuovo Cim., 34, 1790 (1964).

/18/ A.A.Slavnov, Teor.Mat.Fiz., 10, 152 (1972).

/19/ J.C.Taylor, Nucl.Phys., B33, 436 (1971).

/20/ G. 't Hooft and M.Veltman, Nucl.Phys., B44, 189 (1972).

/21/ S.L.Adler, Phys.Rev., 177, 2426 (1969).
J.S.Bell and R.Jackiw, Nuovo Cim., 60, 47 (1969).
D.J.Gross and R.Jackiw, Phys.Rev., D6, 477 (1972).

/22/ Н.Н.Боголюбов и Д.В.Ширков. Введение в теорию квантованных полей, Москва , 1957.

/23/ G. 't Hooft, Nucl.Phys., B.61, 455 (1973).

/24/ C.Geinberg, P.Kabir and S.Feinberg, Phys.Rev.Letters, 3, 527 (1959).

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